ELL 225
COURSE PROJECT

RIDERLESS BICYCLE CONTROL

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1 OBJECTIVE

To develop an autopilot for a rider-less bicycle and to keep it in upright position with respect to some specified speeds.
Considering following quantities as:
i) states: roll angle, roll rate, steer angle and steer rate
ii) input: steering torque
iii) output: roll angle
The dynamics of the bicycle can be expressed as a fourth order linear model.The linear model is velocity dependent (often referred to as linear parameter varying model), where one can obtain linear time-invariant state space models corresponding to the different fixed velocities.
Tasks:

- Obtain state space models and transfer functions of the bicycle at velocities: v1=0m/s, v2=3.5m/s and v3=5m/s. Compute poles and zeros, and the eigenvalues of system matrix A, for the obtained three state space models and transfer functions.
- Show the time response of system, for: i) zero input with (any) non-zero initial states and ii) unit step input.
- Analyze stability (asymptotic stable, marginally stable, BIBO stable) of bicycle corresponding to the velocities: v1, v2 and v3.
- Draw Nyquist plots, Bode plots and Root-locus, considering the open loop transfer functions that are obtained for the velocities: v1, v2 and v3. Describe in your report, which plots are for closed loop system and which are for open loop systems. Give some explainations/concluding remarks (stability, phase and gain margins) that you observe for different plots.
- A control system (autopilot) needs to be designed to keep the bicycle in vertical upright position. Is it possible to stabilize the bicycle (keep the bicycle in vertical upright position), with the help of appropriate automatic control action (steering torque

provided by the actuator), at velocities: v1, v2 and v3. If yes, then design such feedback controllers. Take your own design specifications in terms steady state error, damping ratio and settling time, and then, design controllres to achive these objectives. Three different controllers may be proposed for velocities: v1, v2 and v3. Implement the designed controllers, and show the step input response of the closed loop system. You may use “sisotool” available in MATLAB (or similar simulation software) for designing controllers.

2 INTRODUCTION

Like an inverted pendulum, Bicycles are statically unstable, but can, under certain conditions, be stable. A detailed model of a bicycle is complex because the system has many degrees of freedom and the geometry is intricate. In this project, We assume that the bicycle consists of four rigid parts, specifically, two wheels, a frame, and a front fork with handlebars. The influence of other moving parts, such as pedals, chain, and brakes, on the dynamics is disregarded. We simply assume that the bicycle moves on a horizontal plane and that the wheels always maintain contact with the ground.

3 MATHEMATICAL MODELLING OF THE BICYCLE

In order to simulate the model we will consider three degrees of freedom, namely, the roll angle $\phi(t)$, the steering angle $\delta(t)$, and the speed $v(t)$. The linearized equations of motion are two coupled second-order ordinary differential equations written in matrix form as

$$M\ddot{q}(t) + v(t)C_1\dot{q}(t) + \left(K_\circ + v(t)^2 K_2\right)q(t) = f(t) \qquad (1)$$

$$q(t) = \begin{bmatrix}\phi(t)\\ \delta(t)\end{bmatrix},\ f(t) = \begin{bmatrix}T_\phi(t)\\ T_\delta(t)\end{bmatrix}$$

where $T\phi(t)$ is an exogenous roll-torque disturbance and $T\delta(t)$ is the steering torque provided by the actuator on the handlebar axis. The remaining quantities are the symmetric mass matrix M, the speed-dependent damping matrix $v(t)C1$, and the stiffness matrix, which is the sum of a constant symmetric part K0 and a quadratically speed-dependent part v^2K_2.

The linearized ordinary differential equation (1) is rewritten in state-space form choosing $\phi(t)$the roll angle , the $\delta(t)$steering angle , and their derivatives
$\dot{\phi}(t)\ and\ \dot{\delta}(t)$, respectively, as state variables. The control input u(t) is the torque $T\delta(t)$ applied to the handlebar axis. The measured output y(t) includes all of the state variables.

$$\dot{x}(t) = A(v(t))x(t) + Bu(t) \qquad (2)$$

$$y(t) = Cx(t) + Du(t) \qquad (3)$$

$$x(t) = \begin{bmatrix}\phi(t)\\ \delta(t)\\ \dot{\phi}(t)\\ \dot{\delta}(t)\end{bmatrix}$$

$$u(t) = T_\delta(t),\ y(t) = \phi(t) \qquad (4)$$

The entries of the matrices A(v(t)), B, C, and D depend on the geometry as well as the physical parameters of the bicycle.For this project, We take the following values.

$$A(v) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & 0.225 - 1.319v^2(t) & -0.164v(t) & -0.552v(t) \\ 4.857 & 10.81 - 1.125v^2(t) & 3.621v(t) & -2.388v(t) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The dependence of A(v(t)) on the speed v(t) qualifies the system as an LPV model, that is, for each constant value of the time-varying parameter v(t), (2) and (3) describe a linear time-invariant system.

4 Analysis of Bicycle Dynamics and Stability Control (TASK1)

Transfer function can be calculated by $T(s) = C(sI - A)^{-1}B + D$

<u>Case I- v(t)=0m/s</u>

STATE SPACE REPRESENTATION

$$\begin{bmatrix} \dot{\phi}(t) \\ \dot{\delta}(t) \\ \ddot{\phi}(t) \\ \ddot{\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & 0.225 & 0 & 0 \\ 4.857 & 10.81 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi(t) \\ \delta(t) \\ \dot{\phi}(t) \\ \dot{\delta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{bmatrix} T_\delta(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi(t) \\ \delta(t) \\ \dot{\phi}(t) \\ \dot{\delta}(t) \end{bmatrix}$$

TRANSFER FUNCTION REPRESENTATION

$$\frac{-0.339s^2 + 1.505 \cdot 10^{-16}s + 5.342}{s^4 + 3.553 \cdot 10^{-15}s^3 - 24.48s^2 - 4.263 \cdot 10^{-14}s + 146.7}$$

ZEROES

+3.9698i,-3.9698i

POLES

-3.7432, -3.2355, 3.7432, 3.2355

EIGENVALUES

-3.7432, -3.2355, 3.7432, 3.2355

5 TIME RESPONSE(TASK 2)

For zero input, with initial state

$$x_\circ = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

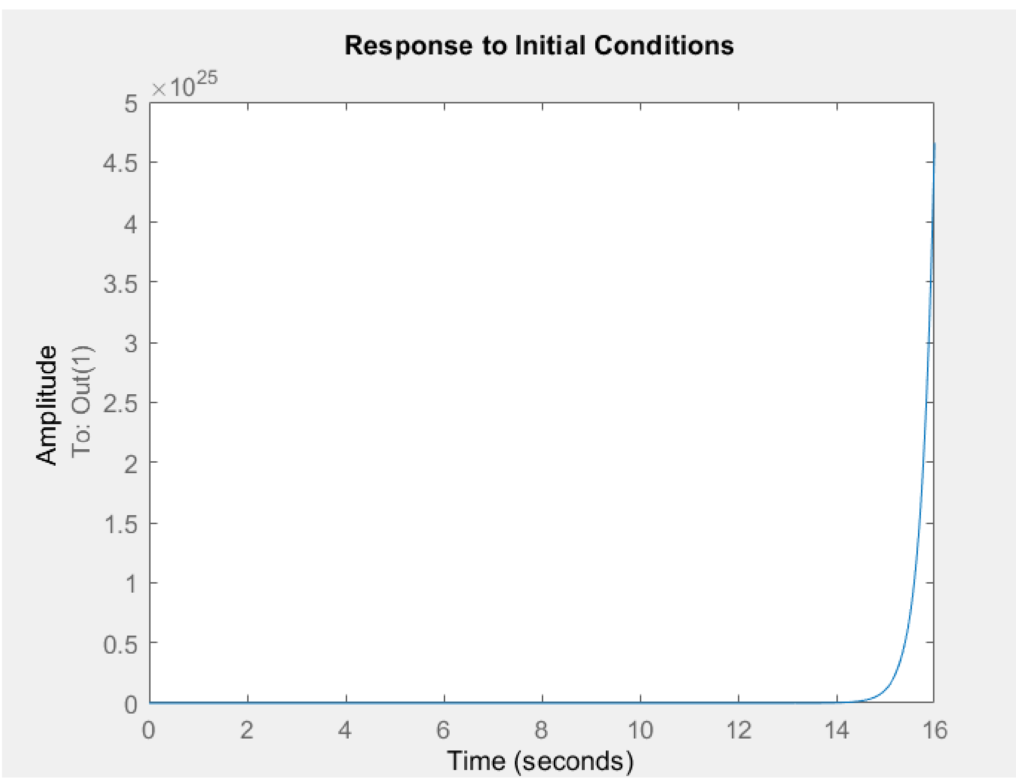
Time response of the system:

