# NUCLEAR PHYSICS

branch of physics which deals with the study of nucleus.

## NUCLEUS :

(a) *Discoverer*: Rutherford

(b) *Constituents* : neutrons (n) and protons (p) [collectively known as nucleons]

1. **Neutron** : It is a neutral particle. It was discovered by J. Chadwick. Mass of neutron, $m_n = 1.6749286 \times 10^{-27}$ kg.

2. **Proton** : It has a charge equal to +e. It was discovered by Goldstein. Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg $\therefore m_p \lesssim m_n$

* electron : $m_e \doteq 9.109438356 \times 10^{-31}$ kg

(c) *Representation*:

$_ZX^A$ or $^A_ZX$

where X $\Rightarrow$ symbol of the atom
Z $\Rightarrow$ Atomic number = number of protons
A $\Rightarrow$ Atomic mass number = total number of nucleons.
= no. of protons + no. of neutrons.

**Atomic mass number :**

*It is nearest integer value of mass represented in a.m.u. (atomic mass unit).*

1 a.m.u. = $\frac{1}{12}$ [mass of one atom of $_6C^{12}$ atom at rest and in ground state]

$1.6603 \times 10^{-27}$ kg ; 931.478 MeV/$c^2$

Mass of proton ($m_p$) = mass of neutron ($m_n$) = 1 a.m.u.

**Some Definitions :**

(1) *Isotopes* :
The nuclei having the same number of protons but different number of neutrons are called isotopes.

(2) *Isotones* :
Nuclei with the same neutron number N but different atomic number Z are called isotones.

(3) *Isobars* :
The nuclei with the same mass number but different atomic number are called isobars.

(4) *Size* of nucleus : Order of $10^{-15}$ m (fermi)
Radius of nucleus ; $R = R_0A^{1/3}$
where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)
A = Atomic mass number of atom.

(5) *Density* : density = $\frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi(R_0A^{1/3})^3} = \frac{3m_p}{4\pi R_0^3}$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3} = 3 \times 10^{17} \text{ kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

MASS DEFECT: When the nucleons comprising a nucleus are weighed individually (not literally, but by calculating), their weight (Total) is slightly greater than the mass of nucleus formed by them. This small loss in mass after making the nucleus is called mass defect.

$$M_{exp.} = Zm_p + (A-Z)m_n$$

$$M_{obs} = M_{atom} - Zm_e$$ (nucleus cannot be weighed directly. only the atom can.)

$$M_{obs} < M_{exp.}$$

$$\text{mass defect } (\Delta m) = M_{exp} - M_{obs} = Z(m_p + m_e - m_n) + Am_n - M_{atom}$$

$$= Am_n - 1.394561 \times 10^{-30} \times Z - M_{atom}$$

BINDING ENERGY.

min. energy req. to break the nucleus into its nucleons (or) amount of energy released during formation of nucleus by its constituent particles from $\infty$ sep.

$$BE = \Delta mc^2$$

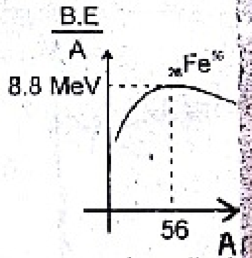
$$= \Delta m \times 931.5 \text{ MeV}$$

NOTE. BE/nucleon is more for a nucleus which is more stable.

$\therefore$ $\frac{BE_1}{A_1} > \frac{BE_2}{A_2}$ then nucleus (1) is more stable.

### 3.1 Variation of binding energy per nucleon with mass number :

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.8 MeV for $A \approx 50 \rightarrow 80$. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases. Binding energy per nucleon is maximum for $_{26}Fe^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.




* *The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called* Fission
* *The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process* is called : *Fusion.*

## 4 RADIOACTIVITY :

It was discovered by Henry Becquerel.

*Spontaneous emission of radiations ($\alpha$, $\beta$, $\gamma$) from unstable nucleus is called* ***radioactivity.***

Substance *which show radioactivity are known as* ***radioactive substance.***

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits $\alpha$ particle or $\beta$ particle. After emission of $\alpha$ or $\beta$ the remaining nucleus may emit $\gamma$-particle, and converts into more stable nucleus.

### $\alpha$-particle :

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of $\alpha$-particle = Mass of $_2He^4$ atom $- 2m_e \approx 4\, m_p$

Charge of $\alpha$-particle = + 2 e

### $\beta$-particle :

(a) **$\beta^-$ (electron) :**

Mass = $m_e$ ; Charge = $-e$

(b) **$\beta^+$ (positron) :**

Mass = $m_e$ ; Charge = $+e$

positron is an antiparticle of electron.

### Antiparticle :

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and convert into energy. For example : (i) electron ($-e$, $m_e$) and positron ($+e$, $m_e$) are anti particles. (ii) neutrino ($\nu$) and antineutrino ($\overline{\nu}$) are anti particles.

**$\gamma$-particle :** They are energetic photons of energy of the order of Mev and having rest mass zero.

## 5. RADIOACTIVE DECAY (DISPLACEMENT LAW) :

### 5.1 $\alpha$-decay :

Q value = $\Delta mc^2$

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

***Q value :*** *It is defined as energy released during the decay process.*

Q value = rest mass energy of reactants − rest mass energy of products.

This energy is available in the form of increase in K.E. of the products.

Let, $M_x$ = mass of atom $_ZX^A$

$M_y$ = mass of atom $_{Z-2}Y^{A-4}$

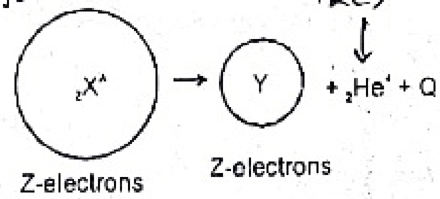
$M_{He}$ = mass of atom $_2He^4$.

$$Q \text{ value} = [(M_x - Zm_e) - \{(M_y - (Z-2)\, m_e) + (M_{He} - 2m_e)\}]c^2$$
$$= [M_x - M_y - M_{He}]\, c^2$$

α particle (It has charge +2e)

Considering actual number of electrons in $\alpha$-decay

$$Q \text{ value} = [M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]c^2$$
$$= [M_x - M_y - M_{He}]\, c^2$$



## Calculation of kinetic energy of final products :

As atom X was initially at rest and no external forces are acting, so final momentum also has to be zero. Hence both Y and α-particle will have same momentum in magnitude but in opposite direction.


$p_\alpha^2 = p_Y^2$ $\qquad$ $2m_\alpha T_\alpha = 2m_Y T_Y$ $\qquad$ (Here we are representing T for kinetic energy)

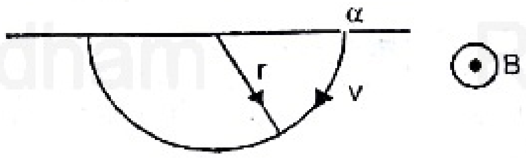
$Q = T_Y + T_\alpha$ $\qquad$ $m_\alpha T_\alpha = m_Y T_Y$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q; \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q$$

$$T_\alpha = \frac{A-4}{A} Q \quad ; \qquad T_Y = \frac{4}{A} Q$$

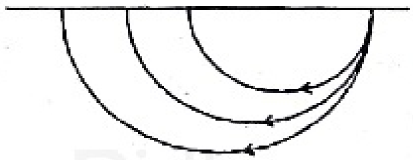
From the above calculation, one can see that all the α-particles emitted should have same kinetic energy. Hence, if they are passed through a region of uniform magnetic field having direction perpendicular to velocity, they should move in a circle of same radius.

