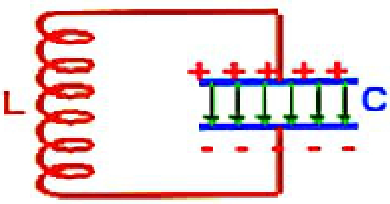
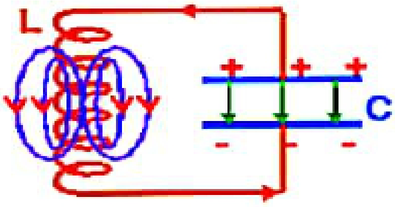
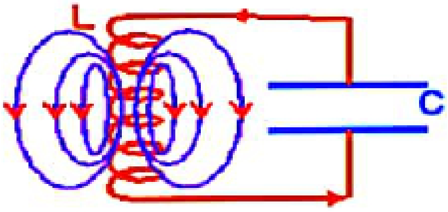
L C Oscillations:



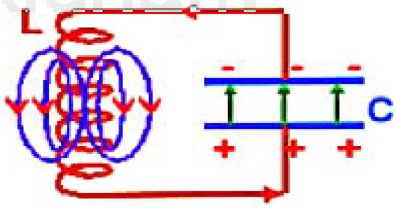
At $t = 0$, U_E=Max. & U_B=0



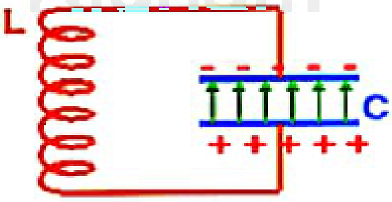
At $t = T/8$, $U_E = U_B$



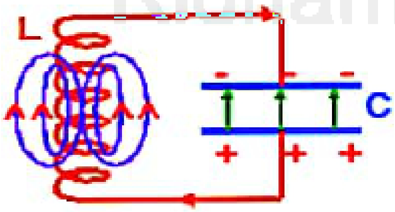
At $t = 2T/8$, U_E=0 & U_B=Max.



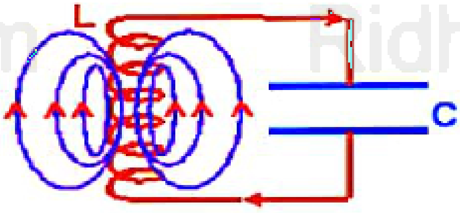
At $t = 3T/8$, $U_E = U_B$



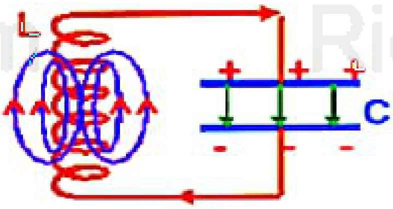
At $t = 4T/8$, U_E=Max. & U_B=0



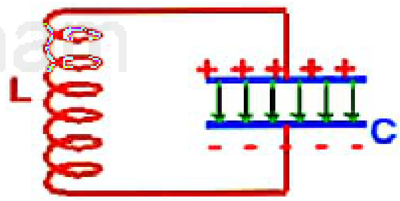
At $t = 5T/8$, $U_E = U_B$



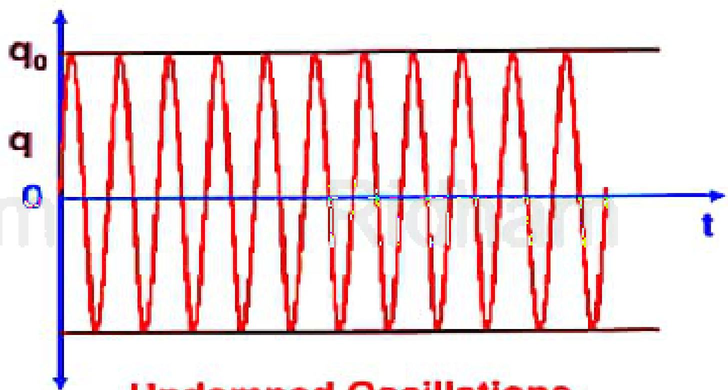
At $t = 6T/8$, U_E=0 & U_B=Max.



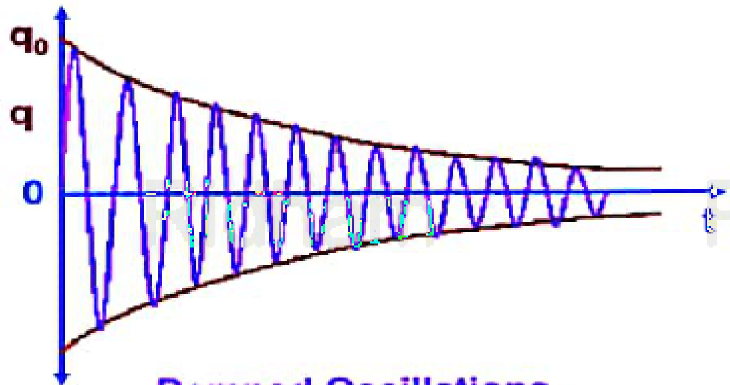
At $t = 7T/8$, $U_E = U_B$



At $t = T$, U_E=Max. & U_B=0



Undamped Oscillations



Damped Oscillations

If q be the charge on the capacitor at any time t and dI / dt the rate of change of current, then

$$L\,dI/dt + q/C = 0$$

or $L\,(d^2q/dt^2) + q/C = 0$

or $d^2q/dt^2 + q/(LC) = 0$

Putting $1/LC = \omega^2$

$$d^2q/dt^2 + \omega^2 q = 0$$

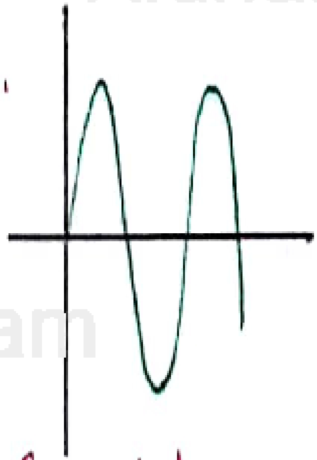
The final equation represents **Simple Harmonic Electrical Oscillation** with ω as angular frequency.

So, $\omega = 1/\sqrt{LC}$

or $$f = \frac{1}{2\pi\sqrt{LC}}$$

NOTE: 1. AC changes its dirn. periodically.

$I = I_0 \sin(\omega t + \phi)$
$\mathcal{E} = \mathcal{E}_0 \sin(\omega t + \phi)$
} Sinosudal AC



Sinosudal AC.

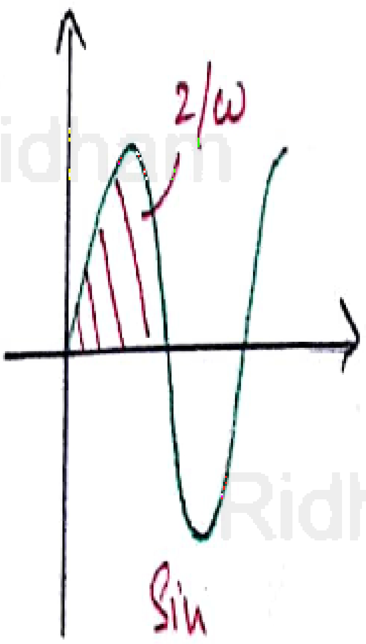
clock AC.

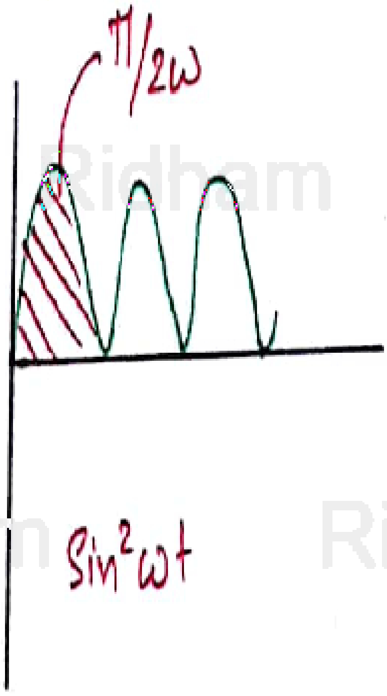
2. $Y_{avg} = \dfrac{\int_{x_i}^{x_f} y\,dx}{\int_{x_i}^{x_f} dx}$ $\quad Y_{rms} = \sqrt{\dfrac{\int_{x_i}^{x_f} y^2\,dx}{\int_{x_i}^{x_f} dx}}$

$V = V_0 \sin(\omega t + \phi)$

AC source.

3.





$I_{max} = \sqrt{2}\, I_{rms}$

$\omega T = 2\pi$

4. $I = a + b\sin\omega t$.

$I_{avg} = I$. $\quad I_{rms} = \sqrt{a^2 + \dfrac{b^2}{2}}$

Where $n=\frac{N}{l}$ is the number of turns per metre length.

8. MUTUAL INDUCTION

When two coils are palced nearby and the current in one coil (often called primary coil) is changed, the magnetic flux linked with the neigh bouring coil (often called secondary coil)
Changes; due to which an emf of two coils, then

Definition of mutual inductance: $M=\frac{\phi_2}{I_2}$

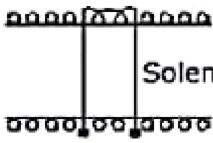
The mutual infuctance of two coils is defined as the emf induced in the secondary coil when the rate of chage of current in the primary coil is 1A/S.
The unit of mutual inductance is also henry (H). The mutual inductance of two coils does not depend on the fact which oil carries the current and in which coil emf is induced i.e., $M_{12}=M_{21}=M$

If L_1 AND L_2 are self inductances two coils with 100% fulx linkage between them, then

$M=\sqrt{L_1L_2}$, otherwise $M=k\sqrt{L_1L_2}$

Where K is coefficient of flux linkage between the coils. Mutual inductance of solenoid-coil system

$$M=\frac{u_0N_1N_2A}{l}$$



Where A is area of coil, l is length of solenoid. N_1 is nuber of turns in solenoid and N_2 is number of turns in coil.