$$x(t) = e^{At}x_\circ(t)$$

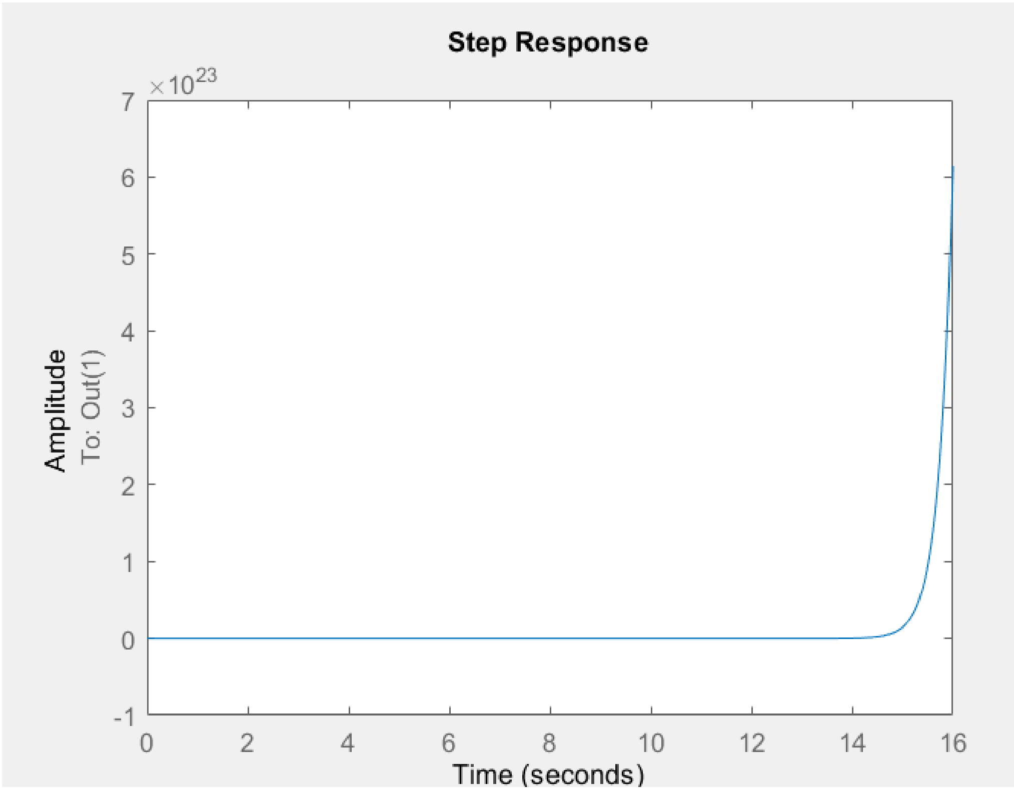
$$x(t) = V\,e^{\Lambda t}\,V^{-1}x_\circ(t)$$

$$y(t) = Cx(t)$$

ZERO INPUT RESPONSE



UNIT STEP RESPONSE

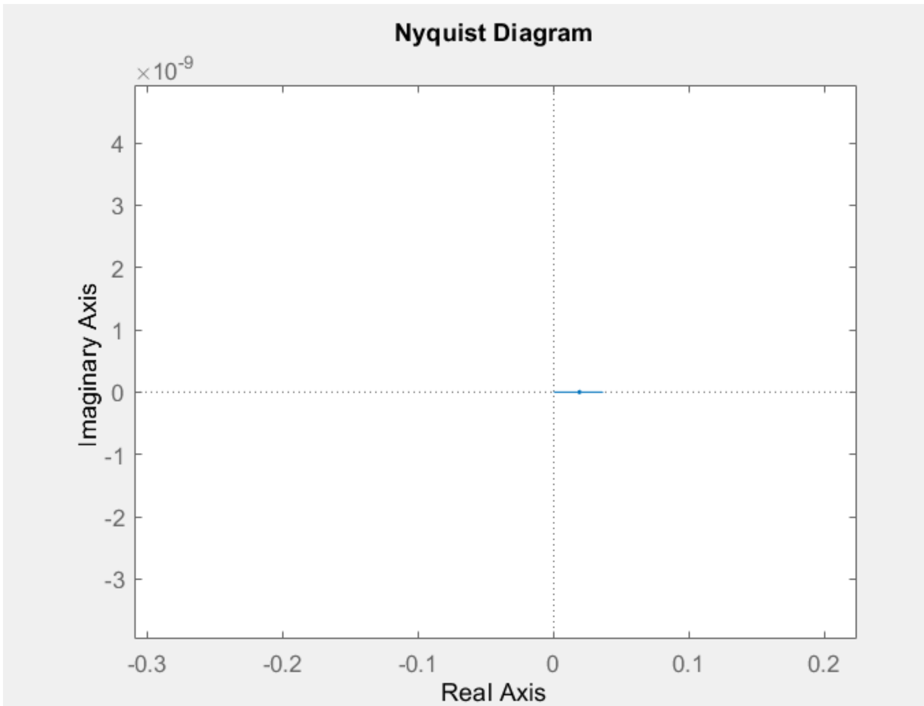


6 Stability (TASK 3)

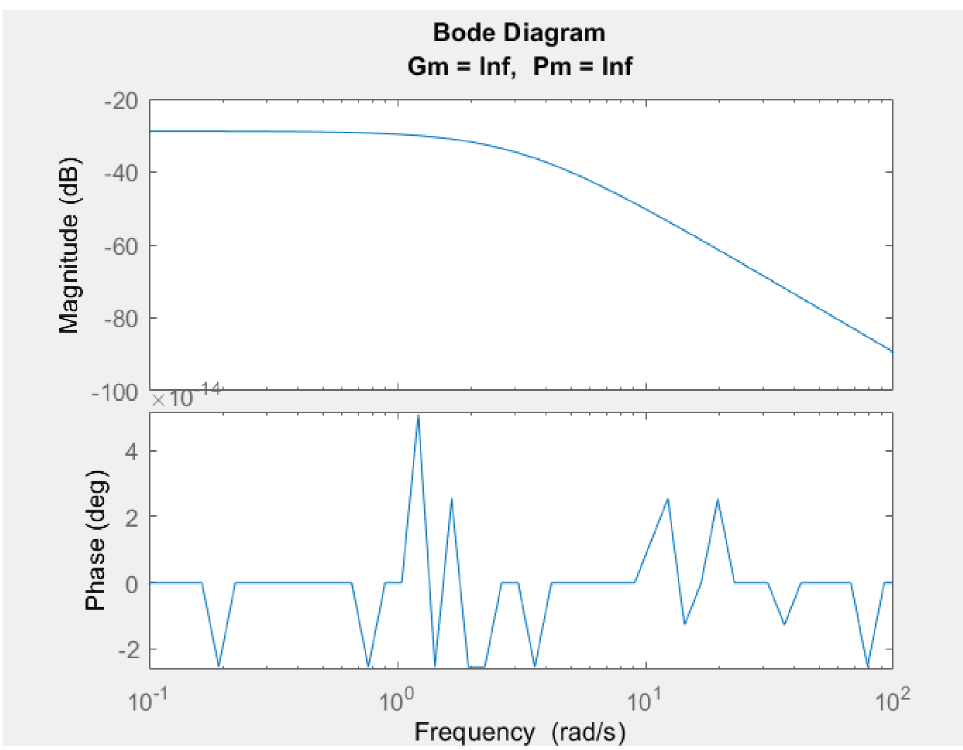
Since the eigenvalues are also in RHP. So the system is unstable for v=0m/s. And since for bounded input (step) we are getting unbounded output, thus the system is BIBO unstable.

