



# Politecnico di Torino

Fingerprint spoofing detection

Riccardo Cardona, Nicholas Berardo

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# Chapter 1

## Introduction

### 1.1 Abstract

The goal is to build the best model for the Fingerprint-Spoofing Detection task, using the models developed during the laboratory and the lessons.

### 1.2 Overview

The data set and test set are composed of 10 dimensional features. The last column is the class label which is **1** for the *authentic fingerprint* and **0** for the *spoofed fingerprint*. We have that the spoofed fingerprints have 6 sub-classes corresponding to the different spoofing ways.

The datasets are imbalanced, we have more spoofed fingerprint than the authentic fingerprint. The Training set is composed of 2325 samples and the Test set is composed of 7704 samples.

The application we are going to build has  $\pi = 0.5$  meaning that we have the same prior probability for both classes, but we have different costs. The cost of False Negative  $C_{fn}$  is 1 but the cost of False Positive  $C_{fp}$  is 10, so classifying a spoofed fingerprint as an authentic one has an higher cost due to security vulnerability.

### 1.3 Features Analysis

As we said, the Fingerprint Spoofing Detection task has 10 features. Here below we are going to analyze them.

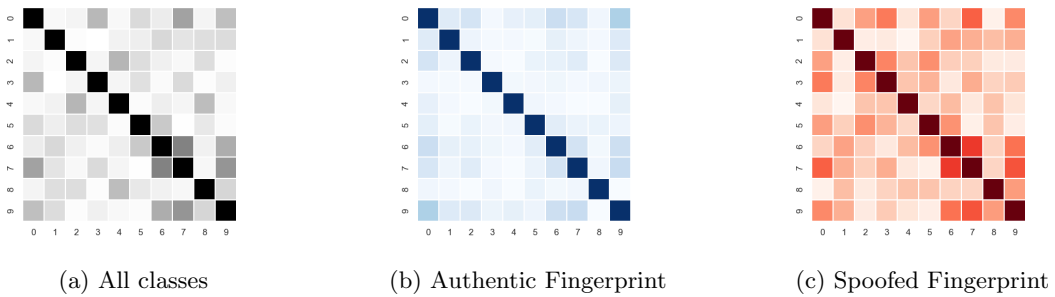


Figure 1.1: Heatmap

As we can see from the Figure 1.1, the correlation between the features is very small. This lead us to think that using PCA for dimensionality reduction is useless (it could also remove some important information!), but we will try using it.

Now we are going to see the histograms for each of the 10 features. It's easy to see that each feature has a Gaussian distribution.

We can see that the feature 4 is the most discriminant feature.

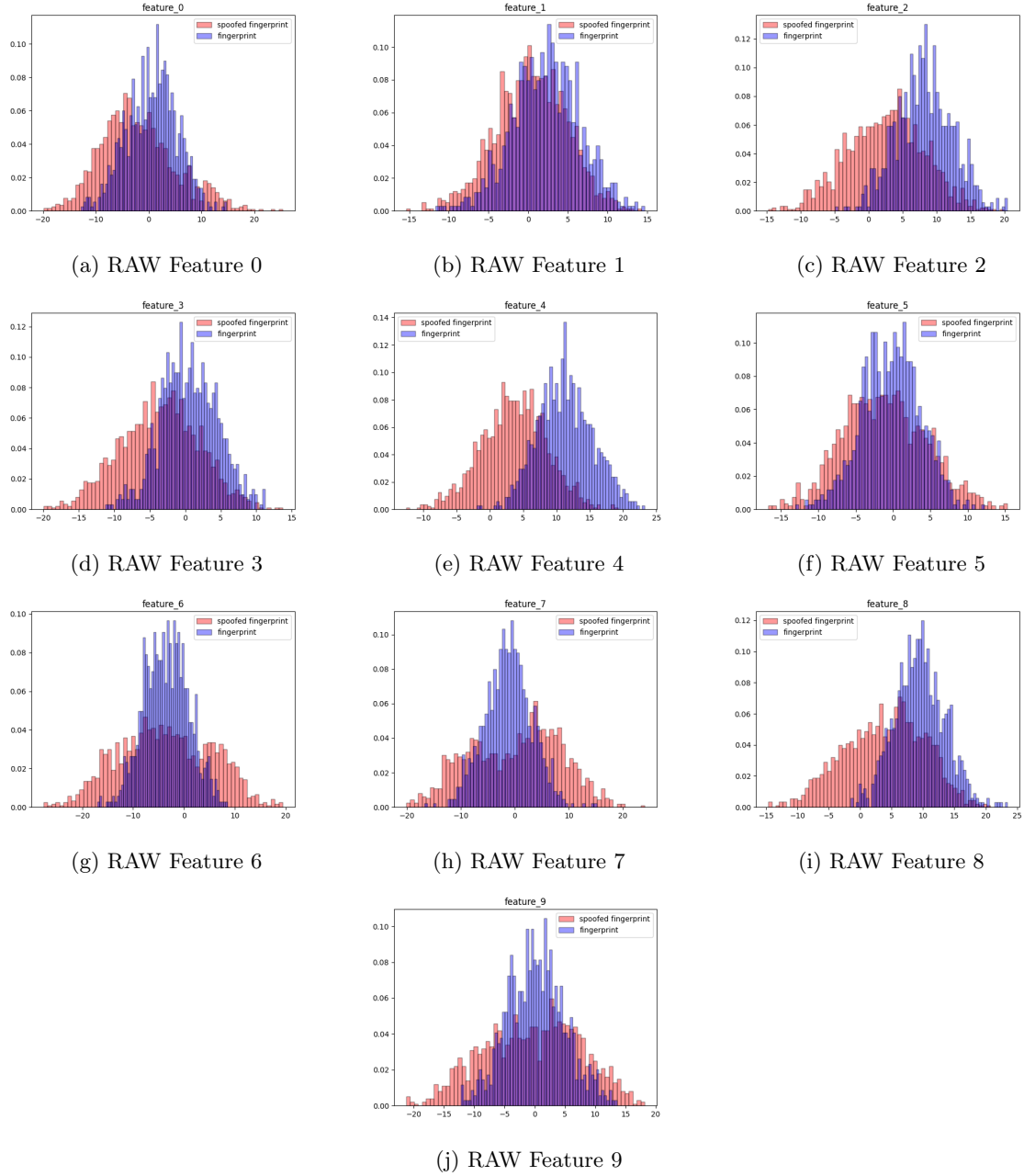


Figure 1.2: RAW Feature Histogram

## 1.4 PCA Usage

PCA (Principal Component Analysis) is a Dimensionality Reduction technique that can be used to remove noise, simplify classification and data visualization. It can be also used to avoid overfitting, but it may produce underfitting if too many dimensions are removed (removing too much useful information). So it is a useful technique, but sometimes it can be useless because if used could lead only to bad models.

What does PCA do?

First of all we need to compute the mean of the Dataset  $\mu$ , then we need to compute the covariance matrix

$$C = \frac{1}{K} \sum_{i=1}^K (x_i - \mu)(x_i - \mu)^T$$

From  $C$  we need the eigen-decomposition  $C = U\Sigma U^T$ , where  $U$  is a matrix that contains the eigen-vectors and  $\Sigma$  is a diagonal matrix that contains the eigen-values in descending order. The solution are the  $m$  eigen-vectors of  $U$  corresponding to the  $m$  highest eigen-values of  $\Sigma$ .

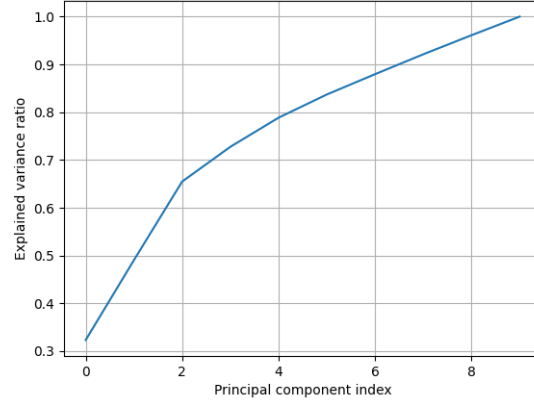


Figure 1.3: PCA variance

As we can see, if we use 9 dimensions (Number 8 on the principal component index), we are going to reach a 96% of explainance, then if we use 8 dimensions we will explain 91% of the data variance. Starting from 7 to 1 dimensions, we are lower than 90%. We will consider only 9 and 8 as PCA parameter.

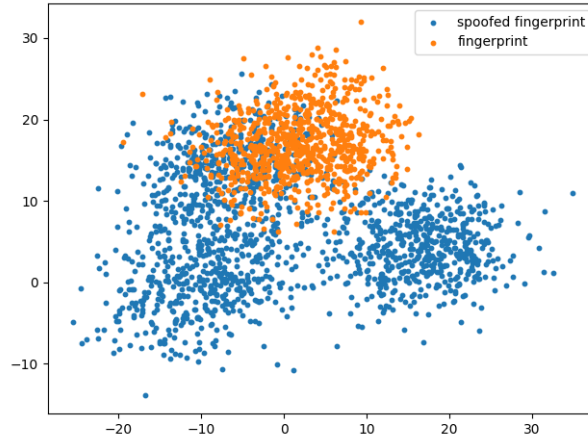


Figure 1.4: PCA

From this plot (Figure 1.4) we can see the distribution of the 2 main features. It's also easy to see that linear models won't be able to find a good separation, meanwhile the quadratic models will find it. So models like MVG, Quadratic SVM and RBF SVM will perform well.