9. EDDY CURRENTS

When a conductor is placed in a varying magnetic field the magnetic flux linked with the conductor changes, so induced current are induced in the body of conductor,

Which causes heating of conductor.

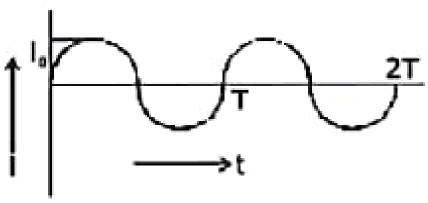
The current in induced in the comductor are called the eddy currents. In varying magnetic field, the free electrons of conductor experience Lorentz force and traverse closed paths; which are equivalent to small current loops. |These currents are the eddy currents; they cause heating effect and sometimes the conductor becomes red-hot.

Eddy current losses may be reduced by using laminated soft iron cores in galvanometers, transformers. Etc. making holes in conductor.

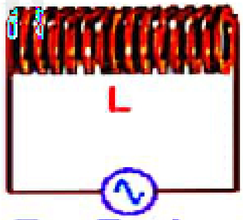
10. NEED FOR DISPLACEMENT CURRENT

Ampere s circuital law for conduction current during charging of a capacitor was found inconsistent. Therefore Maxwell modified Ampere s circuital law by introducing displacement current. It is given by

$I_d=\varepsilon_0\frac{d\phi_E}{dt}$ dodified Ampere s Circuital law is $\oint \vec{B}.d\vec{l}=\mu_0\left(I+\varepsilon_0\frac{d\phi_E}{dt}\right)$ where f_E = electric flux.



AC Circuit with a Pure Inductor:



$E = E_0 \sin \omega t$

Induced emf in the inductor is $-L\,(dI/dt)$

In order to maintain the flow of current, the applied emf must be equal and opposite to the induced emf.

$E = E_0 \sin \omega t$

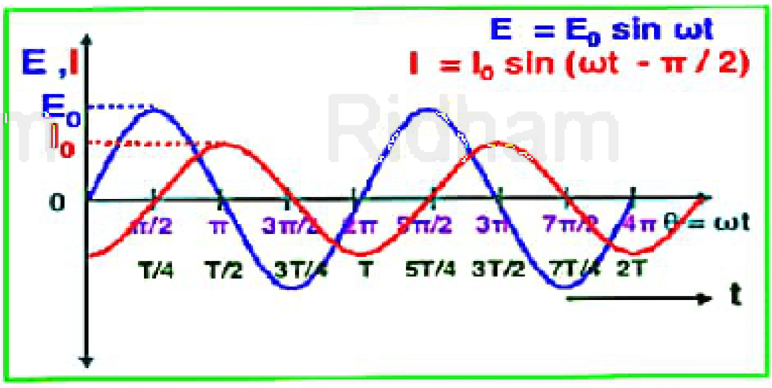
$$\therefore \; E = L\,(dI/dt)$$
$$E_0 \sin \omega t = L\,(dI/dt)$$
$$dI = (E_0/L) \sin \omega t\, dt$$

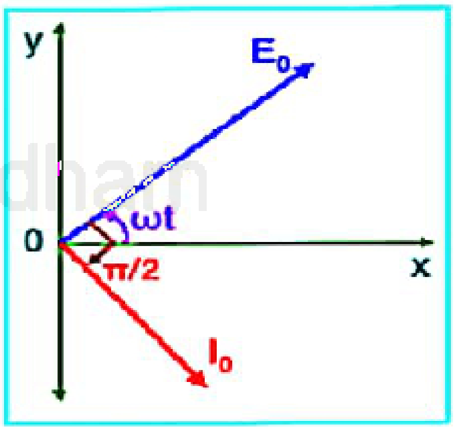
$$I = \int (E_0/L) \sin \omega t\, dt$$
$$I = (E_0/\omega L)\,(-\cos \omega t)$$
$$I = I_0 \sin(\omega t - \pi/2)$$

(where $I_0 = E_0/\omega L$ and $X_L = \omega L = E_0/I_0$)

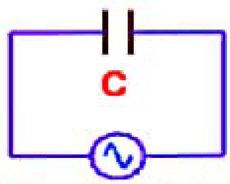
Current lags behind emf by $\pi/2$ rad.

X_L is Inductive Reactance. Its SI unit is ohm.





AC Circuit with a Capacitor:



$E = E_0 \sin \omega t$

$E = E_0 \sin \omega t$

$q = CE = CE_0 \sin \omega t$

$i = dq/dt$

$= (d/dt)\,[CE_0 \sin \omega t]$

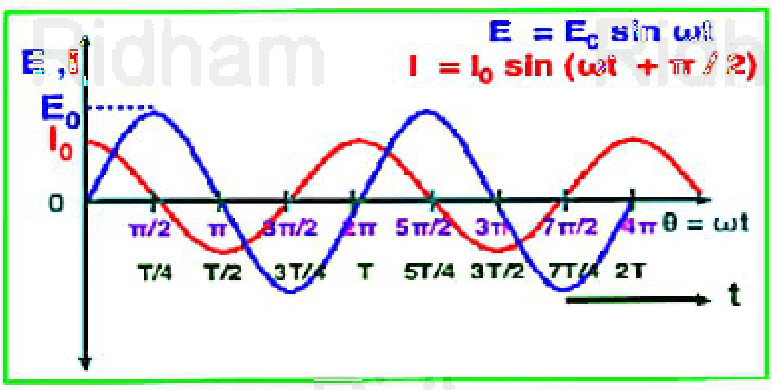
$I = [E_0/(1/\omega C)]\,(\cos \omega t)$

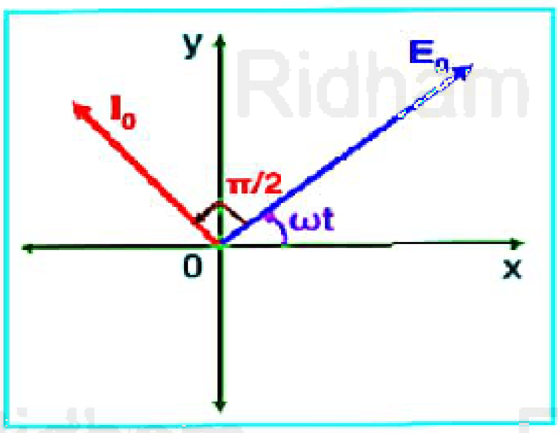
$I = I_0 \sin(\omega t + \pi/2)$

(where $I_0 = E_0/(1/\omega C)$ and $X_C = 1/\omega C = E_0/I_0$)

X_C is Capacitive Reactance. Its SI unit is ohm.

Current leads the emf by $\pi/2$ radians.





Power in AC Circuit with L, C, R:

$E = E_0 \sin \omega t$

$I = I_0 \sin (\omega t + \Phi)$ (where Φ is the phase angle between emf and current)

Instantaneous Power $= E\,I$

$= E_0 I_0 \sin \omega t \sin (\omega t + \Phi)$

$= E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \ \cos\Phi]$

If the instantaneous power is assumed to be constant for an infinitesimally small time dt, then the work done is

$dW = E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \ \cos\Phi]$

Work done over a complete cycle is

$$W = \int_0^T E_0 I_0 [\sin^2 \omega t \cos\Phi + \sin \omega t \cos\omega t \ \cos\Phi]\, dt$$

$W = E_0 I_0 \cos \Phi \times T / 2$

Average Power over a cycle is $P_{av} = W / T$

$P_{av} = (E_0 I_0 / 2) \cos \Phi$

$P_{av} = (E_0/\sqrt{2})\,(I_0/\sqrt{2}) \cos \Phi$

$$\boxed{P_{av} = E_v\, I_v \cos \Phi}$$

(where $\cos \Phi = R / Z$

$= R / \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$

is called Power Factor)

$P_{av} = E_v\, I_v \cos \Phi$

Power in AC Circuit with R:

In R, current and emf are in phase.

$\Phi = 0°$

$P_{av} = E_v\, I_v \cos \Phi = E_v\, I_v \cos 0° = E_v\, I_v$

Power in AC Circuit with L:

In L, current lags behind emf by $\pi/2$.

$\Phi = -\ \pi/2$

$P_{av} = E_v\, I_v \cos (-\pi/2) = E_v\, I_v\, (0) = 0$

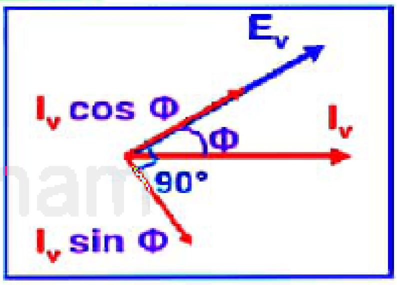
Power in AC Circuit with C:

In C, current leads emf by $\pi/2$.

$\Phi = +\ \pi/2$

$P_{av} = E_v\, I_v \cos (\pi/2) = E_v\, I_v\, (0) = 0$

Wattless Current or Idle Current:



The component $I_v \cos \Phi$ generates power with E_v.

However, the component $I_v \sin \Phi$ does not contribute to power along E_v and hence power generated is zero. This component of current is called wattless or idle current.

$P = E_v\, I_v \sin \Phi \cos 90° = 0$

Note:

Power (Energy) is not dissipated in Inductor and Capacitor and hence they find a lot of practical applications and in devices using alternating current.

radian, calculate the value of L.