7 PLOTS (TASK 4)

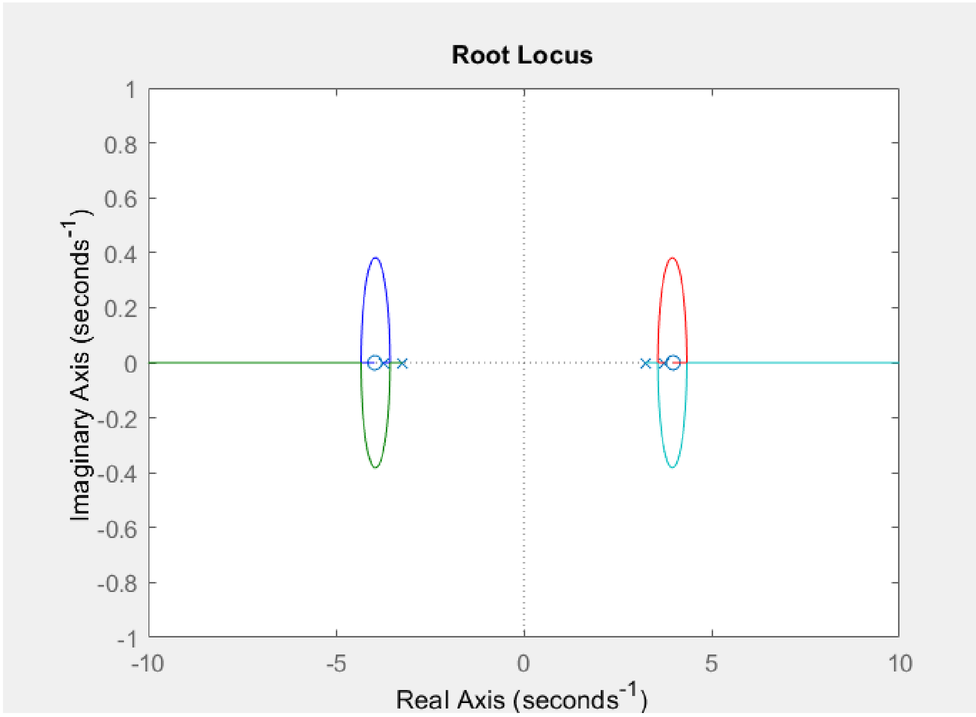
Nyquist Plot



Bode Plot



Root Locus



From the Root locus,we can conclude that the system is unstable for v=0m/s due to the presence of zeroes and poles in RHP.
From the BODE plot, we can conclude that the the system has inf gain and phase margin.

<u>Case II- v(t)=3.5m/s</u>

STATE SPACE REPRESENTATION

$$\begin{bmatrix} \dot{\phi}(t) \\ \dot{\delta}(t) \\ \ddot{\phi}(t) \\ \ddot{\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 13.67 & -15.9328 & -0.5740 & -1.9320 \\ 4.857 & -2.9712 & 12.6735 & -8.3580 \end{bmatrix} \begin{bmatrix} \phi(t) \\ \delta(t) \\ \dot{\phi}(t) \\ \dot{\delta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.339 \\ 7.457 \end{bmatrix} T_{\delta}(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi(t) \\ \delta(t) \\ \dot{\phi}(t) \\ \dot{\delta}(t) \end{bmatrix}$$

TRANSFER FUNCTION REPRESENTATION

$$\frac{-0.339s^2 + 1.505 \cdot 10^{-16}s + 5.342}{s^4 + 3.553 \cdot 10^{-15}s^3 - 24.48s^2 - 4.263 \cdot 10^{-14}s + 146.7}$$

ZEROES

-42.5497, -8.3066

POLES

-8.0753 + 0.0000i , -0.2301 + 3.3809i , -0.2301 - 3.3809i ,
-0.3965 +0.0000i

EIGENVALUES

-8.0753 + 0.0000i
-0.2301 + 3.3809i
-0.2301 - 3.3809i
-0.3965 + 0.0000i

5 TIME RESPONSE(TASK 2)

For zero input , with initial state

$$x_\circ = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

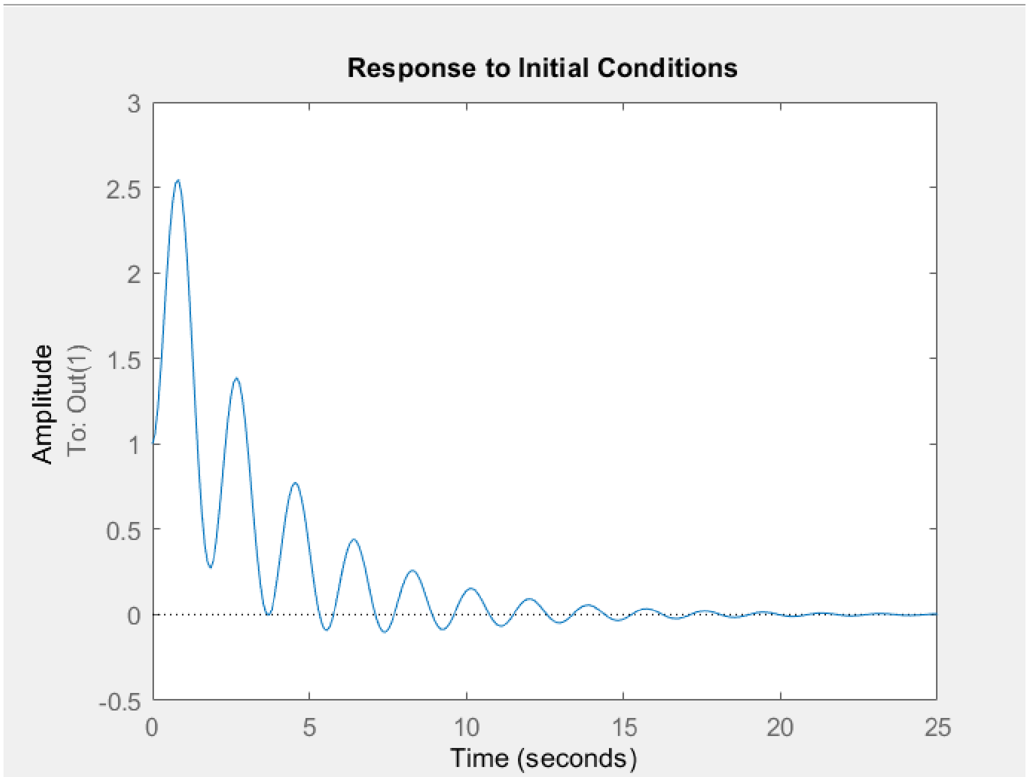
Time response of the system:

$$x(t) = e^{At}x_\circ(t)$$

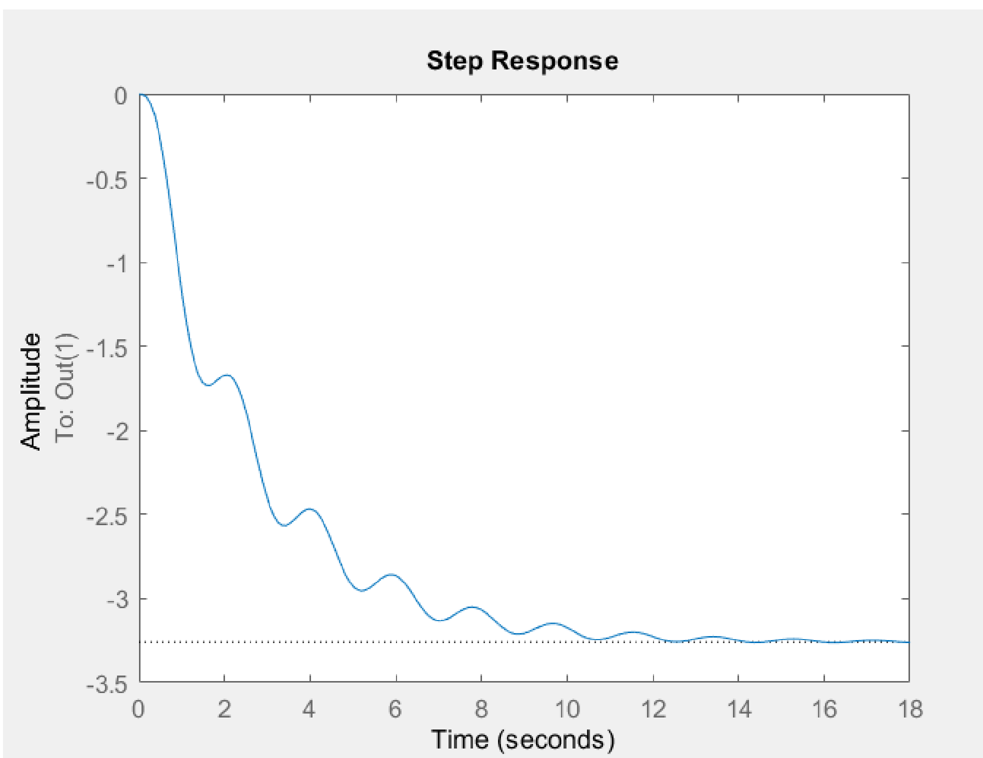
$$x(t) = V\, e^{\Lambda t}\, V^{-1}x_\circ(t)$$

$$y(t) = Cx(t)$$

ZERO INPUT RESPONSE



UNIT STEP RESPONSE

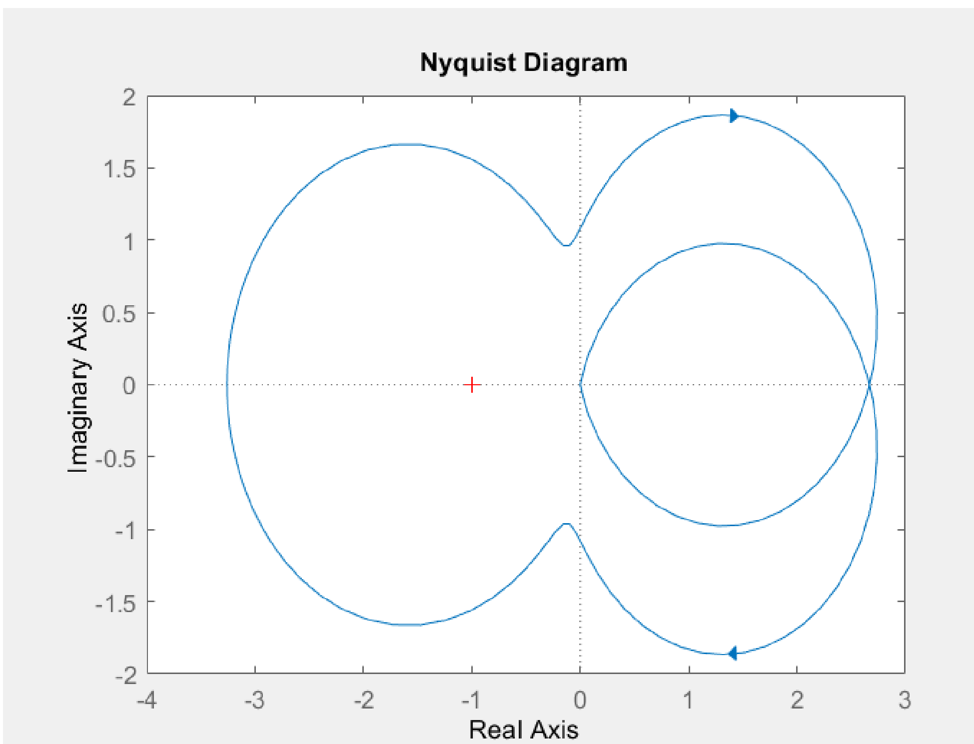


6 Stability (TASK 3)

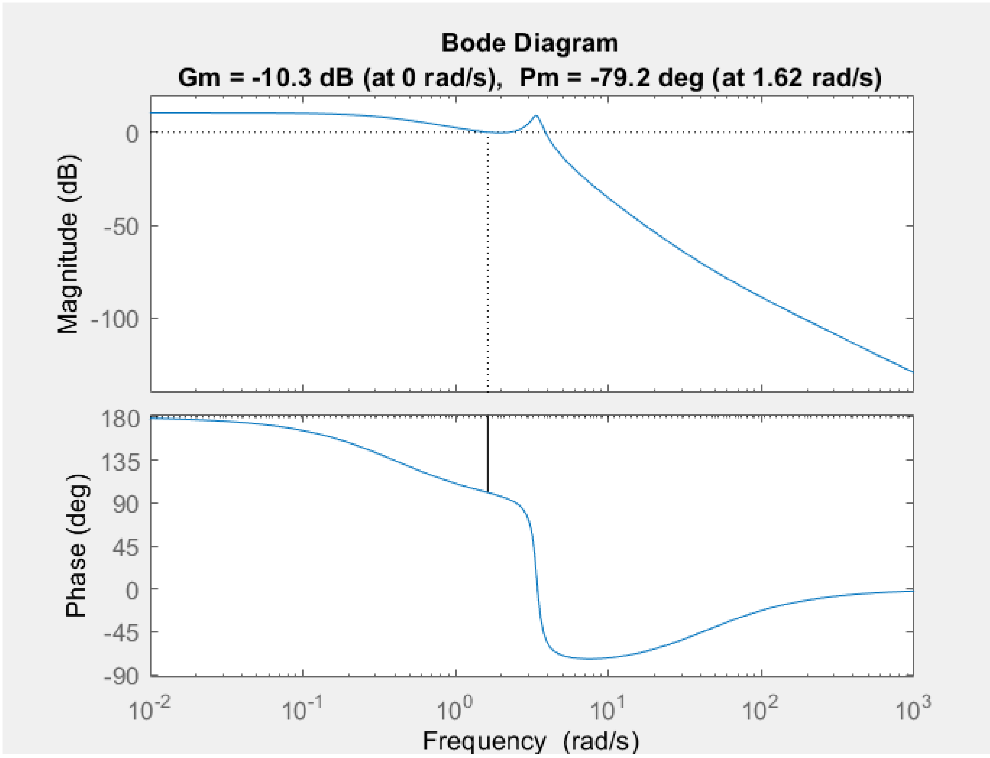
Since all the eigenvalues are in LHP. So the system is asymptotically stable for v=3.5 m/s.And BIBO stable as bounded input(step response) is giving bounded output as could be seen from the plot.