$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2Km}}{2eB}$$




## Experimental Observation :

Experimentally it has been observed that all the α-particles do not move in the circle of same radius, but they move in circles having different radii.



This shows that they have different kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e. yet the energy of emitted α-particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emits photon to acquire their ground state.

$X \longrightarrow Y^*$(excited state) $+ \alpha + Q$

$\downarrow$

$Y$ + photon (γ particle)

The only difference between Y and Y* is that Y* is in excited state and Y is in ground state.

Let, the energy of emitted γ-particles be E

$Q = T_\alpha + T_Y + E$

where $Q = [M_x - M_y - M_{He}]\, c^2$

$T_\alpha + T_Y = Q - E$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q - E); \quad T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q - E)$$

## 5.2 $\beta^-$- decay :

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

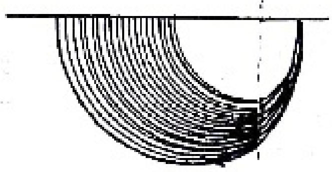
${}_{-1}e^0$ can also be written as ${}_{-1}\beta^0$.

Here also one can see that by momentum and energy conservation, we will get

$$T_e = \frac{m_Y}{m_e + m_Y} Q;$$

$$T_Y = \frac{m_e}{m_e + m_Y} Q$$

as $m_e << m_Y$, we can consider that all the energy is taken away by the electron.



From the above results, we will find that all the β-particles emitted will have same energy and hence they have same radius if passed through a region of perpendicular magnetic field. But, experimental observations were completely different. On passing through a region of uniform magnetic field perpendicular to the velocity, it was observed that β-particles take circular paths of different radius having a continuous spectrum.

To explain this, Paulling has introduced the extra particles called neutrino and antineutrino (antiparticle of neutrino).

$\bar{\nu} \rightarrow$ antineutrino, $\nu \rightarrow$ neutrino

**Properties of antineutrino($\bar{\nu}$) & neutrino(n) :**

(1) They are like photons having rest mass = 0
speed = c
Energy, $E = mc^2$

(2) They are chargeless (neutral)

(3) They have spin quantum number, $s = \pm \frac{1}{2}$

Considering the emission of antineutrino, the equation of $\beta^-$ - decay can be written as

$$_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum no. balance (p, n and ± e each has spin quantum number ± 1/2).

During $\beta^-$ - decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$$

Let, $M_x$ = mass of atom $_ZX^A$
$M_y$ = mass of atom $_{Z+1}Y^A$
$m_e$ = mass of electron

Q value = $[(M_X - Zm_e) - \{(M_Y - (z+1)\,m_e) + m_e\}]\,c^2$ $= [M_X - M_Y]\,c^2$

Considering actual number of electrons.

Q value = $[M_X - \{(M_Y - m_e) + m_e\}]\,c^2$ $= [M_X - M_Y]\,c^2$

## Solved Examples

**Example 8.** Consider the beta decay

$^{198}Au \rightarrow {}^{198}Hg^* + \beta^- + \bar{\nu}$

where $^{198}Hg^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV ab ground state. What can be the maximum kinetic energy of the electron emitted? The atom $^{198}Au$ is 197.968233 u and that of $^{198}Hg$ is 197.966760 u.

**Solution :** If the product nucleus $^{198}Hg$ is formed in its ground state, the kinetic energy available to the e and the antineutrino is

$$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 .$$

As $^{198}Hg^*$ has energy 1.088 MeV more than $^{198}Hg$ in ground state, the kinetic energy actually ava

$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1.088$ MeV

$$= (197.968233\text{ u} - 197.966760\text{ u})\left(931\frac{\text{MeV}}{\text{u}}\right) - 1.088\text{ MeV}$$

$= 1.3686$ MeV $- 1.088$ MeV $= 0.2806$ MeV.

This is also the maximum possible kinetic energy of the electron emitted.

**$\beta^+$- decay :**

$$_ZX^A \to {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$$

In $\beta^+$ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$$p \to n + {}_{+1}e^0 + \nu$$

As mass increases during conversion of proton to a neutron, hence it requires energy for $\beta^+$ decay to take place, $\therefore$ $\beta^+$ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

$$\text{Q value} = [(M_X - Zm_e) - \{(M_Y - (Z-1)\,m_e) + m_e\}]\,c^2$$
$$= [M_X - M_Y - 2m_e]\,c^2$$

Considering actual number of electrons.

$$\text{Q value} = [M_X - \{(M_Y + m_e) + m_e\}]\,c^2$$
$$= [M_X - M_Y - 2m_e]\,c^2$$

## *Solved Examples*

Calculate the Q-value in the following decays :

(a) $^{19}O \to {}^{19}F + e^- + \bar{\nu}$

(b) $^{25}Al \to {}^{25}Mg + e^+ + \nu$.

The atomic masses needed are as follows:

| $^{19}O$ | $^{19}F$ | $^{25}Al$ | $^{25}Mg$ |
|---|---|---|---|
| 19.003576 u | 18.998403 u | 24.990432 u | 24.985839 u |

Solu: (a) The Q-value of $\beta^-$-decay is

$$Q = [m(^{19}O) - m(^{19}F)]c^2$$
$$= [19.003576\text{ u} - 18.998403\text{ u}]\,(931\text{ MeV/u})$$
$$= 4.816\text{ MeV}$$

(b) The Q-value of $\beta^+$-decay is

$$Q = [m(^{25}Al) - m(^{25}Mg) - 2m_e]c^2$$

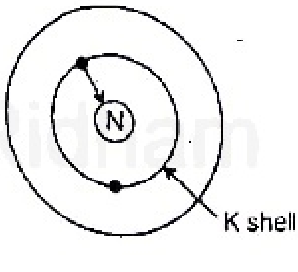
$$= \left[24.99032\,\text{u} - 24.985839\,\text{u} - 2 \times 0.511\frac{\text{MeV}}{c^2}\right]c^2$$

$$= (0.004593\text{ u})\,(931\text{ MeV/u}) - 1.022\text{ MeV}$$
$$= 4.276\text{ MeV} - 1.022\text{ MeV} = 3.254\text{ MeV}.$$

## K capture :

It is a rare process which is found only in few nucleus. In this process the nucleus captures one of the atomic electrons from the K shell. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.

$$p + {}_{-1}e^0 \to n + \nu$$




If X and Y are atoms then reaction is written as :

$$_ZX^A \to {}_{Z-1}Y^A + \nu + Q + \text{characteristic x-rays of Y.}$$

If X and Y are taken as nucleus, then reaction is written as :

$$_ZX^A + {}_{-1}e^0 \to {}_{Z-1}Y^A + \nu$$

Note: (1) Nuclei having atomic numbers from Z = 84 to 112 shows radioactivity.

(2) Nuclei having Z = 1 to 83 are stable (only few exceptions are there)

(3) Whenever a neutron is produced, a neutrino is also produced.