Sol. Given R=10 Ω E=100V , f=50Hz, $\phi=\frac{\pi}{4}$

In Rl circuit

Phase angle ϕ is given by

$$\tan f=\frac{X_L}{R}=\frac{\omega L}{R}=\frac{2\pi f L}{R}$$

$$\Rightarrow \quad L=\frac{R\tan f}{2\pi f}=\frac{10\ \tan\frac{\pi}{4}}{2\ 3.14\ 50}=\frac{10\ 1}{314}=0.0318A$$

7. An inductor of unknown value, a capacitor of 100uF and a resistor of 10 Ω are connected in series to a 200 V, 50 Hz a.c. source. It is found that the power factor of the circuit is unity. Calcualte the inductance of the inductor and current amplitude.

Sol. For power factor unity,

$$X_L=X_C\Rightarrow\omega L=\frac{1}{\omega C}$$

$$\Rightarrow L=\frac{1}{\omega^2 C}=\frac{1}{(2\pi f)^2 C}=\frac{1}{4\pi^2 f^2 C}$$

Given f = 50Hz, C=100uF= 100 10^{-6}F= 10^{-4}F

$$L=\frac{1}{4\times(3.14)^2\times(50)^2\times10^{-4}}H=0.10H\ \mathrm{s}$$

Current amplitude, $I_0=\frac{V_0}{Z}$

At resonance, $I_0=\frac{V_0}{R}=\frac{200\sqrt{2}}{10}=20\sqrt{2A}=20\times1.414A=28.3A$

EMI & AC

BASIC CONCEPTS AND FORMULAE

1. MAGENTIC FLUX

Magnetic Flux through a surface of area A placed in a uniform magnetic field is $\phi_m = \vec{B}.\vec{A} = BA\cos\theta, \theta$ being angle between $\vec{B}$ and $\vec{A}$. If magnetic field is not uniform, then

$\phi_m = \int \vec{B}.\vec{d}A$. where integral extends for whole area A.

The S.I. unit of magnetic flux is weber. Magnetic flux is a **scalar quantity;** because of being scalar product of two vectors $\vec{B}$ and $\vec{A}$.

2. ELECTROMAGNETIC INDUCTION

a. Whenever the magnetic flux linked with a coil changes, an emf is induced in the coil. The Induced emf lasts so long as the change in magnetic flux lasts. This phenomenon is called **electromagnetic induction.**

3. FARADAY S LAWS

(i) Whenever there is a change in magnetic flux linked with of a coil, an emf is induced in the coil. The induced emf is proportional to the rate of change of magnetic flux linked with the coil.

(ii) Emf induced in the coil opposes the change in flux, i.e.,

Where k is a constant of proportionality.

In S.I. system ϕ is in weber, t in second, e in volt, then k=1,so $e=-\frac{\Delta\phi}{\Delta t}$

If all coil contains N-turns, then $e=-N\frac{\Delta\phi}{\Delta t}$

4. INDUCED CURRENT AND INDUCED CHARGE

If a coil is closed and has resistance R, then current induced in the coil,

$$i=\frac{e}{R}=-\frac{N\Delta\phi}{R\Delta T} \text{ ampere}$$

Electromagnetic induction and Alternating Current

$$\text{Induced charge, } q=I\Delta T=\frac{N\Delta\phi}{R}=\frac{\text{Totalfluxlinkage}}{\text{Resistance}}$$

5. LENZ S LAW

If sates that the direction of induced emf is such that it tends to produce a current which opposes the change in magnetic flux producing it.

6. EMF INDUCED IN A MOVING CONDUCTING ROD

EMF induced in a conducting rod of length I moving with velocity v in a magnetic field of induction B. such that B, I and v are mutually perpendicular, is given by

7. SELF INDUCTANCE

When the current in a coil is changed, a back emf is induced in the same coil, Tis phenomenon is called self-induction. If L is self-inductance of coil, then

Symbol of inductor

Also induced emf $e=-L\frac{\Delta I}{\Delta t}$

The unit of self inductance is henry (H). the self inductance acts as intertia in electrical circuits; so it is also called **electrical inertia.**

The self inductance of a solenoid consisting core of relative permeability u_r is

$$L=u_r u_0 n^2 A l$$

11. ALTERNATING CURRENT

Alternating current is one which changes in magnitude and direction periodically. The maximum value of current.
is called **current-amplitude or peak value of current**

If $f = \frac{\omega}{2\pi}$ is frequency of alternating current, then it is expressed as Similarly alternating voltage (or emf) is

$$I = I_0 \sin \omega t$$

Similarly alternating voltage (or emf) is

$$V = V_0 \sin\omega t$$

12. MEAN AND RMS VALUE OF ALTERNATING CURRENTS

The mean value of alternating current over complete cycle is zero

$$(I_{mean})_{full\ cycle} = 0$$

While for half cycle it is

$$(I_{mean})_{half\ cycle} = \frac{2I_0}{\pi} = 0.636I_0$$

An instrument read root mean square values as

$$I_{rms} = = \sqrt{(I^2)_{mean}} = \frac{I_0}{\sqrt{2}} = 0.707I_0$$

Electromagnetic induction and alternating current

13. PHASE DIFFERENCE BETWEEN VOLTAGE AND CURRENT

Area may be controlled by resistance, inductance and capacitance. Due to the presence of inductance and capacitance, current is usually not in phase with applied voltage. In general

$$V = V_0 \sin \omega t$$
$$I = I_0 \sin (\omega t + \phi)$$

Where ϕ is the phase difference.

14. IMPEDANCE AND REACTANCE

The hindrace offered by a circuit to flow of AC is called impedance. It is denoted by z.

$$z = \frac{V}{I} = \frac{V_0}{I_0} \text{ ohm}$$

Reactance: the hindrance offered by inductance and capacitance in ac circuit is called reactance. It is demoted by X.
The hindrance due to inductance alone is called the inductive reactance while that due to capacitance alone is called the capacitive reactance.

Inductive reactance, $X_L = \omega t$

Capacitive reactance, $X_c = \frac{1}{\omega C}$

15. PURELY RESISTIVE CIRCUIT

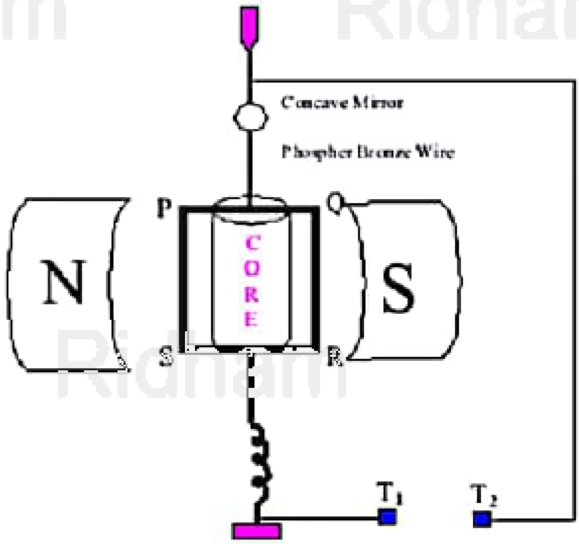
If a circuit contains pure resistance, then phase ϕ=oi.c., current and voltage are always in the same phase. Impedance, $Z = R$

16. PURELY INDUCTIVE CIRCUIT

If a circuit contains pure inductance, then $\phi = -\frac{\pi}{2}$, *ie.* current lags behind the applied voltage by an angle $\frac{\pi}{2}$ *i.e.*, $V = V_0 \sin \omega t$

$$I = I_0 \sin\left(\omega t \frac{\pi}{2}\right)$$

Question: - Write down the principle construction and working of a moving coil galvanometer?
Answer: - It is a device used to detect and measure small current in the circuit.
Principle: -A current carrying coil placed in a magnetic field experiences a torque.
Construction: - It consists of a narrow rectangular coil having a large number of turns of copper wound on a non-metallic frame. It is suspended between the semicircular poles pieces of powerful horseshoe magnet by mean of a suspension wire made of phosphor bronze with a plane mirror attached to it. A cylindrical soft iron core is fixed within the coil. This makes the magnetic field linked with coil to be radial. At the lower end, coil is attached to a fine spring. Lamp and Scale arrangement is used to measure deflection.



Working: When the current I is passed through the coil, it experiences a torque which is given as

$$\tau = N I A B \sin\theta$$

Where N = no of turns
A = area of coil
B = magnetic field, and
θ = Angle between normal of coil and magnetic field

Since magnetic field is radial, so $\theta = 90^0$ Therefore

$$\tau = N I A B$$

Due to this torque, coil starts rotating. So the suspension wire gets twisted and a restoring torque is developed in the suspension wire made of Phosphor bronze. The restoring torque continues to increase and stage reaches when restoring torque becomes equal to the deflecting torque.

At equilibrium **Deflecting Torque** = Restoring Torque

$$N I A B = k\alpha$$

Where **α** is the angle by which the coil is rotated and k is the restoring torque per unit angle.
Therefore $I = k\alpha / NBA = G\alpha$
Where $G = k / NBA$ is called galvanometer constant.

$$I \propto \alpha$$

Hence current is directly proportional to the angle by which the coil rotated. Also light from lamp falls on the mirror gets deflected and falls on the scale. When current passes the mirror moves and light spot moves on the scale.
Sensitivity: - A galvanometer is said to be sensitive, if it give large deflection, even when a small current is passed through it or a small voltage is applied across its coil.
Current Sensitivity:-It is defined as the deflection produced in the galvanometer on passing a current of one ampere. i.e.

Current Sensitivity $\dfrac{\alpha}{I} = \dfrac{NBA}{k}$

Voltage Sensitivity: -It is defined as the deflection produced in the galvanometer when a potential difference of one volt is applied its coil.

Voltage Sensitivity $\dfrac{\alpha}{V} = \dfrac{NBA}{kR}$ since V=IR.

