Fall 2018

California State University, Northridge
Department of Electrical & Computer Engineering

Experiment 11
2nd Order Systems
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ECE 240L
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Group 2
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Introduction:

The objective of this experiment is to understand how to build and analyze circuits that contain two different storage elements or storage elements that cannot be simplified. The experiment covers both RLC circuits in parallel and in series. The experiment also covers the use of differential equations and goes over the various conditions that can occur in RLC circuits.

Equipment used:

Type	Model	Serial No.	Calibration Date
Oscilloscope	Tektronix 2213A	N/A	N/A
Function Generator	Agilent 33220A	N/A	N/A
Proto Board	Tektronix	N/A	N/A

Parts Used:

QTY	Component	Value	Tolerance	Type
1	Resistor	560 Ω	+/- 5%	carbon
1	Resistor	6.8 kΩ	+/- 5%	carbon
1	Inductor	6.8 mH	+/- 5%	n/a
1	Capacitor	.01 µF	+/- 5%	n/a

Software Used:

1. Google Docs
2. Krita
3. Snipping Tool

Theory:

The differential equations can be found using KVL and KCL in addition to the equations for the voltages or current of inductors. (Equations 9.1, 9.2, 9.6, and 9.7). The KVL or KCL equations should have their derivatives taken on both sides to get a second order differential equation. The general form of a differential equation and its roots are the following

$$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=0$$ (Equation 11.1)

$$r_{1,2}=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$ (Equation 11.2)

Three cases can occur in an RLC circuit

Case 1 is when the circuit is overdamped in which the equations roots are real and distinct. The resulting current or voltage will take on the form of decaying exponentials

$$y=A_1e^{r_1t}+A_2e^{r_2t}$$ (Equation 11.12a)

Case 2 is when the circuit is critically damped in which its roots are real and equal and has the following form.

$$y=e^{rt}\left(A_1+A_2t\right)$$ (Equation 11.12b)

Case 3 is underdamped in which r1 and r2 are complex.

$$y=e^{\alpha t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$ (Equation 11.12c)

Where

$$\alpha=-\frac{b}{2a}$$ (Equation 11.3)

$$\omega_d=\frac{\sqrt{4ac-b^2}}{2a}$$ (Equation 11.4)

To find ω_d and $\boldsymbol{\alpha}$ in the experiment the following functions are used.

$$t_2-t_1=T_D \quad\rightarrow\quad T_D=\frac{2\pi}{\omega_d}$$ (Equation 11.9)

$$\alpha=\frac{\omega_d}{2\pi}\ln\left(\frac{v_1}{v_2}\right)$$ (Equation 11.11)

Procedure & Results:

1. The ω_d and $\boldsymbol{\alpha}$ values for Figure 11.2 were calculated for values of R at 560 Ω and 6.8kΩ and recorded in Table 11.1

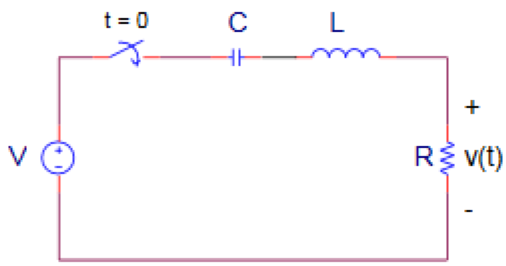


Figure 11.2

$$d^2 i + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i(t) = 0$$

$$b = \frac{R}{L} \qquad c = \frac{1}{LC} \qquad \omega_0^2 = \frac{1}{LC} = \frac{1}{(6.8\times10^{-3})(.01\times10^{-6})} = 1470588235$$

$$\alpha = \frac{R}{2L} \qquad R = 560, \ \alpha = 4117.64$$
$$R = 6.8K, \ \alpha = 50,000$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \ ; \ R = 560, \ \omega_d = 38126.5 \text{ rad/s}$$

The circuit Figure 11.4 was constructed and measured using the digital oscilloscope using R at 560 Ω and 6.8kΩ. The values of ω_d and $\boldsymbol{\alpha}$ were recorded in Table 11.1.