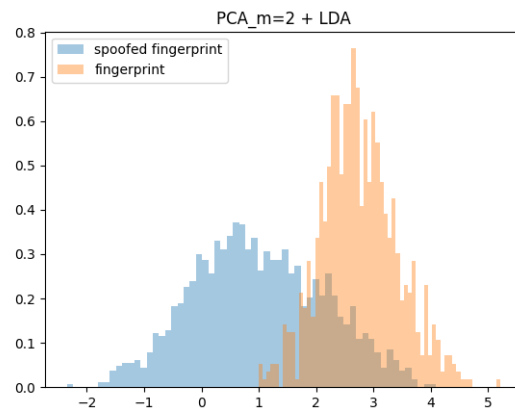


Figure 1.5: LDA + PCA

From this plot (Figure 1.5) we can see the distribution of the 2 main features using both PCA and LDA. Also from here we can see that LDA can't find a good separation between the 2 classes, so linear models WON'T perform well.

## 1.5 Training

- We are going to use a K-Fold Cross Validation approach, with  $K = 5$
- The minDCF computation is computed based on our application  $(\pi, C_{fn}, C_{fp}) = (0.5, 1, 10)$ , but we have computed the effective prior, which is approximated to  $(\pi, C_{fn}, C_{fp}) = (0.09, 1, 1)$ .

We have also tried this two different application:

- $(\pi, C_{fn}, C_{fp}) = (0.1, 1, 1)$
- $(\pi, C_{fn}, C_{fp}) = (0.9, 1, 1)$

# Chapter 2

## Classification and Validation

### 2.1 Introduction

We are going to develop the following models:

- Generative Models
  - Multivariate Gaussian Classifier (**MVG**)
  - MVG Tied Covariance (Only one covariance matrix for everything)
  - MVG Naive (For each covariance matrix takes out only the diagonal)
  - MVG Tied Naive (Only one covariance matrix, but taking only the diagonal)
- Logistic Regression (**LR**)
  - Linear Logistic Regression
- Support Vector Machine (**SVM**)
  - Linear SVM
  - Polynomial SVM
  - Radial Basis Function SVM
- Gaussian Mixture Models (**GMM**)
  - Full Covariance Matrix
  - Diagonal Covariance Matrix
  - Tied Full Covariance Matrix
  - Tied Diagonal Covariance Matrix

## 2.2 Multivariate Gaussian Classifiers

We are going to analyze all the Gaussian Classifiers, using the Multivariate Gaussian Classifier and all its variants:

- Tied Covariance
- Naive Bayes
- Tied Naive Bayes

For all these 4 models we have 2 parameters:  $\theta = (\mu, \Sigma)$ . We can easily estimate them starting from the likelihood w.r.t.  $\theta$  for the **M.V.G.**

$$\mathcal{L}(\theta) = \prod_c^K \prod_{i|c_i=c} \mathcal{N}(x_i | \mu_c, \Sigma_c)$$

Then we can build the log-likelihood because it's easier to work with.

$$l(\theta) = \sum_c^K \sum_{i|c_i=c} \log \mathcal{N}(x_i | \mu_c, \Sigma_c)$$

Now we define  $l_c(\theta) = \sum_{i|c_i=c} \log \mathcal{N}(x_i | \mu_c, \Sigma_c)$  and compute the  $\nabla_{\mu} l_c(\mu_c, \Sigma_c) = 0$  and the  $\nabla_{\Sigma} l_c(\mu_c, \Sigma_c) = 0$  obtaining

$$\mu_c^* = \frac{1}{N_c} \sum_{i|c_i=c} x_i$$

$$\Sigma_c^* = \frac{1}{N_c} \sum_{i|c_i=c} (x_i - \mu_c^*)(x_i - \mu_c^*)^T$$

We can do the same thing for the **Naive Bayes** model (that has diagonal covariance matrix)

$$l(\theta) = \sum_c^K \sum_{i|c_i=c} \sum_{j=1}^D \log \mathcal{N}(x_{i,[j]} | \mu_{c,[j]}, \sigma_{c,[j]}^2)$$

which solutions are

$$\mu_{c,[j]}^* = \frac{1}{N_c} \sum_{i|c_i=c} x_{i,[j]}$$

$$\sigma_{c,[j]}^{2*} = \frac{1}{N_c} \sum_{i|c_i=c} (x_{i,[j]} - \mu_{c,[j]}^*)(x_{i,[j]} - \mu_{c,[j]}^*)^T$$

Finally, the **Tied** model can be built in the same way (has a single covariance matrix for all classes)

$$l(\theta) = \sum_c^K \sum_{i|c_i=c} \log \mathcal{N}(x_i | \mu_c, \Sigma)$$

which solutions are

$$\mu_c^* = \frac{1}{N_c} \sum_{i|c_i=c} x_i$$

$$\Sigma^* = \frac{1}{N} \sum_c^K \sum_{i|c_i=c} (x_i - \mu_c^*)(x_i - \mu_c^*)^T$$

The tables below show the computation of the minDCF on the validation set (extracted using K-Fold Cross Validation with  $K = 5$ ). As said before, we are going to consider the main application (0.5,1,10) and 2 other applications (0.1,1,1) and (0.9,1,1). There are also some computation of PCA with  $m=9$  and  $m=8$ .



Classifier	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
RAW Features - NO PCA			
MVG	0.616	<b>0.331</b>	0.111
Naive Gaussian	0.806	0.472	0.144
Tied Gaussian	0.706	0.486	0.184
Naive Tied Gaussian	0.79	0.551	0.198

Classifiers	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
PCA (m=9)			
MVG	0.629	<b>0.33</b>	0.109
Naive Gaussian	0.74	0.369	0.113
Tied Gaussian	0.696	0.492	0.18
Naive Tied Gaussian	0.772	0.543	0.202

Classifiers	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
PCA (m=8)			
MVG	0.612	<b>0.333</b>	0.109
Naive Gaussian	0.712	0.36	0.112
Tied Gaussian	0.686	0.485	0.181
Naive Tied Gaussian	0.774	0.544	0.201

As we can see, the MVG model, that consider all the covariance matrices, is the best either with PCA or without. As we said previously, the quadratic models do a good job on these data, in fact the MVG is the best, meanwhile the Tieds are the worst. PCA is useful only in the Naive Gaussian model. Tied models perform worse.

Overall the best candidate is the **MVG classifier** without PCA. However, we will see that other models perform better.

## 2.3 Logistic Regression

We are going to analyze the Linear Logistic Regression and the Quadratic Logistic Regression.

### 2.3.1 Linear Logistic Regression

Logistic Regression is a discriminative model that directly computes the posterior probability  $P(C = c|X = x)$ . The model parameters are  $(w, b)$  that can be estimated using a frequentist approach. There's also an hyper-parameter  $\lambda$  that must be estimated by cross-validation.  $\lambda$  is needed to avoid overfitting, if it's too high then the model will underfit but if it's too low then the model will overfit.

We need to minimize:

$$\operatorname{argmin}_{w,b} J(w, b) = \frac{\lambda}{2} \|w\|^2 + \frac{\pi_T}{n_T} \sum_{i|z_i=1}^n \log(1 + e^{-z_i(w^T x_i + b)}) + \frac{1 - \pi_T}{n_F} \sum_{i|z_i=-1}^n \log(1 + e^{-z_i(w^T x_i + b)})$$

where  $\frac{\lambda}{2} \|w\|^2$  is a penalizer term,  $\log(1 + e^{-z_i(w^T x_i + b)})$  is the logistic loss function,  $\pi_T$  is the prior of class true and  $n_T$  ( $n_F$ ) is the number of elements of class true (false).

The plots show how minDCF is affected by different values of  $\lambda$ . They are exploited to calibrate  $\lambda$ , which is the regularization coefficient.

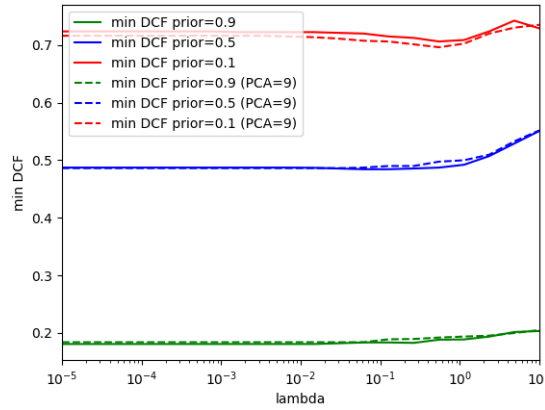


Figure 2.1: DCF - Linear LogReg

The Figure 2.1 shows that using a  $\lambda = 10^{-5}$  or  $\lambda = 10^{-1}$  is more or less the same, the minDCF doesn't change that much. From  $\lambda = 10^{-1}$  to  $+\infty$  the minDCF will be too high.