Note: - A galvanometer is said to be highly sensitive, if
(i) N is large (ii) B is large; (iii) A is large
(iv) k is small (v) R is small
However, N and A cannot be increased beyond a certain limit as otherwise the size and resistance of galvanometer will become large. Therefore B is made as large as possible and k is made as small as possible. To decrease k the suspension wire is made of phosphor bronze. Restoring torque per unit twist may further be decrease by taking the suspension wire in the form of strip.

$$\tan\Phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \tan\Phi = \frac{\omega L - 1/\omega C}{R}$$

Special Cases:

Case I: When $X_L > X_C$ i.e. $\omega L > 1/\omega C$,

$\tan\Phi = +ve$ or Φ is +ve

The current lags behind the emf by phase angle Φ and the LCR circuit is inductance - dominated circuit.

Case II: When $X_L < X_C$ i.e. $\omega L < 1/\omega C$,

$\tan\Phi = -ve$ or Φ is -ve

The current leads the emf by phase angle Φ and the LCR circuit is capacitance - dominated circuit.

Case III: When $X_L = X_C$ i.e. $\omega L = 1/\omega C$,

$\tan\Phi = 0$ or Φ is 0°

The current and the emf are in same phase. The impedance does not depend on the frequency of the applied emf. LCR circuit behaves like a purely resistive circuit.

Resonance in AC Circuit with L, C, R:

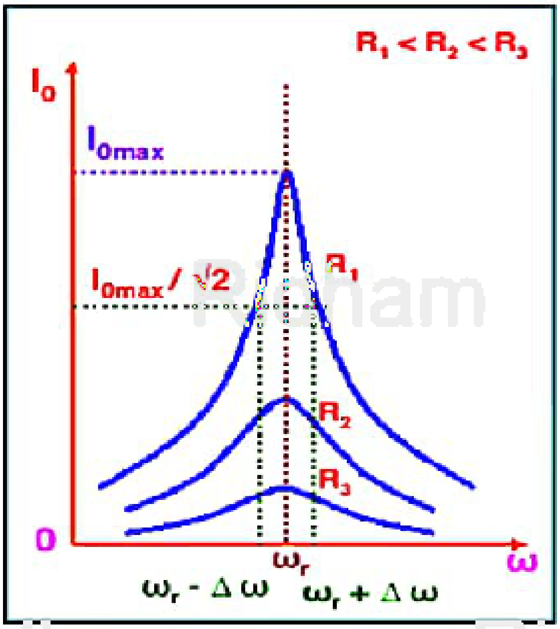
When $X_L = X_C$ i.e. $\omega L = 1/\omega C$, $\tan\Phi = 0$ or Φ is 0° and

$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$ becomes $Z_{min} = R$ and $I_{0max} = E / R$

i.e. The impedance offered by the circuit is minimum and the current is maximum. This condition is called resonant condition of LCR circuit and the frequency is called resonant frequency.

At resonant angular frequency ω_r,

$\omega_r L = 1/\omega_r C$ or $\omega_r = 1/\sqrt{LC}$ or $f_r = 1/(2\pi\sqrt{LC})$



Resonant Curve & Q - Factor:

Band width = $2\Delta\omega$

Quality factor (Q – factor) is defined as the ratio of resonant frequency to band width.

$$\boxed{Q = \omega_r / 2\Delta\omega}$$

It can also be defined as the ratio of potential drop across either the inductance or the capacitance to the potential drop across the resistance.

$Q = V_L / V_R$ or $Q = V_C / V_R$

or $Q = \omega_r L / R$ or $Q = 1 / \omega_r CR$

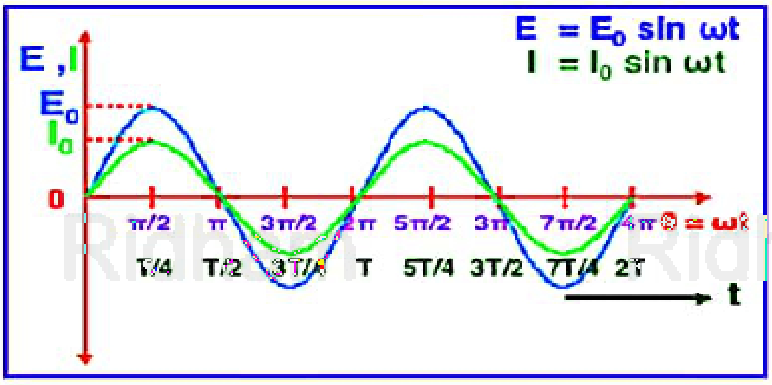
ALTERNATING CURRENTS

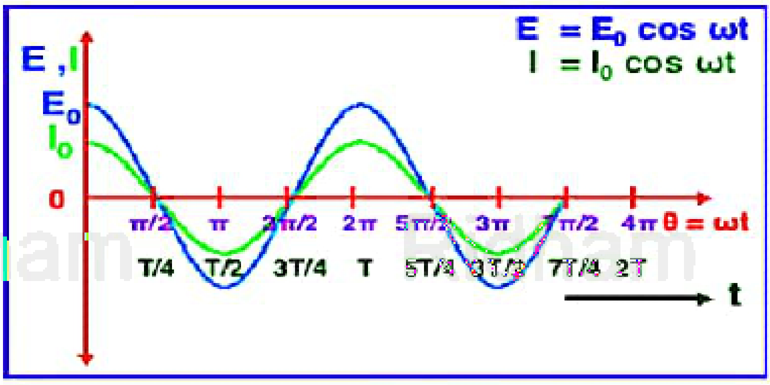
Alternating emf:

Alternating emf is that emf which continuously changes in magnitude and periodically reverses its direction.

Alternating Current:

Alternating current is that current which continuously changes in magnitude and periodically reverses its direction.



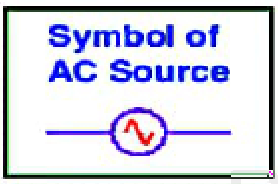


E, I – Instantaneous value of emf and current

E_0, I_0 – Peak or maximum value or amplitude of emf and current

ω – Angular frequency t – Instantaneous time

ωt – Phase



Average or Mean Value of Alternating Current:

Average or Mean value of alternating current over half cycle is that steady current which will send the same amount of charge in a circuit in the time of half cycle as is sent by the given alternating current in the same circuit in the same time.

$dq = I\,dt = I_0 \sin \omega t\,dt$

$q = \int_0^{T/2} I_0 \sin \omega t\,dt$

$q = 2\,I_0 / \omega \;=\; 2\,I_0\,T / 2\pi \;=\; I_0\,T / \pi$

Mean Value of AC, $I_m = I_{av} = q / (T/2)$

$$\boxed{I_m = I_{av} = 2\,I_0 / \pi = 0.637\,I_0 = 63.7\,\%\,I_0}$$

Average or Mean Value of Alternating emf:

$$\boxed{E_m = E_{av} = 2\,E_0 / \pi = 0.637\,E_0 = 63.7\,\%\,E_0}$$

Note: Average or Mean value of alternating current or emf is zero over a cycle as the + ve and – ve values get cancelled.

Root Mean Square or Virtual or Effective Value of Alternating Current:

Root Mean Square (rms) value of alternating current is that steady current which would produce the same heat in a given resistance in a given time as is produced by the given alternating current in the same resistance in the same time.

$dH = I^2R\,dt = I_0^2\,R \sin^2 \omega t\,dt$

$H = \int_0^{T} I_0^2\,R \sin^2 \omega t\,dt$

$H = I_0^2\,RT / 2$ (After integration, ω is replaced with $2\,\pi / T$)

If I_v be the virtual value of AC, then

$H = I_v^{\,2}\,RT$ $\therefore$ $\boxed{I_v = I_{rms} = I_{eff} = I_0 / \sqrt{2} = 0.707\,I_0 = 70.7\,\%\,I_0}$

Root Mean Square or Virtual or Effective Value of Alternating emf:

$$\boxed{E_v = E_{rms} = E_{eff} = E_0 / \sqrt{2} = 0.707\,E_0 = 70.7\,\%\,E_0}$$

Note:

1. Root Mean Square value of alternating current or emf can be calculated over any period of the cycle since it is based on the heat energy produced.
2. *Do not use the above formulae if the time interval under the consideration is less than one period.*

3. **Iron Losses:**

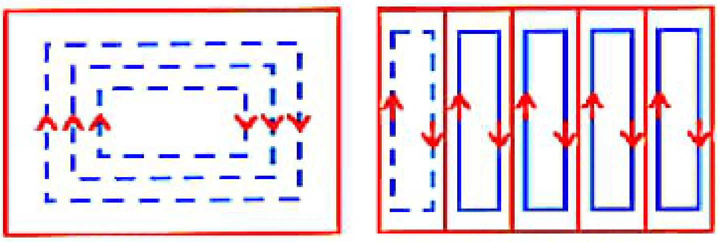
a) Eddy Currents Losses:

When a changing magnetic flux is linked with the iron core, eddy currents are set up which in turn produce heat and energy is wasted.

Eddy currents are reduced by using laminated core instead of a solid iron block because in laminated core the eddy currents are confined with in the lamination and they do not get added up to produce larger current. In other words their paths are broken instead of continuous ones.

b) Hysteresis Loss:

When alternating current is passed, the iron core is magnetised and demagnetised repeatedly over the cycles and some energy is being lost in the process.

