NON-LINEAR EQUATION OPEN METHOD

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Non-linear equations

Open Method

- Fixed Point
- Newton Rhapson
- Secant

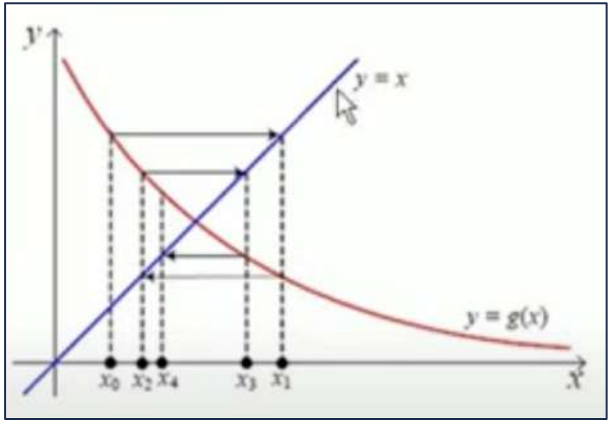
Closed Method

- Table
- Bisection
- Regula Falsi

1. Fixed Point Method

The Root and **Fixed Point** of a function f are two different things.
The root of a function is the intersection of the function f with the x-axis
The fixed point of a function is the intersection of the function f with the line y = x

When we use a fixed-point iteration to find the root of a function f, then the root is the fixed point of another function g, not the function f.



Arrange the equation $f(x) = 0$ into the form $x = g(x)$, and then form it into the following iteration process: $\mathbf{X_{r+1} = g(X_r)}$

Example of Fixed Point Iteration

Determine the approximate root in the 3rd iteration for the equation $x^2 - 2x - 6 = 0$ using the fixed-point iteration method, then determine the approximate relative error!

There are 3 Alternative Ways to solve the problem, namely:

Cara 1

$$x^2 - 2x - 6 = 0$$
$$2x = x^2 - 6$$
$$x = \frac{x^2 - 6}{2}$$
$$\boldsymbol{x_{r+1} = \frac{x_r^2 - 6}{2}}$$

Cara 2

$$x^2 - 2x - 6 = 0$$
$$x^2 = 2x + 6$$
$$x = \sqrt{2x + 6}$$
$$\boldsymbol{x_{r+1} = \sqrt{2x_r + 6}}$$

Cara 3

$$x^2 - 2x - 6 = 0$$
$$x(x - 2) = 6$$
$$x = \frac{6}{x - 2}$$
$$\boldsymbol{x_{r+1} = \frac{6}{x_r - 2}}$$

Example of Fixed Point Iteration

Determine the approximate root in the 3rd iteration for the equation $x^2 - 2x - 6 = 0$ using the fixed-point iteration method, then determine the approximate relative error!

<u>**Solution in Way 2:**</u>

1. Define iteration procedures $x_{r+1} = g(x_r)$

$x^2 - 2x - 6 = 0$

$x^2 = 2x + 6$

$x_{r+1} = \sqrt{2xr + 6}$

2. Iteration Process

Iterasi ke-1 $x_1 = \sqrt{2x_0 + 6} = \sqrt{2(0) + 6} = \sqrt{6} = 2{,}4495$

Iterasi ke-2 $x_2 = \sqrt{2x_1 + 6} = \sqrt{2(2{,}4495) + 6} = \sqrt{10{,}8990} = 3{,}3014$

Iterasi ke-3 $x_3 = \sqrt{2x_2 + 6} = \sqrt{2(3{,}3014) + 6} = \sqrt{12{,}6027} = 3{,}5500$

3. Conclusion: Since the 2nd and 3rd iterations are 3.3014 and 3.5500, the approximate relative error in the 3rd iteration is

$$\varepsilon_{RA} = \left|\frac{3{,}5500 - 3{,}3014}{3{,}5500}\right| = 0{,}0700 = 7\%$$