For our main application, it's hard to see, but  $\lambda = 0.4$  is the best choice. By the way, the LR computes a linear separation, but we said that the separation can't be found using a linear function, so we expect higher minDCF than the MVG.

	pi=0.1	pi=0.5	pi=0.9
		RAW	
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.1$ )	0.705	0.486	0.187
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.5$ )	0.709	0.488	0.186
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.9$ )	0.726	<b>0.506</b>	0.194
		PCA m=9	
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.5$ )	0.691	0.489	0.189
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.5$ )	0.7	0.497	0.191
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.9$ )	0.713	<b>0.506</b>	0.194
		PCA m=8	
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.9$ )	0.68	<b>0.48</b>	0.189
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.5$ )	0.691	<b>0.483</b>	0.191
Log Reg ( $\lambda = 0.4$ $\pi_T = 0.9$ )	0.708	0.51	0.192

Overall, the MVG without PCA has a **minDCF** = **0.331** and this value is better than all the Linear Logistic Regression values (for our main application).

### 2.3.2 Quadratic Logistic Regression

The Quadratic Logistic Regression allows the computation of a linear separation in the expanded feature space that correspond to a non-linear separation in the original feature space. From the MVG we have that the llr is a quadratic function:  $xAx^T + x^Tb + c$

$$\Phi(x) = \begin{bmatrix} \text{Vec}(xx^T), x \end{bmatrix}, w = \begin{bmatrix} \text{Vec}(A), b \end{bmatrix}$$

where  $\text{Vec}(T)$  just stack the columns of  $T$ . We can now compute a linear separation  $w^T\Phi(x) + b$ .

The Figure 2.2 shows how minDCF is affected by different values of  $\lambda$ . Also in this case, for our main application, taking a  $\lambda=10^{-5}$  or  $\lambda=10^{-1}$  doesn't change the minDCF that much.

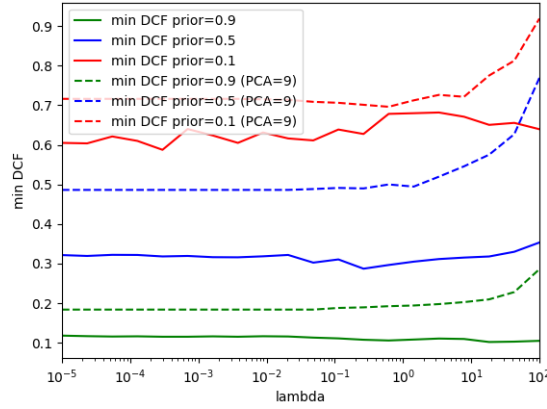


Figure 2.2: DCF - Quadratic LogReg

It is also easy to see that PCA is pretty useless for all applications. The minDCF it's always worse than the model without PCA.

	pi=0.1	pi=0.5	pi=0.9
RAW			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.616	0.312	0.118
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.664	0.297	0.109
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.708	0.333	0.105
PCA m=9			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.691	0.489	0.189
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.7	0.497	0.191
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.713	0.506	0.194
PCA m=8			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.68	0.48	0.189
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.691	0.483	0.191
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.708	0.51	0.192

We can see that, since it finds a non-linear separation, the results are good. From the minDCF = 0.331 of the MVG we got a 0.297 on the Quadratic LR.

## 2.4 Support Vector Machine

The SVM has a non probabilistic interpretation, in fact, as we will see, we'll have to do some score calibration in order to obtain a good DCF. In this section we are going to see how Linear SVM, Polynomial SVM and RBF SVM will handle our application.

### SVM - Linear

**Linear SVM** can be obtained by the minimization of the primal problem:

$$\hat{J}(\hat{w}) = \frac{1}{2} \|\hat{w}\|^2 + C \sum_{i=1}^n \max_i(0, 1 - z_i(\hat{w}^T \hat{x}_i))$$

where

$$\hat{x}_i = \begin{bmatrix} x_i, K \end{bmatrix}, \hat{w}_i = \begin{bmatrix} w, b \end{bmatrix}$$

and K is a regularization term.

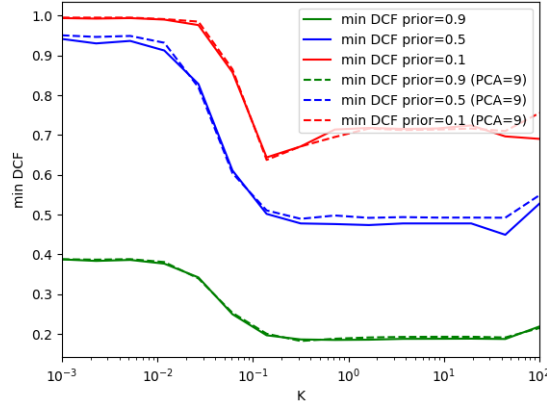


Figure 2.3: Linear SVM with C=1

From Figure 2.3 we can see how K is affected (using  $C = 1$ ). It can be seen that values of K between  $10^{-1}$  and  $10^1$  have a low DCF. In our cases we are going to try 3 different values of  $K = [0.1, 1, 10]$ .

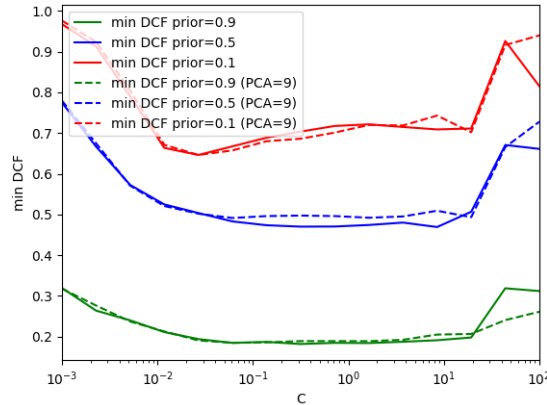


Figure 2.4: Linear SVM with K=1

Figure 2.4 shows the different values of C while setting  $K = 1$ . Again, it can be seen that the optimal values of C are the ones between  $10^{-1}$  and  $10^1$ . We are going to test 4 different values of  $C = [0.01, 0.1, 1, 10]$ . We also decided to test 0.01 even if the minDCF is "optimal" only for the  $\pi = 0.1$

Here the table containing the results:

### SVM - Linear - RAW Features

	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.99	0.68	0.71
C = 0.1	0.964	0.672	0.713
C = 1.0	0.67	0.722	0.715
C = 10.0	0.956	0.71	0.7
	$\pi=0.5$		
C = 0.01	0.921	0.532	0.473
C = 0.1	0.78	0.476	0.479
C = 1.0	0.532	0.466	0.473
C = 10.0	0.666	0.492	0.47
	$\pi=0.9$		
C = 0.01	0.379	0.216	0.187
C = 0.1	0.321	0.186	0.188
C = 1.0	0.216	0.184	0.188
C = 10.0	0.263	0.197	0.187

### SVM - Linear - PCA with m = 9

We also tried to apply PCA with  $m = 9$  since from Figure 2.3 and Figure 2.4 we can see that we can achieve also a good minDCF.

	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.991	0.704	0.701
C = 0.1	0.971	0.669	0.718
C = 1.0	0.694	0.709	0.709
C = 10.0	0.685	0.672	0.73
	$\pi=0.5$		
C = 0.01	0.973	0.538	0.496
C = 0.1	0.776	0.493	0.492
C = 1.0	0.54	0.494	0.489
C = 10.0	0.518	0.491	0.492
	$\pi=0.9$		
C = 0.01	0.382	0.216	0.187
C = 0.1	0.321	0.182	0.191
C = 1.0	0.214	0.19	0.194
C = 10.0	0.223	0.196	0.195

From these results we can conclude that using or not using PCA (with  $m = 9$ , so with almost all features) doesn't change that much.

## SVM - Quadratic

We are going to apply a non-linear separation algorithm. We have seen two kind of non-linear separation. In this section we are going to see the Quadratic one, in the next section there's the RBF.

The kernel is a function that allows the computation of a linear separation in the expanded feature space that corresponds to a non-linear separation in the original feature space. For the Quadratic kernel we have:

$$k(x_1, x_2) = (x_1^T x_2 + c)^d$$

where d identifies the degree, in our case d = 2 and c is a Constant that in our case will take values 1 and 0.

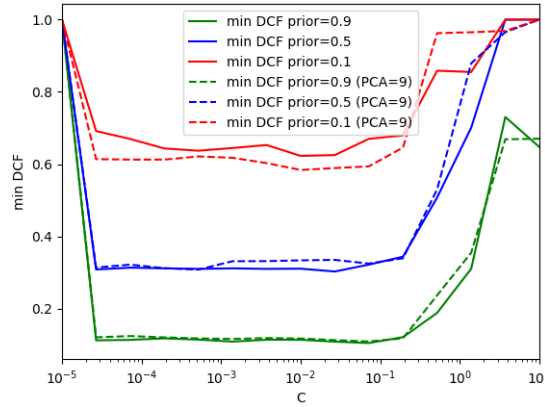


Figure 2.5: Poly K=1, c=1, d=2

Figure 2.5 shows the different values of C with K=1, c=1 e d=2. We are going to try  $C = [0.01, 0.1, 1, 10]$ , but from the plot it's easy to see that C=1 and C=10 won't be good.

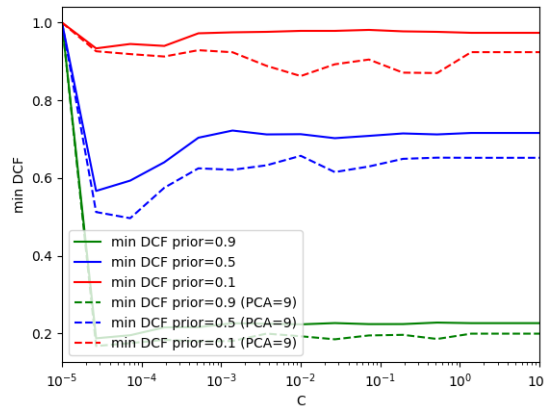


Figure 2.6: Poly K=1, c=1, d=3

Figure 2.6 shows the different values of C with K=1, c=1 but d=3. It's much worse than using d=2, so we decided to don't even show the results of this kernel.