7 Plots (TASK 4)

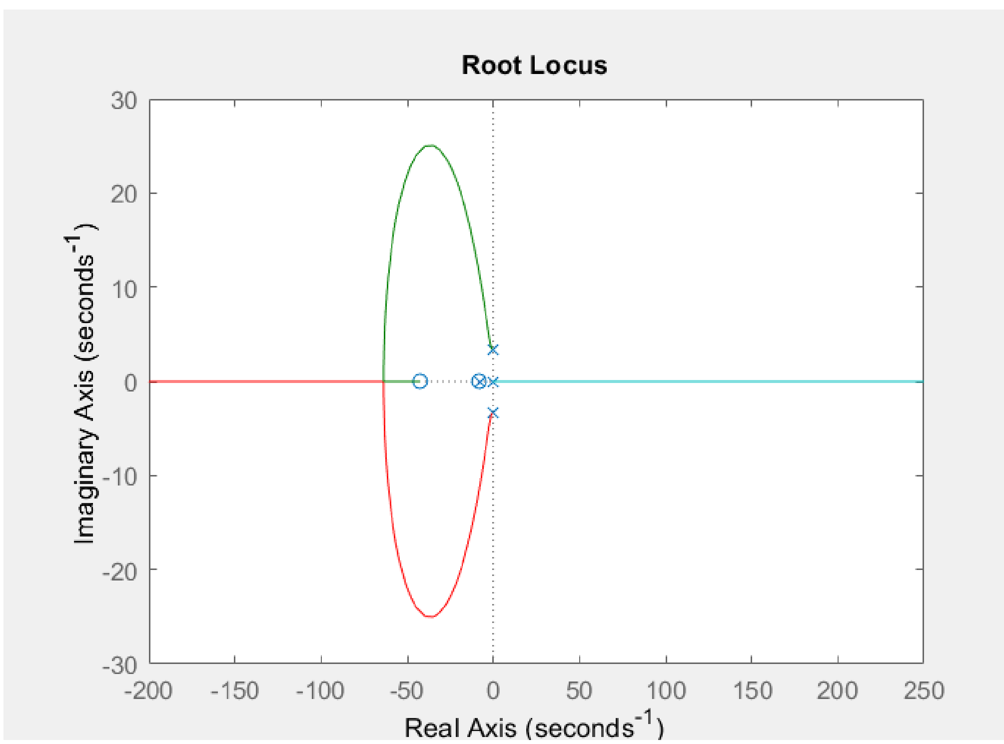
Nyquist Plot



Bode Plot



Root locus



From the Root locus,we can conclude that the system is stable for v=3.5m/s due to the presence of zeroes and poles in LHP.
From the BODE plot, we can conclude that the system has gain of -10.3 at gain crossover frequency of 0rad/s and phase margin of -79.2degrees at phase crossover frequency of 1.62 rad/s.

Case III- v(t)=5m/s

STATE SPACE REPRESENTATION

$$\begin{bmatrix}\dot{\phi}(t)\\ \dot{\delta}(t)\\ \ddot{\phi}(t)\\ \ddot{\delta}(t)\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 13.67 & -32.75 & -0.82 & -2.760\\ 4.857 & -17.315 & 18.105 & -11.940\end{bmatrix}\begin{bmatrix}\phi(t)\\ \delta(t)\\ \dot{\phi}(t)\\ \dot{\delta}(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -0.339\\ 7.457\end{bmatrix}T_\delta(t)$$

$$y(t) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi(t)\\ \delta(t)\\ \dot{\phi}(t)\\ \dot{\delta}(t)\end{bmatrix}$$

TRANSFER FUNCTION REPRESENTATION

$$\frac{0.339s^2 - 24.63s - 250.1}{s^4 + 12.76s^3 + 63.41s^2 + 457.3s - 77.63}$$

ZEROES

-60.4476,-12.2043

POLES

-10.8598 ,-1.0330 + 6.4842i , -1.0330 - 6.4842i , 0.1658

EIGENVALUES

-10.8598 ,-1.0330 + 6.4842i , -1.0330 - 6.4842i , 0.1658

5 TIME RESPONSE(TASK 2)

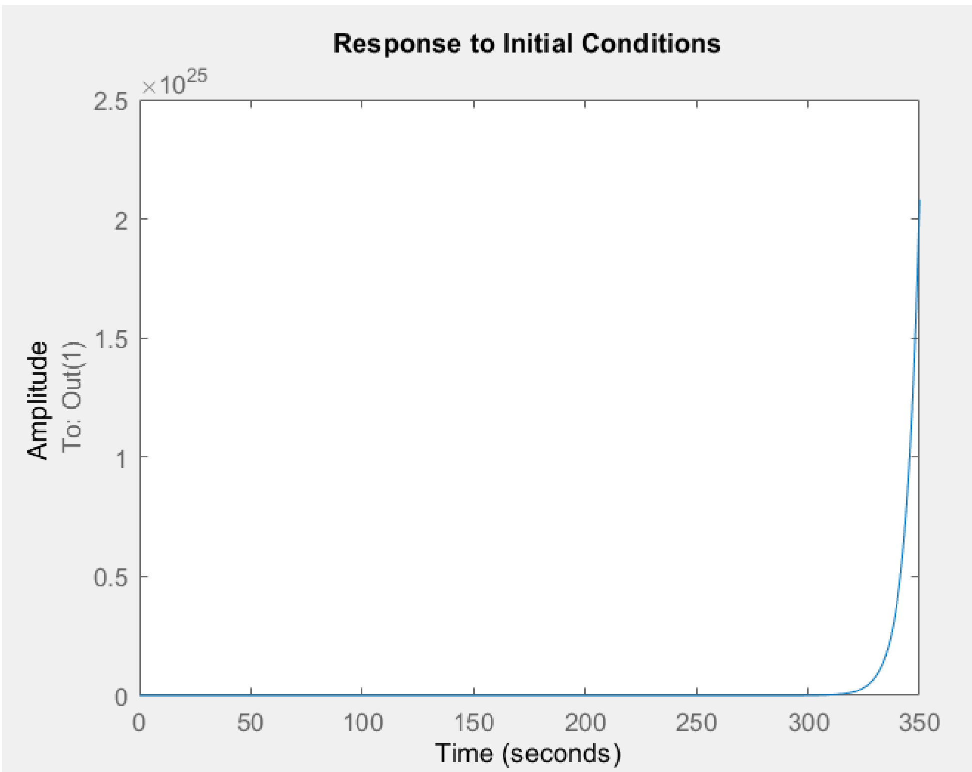
For zero input , with initial state

$$x_\circ = \begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix}$$

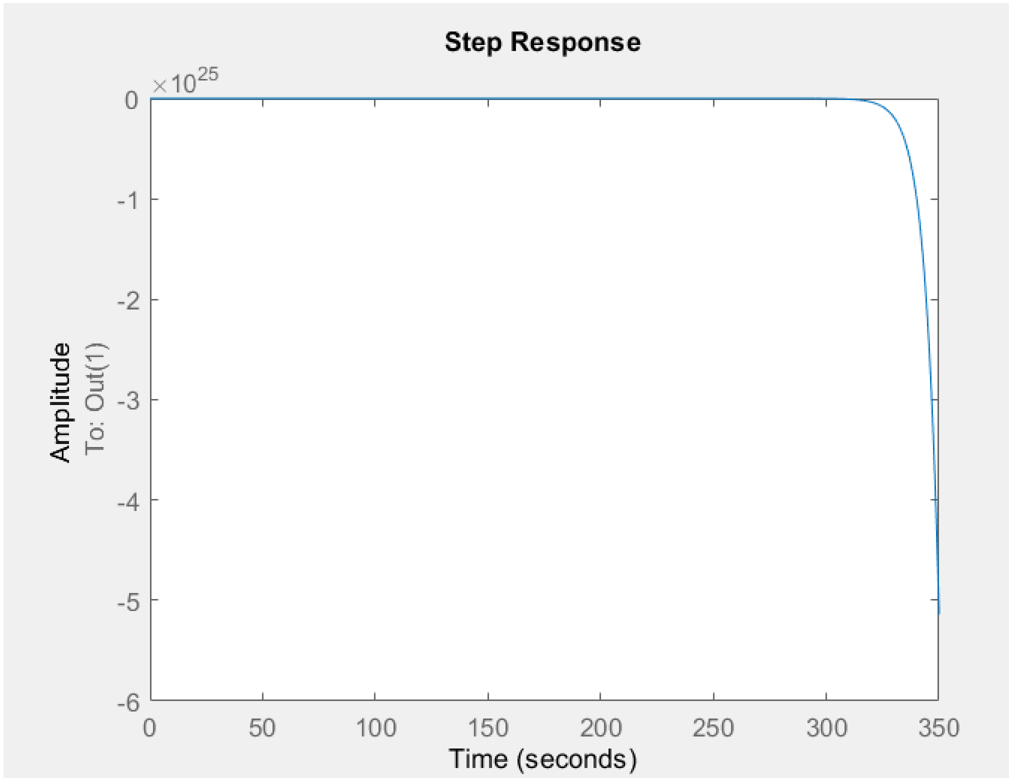
Time response of the system:

$$x(t) = e^{At}x_{\circ}(t)$$
$$x(t) = V\,e^{\Lambda t}\,V^{-1}x_{\circ}(t)$$
$$y(t) = Cx(t)$$