(4) Whenever a neutron is converted into a proton, a antineutrino is produced.

## 6. NUCLEAR STABILITY :

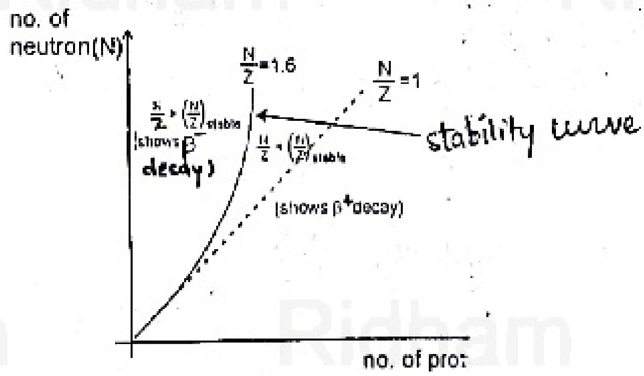



Figure shows a plot of neutron number N versus proton number Z for the nuclides found in nature. The solid line in the figure represents the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for the heaviest stable nuclides. The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability belt as well as to the right of it. The nuclides to the left of the stability region have excess neutrons, whereas, those to the right of the stability belt have excess protons. These nuclides are unstable and decay with time according to the laws of radioactive disintegration. Nuclides with excess neutrons (lying above stability belt) show $\beta^-$ decay while nuclides with excess protons (lying below stability belt) show $\beta^+$ decay and K - capture

## 7. NUCLEAR FORCE :

(i) Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus in spite of repulsion between the positively charge protons.

(ii) It is strongest force with in nuclear dimensions ($F_n$ ; 100 $F_e$)

(iii) It is short range force (acts only inside the nucleus)

(iv) It acts only between neutron-neutron, neutron-proton and proton-proton i.e. between nucleons

(v) It does not depend on the nature of nucleons.

(vi) An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the nucleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have $m_s = +1/2$ or $-1/2$) and is weaker if the spins are antiparallel (i.e., one nucleon has $m_s = +1/2$ and the other has $m_s = -1/2$). Here $m_s$ is spin quantum number.

## 8. RADIOACTIVE DECAY : STATISTICAL LAW :

(Given by Rutherford and Soddy)

Rate of radioactive decay $\propto$ N

where N = number of active nuclei

$= \lambda N$

where $\lambda$= decay constant of the radioactive substance.

Decay constant is different for different radioactive substances, but it does not depend on amount of Substance and time.

SI unit of $\lambda$ is $s^{-1}$

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant $\lambda$

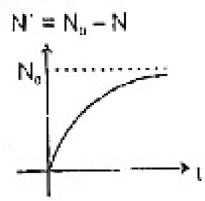
| | $A \xrightarrow{\lambda}$ | B | |
|---|---|---|---|
| t = 0 | $N_0$ | 0 | where $N_0$ = number of active nuclei of A at t = 0 |
| t = t | N | N' | where N = number of active nuclei of A at t = t |

Rate of radioactive decay of A $= -\dfrac{dN}{dt} = \lambda N$


$$-\int_{N_0}^{N}\frac{dN}{N} = \int_0^t \lambda dt \Rightarrow \qquad N = N_0 e^{-\lambda t} \qquad \text{(it is exponential decay)}$$

Number of nuclei decayed (i.e. the number of nuclei of B formed)

$$N' = N_0 - N$$
$$= N_0 - N_0 e^{-\lambda t}$$
$$N' = N_0(1 - e^{-\lambda t})$$



Half life.

All nuclear reax. are of $1^{st}$ order.

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

nuclei present after n half life ($t = nt_{1/2}$)

$N = N_0 e^{-\lambda t}$ $\therefore N = \frac{N_0}{2^n}$ $n \to \frac{t}{t_{1/2}}$ may be integral/rational.

Average life

$$T_{avg} = \frac{\text{Sum of ages of all nuclei}}{N_0} = \frac{t\int_0^{\infty} \lambda N_0 e^{-\lambda t} dt}{N_0} = \frac{1}{\lambda}$$

Note: If a radioactive decay undergoes through two diff. process, where $(t_{1/2})_1 = t_1$ $(t_{1/2})_2 = t_2$ then $t_{eff} \Rightarrow$ given by form -

$$\frac{1}{t_{eff}} = \frac{1}{t_1} + \frac{1}{t_2} \quad (\lambda = \lambda_1 + \lambda_2)$$

Activity: rate of radioactive decay of nuclei

$A = \lambda N$

$A = A_0 e^{-\lambda t}$ unit : Bq (Becquerel)

Arrhenius' eqn

1dps (disintegration per sec.) } same.

1 curie = $3.7 \times 10^{10}$ dps

(activity of 1g Radium).

Nuclear Fission.

$$_{92}U^{235} + {}_0n^1 \xrightarrow[\text{(slow)}]{} {}_{92}U^{236} \longrightarrow X + Y + 2\,{}_0n^1$$

$$\qquad\qquad \longrightarrow X' + Y' + 3\,{}_0n^1$$

Note: On an avg 2.5 neutron are emitted in fission $R^x$

In each fission $R^x$ 200 MeV of energy is released.

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{92}U^{236} \longrightarrow {}_{56}Ba^{141} + {}_{92}Kr^{236} + 3\,{}_0n^1 + Q.$$

Mass lost per fission $R^X$ : 0.2 amu (approx.)

Temp. very High. (ultra high $\approx 10^7 - 10^9$) / High Pr. lower temp.

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1 + 3.3\text{ MeV} \quad (D-D\ R^x)$$

$$_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + 4\text{ MeV} \quad (D-D\ R^x)$$

$$_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1 + 17.6\text{ MeV} \quad (D-T\ R^x)$$

nuclei should be brought upto 1 fermi dist.

Energy released / unit mass of fusion > fission.

Total energy released in fusion < fission. (for 1 $R^x$.)

* Work function is minimum for cesium (1.9 eV)

* work function $W = h\nu_0 = \dfrac{hc}{\lambda_0}$

* Photoelectric current is directly proportional to intensity of incident radiation. ($\nu$ = constant)

* Photoelectrons ejected from metal have kinetic energies ranging from 0 to $KE_{max}$

  Here $KE_{max} = eV_s$ $V_s$ - stopping potential

* Stopping potential is independent of intensity of light used ($\nu$-constant)

* Intensity in the terms of electric field is

$$I = \frac{1}{2}\epsilon_0 E^2.c$$

* Momentum of one photon is $\dfrac{h}{\lambda}$.

* Einstein equation for photoelectric effect is

$$h\nu = w_0 + k_{max} \qquad \frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV_s$$

* Energy $\Delta E = \dfrac{12400}{\lambda(A^0)}$ eV

* Force due to radiation (Photon) (no transmission)

  When light is incident perpendicularly

  (a) $a = 1,\ r = 0$

$$F = \frac{IA}{c}, \quad \text{Pressure} = \frac{I}{c}$$

  (b) $r = 1,\ a = 0$

$$F = \frac{2IA}{c}, \qquad P = \frac{2I}{c}$$

  (c) when $0 < r < 1$ and $a + r = 1$

$$F = \frac{IA}{c}(1 + r), \qquad P = \frac{I}{c}(1 + r)$$

When light is incident at an angle θ with vertical.