### SVM - Quadratic - Raw Feature with degree = 2

Constant = 0	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.71	0.704	0.682
C = 0.1	0.713	0.674	0.678
C = 1.0	0.831	0.922	0.84
C = 10.0	0.982	0.994	0.975
	$\pi=0.5$		
C = 0.01	0.337	0.346	0.342
C = 0.1	0.324	0.344	0.346
C = 1.0	0.528	0.551	0.49
C = 10.0	0.947	0.957	0.946
	$\pi=0.9$		
C = 0.01	0.123	0.121	0.187
C = 0.1	0.125	0.11	0.115
C = 1.0	0.21	0.203	0.223
C = 10.0	0.586	0.769	0.583

Constant = 1	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.63	0.623	0.598
C = 0.1	0.66	0.624	0.677
C = 1.0	0.96	0.931	0.95
C = 10.0	0.996	1.0	1.0
	$\pi=0.5$		
C = 0.01	0.316	0.311	0.313
C = 0.1	0.348	0.347	0.323
C = 1.0	0.701	0.631	0.644
C = 10.0	0.966	1.0	1.0
	$\pi=0.9$		
C = 0.01	0.113	0.114	0.104
C = 0.1	0.114	0.112	0.11
C = 1.0	0.281	0.207	0.233
C = 10.0	0.656	0.647	0.569

### SVM - Quadratic - PCA with m = 9 and degree = 2

We also show the results of PCA with m = 9 (always with d=2). From Figure 2.5 we can see that using PCA = 9 or don't use it doesn't change that much. Just for  $\pi = 0.1$  there's a slight improvement.

Constant = 0	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.645	0.645	0.638
C = 0.1	0.659	0.659	0.656
C = 1.0	0.92	0.871	0.916
C = 10.0	1.0	0.985	0.99
	$\pi=0.5$		
C = 0.01	0.33	0.344	0.354
C = 0.1	0.345	0.345	0.391
C = 1.0	0.449	0.575	0.574
C = 10.0	1.0	0.975	0.951
	$\pi=0.9$		
C = 0.01	0.127	0.124	0.109
C = 0.1	0.125	0.113	0.125
C = 1.0	0.181	0.203	0.172
C = 10.0	0.605	0.66	0.527



<b>Constant = 1</b>	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.576	0.584	0.59
C = 0.1	0.586	0.624	0.667
C = 1.0	0.894	0.992	0.975
C = 10.0	0.998	1.0	0.982
	$\pi=0.5$		
C = 0.01	0.335	0.334	0.306
C = 0.1	0.35	0.302	0.346
C = 1.0	0.688	0.772	0.56
C = 10.0	0.993	1.0	0.975
	$\pi=0.9$		
C = 0.01	0.118	0.117	0.105
C = 0.1	0.132	0.114	0.107
C = 1.0	0.22	0.236	0.189
C = 10.0	0.677	0.67	0.594

### SVM - Quadratic - PCA with m = 8 and degree = 2

We also tried PCA with m = 8 because it's even better for some cases.

<b>Constant = 0</b>	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.624	0.654	0.666
C = 0.1	0.66	0.648	0.658
C = 1.0	0.848	0.989	0.798
C = 10.0	0.981	0.999	0.97
	$\pi=0.5$		
C = 0.01	0.324	0.331	0.333
C = 0.1	0.321	0.336	0.317
C = 1.0	0.593	0.934	0.494
C = 10.0	0.97	0.999	0.91
	$\pi=0.9$		
C = 0.01	0.127	0.12	0.113
C = 0.1	0.124	0.11	0.116
C = 1.0	0.285	0.359	0.203
C = 10.0	0.511	0.74	0.563

<b>Constant = 1</b>	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 0.01	0.561	0.541	0.53
C = 0.1	0.546	0.592	0.634
C = 1.0	0.985	0.865	0.98
C = 10.0	1.0	0.996	0.994
	$\pi=0.5$		
C = 0.01	0.323	0.32	0.308
C = 0.1	0.349	0.316	0.336
C = 1.0	0.657	0.649	0.7
C = 10.0	1.0	0.985	0.989
	$\pi=0.9$		
C = 0.01	0.114	0.108	0.101
C = 0.1	0.111	0.113	0.102
C = 1.0	0.262	0.217	0.211
C = 10.0	0.697	0.743	0.647

### SVM - Radial Basis kernel Function

As we introduced in the Quadratic SVM section, there's another kernel function we are going to see. This one is called Gaussian Radial Basis Function or RBF:

$$k(x_1, x_2) = e^{-\gamma \|x_1 - x_2\|^2}$$

where  $\|x_1 - x_2\|^2$  represent the distance between  $x_1$  and  $x_2$  and the closer it is, the bigger is the kernel, meanwhile  $\gamma$  is an indicator of the width of the kernel. If  $\gamma$  is low then the kernel will be large, otherwise it will be small (if  $\gamma \gg 0$  then this will be equal to 1-NN).

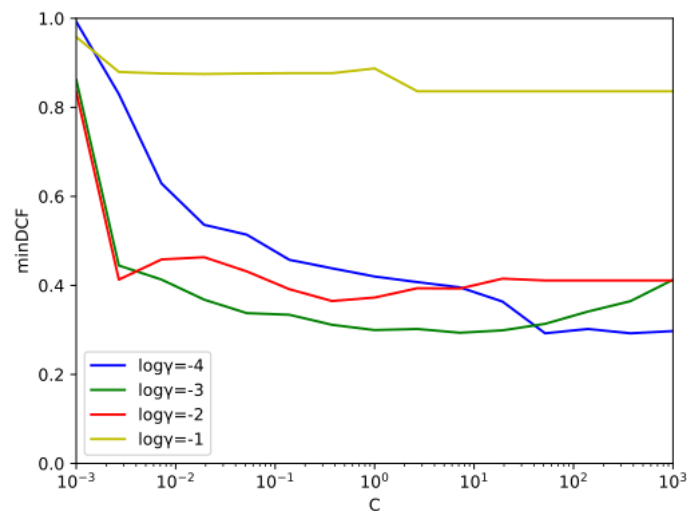


Figure 2.7: RBF

Figure 2.7 shows the different values of  $C$  with different values of  $\gamma$ . We are going to test  $C = [1, 10, 100]$  with a  $\gamma = [0.001, 0.0001]$ , those are the best parameter we can get.

### SVM - Radial Basis kernel Function - Raw Feature

$\gamma = \mathbf{0.001}$	$K = 0.1$	$K = 1.0$	$K = 10$
	$\pi=0.1$		
$C = 1.0$	0.592	0.575	0.578
$C = 10.0$	0.58	0.599	0.601
$C = 100.0$	0.586	0.579	0.601
	$\pi=0.5$		
$C = 1.0$	0.302	0.3	<b>0.301</b>
$C = 10.0$	<b>0.293</b>	<b>0.298</b>	0.302
$C = 100.0$	0.332	0.331	0.335
	$\pi=0.9$		
$C = 1.0$	0.097	0.1	0.101
$C = 10.0$	0.089	0.089	0.089
$C = 100.0$	0.101	0.103	0.102

$\gamma = \mathbf{0.0001}$	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 1.0	0.671	0.662	0.659
C = 10.0	0.574	0.58	0.584
C = 100.0	0.568	0.575	0.725
	$\pi=0.5$		
C = 1.0	0.473	0.42	0.419
C = 10.0	0.406	0.381	<b>0.354</b>
C = 100.0	<b>0.289</b>	<b>0.306</b>	0.472
	$\pi=0.9$		
C = 1.0	0.163	0.159	0.158
C = 10.0	0.137	0.133	0.13
C = 100.0	0.097	0.103	0.276

### SVM - Radial Basis kernel Function - PCA with m=9

We also tried to use PCA both with m=9 and m=8.