ZERO INPUT RESPONSE



UNIT STEP RESPONSE

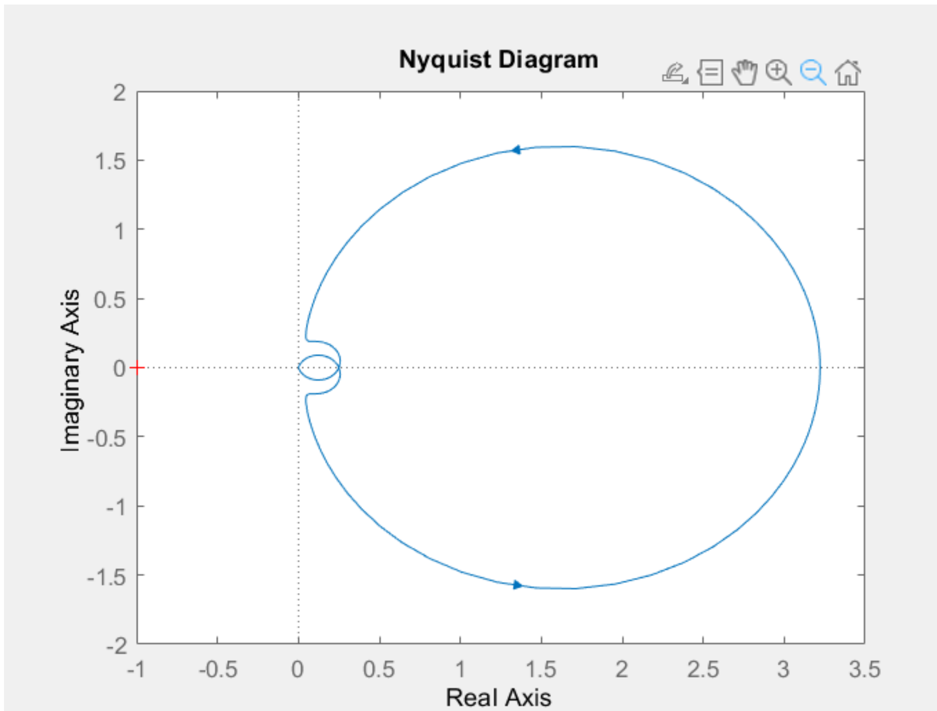


6 Stability (TASK 3)

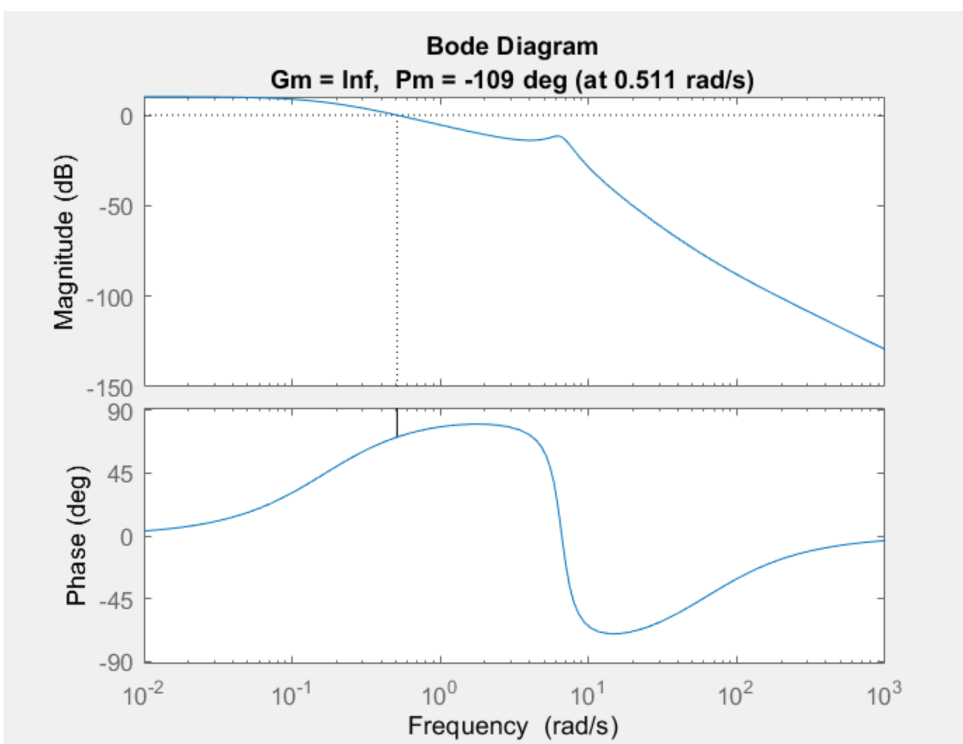
Since the eigenvalues are also in RHP. So the system is unstable for v=5m/s. And BIBO unstable as bounded input(step response) is giving unbounded output as could be seen from the plot.

7 Plots (TASK 4)

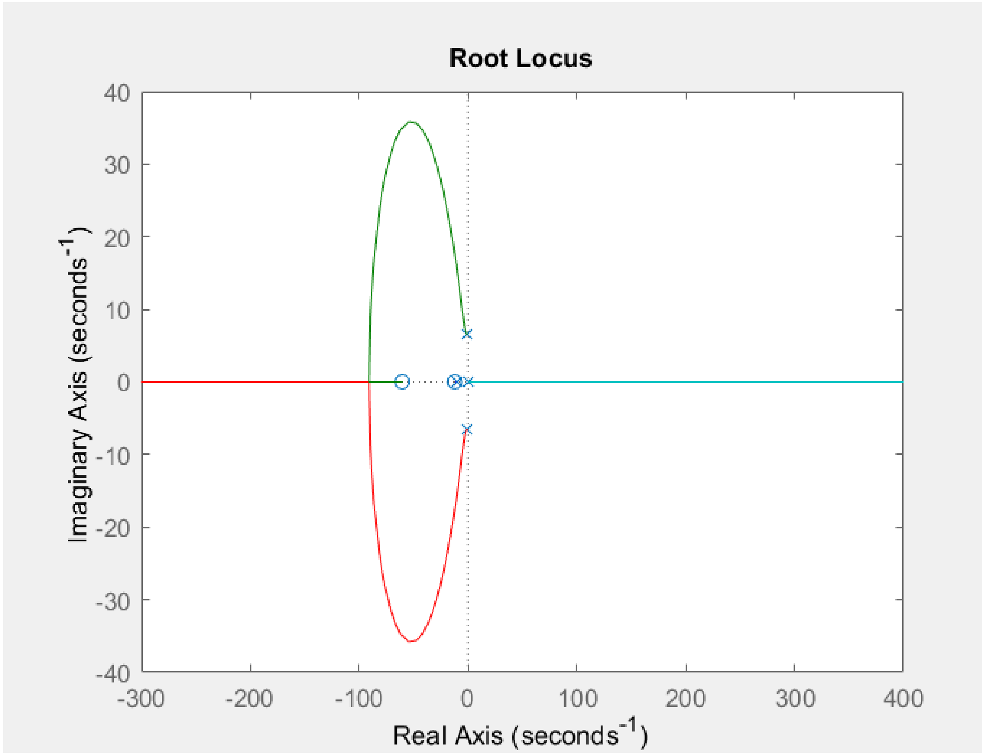
Nyquist Plot



BODE PLOT



Root Locus



From the Root locus,we can conclude that the system is unstable for v=5m/s due to the presence of poles in RHP.
From the BODE plot, we can conclude that the system has inf gain margin and phase margin of -109degrees at phase crossover frequency of 0.511 rad/s.