(a) $a = 1, r = 0$

$$F = \frac{IA\cos\theta}{c}, \qquad P = \frac{F\cos\theta}{A} = \frac{I}{c}\cos 2\theta$$

(b) $r = 1, a = 0$

$$F = \frac{2IA\cos^2\theta}{c}, \qquad P = \frac{2I\cos^2\theta}{c}$$

(c) $0 < r < 1, \quad a + r = 1$

$$P = \frac{I\cos^2\theta}{c}(1 + r)$$

* De Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{P} = \frac{h}{\sqrt{2km}}$$

* Radius and speed of electron in hydrogen like atoms.

$$r_n = \frac{n^2}{Z}a_0 \qquad a_0 = 0.529\ \text{Å}$$

$$v_n = \frac{Z}{n}v_0 \qquad v_0 = 2.19 \times 106\ \text{m/s}$$

* Energy in nth orbit

$$E_n = E_1 \cdot \frac{Z^2}{n^2} \qquad E_1 = \quad 13.6\ \text{eV}$$

* Wavelength corresponding to spectral lines

$$\frac{1}{\lambda} = R\ \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

| | | |
|---|---|---|
| for Lyman series | $n_1 = 1$ | $n_2 = 2, 3, 4$.......... |
| Balmer | $n_1 = 2$ | $n_2 = 3, 4, 5$.......... |
| Paschen | $n_1 = 3$ | $n_2 = 4, 5, 6$.......... |

* The lyman series is an ultraviolet and Paschen, Brackett and Pfund series are in the infrared region.

* Total number of possible transitions, is $\frac{n(n-1)}{2}$, (from nth state)

* If effect of nucleus motion is considered,

$$r_n = (0.529\ \text{Å})\ \frac{n^2}{Z} \cdot \frac{m}{\mu}$$

$$E_n = (\ 13.6\ \text{eV})\ \frac{Z^2}{n^2} \cdot \frac{\mu}{m}$$

Here μ - reduced mass

$$\mu = \frac{Mm}{(M+m)}, \quad M \text{ - mass of nucleus}$$

- Minimum wavelength for x-rays

$$\lambda_{min} = \frac{hc}{eV_0} = \frac{12400}{V_0(volt)} A$$

- Moseley s Law

$$\sqrt{v} = a(z - b)$$

a and b are positive constants for one type of x-rays (independent of Z)

- Average radius of nucleus may be written as

$R = R_0 A^{1/3}$, $\quad R_0 = 1.1 \times 10^{15}$ M

A - mass number

- Binding energy of nucleus of mass M, is given by $B = (ZM_p + NM_N \quad M)C^2$
- Alpha - decay process

$$^{A}_{Z}X \quad ^{A-4}_{Z-2}Y + ^{4}_{2}He$$

Q-value is

$$Q = \left[m\left(^{A}_{Z}X\right) - m\left(^{A-4}_{Z-2}Y\right) - m\left(^{4}_{2}He\right)\right]C^2$$

- Beta- minus decay

$$^{A}_{Z}X \quad ^{A}_{Z+1}Y + \beta^- + v^-$$

Q- value = $[m(^{A}_{Z}X) - m(^{A}_{Z+1}Y)]c^2$

- Beta plus-decay

$$^{A}_{Z}X \qquad ^{A}_{Z-1}Y + \beta+ + v$$

$$\text{Q- value} = [m(^{A}_{Z}X) - m(^{A}_{Z-1}Y) - 2me]c^2$$

- Electron capture : when atomic electron is captured, X-rays are emitted.

$$^{A}_{Z}X + e \qquad ^{A}_{Z-1}Y + v$$

$$\text{Q - value} = [m(^{A}_{Z}X) - m(^{A}_{Z-1}Y)]c^2$$

- In radioactive decay, number of nuclei at instant t is given by $N = N_0 e^{\lambda t}$, $\lambda$-decay constant.

- Activity of sample : $\quad A = A_0 e^{-\lambda t}$

- Activity per unit mass is called specific activity.

- Half life : $T_{1/2} = \dfrac{0.693}{\lambda}$

- Average life : $T_{av} = \dfrac{T_{1/2}}{0.693}$

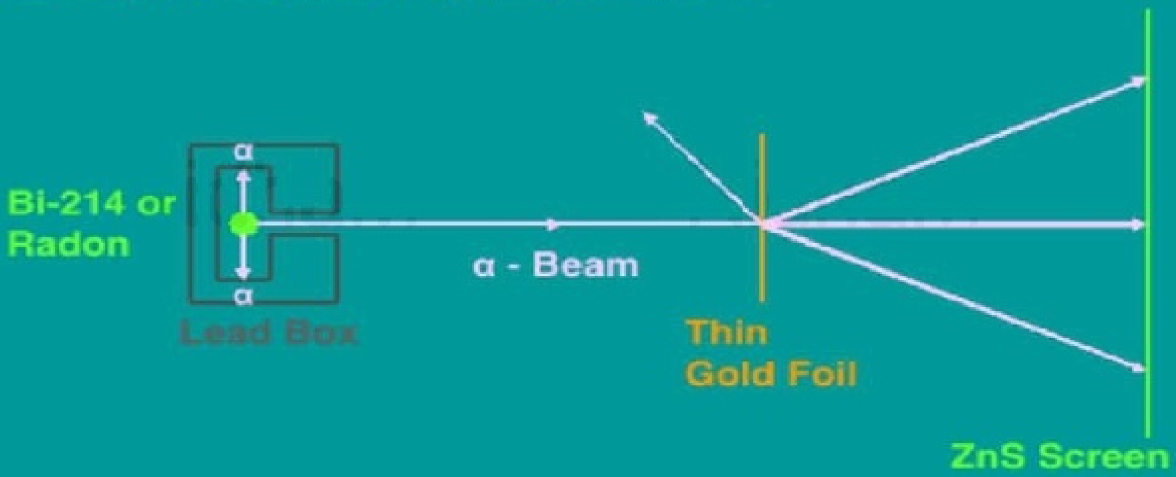
- A radioactive nucleus can decay by two different processes having half lives $t_1$ and $t_2$ respectively. Effective half-life of nucleus is given by $\dfrac{1}{t} = \dfrac{1}{t_1} + \dfrac{1}{t_2}$.