Fixed Point Simulation in Excel

Iterasi ke-i	xr+1 = (xr^2 - 6)/2		xr+1 = sqrt(2*xr+6)		xr+1 = 6/(xr-2)	
	xi	Galat	xi	Galat	xi	Galat
0	0		0		0	
1	-3	1	2,44949	1	-3	1
2	1,5	3	3,30136	0,258036	-1,2	1,5
3	-1,875	1,8	3,550031	0,070048	-1,875	0,36
4	-1,24219	0,509434	3,619401	0,019166	-1,54839	0,210938
5	-2,22849	0,442587	3,638516	0,005254	-1,69091	0,084287
6	-0,51693	3,311024	3,643766	0,001441	-1,62562	0,040165
7	-2,86639	0,819659	3,645207	0,000395	-1,65489	0,01769
8	1,108105	3,586752	3,645602	0,000108	-1,64164	0,008075
9	-2,38605	1,464409	3,64571	2,97E-05	-1,64761	0,003627
10	-0,15338	14,5566	3,64574	8,15E-06	-1,64491	0,001641
11	-2,98824	0,948672	3,645748	2,24E-06	-1,64613	0,00074
12	1,464782	3,040057	3,64575	6,14E-07	-1,64558	0,000334
13	-1,92721	1,760054	3,645751	1,68E-07	-1,64583	0,000151
14	-1,14294	0,68619	3,645751	4,62E-08	-1,64572	6,81E-05
15	-2,34685	0,512991	3,645751	1,27E-08	-1,64577	3,07E-05

2. Newton Rhapson Method

For example, given an initial guess, namely x_0

Function tangent at a point x_0 has a slope $f'(x_0)$

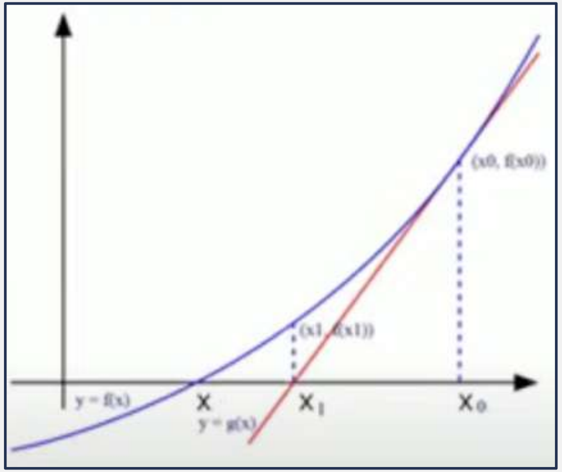
The tangent equation is: $f(x) - f(x_0) = f'(x_0)(x - x_0)$

Approximate root is obtained when the tangent crosses the x-axis, thus obtaining:

$$\frac{0 - f(x_0)}{f'(x_0)} + x_0 = x \longrightarrow x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Prosedur Iterasi

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}, \quad dimana\ f'(x_r) \neq 0$$



Newton Rhapson's Example

Determine the approximate root in the 3rd iteration for the equation $x^2 - 2x - 6 = 0$ using the Netwon Rhapson method, then determine the approximate relative error!

Solution:

1. Defining the first derivative of a function

$f(x) = x^2 - 2x - 6$

$f'(x) = 2x - 2$

2. Specifies the initial guess, e.g. x0 = 0

3. Iteration Process

1st iteration

$f(0) = 0^2 - 2(0) - 6 = -6$

$f'(0) = 2(0) - 2 = -2$

$x_1 = 0 - (-6/-2) = -3$

2nd iteration

$f(-3) = (-3)^2 - 2(-3) - 6 = 9$

$f'(-3) = 2(-3) - 2 = -8$

$x_2 = -3 - (9/-8) = -1{,}875$

3rd iteration

$f(-1{,}875) = (-1{,}875)^2 - 2(-1{,}875) - 6 = 1{,}2656$

$f'(-1{,}875) = 2(-1{,}875) - 2 = -5{,}75$

$x_3 = (-1{,}875) - (1{,}2656/-5{,}75) = -1{,}6549$

Conclusion:

The approximate root in the 3rd iteration is -1.6549. Since the approximate roots in the 2nd and 3rd iterations are -1.875 and -1.6549, the approximate relative error is

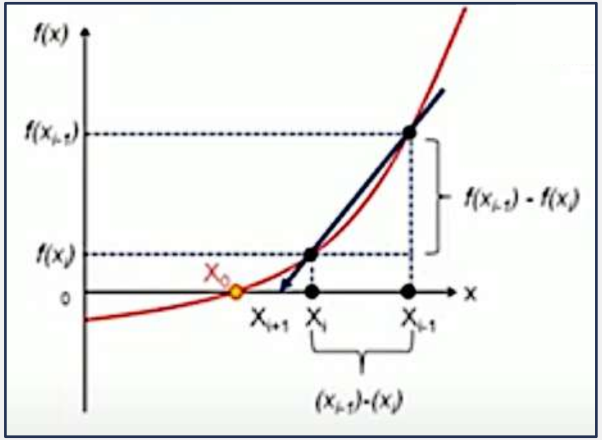
$$\varepsilon_{RA} = \left|\frac{-1{,}6549 - (-1{,}875)}{-1{,}6549}\right| = 0{,}1330 = 13{,}30\%$$