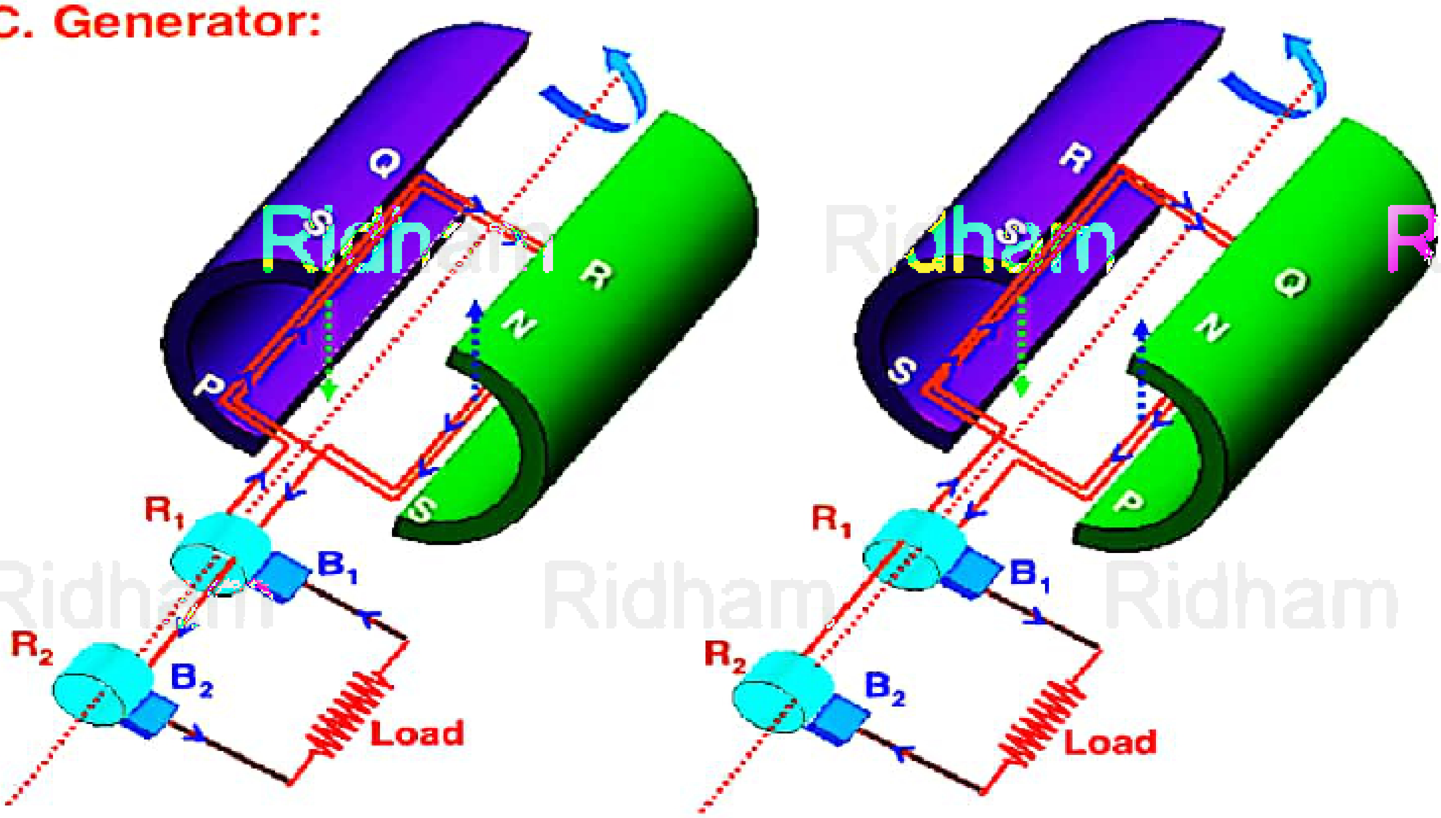


Solid Core Laminated Core

This can be minimised by using suitable material with thin hysteresis loop.

4. **Losses due to vibration of core:** Some electrical energy is lost in the form of mechanical energy due to vibration of the core and humming noise due to magnetostriction effect.

A.C. Generator:



A.C. Generator or A.C. Dynamo or Alternator is a device which converts mechanical energy into alternating current (electrical energy).

Principle:

A.C. Generator is based on the principle of Electromagnetic Induction.

Construction:

(i) Field Magnet with poles N and S
(ii) Armature (Coil) PQRS
(iii) Slip Rings (R_1 and R_2)
(iv) Brushes (B_1 and B_2)
(v) Load

Working:

Let the armature be rotated in such a way that the arm PQ goes down and RS comes up from the plane of the diagram. Induced emf and hence current is set up in the coil. By Fleming's Right Hand Rule, the direction of the current is $PQRSR_2B_2B_1R_1P$.

After half the rotation of the coil, the arm PQ comes up and RS goes down into the plane of the diagram. By Fleming's Right Hand Rule, the direction of the current is $PR_1B_1B_2R_2SRQP$.

If one way of current is taken +ve, then the reverse current is taken –ve.

Therefore the current is said to be alternating and the corresponding wave is sinusoidal.

Theory:

$\Phi = N B A \cos \theta$

At time t, with angular velocity ω,

$\theta = \omega t$ (at t = 0, loop is assumed to be perpendicular to the magnetic field and θ = 0°)

$\therefore \Phi = N B A \cos \omega t$

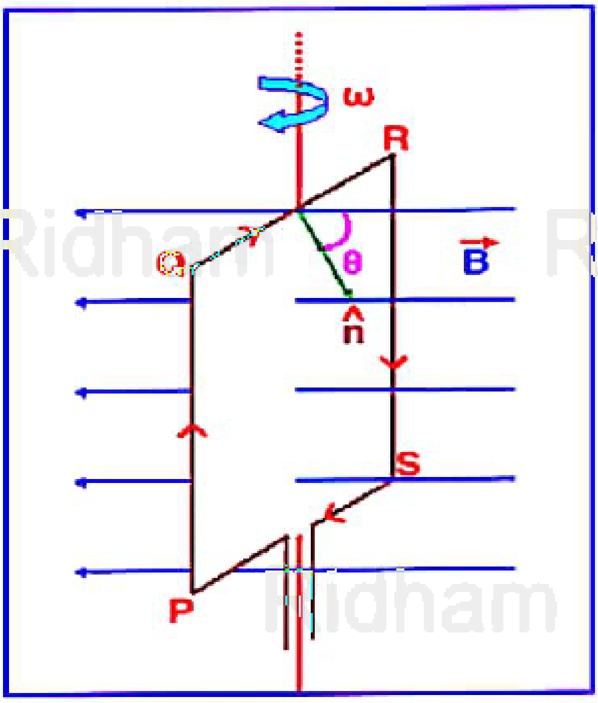
Differentiating w.r.t. t,

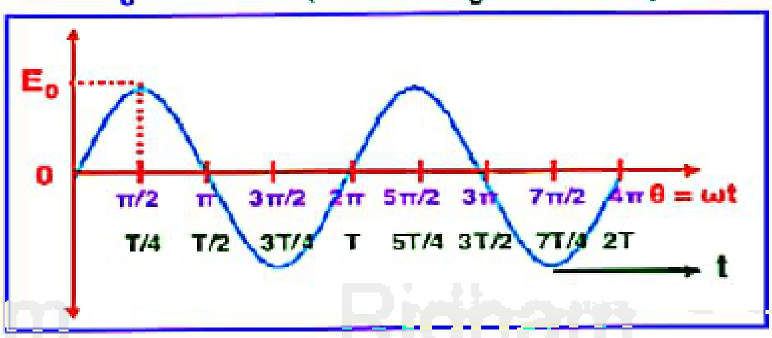
$d\Phi / dt = - NBA\omega \sin \omega t$

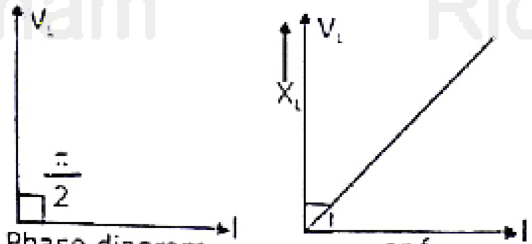
$E = - d\Phi / dt$

$\boxed{E = NBA\omega \sin \omega t}$

$E = E_0 \sin \omega t$ (where $E_0 = NBA\omega$)







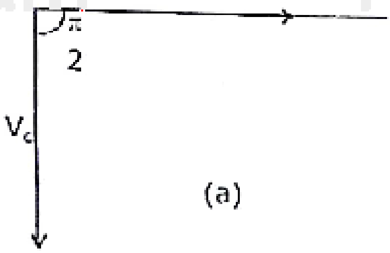
Phase diagram (a) (b)

In this case z=inductive reactance, $X_L=\omega L$. The inductive reactance increases with the increase of frequency of AC linearly (fig.b.)

17. PURELY CAPACITIVE CIRUIT

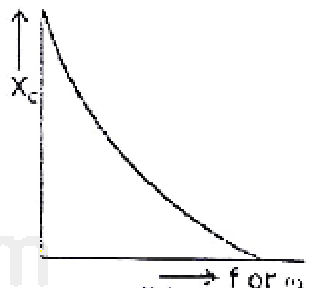
If circuit contains pure capacitance, the $\phi=\frac{\pi}{2}$, i.e. current leads the applied voltage by angle $\pi/2$.

i.e. $V=V_0\sin\omega t, I=I_0\sin\left(\omega t+\frac{\pi}{2}\right)$



(a)

Impedance =capacitance reactance, $X_C=\frac{1}{\omega C}$ Clearly capacitance reactance (X_c) is inversely proportional to the frequency f (fig.b).