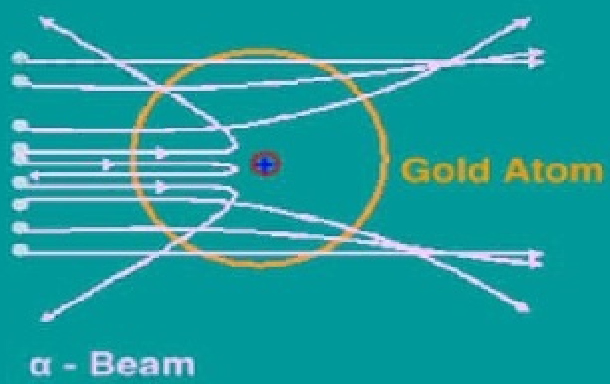
# ATOMS & NUCLEI

1. Rutherford's Alpha Scattering Experiment
2. Distance of Closest Approach (Nuclear Size)
3. Impact Parameter
4. Composition of Nucleus
5. Atomic Number, Mass Number and Atomic Mass Unit
6. Radius of the Nucleus and Nuclear Density
7. Mass Energy Relation and Mass Defect
8. Binding Energy and Binding Energy per Nucleon
9. Binding Energy Curve and Inferences
10. Nuclear Forces and Meson Theory
11. Radioactivity and Soddy's Displacement Law
12. Rutherford and Soddy's Laws of Radioactive Decay
13. Radioactive Disintegration Constant and Half-Life Period
14. Units of Radioactivity
15. Nuclear Fission and Fusion



## Rutherford's Alpha Scattering Experiment




Alpha – particle is a nucleus of helium atom carrying a charge of '+2e' and mass equal to 4 times that of hydrogen atom. It travels with a speed nearly $10^4$ m/s and is highly penetrating.

| | Rutherford Experiment | Geiger & Marsden Experiment |
|---|---|---|
| Source of α-particle | Radon $_{86}Rn^{222}$ | Bismuth $_{83}Bi^{214}$ |
| Speed of α-particle | $10^4$ m/s | $1.6 \times 10^7$ m/s |
| Thickness of Gold foil | $10^{-6}$ m | $2.1 \times 10^{-7}$ m |

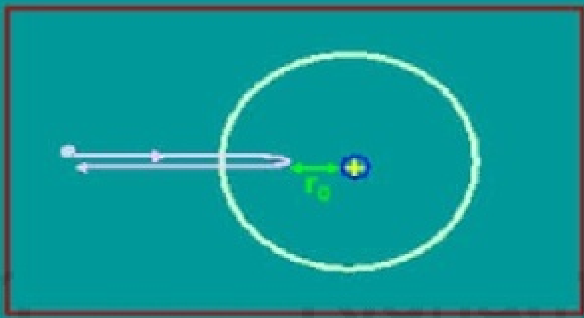


| S. No. | Observation | Conclusion |
|---|---|---|
| 1 | Most of the α-particles passed straight through the gold foil. | It indicates that most of the space in an atom is empty. |
| 2 | Some of the α-particles were scattered by only small angles, of the order of a few degrees. | α-particles being +vely charged and heavy compared to electron could only be deflected by heavy and positive region in an atom. It indicates that the positive charges and the most of the mass of the atom are concentrated at the centre called 'nucleus'. |
| 3 | A few α-particles (1 in 9000) were deflected through large angles (even greater than 90°). Some of them even retraced their path. i.e. angle of deflection was 180°. | α-particles which travel towards the nucleus directly get retarded due to Coulomb's force of repulsion and ultimately comes to rest and then fly off in the opposite direction. |

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

## Distance of Closest Approach (Nuclear size):

When the distance between α-particle and the nucleus is equal to the distance of the closest approach ($r_0$), the α-particle comes to rest.




At this point or distance, the kinetic energy of α-particle is completely converted into electric potential energy of the system.

$$\frac{1}{2} mu^2 = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{\frac{1}{2} mu^2}$$



## Impact Parameter (b):

The perpendicular distance of the velocity vector of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter.


$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0 (\frac{1}{2} mu^2)}$$

i) For large value of b, cot θ/2 is large and θ, the scattering angle is small.

i.e. α-particles travelling far away from the nucleus suffer small deflections.

ii) For small value of b, cot θ/2 is also small and θ, the scattering angle is large.
i.e. α-particles travelling close to the nucleus suffer large deflections.

iii) For b = 0 i.e. α-particles directed towards the centre of the nucleus,

$\cot \theta/2 = 0$ or $\theta/2 = 90°$ or $\theta = 180°$

The α-particles retrace their path.

## Composition of Nucleus:

Every atomic nucleus except that of Hydrogen has two types of particles – protons and neutrons. (Nucleus of Hydrogen contains only one proton)

Proton is a fundamental particle with positive charge $1.6 \times 10^{-19}$ C and mass $1.67 \times 10^{-27}$ kg (1836 times heavier than an electron).

Neutron is also a fundamental particle with no charge and mass $1.675 \times 10^{-27}$ kg (1840 times heavier than an electron).

## Atomic Number (Z):

The number of protons in a nucleus of an atom is called atomic number.

## Atomic Mass Number (A):

The sum of number of protons and number of neutrons in a nucleus of an atom is called atomic mass number.

$$A = Z + N$$

## Atomic Mass Unit (amu):

Atomic Mass Unit (amu) is (1 / 12)th of mass of 1 atom of carbon.

$$1 \text{ amu} = \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} \text{ g} = 1.66 \times 10^{-27} \text{ kg}$$



## Size of Nucleus:

Nucleus does not have a sharp or well-defined boundary.

However, the radius of nucleus can be given by

$R = R_0 A^{1/3}$ where $R_0 = 1.2 \times 10^{-5}$ m is a constant which is the same for all nuclei and A is the mass number of the nucleus.

Radius of nucleus ranges from 1 fm to 10 fm.

Nuclear Volume, $V = (4/3)\,\pi R^3 = (4/3)\,\pi R_0^3 A$

$$V \propto A$$

## Nucleus Density:

Mass of nucleus, $M = A \text{ amu} = A \times 1.66 \times 10^{-27}$ kg

Nuclear Volume, $V = (4/3)\,\pi R^3 = (4/3)\,\pi R_0^3 A$

$$= \frac{4}{3} \times \frac{22}{7} \times (1.2 \times 10^{-15})^3 A \text{ m}^3$$

$$= 7.24 \times 10^{-45} A \text{ m}^3$$

Nucleus Density, $\rho = M / V = 2.29 \times 10^{17}$ kg / m$^3$

**Discussion:**

1. The nuclear density does not depend upon mass number. So, all the nuclei possess nearly the same density.
2. The nuclear density has extremely large value. Such high densities are found in white dwarf stars which contain mainly nuclear matter.
3. The nuclear density is not uniform throughout the nucleus. It has maximum value at the centre and decreases gradually as we move away from the centre of the nucleus.
4. The nuclear radius is the distance from the centre of the nucleus at which the density of nuclear matter decreases to one-half of its maximum value at the centre.



**Mass – Energy Relation:**

According to Newton's second law of motion, force acting on a body is defined as the rate of change of momentum.

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

If this force F displaces the body by a distance dx, its energy increases by

$$dK = F.dx = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$dK = m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$$

$$dK = m\,v\,dv + v^2\,dm \qquad \text{............ (1)}$$

According to Einstein's relation of relativistic mass,

$$m = \frac{m_0}{[1 - (v^2 / c^2)]^{1/2}}$$

Squaring and manipulating, $m^2c^2 - m^2v^2 = m_0{}^2c^2$

Differentiating (with $m_0$ and c as constants)

$$c^2\, 2m\, dm - m^2\, 2v\, dv - v^2\, 2m\, dm = 0$$

or $$c^2\, dm - mv\, dv - v^2\, dm = 0$$

$$c^2\, dm = mv\, dv + v^2\, dm \qquad \text{..................(2)}$$

From (1) and (2), $dK = dm\, c^2$

If particle is accelerated from rest to a velocity v, let its mass $m_0$ increases to m.
Integrating,

$$\text{Total increase in K.E.} = \int_0^K dK = c^2 \int_{m_0}^{m} dm$$

$$\therefore \quad K = (m - m_0)\, c^2 \qquad \text{or} \qquad K + m_0\, c^2 = m\, c^2$$

Here $m_0c^2$ is the energy associated with the rest mass of the body and K is the kinetic energy.