Newton Rhapson Simulation in Excel

Iterasi ke-i	xi	f(xi)	f'(xi)	Galat
0	0	-6	-2	
1	-3	9	-8	1
2	-1,875	1,265625	-5,75	0,6
3	-1,65489	0,048448	-5,30978	0,133005
4	-1,64577	8,33E-05	-5,29153	0,005544
5	-1,64575	2,48E-10	-5,2915	9,56E-06
6	-1,64575	0	-5,2915	2,84E-11
7	-1,64575	0	-5,2915	0
8	-1,64575	0	-5,2915	0
9	-1,64575	0	-5,2915	0
10	-1,64575	0	-5,2915	0
11	-1,64575	0	-5,2915	0
12	-1,64575	0	-5,2915	0
13	-1,64575	0	-5,2915	0
14	-1,64575	0	-5,2915	0
15	-1,64575	0	-5,2915	0

In the 6th iteration the true root is found because the value of $f(x) = 0$

3. Secant Method



In the secant method, two initial guess values are required, for example **x_0 dan x_1**
Points will be earned(x_0, f(x_0) dan (x_1, f(x_1)))

The approximate root is obtained from the intersection of lines passing through the (x_0, f(x_0) dan (x_1, f(x_1))) with x-axis

Note!
Line equations $\frac{y - f(x_0)}{f(x_1) - f(x_0)} = \frac{x - x_0}{x_1 - x_0}$

Cutting the x-axis: $(y = 0)$ $\frac{0 - f(x_0)}{f(x_1) - f(x_0)} = \frac{x - x_0}{x_1 - x_0} \rightarrow x = x_0 - \frac{f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)}$

Iteration Procedure:

$$x_{r+1} = x_r - \frac{f(x_r)(x_r - x_{r-1})}{f(x_r) - f(x_{r-1})}$$

Example of the Secant Method

Determine the approximate root in the 3rd iteration for the equation $x^2 - 2x - 6 = 0$ using the secant method, then determine the approximate relative error!

Solution:

1. Determining two initial guesses, e.g. $x_{-1} = 0$ dan $x_0 = 1$

2.

Proses Iterasi

Iterasi ke-1

$$f(x_{-1}) = f(0) = 0^2 - 2(0) - 6 = -6$$

$$f(x_0) = f(1) = 1^2 - 2(1) - 6 = -7$$

$$x_1 = 1 - \frac{(-7)(1-0)}{(-7)-(-6)} = 1 - 7 = -6$$

Iterasi ke-2

$$f(x_1) = f(-6) = (-6)^2 - 2(-6) - 6 = 42$$

$$x_2 = (-6) - \frac{(42)(-6-1)}{(42)-(-7)} = (-6) - (-6) = 0$$

Iterasi ke-3

$$f(x_2) = f(0) = (0)^2 - 2(0) - 6 = -6$$

$$x_3 = 0 - \frac{(-6)(0-(-6))}{(-6)-(42)} = 0 - 0{,}75 = -0{,}75$$

The approximate root in the 3rd iteration is -0.75

Conclusion: Since the approximate roots in the 2nd and 3rd iterations are 0 and -0.75, the approximate relative error is

$$\varepsilon_{RA} = \left|\frac{-0{,}75 - 0}{-0{,}75}\right| = 1 = 100\%$$

Simulation of the Secant Method in Excel

Iterasi ke-i	xi	f(xi)	Galat
-1	0	-6	
0	1	-7	
1	-6	42	1,166667
2	0	-6	#DIV/0!
3	-0,75	-3,9375	1
4	-2,18182	3,123967	0,65625
5	-1,54839	-0,50572	0,409091
6	-1,63664	-0,04811	0,053925
7	-1,64592	0,000905	0,005638
8	-1,64575	-1,6E-06	0,000104
9	-1,64575	-5E-11	1,79E-07
10	-1,64575	0	5,79E-12
11	-1,64575	0	0

Post Test

1. Find the root of the equation from $f(x) = 7x^2 - 21x + e^x$ using the **fixed-point, Newton Raphson, and Secant iteration methods** with $x_0 = 0.3$ dan $x_1 = 0.5$.
2. Find the root of the equation from $f(x) = 3x^3 - 2x - e^x$ using the **fixed-point, Newton Raphson, and Secant iteration methods** with $x_0 = 0.1$ dan $x_1 = 0.7$.