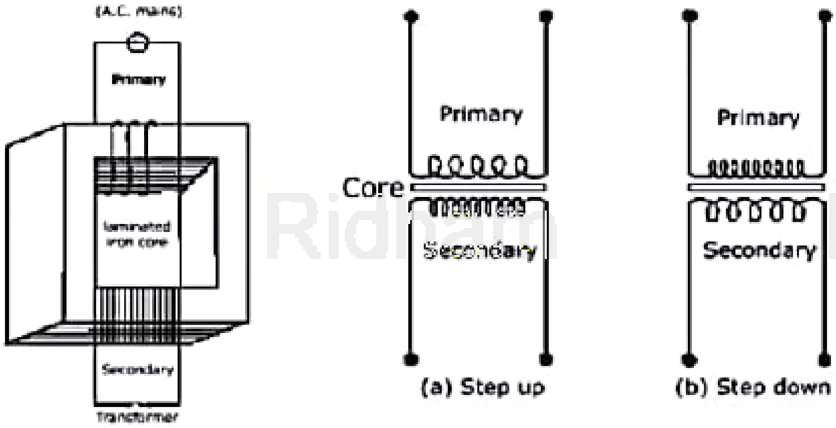
(b)

18. LC OSCILLATION

A circuit containing inductance L and capacitance C is called a resonant ciruit. If capacitance is charged initially and ac source is removed. Then electrostatic energy of capacitor ($q_0^2/2C$)) is converted into magnetic energy of inductor $\left(\frac{1}{2}LI^2\right)$ and vice versa periodically; such oscillation of energy are called LC oscillation. The frequency is given by $\omega_0=\frac{1}{\sqrt{LC}}$

19. LCR SERIES CIRCUIT

If a circuit contains inductance L, capacitance C and resistance R, fed by AC voltage $V=V_0\sin\omega t$ then impedance circuit in which alternating current of desired voltage is required. Transformers are of two types :



1. **Step up Transformer:** It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil.

(i.e., $N_S > N$)

2. **Step down Transformer:** It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (i.e., $N_S < N_P$)

Working: When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_P be the number of turns in primary coil, N_S the number of turns in secondary coil and ϕ The magnetic flux linked with each turn. **We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same.** According to Faraday's laws the emf induced in the primary coil

$$\varepsilon_P = -N_P \frac{\Delta\phi}{\Delta t} \qquad \text{(i)}$$

And emf induced in the secondary coil

$$\varepsilon_S = -N_S \frac{\Delta\phi}{\Delta t} \qquad \text{(ii)}$$

From (1) and (2)

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \qquad \text{(iii)}$$

If the resistance of primary coil is negligible, the emf (ε_P) induced in the primary coil, will be equal to the applied potential difference V_S across its ends will be equal to the emf (ε_S) induced in it; therefore

$$\frac{V_S}{V_P} = \frac{e_S}{e_P} = \frac{N_S}{N_P} = r(\text{say}) \qquad \text{(iv)}$$

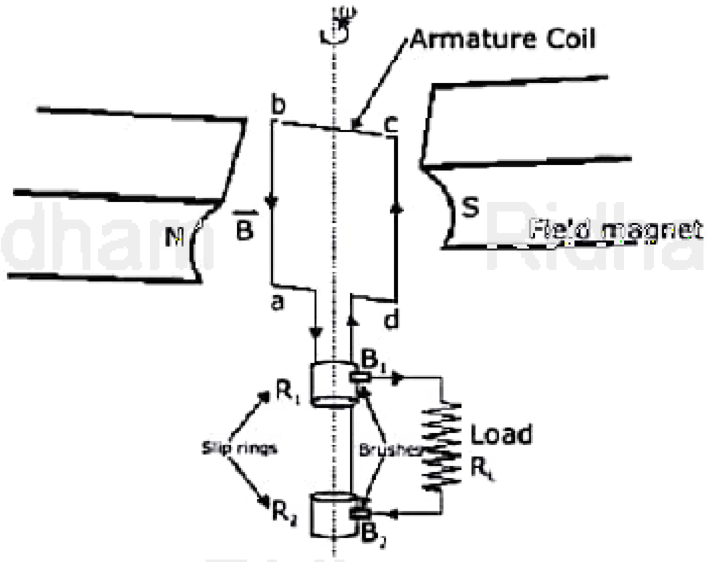
Where r=$\frac{N_S}{N_P}$ is called the transformation ratio. If i_P and i_S are the instantaneous currents in primary and secondary coils and there is no loss of energy; then

Power in primary= Power in secondary

$$V_P i_P = V_S i_S$$

$$\frac{i_S}{i_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{1}{r} \qquad \text{(V)}$$

Working : When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming s right hand rule. Considering the armature to be in vertical position and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in fig. in the external circuit, the current flows along $B_1B_LB_2$.the Direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit if flows along $B_2R_LB_1$. Thus the direction of induced emf and current ch anges in the external circuit after each half Revolution.



If N is the number of turns in coil, f the frequency of rotation. A area of coil and B the magnetic induction, then induced emf

$$e = -\frac{d\phi}{dt} = \frac{d}{dt}(NBA(\cos 2\pi ft)$$

Obviously, the emf produced is alternating and hence the current is also alternating Current produced by an ac generator cannot be measured by moving coil ammeter, because the average value of ac over fully cycle is zero.

5. A jet plane is travelling west at 450 ms^{-1}. If the horizontal component of earth s magnetic field at that place is 4 10^{-4}T and the angle of dip is 30^0, find the emf induced between the ends of wings having a span of 30m.

Sol. The wings of jets plane will cut the vertical component of earth s magnetic field, so emf is induced across the wings. The vertical component of eath s magnetic field

V = H tanθ

Given H = 4.0 10^{-4}T.θ = 30^0

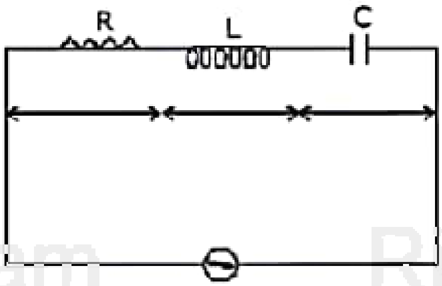
$$V = (4.0 \quad 10^{-4}T)\tan 30^0 = 4 \quad 10^{-4} \quad \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} \quad 10^{-4}T$$

Induced emf across the wing

Given v= 450 $ms^{-1}, l = 30m$

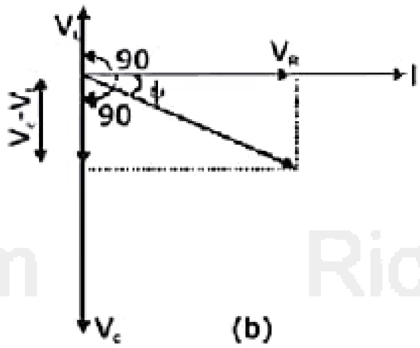
$$\varepsilon = \left(\frac{4}{\sqrt{3}} \quad 10^{-4}\right)(450) \quad 30 = 3.12V$$

6. A bulb of resistance 10 Ω connected to an inductor of inductance L, is in series with an ac source marked 100 V, 50Hz. If the phase angle between the voltage and current is $\frac{\pi}{4}$



$V = V_0 \sin \omega t$

(a)



(b)

Suppose the voltage across resistance R is V_R, voltage across inductance L is V_L and voltage across capacitance C is V_C. The voltage V_R and current I are in the same phase, the voltage V_L will lead the current by angle 90^0 (fig. b) Clearly V_C and V_L are in opposite directions, therefore their resultant difference = V_C-V_L if (V_C>V_L)

Thus V_R and (V_C-V_L) are mutually perpendicular and the phase difference between them is 90^0. As applied voltage across the circuit is V, the resultant of V_R and ($V_C - V_L$) will also be V. From fig.

$$V^2 = V_R^2 + (V_C - V_L)^2 \rightarrow V = \sqrt{V_R^2 + (V_C - V_L)^2} \qquad \text{(i)}$$

But $\quad V_R = Ri, V_C = X_C i \text{ and } V_L = X_L i \qquad$ (ii)

Where $X_C = \dfrac{1}{\omega C}$ = capacitance reactance and $X_L = \omega L$ = inductive reactance

$$\therefore \qquad V = \sqrt{(Ri)^2 + (X_C i - X_L i)^2}$$

$\therefore$ Impedance of circuit, $\quad Z = \dfrac{V}{i} = \sqrt{R^2 + (X_C - X_L)^2}$

i.e., $\quad Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C} - \omega L\right)^2}$

Resonant frequency: The phase difference (ϕ) between current and voltage ϕ is given by

$$\tan \phi = \frac{X_C - X_L}{R}$$

For resonance ϕ = 0, so $X_3 - X_l$ = 0

$$\Rightarrow \qquad \frac{1}{\omega C} = \omega L \Rightarrow \omega^2 = \frac{1}{LC}$$

Resonant frequency $\omega_r = \dfrac{1}{\sqrt{LC}}$

2. Describe briefly the principle, construction and working of a transformer. Why is its core laminated?

Sol. **Transformer:** Transformer is a device by which an alternating voltage may be decreased or increased. This is based on the principle of mutual induction.

Construction: It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils coil. The terminals of primary co8is are connected to AC mains and the terminals of the secondary coil are connected to external

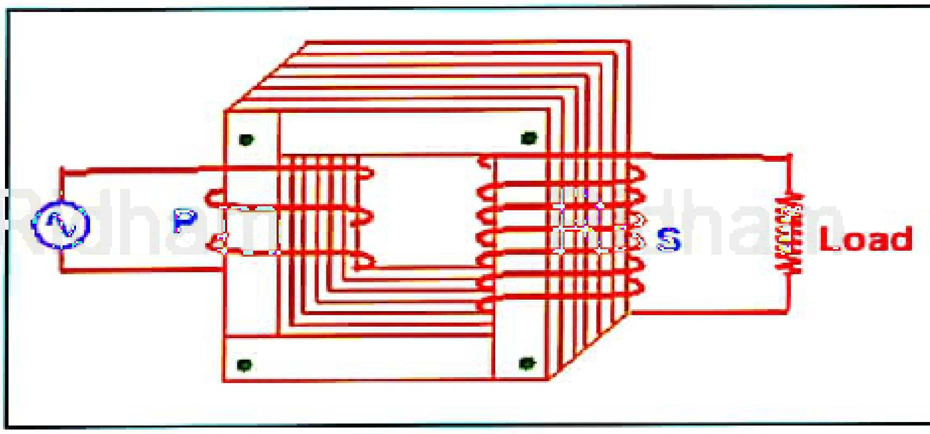
Transformer:

Transformer is a device which converts lower alternating voltage at higher current into higher alternating voltage at lower current.