Thus, the total energy of the body is given by $\boxed{E = m\, c^2}$

This is Einstein's mass - energy equivalence relation.



**Mass Defect:**

It is the difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as mass defect.

$$\Delta m = [\, Zm_p + (A - Z)\, m_n\, ] - M$$

Mass defect per nucleon is called packing fraction.

**Binding Energy:**

It is the energy required to break up a nucleus into its constituent parts and place them at an infinite distance from one another.

$$B.E = \Delta m\, c^2$$

**Nuclear Forces:**

They are the forces between p – p, p – n or n – n in the nucleus. They can be explained by Meson Theory.

There are three kinds of mesons – positive ($\pi^+$), negative ($\pi^-$) and neutral ($\pi^0$).

$\pi^+$ and $\pi^-$ are 273 times heavier than an electron.
$\pi^0$ is 264 times heavier than an electron.

Nucleons (protons and neutrons) are surrounded by mesons.

## Main points of Meson Theory:

1. There is a continuous exchange of a meson between one nucleon and other. This gives rise to an exchange force between them and keep them bound.

2. Within the nucleus, a neutron is never permanently a neutron and a proton is never permanently a proton. They keep on changing into each other due to exchange of $\pi$-mesons.

3. The n – n forces arise due to exchange of $\pi^0$ – mesons between the neutrons.

$n \rightarrow n + \pi^0$ (emission of $\pi^0$)

$n + \pi^0 \rightarrow n$ (absorption of $\pi^0$)

4. The p – p forces arise due to exchange of $\pi^0$ – mesons between the protons.

$p \rightarrow p + \pi^0$ (emission of $\pi^0$)

$p + \pi^0 \rightarrow p$ (absorption of $\pi^0$)



5. The n – p forces arise due to exchange of $\pi^+$ and $\pi^-$ mesons between the nucleons.

$n \rightarrow p + \pi^-$ (emission of $\pi^-$)

$n + \pi^+ \rightarrow p$ (absorption of $\pi^+$)

$p \rightarrow n + \pi^+$ (emission of $\pi^+$)

$p + \pi^- \rightarrow n$ (absorption of $\pi^-$)

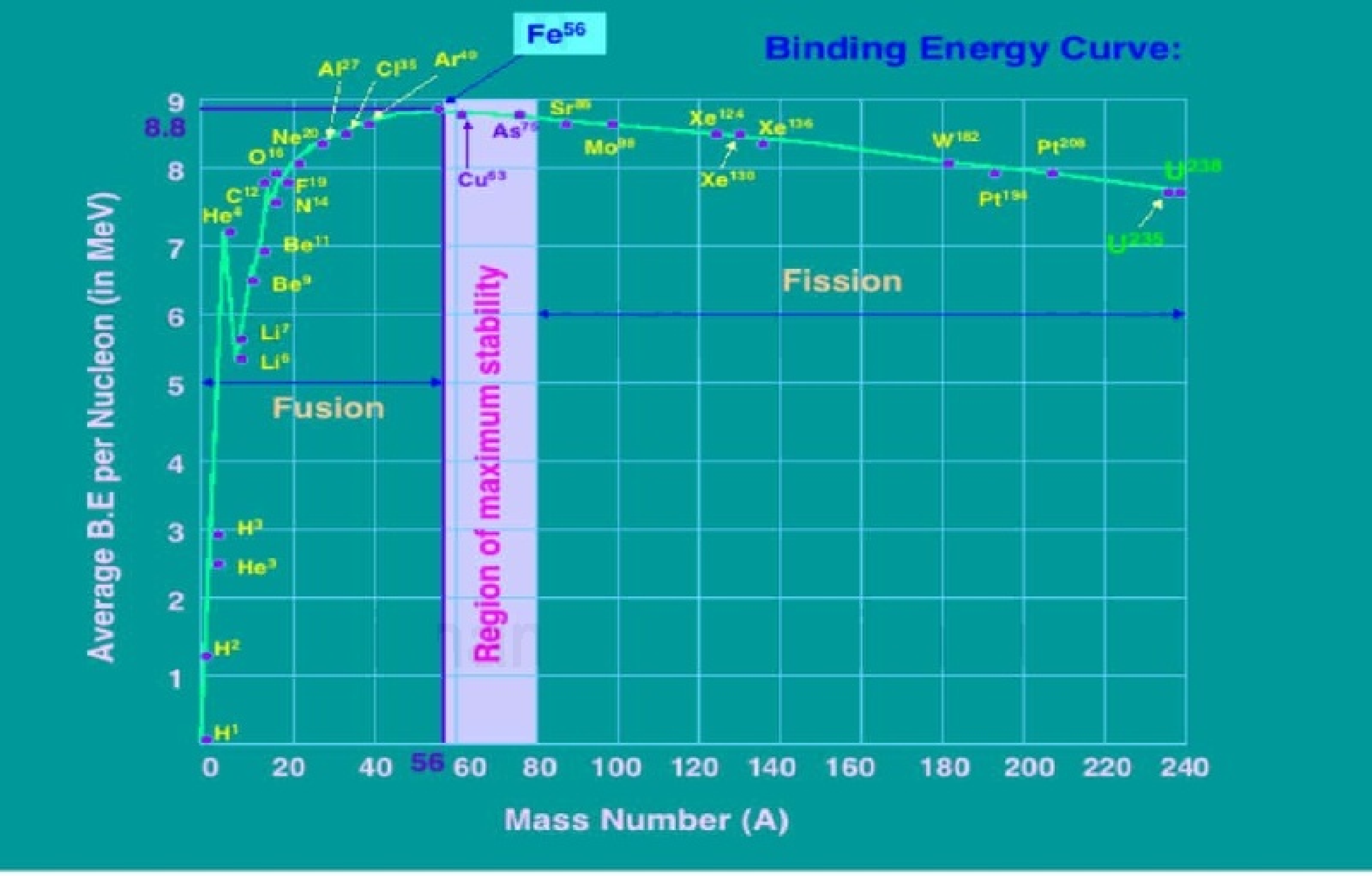
6. The time involved in such an exchange is so small that the free meson particles cannot be detected as such.

## Binding Energy per Nucleon:

It is the binding energy divided by total number of nucleons.