$\gamma = \mathbf{0.001}$	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 1.0	0.588	0.585	0.579
C = 10.0	0.534	0.541	0.542
C = 100.0	0.678	0.671	0.906
	$\pi=0.5$		
C = 1.0	<b>0.295</b>	<b>0.294</b>	<b>0.298</b>
C = 10.0	0.297	0.301	0.299
C = 100.0	0.328	0.333	0.895
	$\pi=0.9$		
C = 1.0	0.103	0.104	0.104
C = 10.0	0.102	0.103	0.102
C = 100.0	0.107	0.108	0.285

$\gamma = \mathbf{0.0001}$	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 1.0	0.672	0.672	0.662
C = 10.0	0.621	0.605	0.624
C = 100.0	0.553	0.502	0.592
	$\pi=0.5$		
C = 1.0	0.439	0.433	0.42
C = 10.0	0.407	0.391	0.376
C = 100.0	<b>0.301</b>	<b>0.302</b>	<b>0.317</b>
	$\pi=0.9$		
C = 1.0	0.165	0.159	0.16
C = 10.0	0.139	0.132	0.128
C = 100.0	0.103	0.106	0.12

With m=9 the results are more or less the same

# SVM - Radial Basis kernel Function - PCA with m=8

$\gamma = \mathbf{0.001}$	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 1.0	0.568	0.569	0.573
C = 10.0	0.528	0.54	0.545
C = 100.0	0.664	0.672	0.666
	$\pi=0.5$		
C = 1.0	0.297	0.297	0.296
C = 10.0	<b>0.281</b>	<b>0.29</b>	<b>0.294</b>
C = 100.0	0.318	0.315	0.305
	$\pi=0.9$		
C = 1.0	0.102	0.104	0.105
C = 10.0	0.101	0.103	0.103
C = 100.0	0.108	0.105	0.104

$\gamma = \mathbf{0.0001}$	K = 0.1	K = 1.0	K = 10
	$\pi=0.1$		
C = 1.0	0.665	0.662	0.645
C = 10.0	0.617	0.607	0.679
C = 100.0	0.55	0.529	0.844
	$\pi=0.5$		
C = 1.0	0.439	0.433	0.43
C = 10.0	0.397	0.389	<b>0.387</b>
C = 100.0	<b>0.306</b>	<b>0.29</b>	0.686
	$\pi=0.9$		
C = 1.0	0.165	0.158	0.156
C = 10.0	0.141	0.135	0.153
C = 100.0	0.106	0.107	0.301

Also with m=8 the results are more or less the same

## 2.5 Gaussian Mixture Models

This is the last model we are going to consider. It's a generative model and it's called Gaussian Mixture Model. We are going to consider 4 models of GMM:

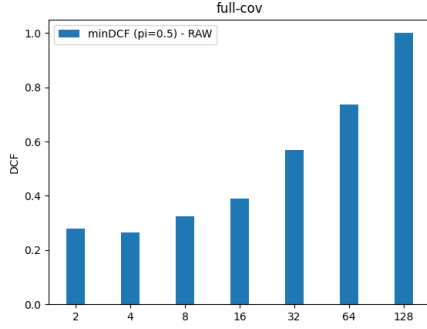
- Full covariance matrix
- Diagonal covariance matrix
- Full Tied covariance matrix
- Diagonal Tied covariance matrix

The GMM is a distribution whose parameters are  $(M, S, W)$  where:

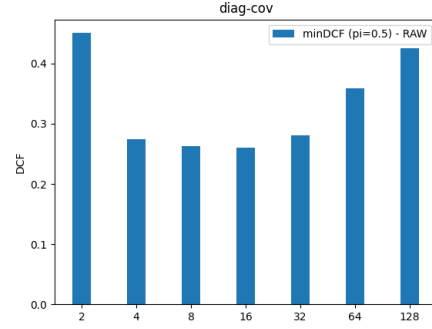
$$M = [\mu_1, \mu_2, \dots, \mu_K], S = [\Sigma_1, \Sigma_2, \dots, \Sigma_K], W = [w_1, w_2, \dots, w_K]$$

The GMM density can be represented as:

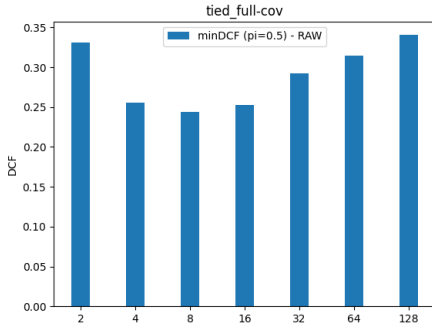
$$f_X(x) = \sum_{c=1}^K \pi_c \mathcal{N}(x | \mu_c, \Sigma_c)$$



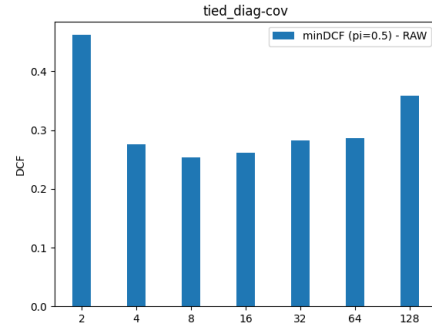
(a) GMM Full Cov



(b) GMM Diag Cov



(c) GMM Tied Full Cov



(d) GMM Tied Full Cov

Figure 2.8: GMM with  $\pi=0.5$  RAW Features

Figure 2.8 shows the different values of minDCF with  $\pi = 0.5$  and the corresponding number of components for each type of GMM. For example, we can see that for the full cov, the best number of components is 4. Here below we are going to see the results of the RAW Features and with PCA with  $m = 9$ .

### GMM - RAW Feature

Components	2	4	8	16	32	64	128
$\pi=0.1$							
Full-Cov	0.497	0.526	0.69	0.63	0.877	0.976	1.0
Diag-Cov	0.719	0.544	0.514	0.531	0.592	0.698	0.741
Full-Cov-Tied	0.616	0.611	0.558	0.542	0.588	0.563	0.71
Diag-Cov-Tied	0.805	0.558	0.501	0.56	0.601	0.673	0.765
$\pi=0.5$							
Full-Cov	0.28	0.264	0.325	0.391	0.568	0.737	1.0
Diag-Cov	0.451	0.275	0.262	0.26	0.281	0.359	0.425
Full-Cov-Tied	0.331	0.255	0.244	0.253	0.292	0.314	0.341
Diag-Cov-Tied	0.462	0.276	0.254	0.262	0.282	0.286	0.358
$\pi=0.9$							
Full-Cov	0.105	0.094	0.108	0.124	0.213	0.306	0.523
Diag-Cov	0.141	0.087	0.09	0.092	0.107	0.122	0.144
Full-Cov-Tied	0.111	0.098	0.083	0.087	0.104	0.108	0.134
Diag-Cov-Tied	0.144	0.086	0.089	0.088	0.093	0.11	0.131

We can see that the Tied Full Covariance is the best in almost all components. Meanwhile, the worst is the Full Covariance that reaches a midDCF = 1.0 with 128 components.

### GMM - PCA with $m = 9$

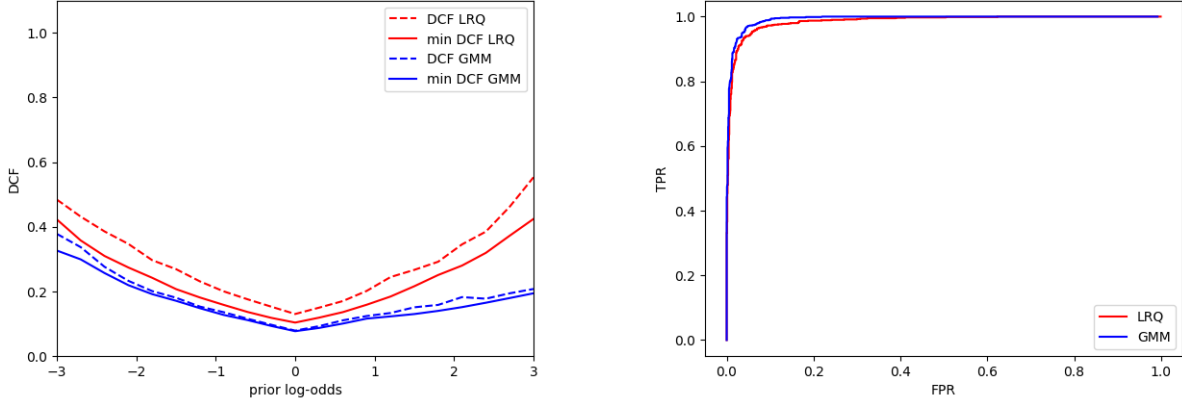
Components	2	4	8	16	32	64	128
$\pi=0.1$							
Full-Cov	0.455	0.599	0.599	0.684	0.837	0.999	1.0
Diag-Cov	0.742	0.599	0.553	0.546	0.513	0.638	0.729
Full-Cov-Tied	0.629	0.608	0.56	0.56	0.494	0.522	0.589
Diag-Cov-Tied	0.741	0.554	0.528	0.538	0.554	0.58	0.624
$\pi=0.5$							
Full-Cov	0.299	0.293	0.322	0.371	0.467	0.704	1.0
Diag-Cov	0.367	0.272	0.253	0.281	0.287	0.34	0.374
Full-Cov-Tied	0.33	0.261	0.269	0.254	0.285	0.324	0.322
Diag-Cov-Tied	0.368	0.268	0.248	0.276	0.306	0.328	0.359
$\pi=0.9$							
Full-Cov	0.099	0.091	0.113	0.125	0.158	0.218	0.376
Diag-Cov	0.114	0.096	0.094	0.093	0.103	0.109	0.135
Full-Cov-Tied	0.109	0.103	0.087	0.095	0.109	0.119	0.138
Diag-Cov-Tied	0.114	0.087	0.095	0.09	0.093	0.101	0.12

## 2.6 Recap

We are now going to compare the best three models with their best parameters:

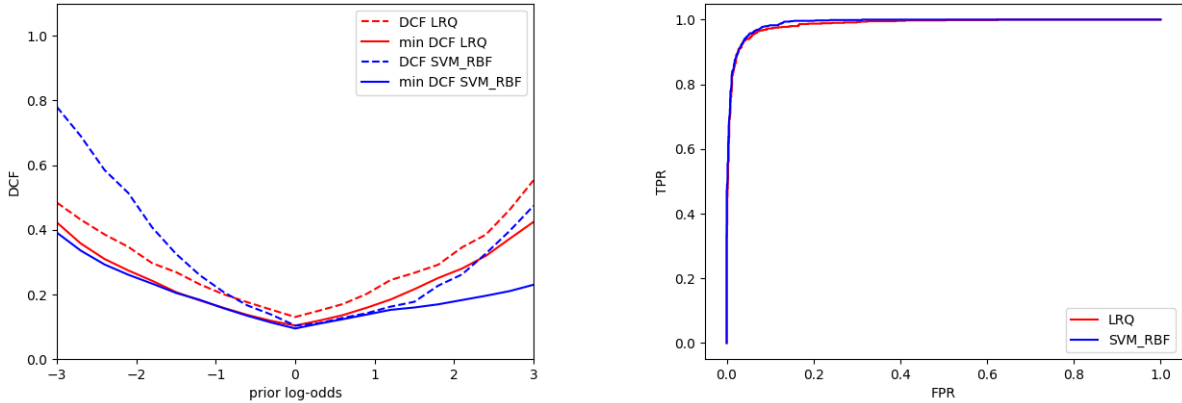
- Quadratic Logistic Regression,  $\lambda = 0.4$
- Radial Basis Function SVM, **PCA**  $\mathbf{m} = 8$ ,  $\mathbf{C} = 10$ ,  $\mathbf{K} = 0.1$ ,  $\gamma = 0.001$
- Gaussian Mixture Model, **RAW**, **Components** = 8

Now let's see the Bayes Error Plot and the ROC curve of each comparison. We start with QLR and GMM.