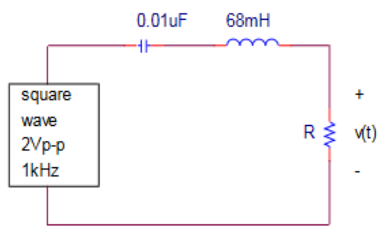


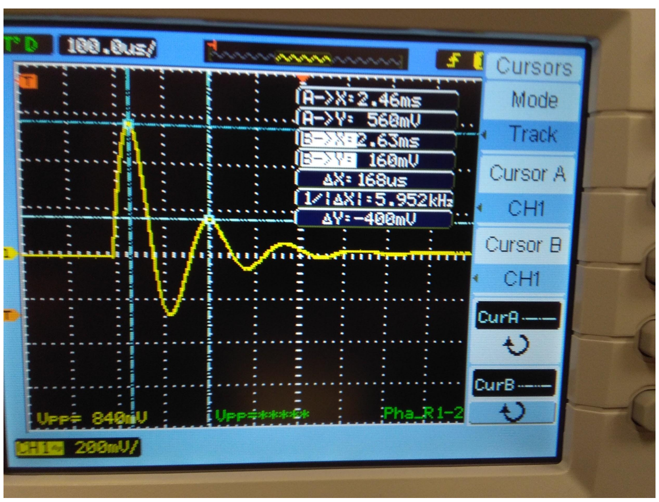
Figure 11.4

	calc			meas			% error	
R	α	ωd	type	α	ωd	type	α	ωd
6.8kΩ	50,000	N/A	over damped	N/A	N/A	over damped	N/A	N/A
56kΩ	4117.14	38126.54 rad/s	under damped	4256.51	38500 rad/s	under damped	2.2%	3.5%

(Table 11.1)

The display is shown in Pictures 11.1.

Picture 11.1a



Picture 11.1b

2. The ω_d and $\boldsymbol{\alpha}$ values for Figure 11.3 were calculated for values of R at 560 Ω and 6.8kΩ and recorded in Table 11.2

   The circuit Figure 11.5 was constructed and measured using the digital oscilloscope using R at 560 Ω and 6.8kΩ. The values of ω_d and $\boldsymbol{\alpha}$ were recorded in Table 11.2.

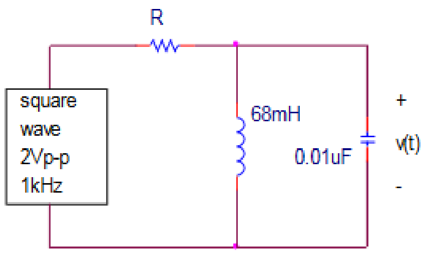
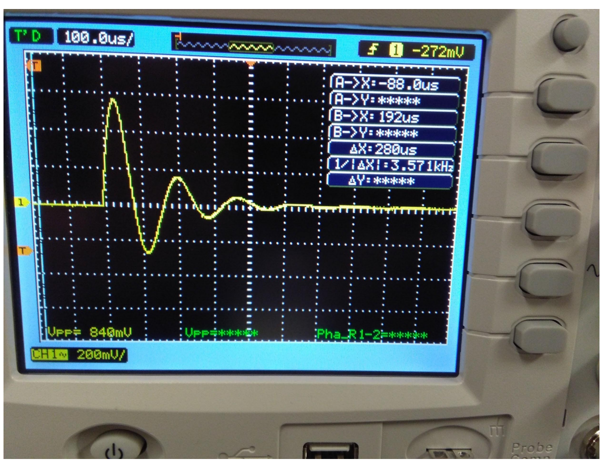


Figure 11.5

	calc			meas			% error	
R	α	ωd	type	α	ωd	type	α	ωd
6.8kΩ	7352.94	37638.7 rad/s	under damped	7451.35	37399.91 rad/s	under damped	2.56%	.62%
560Ω	89285.7	N/A	over damped	N/A	N/A	over damped	N/A	N/A

(Table 11.2)

The display is shown in Picture 11.2.



(Picture 11.2a)

(Picture 11.2b)

Conclusion:

The experiment was a success in determining the various cases in which the resistors would change a parallel and series circuit from from overdamped to underdamped. Higher resistances in a series RLC result in an overdamped circuit, and for parallel RLC circuits higher resistances result in an underdamped circuit.