It is denoted by $\overline{B}$

$\overline{B}$ = B.E / Nucleon = $\Delta m\, c^2 / A$



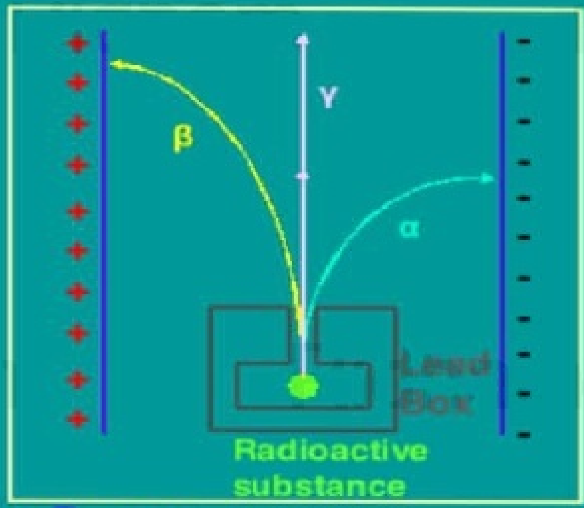



## Special Features of Binding Energy Curve

1. Binding energy per nucleon of very light nuclides such as $_1H^2$ is very small.
2. Initially, there is a rapid rise in the value of binding energy per nucleon.
3. Between mass numbers 4 and 20, the curve shows cyclic recurrence of peaks corresponding to $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. This shows that the B.E. per nucleon of these nuclides is greater than those of their immediate neighbours. Each of these nuclei can be formed by adding an alpha particle to the preceding nucleus.
4. After A = 20, there is a gradual increase in B.E. per nucleon. The maximum value of 8.8 MeV is reached at A = 56. Therefore, Iron nucleus is the most stable.
5. Binding energy per nucleon of nuclides having mass numbers ranging from 40 to 120 are close to the maximum value. So, these elements are highly stable and non-radioactive.
6. Beyond A = 120, the value decreases and falls to 7.6 MeV for Uranium.
7. Beyond A = 128, the value shows a rapid decrease. This makes elements beyond Uranium (trans – uranium elements) quite unstable and radioactive.
8. The drooping of the curve at high mass number indicates that the nucleons are more tightly bound and they can undergo fission to become stable.
9. The drooping of the curve at low mass numbers indicates that the nucleons can undergo fusion to become stable.

## Radioactivity:

Radioactivity is the phenomenon of emitting alpha, beta and gamma radiations spontaneously.




### Soddy's Displacement Law:

1. $_ZY^A \xrightarrow{\alpha} {}_{Z-2}Y^{A-4}$

2. $_ZY^A \xrightarrow{\beta} {}_{Z+1}Y^A$

3. $_ZY^A \xrightarrow{\gamma} {}_ZY^A$ (Lower energy)

### Rutherford and Soddy's Laws of Radioactive Decay:

1. The disintegration of radioactive material is purely a random process and it is merely a matter of chance. Which nucleus will suffer disintegration, or decay first can not be told.
2. The rate of decay is completely independent of the physical composition and chemical condition of the material.
3. The rate of decay is directly proportional to the quantity of material actually present at that instant. As the decay goes on, the original material goes on decreasing and the rate of decay consequently goes on decreasing.



If N is the number of radioactive atoms present at any instant, then the rate of decay is,

$$-\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N$$

where λ is the decay constant or the disintegration constant.

Rearranging,

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating, $\log_e N = -\lambda t + C$ where C is the integration constant.

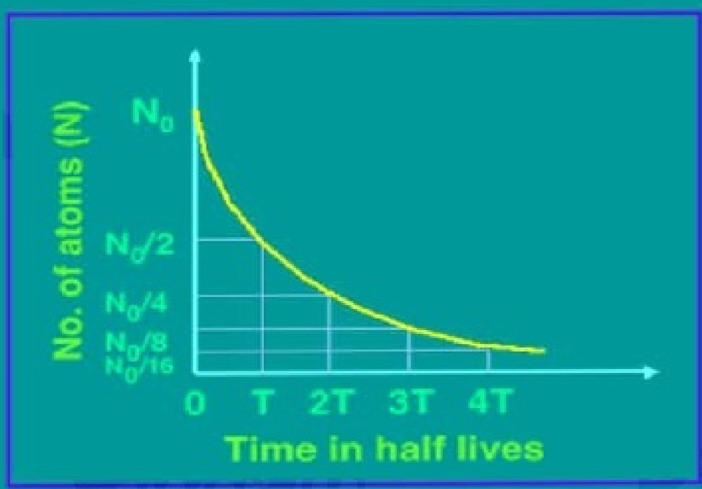
If at t = 0, we had $N_0$ atoms, then

$$\log_e N_0 = 0 + C$$

$\therefore \quad \log_e N - \log_e N_0 = -\lambda t$

or $\quad \log_e (N / N_0) = -\lambda t$

or $\quad \frac{N}{N_0} = e^{-\lambda t} \quad$ or $\quad \boxed{N = N_0 e^{-\lambda t}}$



## Radioactive Disintegration Constant ($\lambda$):

According to the laws of radioactive decay,

$$\frac{dN}{N} = -\lambda\, dt$$

If $dt = 1$ second, then

$$\frac{dN}{N} = -\lambda$$

Thus, $\lambda$ may be defined as the relative number of atoms decaying per second.

Again, since $N = N_0 e^{-\lambda t}$

And if, $t = 1/\lambda$, then $N = N_0 / e$

or $$\frac{N}{N_0} = \frac{1}{e}$$

Thus, $\lambda$ may also be defined as the reciprocal of the time when $N / N_0$ falls to $1 / e$.



## Half – Life Period:

Half life period is the time required for the disintegration of half of the amount of the radioactive substance originally present.

If T is the half – life period, then

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T} \qquad \text{(since } N = N_0 / 2)$$

$$e^{\lambda T} = 2$$

$$\lambda T = \log_e 2 = 0.6931$$

$$T = \frac{0.6931}{\lambda} \quad \text{or} \quad \lambda = \frac{0.6931}{T}$$

**Time t in which material changes from $N_0$ to N:**

$$t = 3.323\, T \log_{10} (N_0 / N)$$

**Number of Atoms left behind after n Half – Lives:**

$$N = N_0 (1/2)^n \quad \text{or} \quad N = N_0 (1/2)^{t/T}$$

## Units of Radioactivity:

1. The curie (Ci): The activity of a radioactive substance is said to be one curie if it undergoes $3.7 \times 10^{10}$ disintegrations per second.

   1 curie = $3.7 \times 10^{10}$ disintegrations / second

2. The rutherford (Rd): The activity of a radioactive substance is said to be one rutherford if it undergoes $10^6$ disintegrations per second.

   1 rutherford = $10^6$ disintegrations / second

3. The becquerel (Bq): The activity of a radioactive substance is said to be one becquerel if it undergoes 1 disintegration per second.