It can be seen that GMM performs better than the QLR for both minDCF and actDCF. Also, the actDCF of both model is close to the minDCF, so no calibration is needed.

Now let's see QLR vs SVM RBF.



Here we clearly need some calibration about the SVM. This is needed because the scores are not calibrated since SVM has no probabilistic interpretation! We are going to apply the prior weighted logistic regression as calibration model. Let's define a function

$$f(s) = \log \frac{f_{S|H}(s|H_T)}{f_{S|H}(s|H_F)} = \alpha s + \beta$$

where  $s$  is the input score,  $f(s)$  computes the  $s_{cal}$  that is the calibrated score. The original score  $s$  are the scores obtained by the SVM.

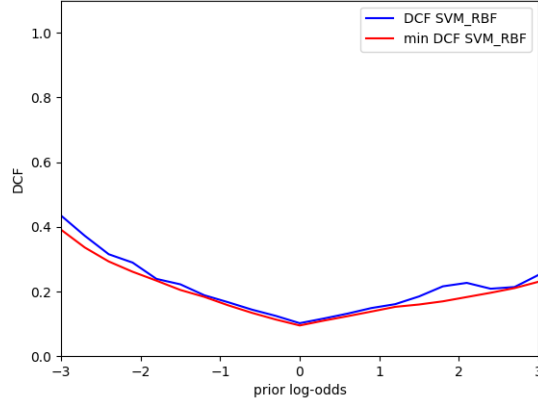
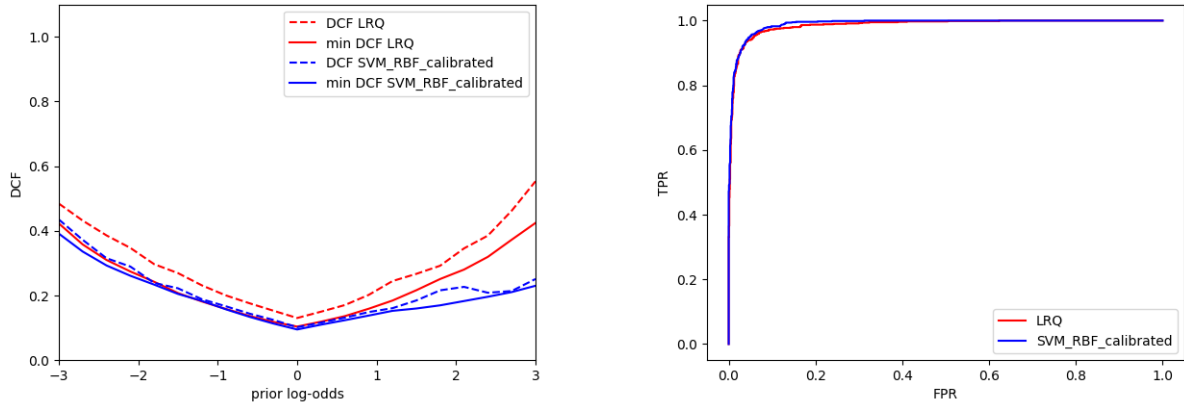


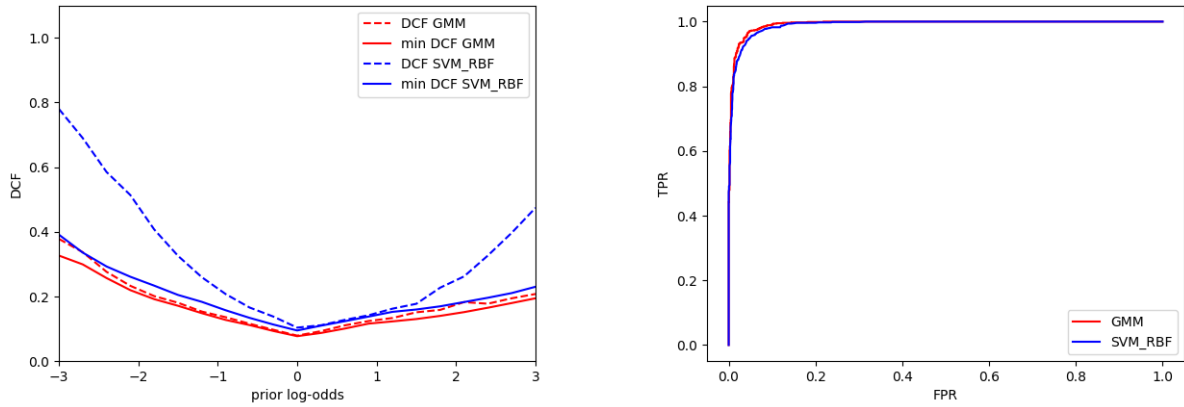
Figure 2.11: SVM RBF Calibrated

Figure 2.11 shows the calibrated SVM, with an actDCF that is closer to the minDCF, so the calibration worked.

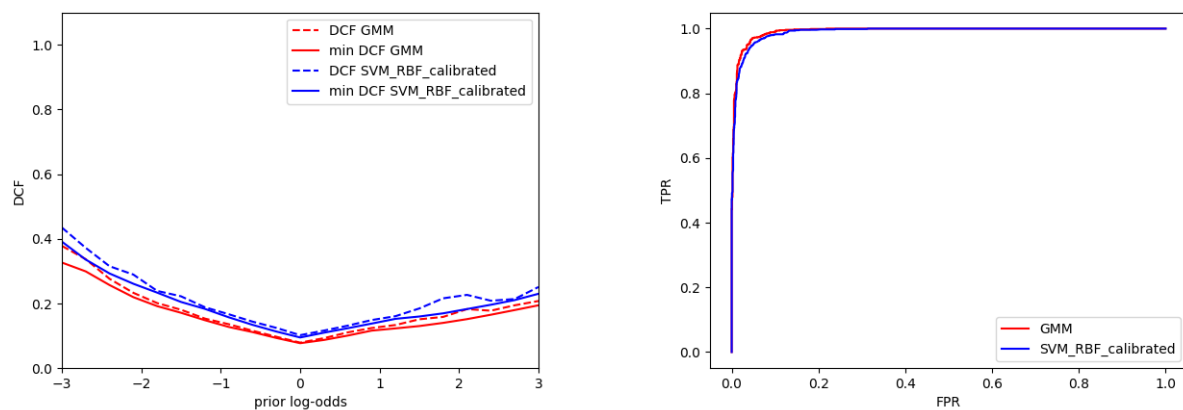
Now let's see the SVM RBF Calibrated vs QLR.



The SVM RBF is slightly better than the QLR for both midDCF and actDCF. Last we have the comparison between the SVM RBF and GMM. Again the SVM's scores are not calibrated so let's calibrate them.







Here we have the calibrated score, we can see that the GMM is better than the SVM.

**LAST DECISION:** Our best model is the **Gaussian Mixture Model, RAW, Components = 8**. Now let's use the Test set and see if the GMM is also the best model for the evaluation.

## Chapter 3

# Evaluation

We are now going to evaluate our models using the evaluation set (or test set). The evaluation will be done thanks to the minDCF. In the validation part we used the k-fold with  $k = 5$ , but in the evaluation part we won't use k-fold because we don't need the validation test, since there's the test set. So we are going to train each model on the full training set then test this model on the test set.

### 3.1 Multivariate Gaussian Model

In this section we are going to show the result on the **Test set** for the Gaussian part, exactly as we did for the validation part (that was done on the **Validation set**). We are going to test on Raw features, PCA with  $m = 8$  and  $m = 9$ .

	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
RAW Features			
MVG	0.545	0.276	0.083
Naive Gaussian	0.718	0.352	0.111
Tied Gaussian	0.789	0.455	0.176
Naive Tied Gaussian	0.829	0.48	0.177

	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
PCA $m = 9$			
MVG	0.569	0.272	0.082
Naive Gaussian	0.635	0.31	0.087
Tied Gaussian	0.783	0.461	0.177
Naive Tied Gaussian	0.864	0.483	0.175

	$\pi = 0.1$	$\pi = 0.5$	$\pi = 0.9$
PCA $m = 8$			
MVG	0.571	0.271	0.083
Naive Gaussian	0.638	0.315	0.087
Tied Gaussian	0.773	0.456	0.178
Naive Tied Gaussian	0.865	0.484	0.176

We can see that for all 3 models (Raw,  $m=8$ ,  $m=9$ ) the MVG is the best. Only for the Naive assumption the model with PCA are better than the Raw model.