Principle:

Transformer is based on Mutual Induction.

It is the phenomenon of inducing emf in the secondary coil due to change in current in the primary coil and hence the change in magnetic flux in the secondary coil.



Theory:

$E_P = - N_P \, d\Phi / dt$

$E_S = - N_S \, d\Phi / dt$

$E_S / E_P = N_S / N_P = K$

(where K is called Transformation Ratio or Turns Ratio)

For an ideal transformer,

Output Power = Input Power

$E_S I_S = E_P I_P$

$E_S / E_P = I_P / I_S$

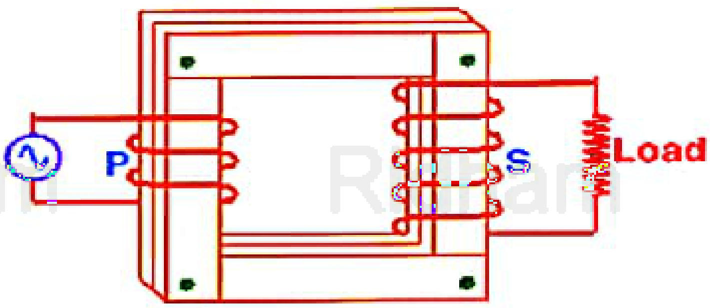
$E_S / E_P = I_P / I_S = N_S / N_P$

Efficiency (η):

$\eta = E_S I_S / E_P I_P$

For an ideal transformer η is 100%

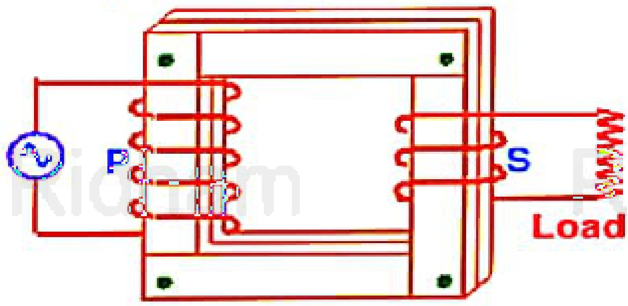
Step - up Transformer:



$N_S > N_P$ i.e. $K > 1$

$E_S > E_P$ & $I_S < I_P$

Step - down Transformer:



$N_S < N_P$ i.e. $K < 1$

$E_S < E_P$ & $I_S > I_P$

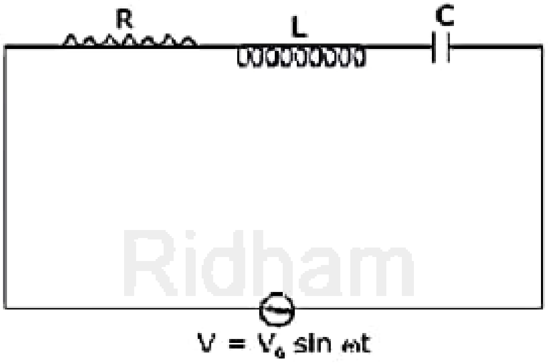
Energy Losses in a Transformer:

1. Copper Loss: Heat is produced due to the resistance of the copper windings of Primary and Secondary coils when current flows through them.

   This can be avoided by using thick wires for winding.

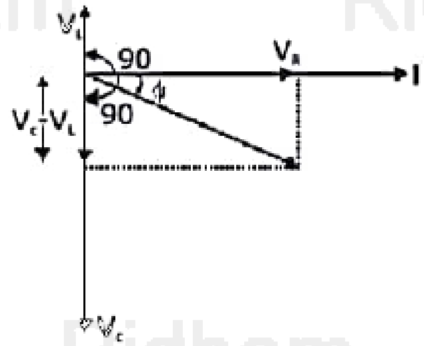
2. Flux Loss: In actual transformer coupling between Primary and Secondary coil is not perfect. So, a certain amount of magnetic flux is wasted.

   Linking can be maximised by winding the coils over one another.



$$Z = \sqrt{R^2 + \left(X_c - X_L\right)^2}$$

And phase $\phi = \tan^{-1}\dfrac{X_c - X_L}{R}$



Phase diagram of LCR circuit

Net voltage $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

20. RESONANT CIRCUITS

(i) Series LCR circuit: In series LCR Circuit, when phase (ϕ) between current and voltage is zero, the circuit is said to be resonant circuit. In resonant circuit

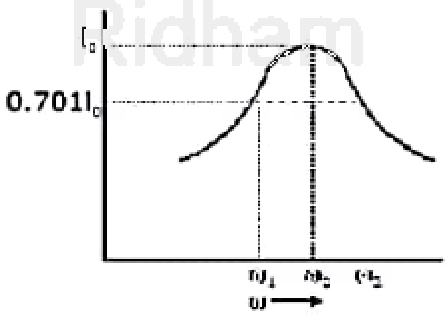
Electromagnetic induction and Alternating Current $\omega = \dfrac{1}{LC}$

Resonant angular frequency $\omega_r = \dfrac{1}{\sqrt{LC}} \Rightarrow \text{(linear) frequency } f_2 = \dfrac{\omega_r}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$

At resonant frequency $\phi = 0$, $V = V_R$

Quality factor (Q)

The quality factor (Q) of a series LCR circuit is given by the ratio of resonant frequency to frequency band width of the resonant 0.707 I_0 curve



i.e. $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$

Clearly, smaller the value of R. larger is the quality factor and sharper the resonance. Thus quality factor. Determines the nature of sharpness of resonance.

(ii) Parallel Resonant circuit: A circuit containing inductance L and capacitance C in parallel and fed by ac voltage is called parallel resonant circuit. In parallel resonant circuit

$$i_C = i_L \Rightarrow X_C = X_L \text{ frequency } \omega_r = \frac{1}{\sqrt{LC}}$$

21. POWER DISSIPATION IN AC CIRCUIT IS

P = $V_{rms} I_{rms} \cos\phi = -\frac{1}{2} V_0 I_0 \cos\phi$ (Where $\cos\phi = \frac{R}{Z}$ is the power factor.)

22. WATTLESS CURRENT

In purely inductive or purely capacitive circuit, power loss is zero. In such a circuit current flowing is called wattles current. In LCR circuit at resonace, the power loss is maximum.

Wattless component of current $= I_{rms} \sin\phi$

Power component of current $= I_{rms} \cos f$

23. AC GENERATOR

It is a device to vonvert mechanical energy into electrical energy based on the phenomenon of electro-magnet induction. If a coil of N turns, area A is rotated with frequency f in uniform magnetic field of induction B, then motional emf in coil (If initially it is perpendicular to field) is

$$e = NBA\,\omega \sin \omega t \text{ with } \omega = 2\pi f$$

Peak emf, $e_0 = NBA\omega$.

24. TRANSFORMER

A transformer is a device which converts low ac voltage into high ac voltage and vice versa. It works on the principle of mutual induction. If N_p and N_s are the number of turns in primary and secondary coils, V_p and I_p are voltage and current in primary coil, then voltage (V_s)and current (I_S) in secondary coil will be

$$V_S = \left(\frac{N_S}{N_P}\right) V_P \text{ and } I_S = \left(\frac{N_P}{N_S}\right) I_P$$

Step up transformer increases the voltage while step down transformer decreases the voltage.

For step up transformer

In step down transformer

<u>SOLVED PROBLEMS</u>

1. What is meant by impedance? Give its unit. Using the phasor diagram or otherwise, derive an expression for the impedance ofan ac circuit containing L,C and R in series. Find the expression for resonant frequency?

Sol. Impedance: The hindrance offered by a circuit to the flow of ac is called impedance. Mathematically it is the ratio of rms voltage applied and rms current produced in circuit i.e., $Z = \frac{V}{I}$ its unit is ohm (Ω).

Expression for Impedance in LCR series circuit: Suppose resistance R, inductance L and capacitance C are connected in series and an alternating source of voltage V= $V_0 \sin \omega t$ is applied across it (fig. a) on account of being in series, the current (f) flowing through all of them is the same.

In step up transformer,

$N_s > N_p \rightarrow r > 2;$

So $V_s > V_p$ and $i_s < i_p$]

i.e., step down transformer decreases the voltage, but increases the current.

Laminated core: The core of transformer is lamented to reduce the energy losses due to eddy currents, so that its efficiency may remain nearly 100%

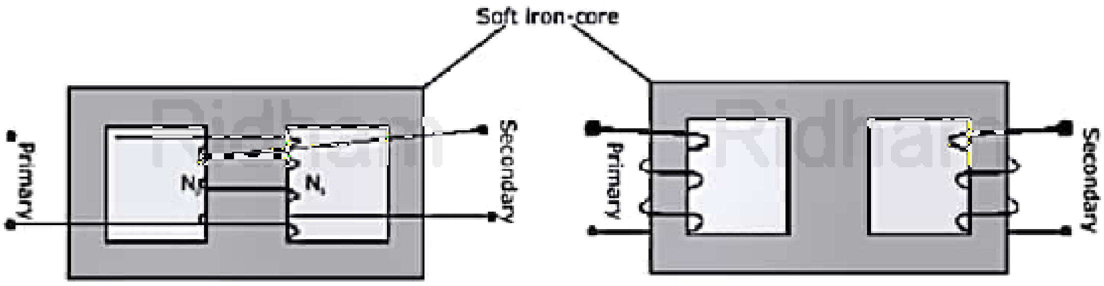
3. Show diagrammatically two different arrangements used for winding the primary and secondary coils in a transformer. Assuming the transformer to be an ideal one, write the expression for the ratio of its

(i) output voltage to input voltage

(ii) output current to input current.