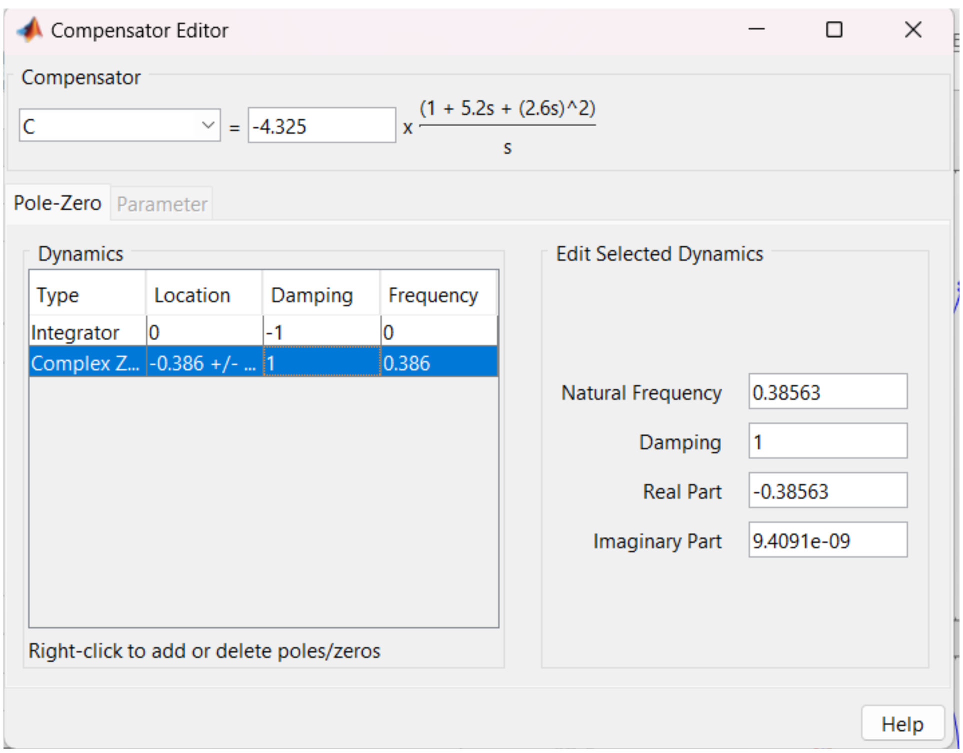
8 Controller Design(Task 5)

Case1 v=0m/s

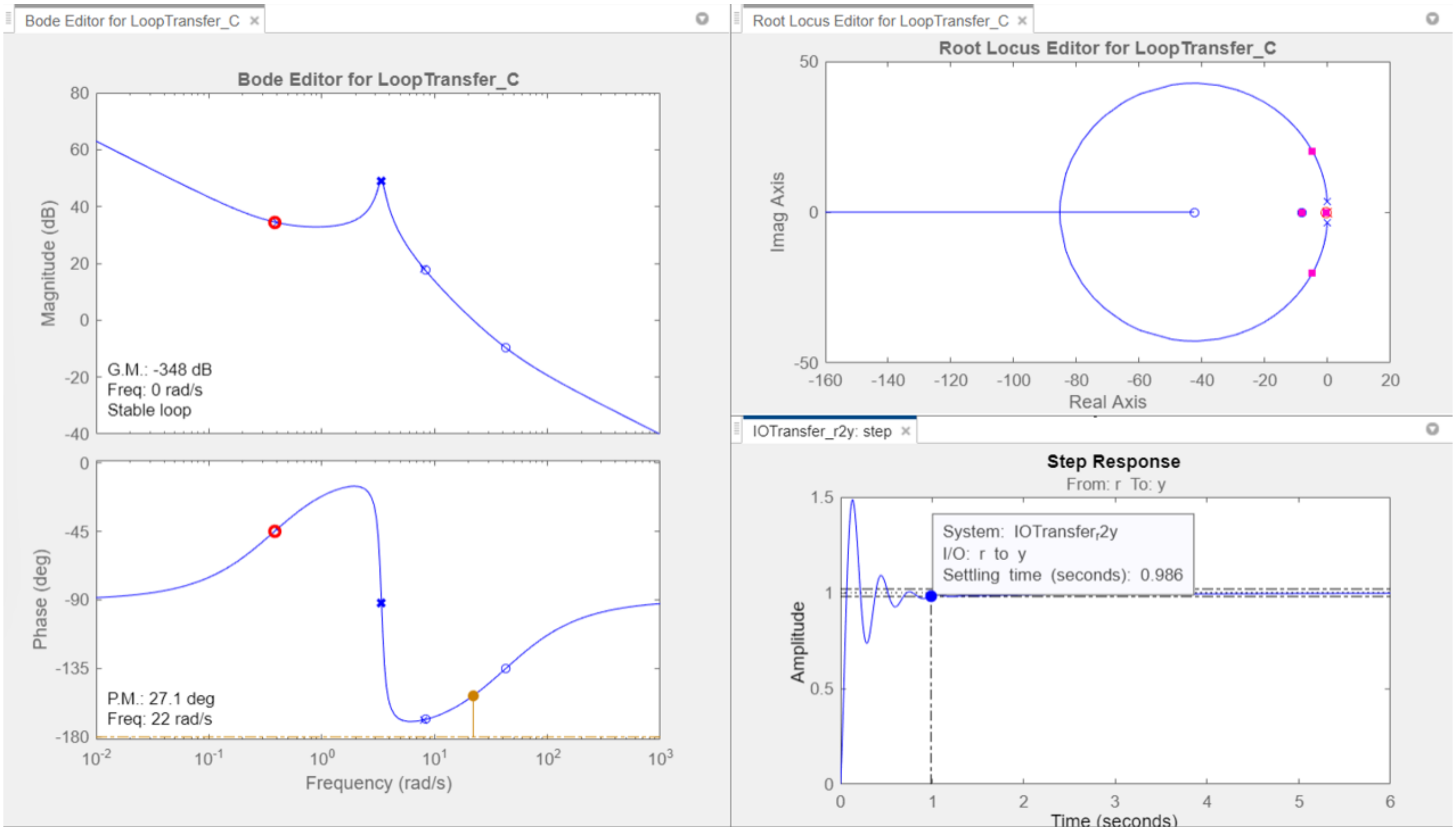
In order to stabilize the system with 0 velocity we have to do pole cancellations. However, practically it is not a good and reliable method. Because in reality if the zeroes added doesn't cancel the RHP pole then, some part of root locus remains in RHP which leads to unstability. Thus, we can't design any efficient PID Controller or Lag-Lead Controller to minimize steady-state error and achieve the required transient conditions.

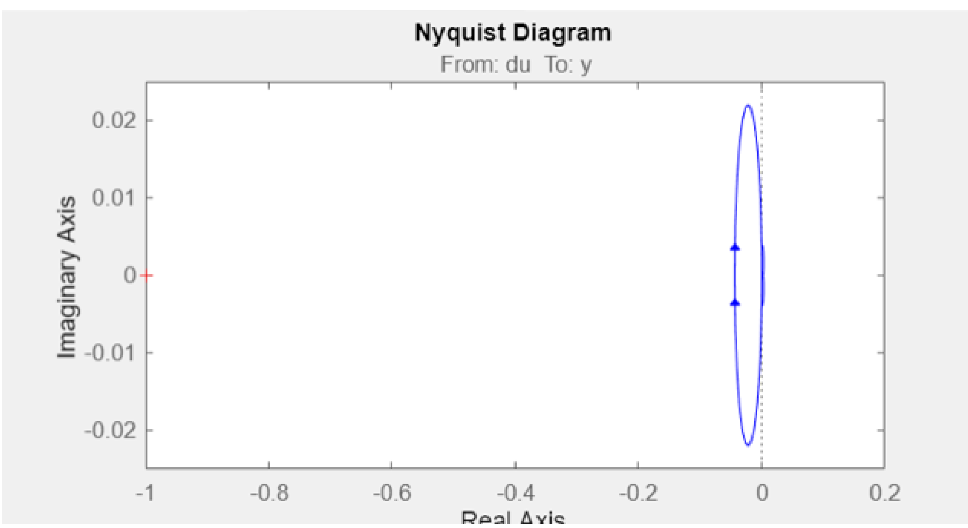
Case2 v=3.5m/s

Design Specifications- Damping Ratio ≥ 0.6   Settling T ime ≤ 1s
We designed a PID controller of the form (k1+k2s+k3/s) with the compensator function C(s) as shown in below dialogue box.
k1=-22.49 k2= -29.237 k3= -4.325