Comparing these results with the one obtained from the Validation part, they seem to be a little bit better, but not that much, so the results are consistent.

## 3.2 Logistic Regression

Again, in this section there are the results for the Logistic Regression part.

### 3.2.1 Linear Logistic Regression

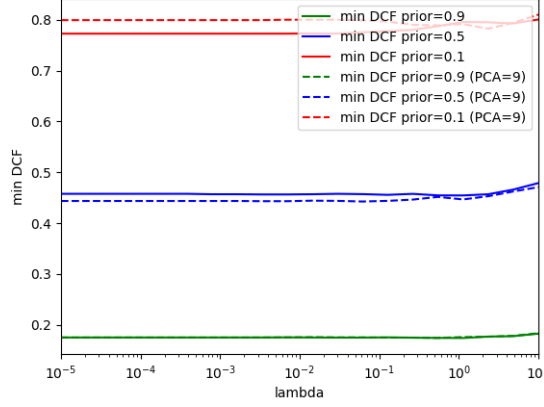


Figure 3.1: LR - compare different  $\lambda$

Figure 3.1 shows the midDCF for each  $\lambda$ . The optimal  $\lambda = 0.4$ . We can also notice that for our application ( $\pi = 0.5$ ) the model with PCA with  $m = 9$  is slightly better. Now let's see the results.

	pi=0.1	pi=0.5	pi=0.9
RAW			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.792	0.458	0.178
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.784	0.457	0.174
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.793	0.455	0.174
PCA m=9			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.787	0.458	0.178
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.79	<b>0.452</b>	0.176
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.795	0.444	0.173
PCA m=8			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.786	<b>0.457</b>	0.179
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.787	<b>0.452</b>	0.176
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.797	<b>0.442</b>	0.174

We can see that using the Raw or PCA models doesn't change that much, so all the three models can be considered the "best" choice for LR, but for our application, from the Gaussian part, the MVG model has midDCF = **0.271**, meanwhile the best I can obtain using LR is minDCF = **0.442**. So the LR model isn't the best choice overall. Again, this is telling us that the linear separation isn't the one we need, so we expect that the RBF, Poly SVM and QLR will perform better! (In fact, those results are similar to the ones given by the Tied Gaussian, that is also linear) By the way, the results are also consistent with the Validation part.

### 3.2.2 Quadratic Logistic Regression

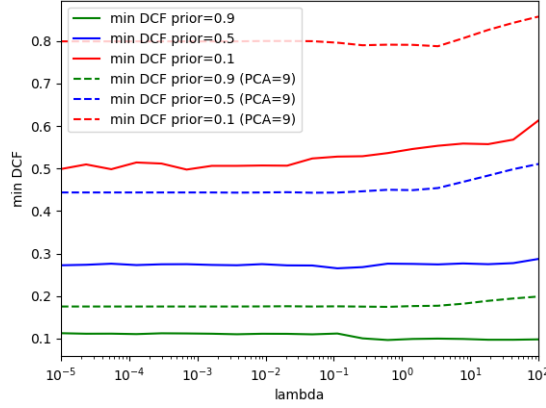


Figure 3.2: minDCF for the QLR

Figure 3.2 show that PCA will perform badly, in fact for all application we have the minDCF related to PCA higher than the minDCF related to the RAW features.

Here the results:

	pi=0.1	pi=0.5	pi=0.9
RAW			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.538	0.292	0.123
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.546	0.277	0.096
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.587	0.28	0.09
PCA m=9			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.787	0.458	0.178
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.79	0.452	0.176
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.795	0.444	0.173
PCA m=8			
Log Reg ( $\lambda = 0.4, \pi_T = 0.1$ )	0.786	0.457	0.179
Log Reg ( $\lambda = 0.4, \pi_T = 0.5$ )	0.787	0.452	0.176
Log Reg ( $\lambda = 0.4, \pi_T = 0.9$ )	0.797	0.442	0.174

These result are consistent with the validation part. As we expected this model is a quadratic one, so the minDCF are good. Those result are also similar to the MVG.

### 3.3 Support Vector Machine

Here there are the best parameter of each SVM:

- Linear SVM:  $K=1.0$ ,  $C=1.0$
- Quadratic SVM:  $K=1.0$ ,  $C=0.1$ ,  $\text{degree}=2$ ,  $\text{constant}=1$
- RBF SVM:  $K=0.1$ ,  $C=10$ ,  $\gamma=0.001$

#### SVM - Raw Features

	$\pi=0.1$	$\pi=0.5$	$\pi=0.9$
Linear SVM	0.778	0.459	0.176
Quadratic SVM	0.529	0.278	0.098
RBF SVM	0.51	0.242	0.086

#### SVM - PCA with $m = 9$

	$\pi=0.1$	$\pi=0.5$	$\pi=0.9$
Linear SVM	0.79	0.456	0.179
Quadratic SVM	0.541	0.274	0.093
RBF SVM	0.494	0.24	0.084

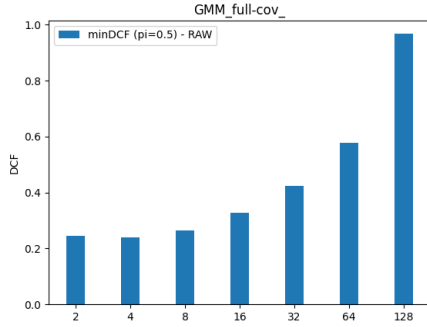
#### SVM - PCA with $m = 8$

	$\pi=0.1$	$\pi=0.5$	$\pi=0.9$
Quadratic SVM	0.51	0.266	0.092
RBF SVM	0.469	0.236	0.084

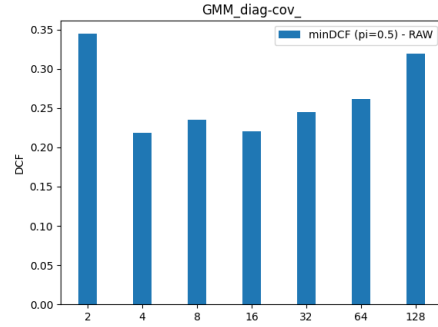
As we said, the RBF and the Quadratic that are non-linear achieve good results. They are pretty similar to the MVG (that is also quadratic), but are way better than the linear ones, for example the LR.

### 3.4 Gaussian Mixture Model

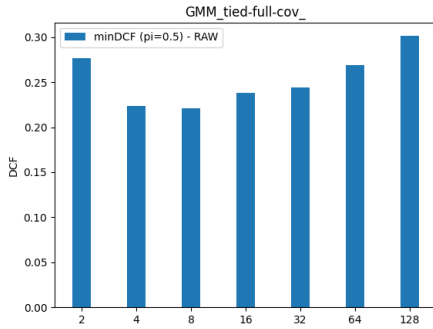
We are now going to see how GMM handles the Test set.



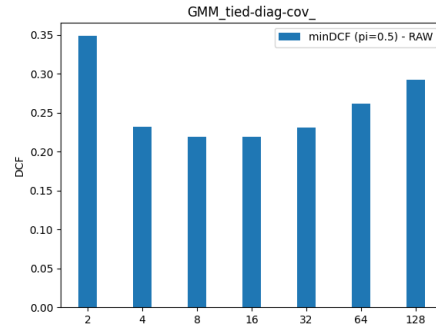
(a) Full Covariance



(b) Diagonal Covariance



(c) Full Tied Covariance



(d) Diagonal Tied Covariance

We have chosen 8 components, which seems to be the best number of components in the validation part.

#### GMM - Raw Features and PCA with $m = 9$

	RAW	PCA $m = 9$
	$\pi=0.1$	
Full-Cov	0.501	0.553
Diag-Cov	0.51	0.467
Full-Cov-Tied	0.503	0.438
Diag-Cov-Tied	0.474	0.484
	$\pi=0.5$	
Full-Cov	0.264	0.268
Diag-Cov	0.235	0.218
Full-Cov-Tied	0.221	0.217
Diag-Cov-Tied	0.219	0.226
	$\pi=0.9$	
Full-Cov	0.086	0.093
Diag-Cov	0.074	0.076
Full-Cov-Tied	0.077	0.074
Diag-Cov-Tied	0.071	0.072

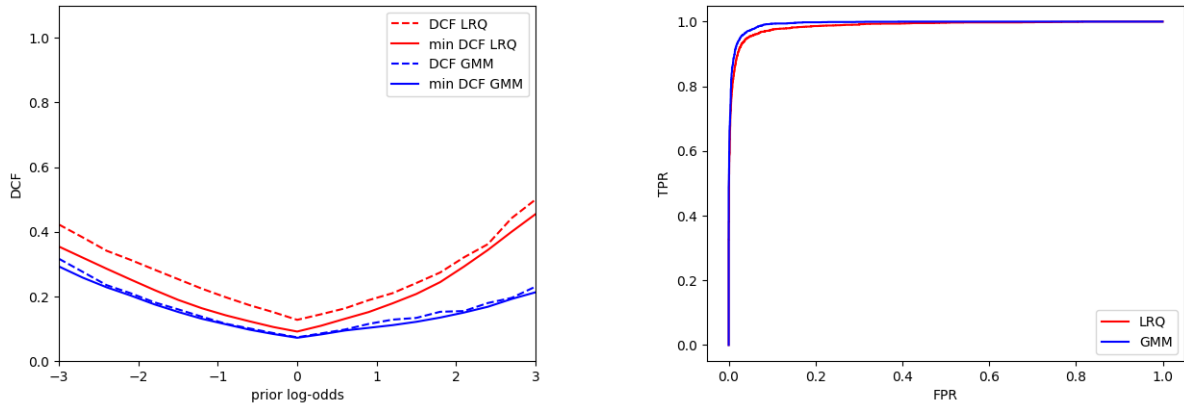
The GMM on the Test set is again consistent with the one on the Validation set. The best we can do for our application is with the Full Tied Covariance Matrix with  $\text{minDCF} = 0.217$ .