Mention two reasons for energy losses in an actual transformer.

Sol. Arrangements of winding of primary and secondary coil in a transformer are shown in figure (a) and (b)



(a) Two coils on top of each other (b) Two coils on separate limbs of the core

(i) Ratio of output voltage to input voltage

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

(ii) Ratio of output current to input current

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Reasons for energy losses in a transformer

(i) Joule Heating: Energy is lost in resistance of primary and secondary winding as heat ($I^2 Rt$).

(ii) Flux Leakage: Energy is lost due to coupling of primary and secondary coils not being perfect. i.e., whole of magnetic flux generated in primary coil is not linked with the secondary coil.

4. Explain with the help of a labeled diagram, the principle and working of an a.c. generator ? write the expression for the emf generated in the coil in terms of speed of rotation. Can the current produced by an a.c. generator be measured with a moving coil galvanometer.

Sol. AC generator: A dynamo or generator is a device which converts mechanical energy into electrical energy. It is based on the principle of electromagnetic induction.

Construction: It consists of the four main parts:

(i) **Field magnet:** It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.

(ii) **Armature:** It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve rebound an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (i) it serves as a support to coils and (ii) it increases the magnetic field due to air core being replaced by an iron core:

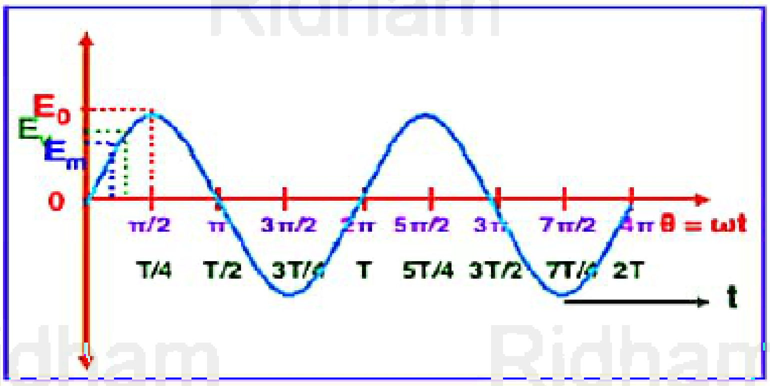
(iii) **Slip Rings:** The slip rings R_1 and R_2 are the two metal rings to which rotates the armature coil so that the rings also rotate along with the armature.

(iv) **Brushes:** These are two flexible metal plates r carbon rods (B_1 and B_2) which are are fixed and constantly touch the revolving rings. The output current in external load R_L is taken through these brushes.

Relative Values Peak, Virtual and Mean Values of Alternating emf:

$$\boxed{E_m = E_{av} = 0.637\, E_0}$$

$$\boxed{E_v = E_{rms} = E_{eff} = 0.707\, E_0}$$



Tips:

1. The given values of alternating emf and current are virtual values unless otherwise specified.

   i.e. 230 V AC means $E_v = E_{rms} = E_{eff} = 230$ V

2. AC Ammeter and AC Voltmeter read the rms values of alternating current and voltage respectively.

   They are called as 'hot wire meters'.

3. The scale of DC meters is linearly graduated where as the scale of AC meters is not evenly graduated because $H \propto I^2$

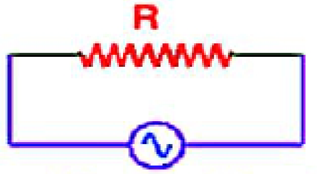
AC Circuit with a Pure Resistor:

$E = E_0 \sin \omega t$

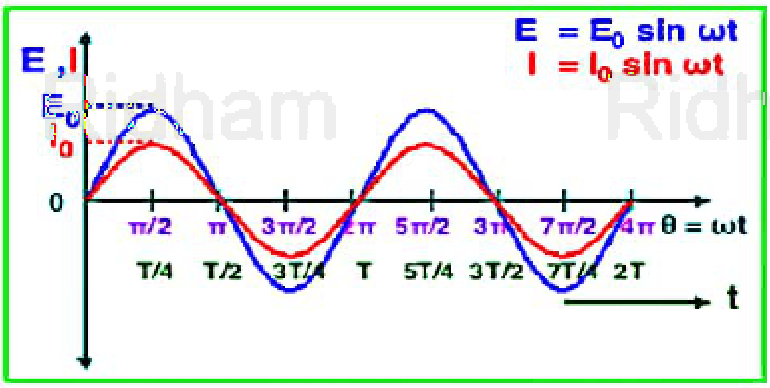
$I = E / R$

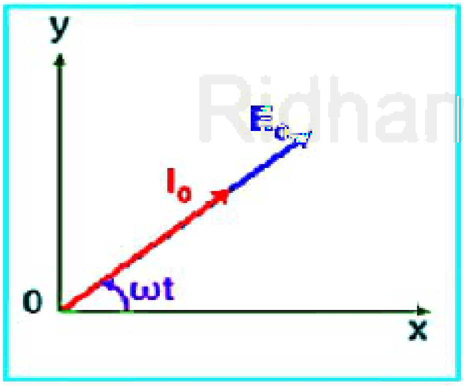
$= (E_0 / R) \sin \omega t$

$I = I_0 \sin \omega t$ (where $I_0 = E_0 / R$ and $R = E_0 / I_0$)



Emf and current are in same phase.



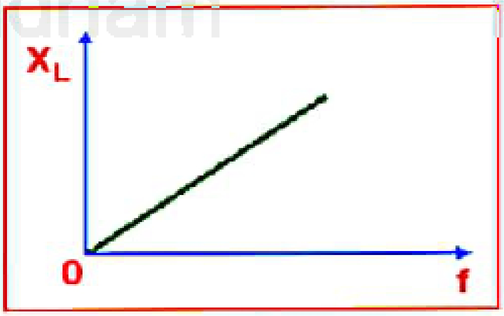


Variation of X_L with Frequency:

$I_0 = E_0 / \omega L$ and $X_L = \omega L$

X_L is Inductive Reactance and $\omega = 2\pi f$

$X_L = 2\pi f L$ i.e. $X_L \; \alpha \; f$

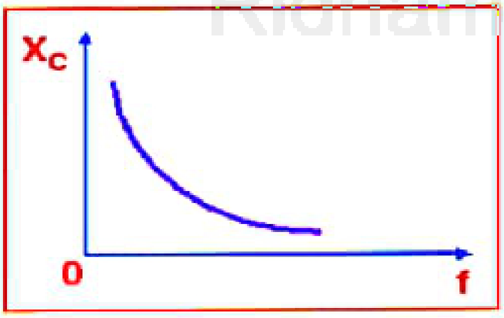


Variation of X_C with Frequency:

$I_0 = E_0 / (1/\omega C)$ and $X_C = 1 / \omega C$

X_C is Inductive Reactance and $\omega = 2\pi f$

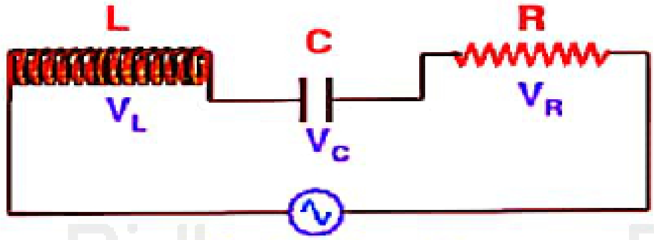
$X_C = 1 / 2\pi f C$ i.e. $X_C \; \alpha \; 1 / f$



TIPS:

1) Inductance (L) can not decrease Direct Current. It can only decrease Alternating Current.
2) Capacitance (C) allows AC to flow through it but blocks DC.

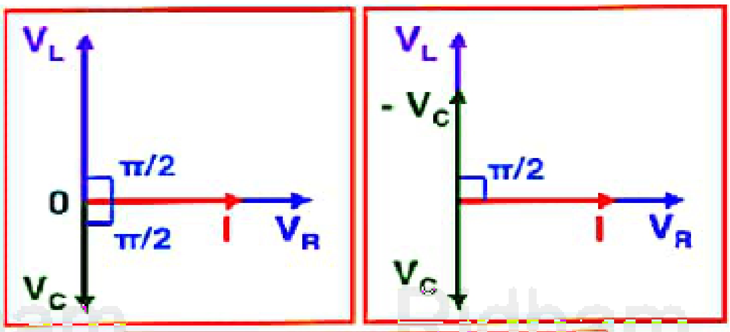
AC Circuit with L, C, R in Series Combination:



$E = E_0 \sin \omega t$

The applied emf appears as Voltage drops V_R, V_L and V_C across R, L and C respectively.

1) In R, current and voltage are in phase.
2) In L, current lags behind voltage by $\pi/2$
3) In C, current leads the voltage by $\pi/2$

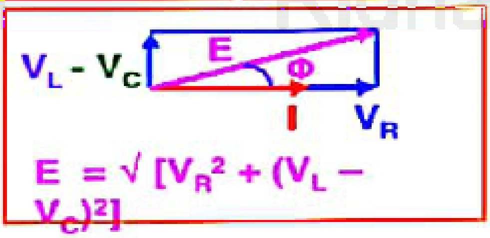


$E = \sqrt{[V_R^2 + (V_L - V_C)^2]}$

$$I = \frac{E}{\sqrt{[R^2 + (X_L - X_C)^2]}}$$

$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}$



$$\tan \Phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \tan \Phi = \frac{\omega L - 1/\omega C}{R}$$