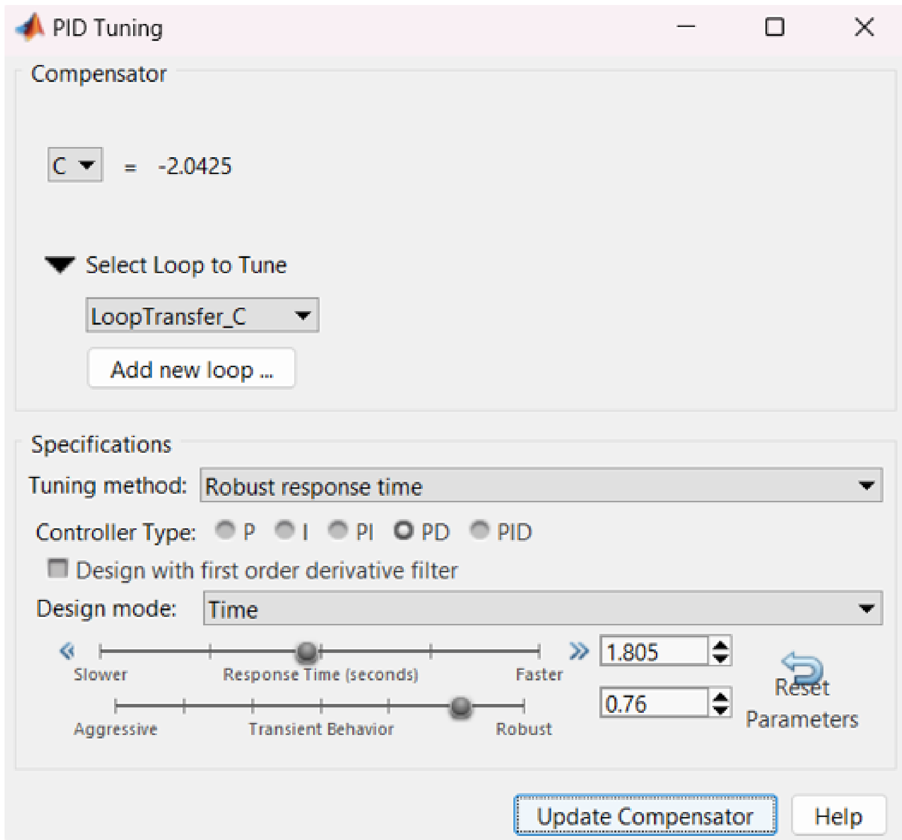
Updated Plots



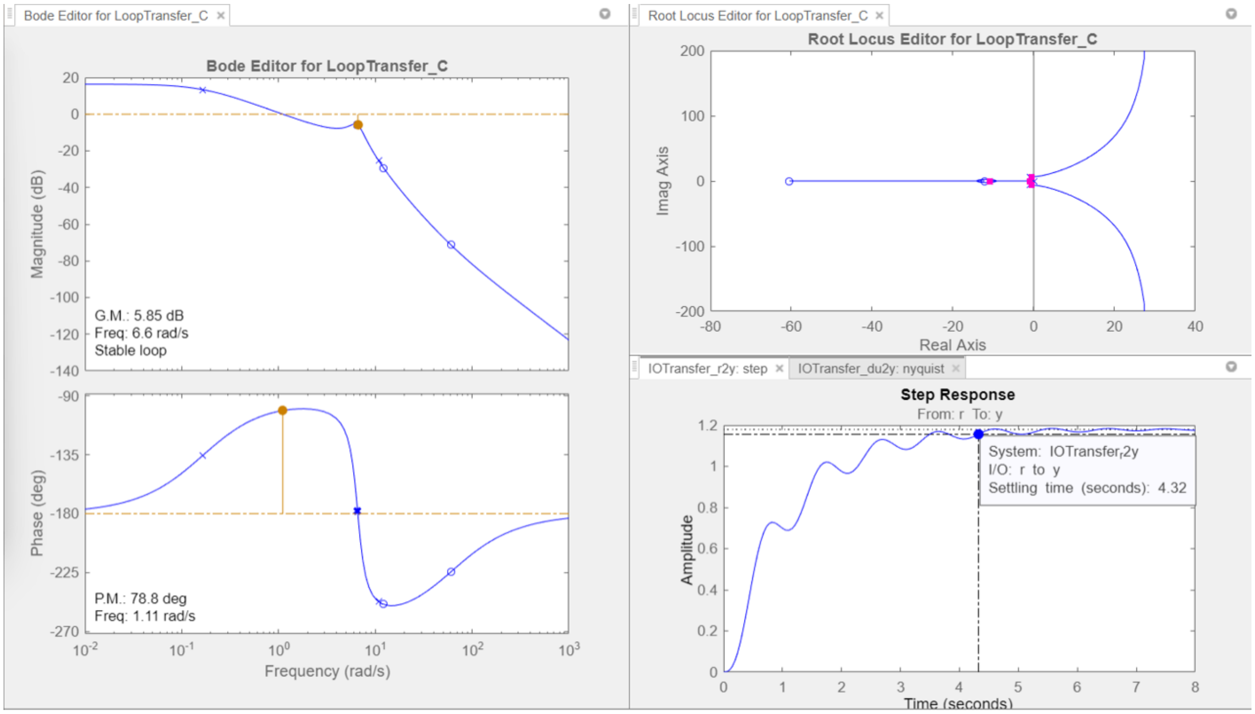


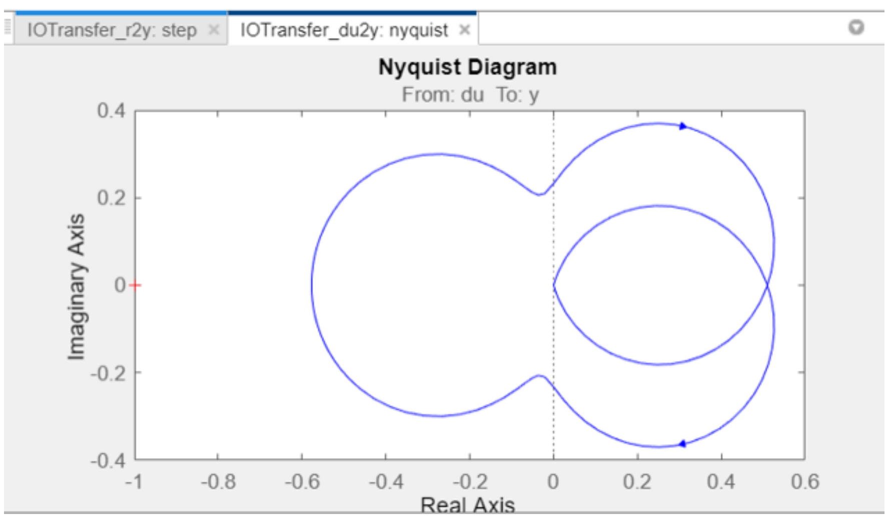
Case3 v=5m/s

We took the specifications as damping ratio >0.6 and settling time<4.5s and designed a Proportional controller.



UPDATED PLOTS





9 MATLAB CODE

```
syms v;
v=5;
a=[0 0 1 0;0 0 0 1;13.67 0.225-1.319*v^2 -0.164*v -0.552*v;4.857
10.81-1.125*v^2 3.621*v -2.388*v];
b=[0;0;-0.339;7.457];
c=[1 0 0 0;0 0 0 0;0 0 0 0;0 0 0 0];
d=[0;0;0;0];
[z,p,k]=ss2zp(a,b,c,d,1)
sys=ss(a,b,c,d);
s=tf(sys);
y=s(1)
margin(y)
rlocus(y);
nyquistplot(y)
```

10 CONCLUSION

- The state space representation of the bicycle model is obtained for the three cases.
- Using MATLAB, the transfer function representation for velocities v1 = 0 m/s, v2 = 3.5 m/s and v3 = 5 m/s were obtained, along with poles, zeroes and eigenvalues for each case.
- Time response was computed for zero input with (any) non-zero initial states and unit step input.
- Using MATLAB, Nyquist plots, Bode plots and Root Locus were drawn for each case. Nyquist plots and Root Locus are for closed loop systems while Bode plots are for open loop systems. Such plots help in commenting on the stability of the system.
- The sisotool in MATLAB was used to design controllers for each case considering unity negative feedback. It was concluded that designing a controller for v1 = 0m/s case is not possible if epole cancellations are excluded.