### 3.5 Recap

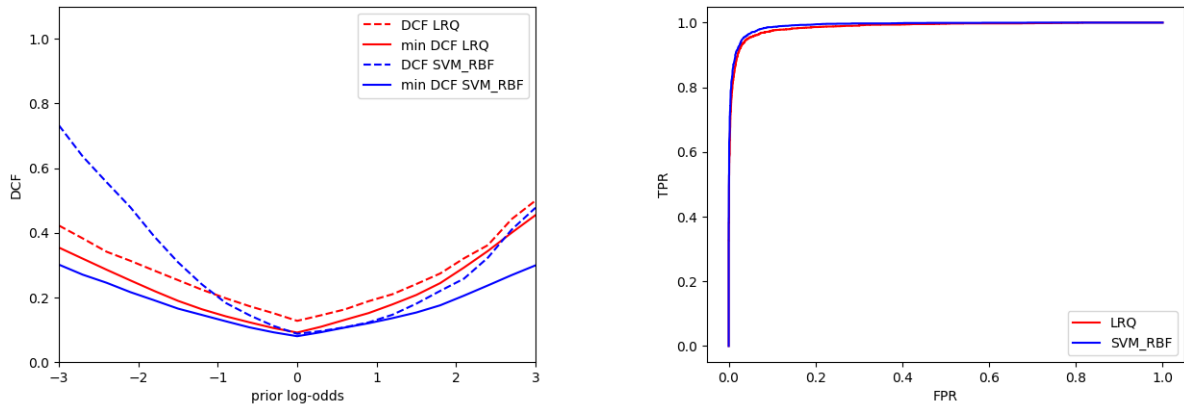
As we did for the evaluation part, we are going to compare the three models:

- Quadratic Logistic Regression,  $\lambda = 0.4$
- Radial Basis Function SVM, **PCA**  $m = 8$ ,  $C = 10$ ,  $K = 0.1$ ,  $\gamma = 0.001$
- Gaussian Mixture Model, **RAW**, **Components** = 8

Let's start with QLR vs GMM.



The minDCF and actDCF are close, so we don't need any calibration. The GMM model is better than the QLR for both actDCF and minDCF. Now let's see the SVM vs QLR.



Again the SVM RBF is not calibrated, so we need to calibrate the scores. We used the prior weighted logistic regression.

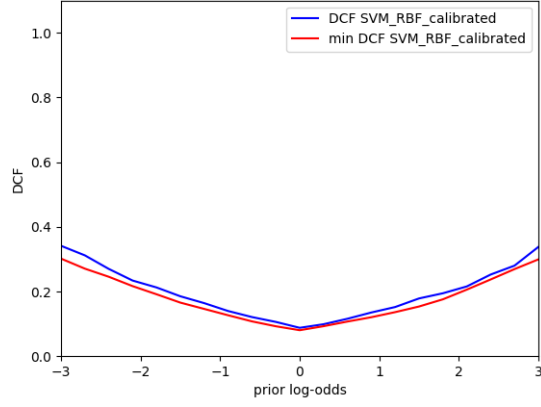
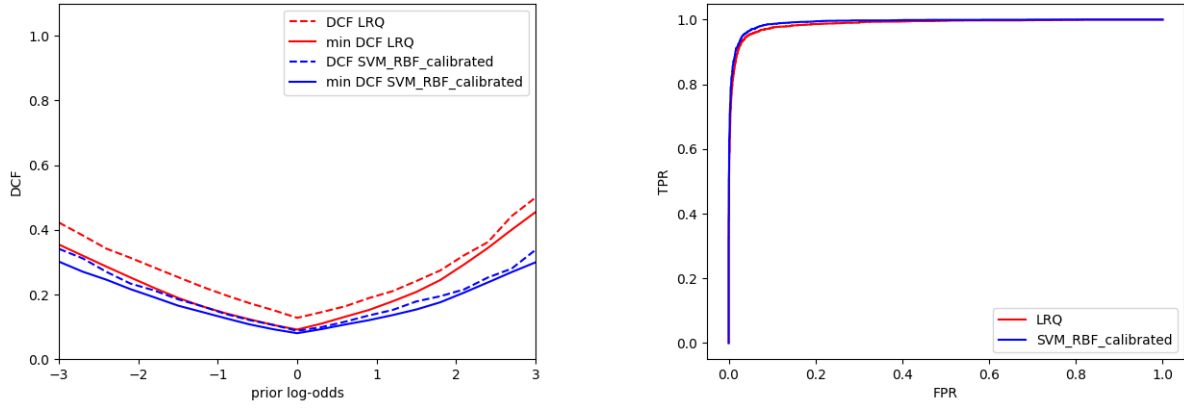
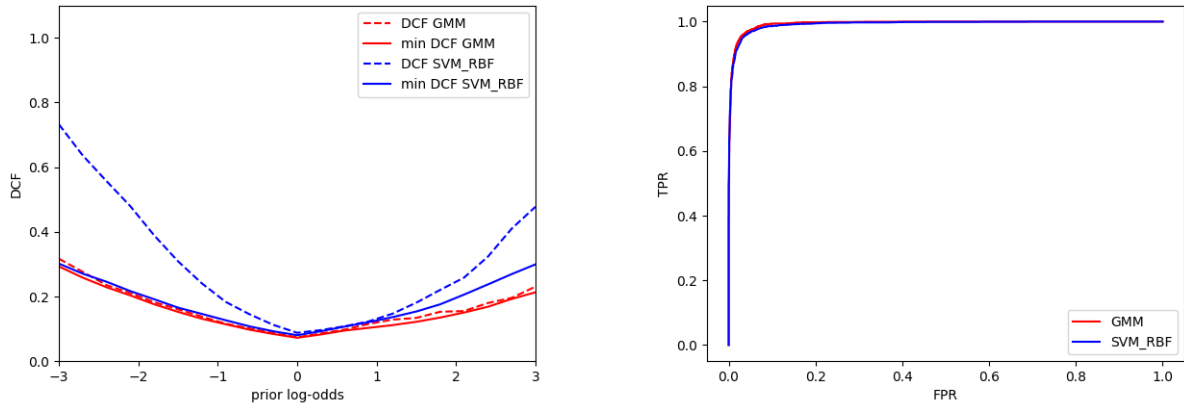


Figure 3.6: SVM RBF Calibrated

Figure 3.6 shows the calibrated SVM.

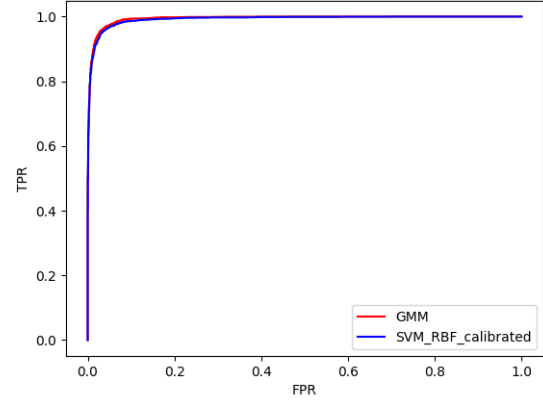
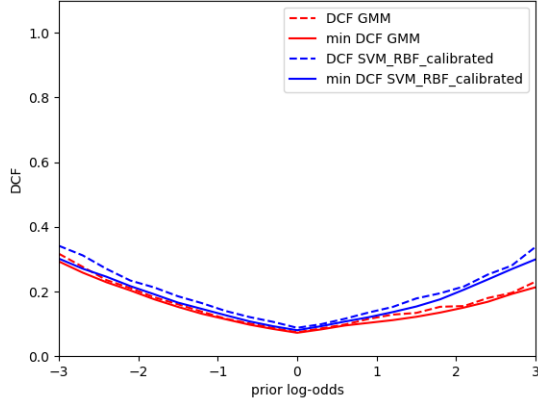


Here the SVM RBF calibrated vs QLR. We can see that SVM is better than the QLR. The last comparison is between SVM RBF and GMM.



Again, the scores are not calibrated, let's calibrate them.





We can see that the GMM is better than the SVM.

**Final Decision** The best model is the Gaussian Mixture Model with 8 components, but the with the RAW we obtain  $\text{midDCF} = 0.219$  and with PCA with  $m = 9$  we obtain  $\text{midDCF} = 0.217$ . So the best of the best would be the GMM PCA with  $m = 9$  instead of GMM RAW. Anyway, the difference is so small that we can consider that the expectation made on the validation part are right.