   1 becquerel = 1 disintegration / second

**1 curie = $3.7 \times 10^4$ rutherford = $3.7 \times 10^{10}$ becquerel**

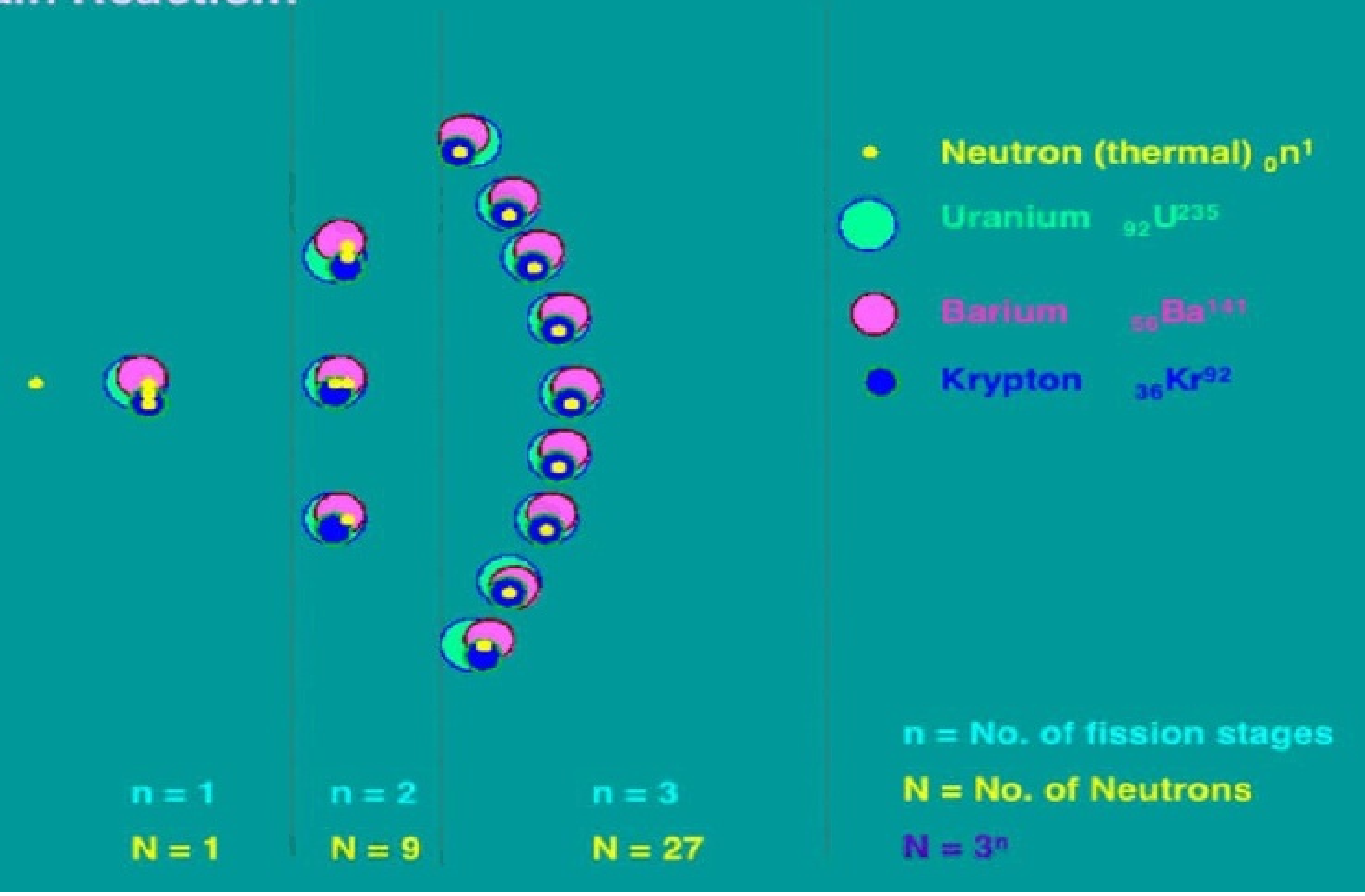
## Nuclear Fission:

Nuclear fission is defined as a type of nuclear disintegration in which a heavy nucleus splits up into two nuclei of comparable size accompanied by a release of a large amount of energy.

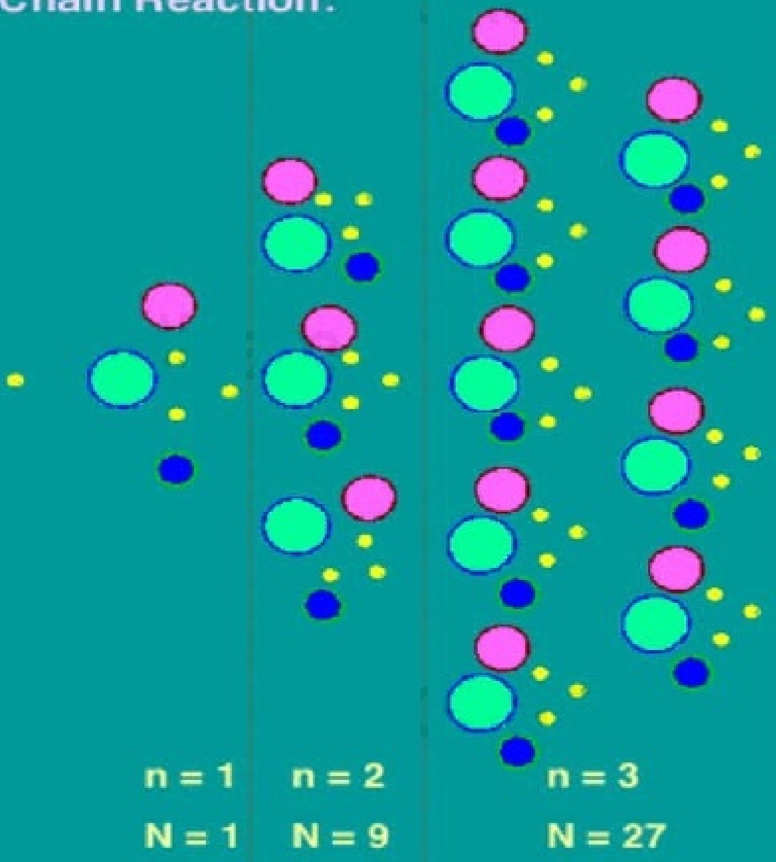
$$ {}_0n^1 + {}_{92}U^{235} \rightarrow ({}_{92}U^{236}) \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + \gamma \text{ (200 MeV)} $$



## Chain Reaction:



## Chain Reaction:




## Critical Size:

For chain reaction to occur, the size of the fissionable material must be above the size called 'critical size'.

A released neutron must travel minimum through 10 cm so that it is properly slowed down (thermal neutron) to cause further fission.

If the size of the material is less than the critical size, then all the neutrons are lost.

If the size is equal to the critical size, then the no. of neutrons produced is equal to the no. of neutrons lost.

If the size is greater than the critical size, then the reproduction ratio of neutrons is greater than 1 and chain reaction can occur.



## Nuclear Fusion:

Nuclear fusion is defined as a type of nuclear reaction in which two lighter nuclei merge into one another to form a heavier nucleus accompanied by a release of a large amount of energy.

### Energy Source of Sun:

**Proton – Proton Cycle:**

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + 0.4\ MeV$$

$${}_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + 5.5\ MeV$$

$${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 12.9\ MeV$$

### Energy Source of Star:

**Carbon - Nitrogen Cycle:**

$${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + \gamma \quad \text{(energy)}$$

$${}_7N^{13} \rightarrow {}_6C^{13} + {}_1e^0 \quad \text{(positron)}$$

$${}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14} + \gamma \quad \text{(energy)}$$

$${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + \gamma \quad \text{(energy)}$$

$${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 \quad \text{(positron)}$$

$${}_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4 + \gamma \quad \text{(energy)}$$

End of Atomic Nucleus