221310332 周立成的人工智能作业 3

1. 设有3个独立的结论H_1,H_2,H_3,以及两个独立的证据E_1,E_2,他们的先验概率和条件概率分别为:

$$P(H_1)=0.4 \qquad P(H_2)=0.3 \qquad P(H_3)=0.3$$
$$P(E_1|H_1)=0.5 \qquad P(E_1|H_2)=0.3 \qquad P(E_1|H_3)=0.5$$
$$P(E_2|H_1)=0.7 \qquad P(E_2|H_2)=0.9 \qquad P(E_2|H_3)=0.1$$

试利用概率方法分别求出:

(1):已知证据E_1出现时$P(H_1|E_1),P(H_2|E_1),P(H_3,E_1)$的概率值;说明$E_1$的出现对结论$H_1,H_2,H_3$的影响;

解:

对于$P(H_i|E_1)$,根据$\boxed{\text{贝叶斯公式}}$:$\boxed{P(H_i|E_1)\times P(E_1)=P(E_1|H_i)\times P(H_i)}$

于是会有:$\boxed{P(H_i|E_1)=\dfrac{P(E_1|H_i)\times P(H_i)}{P(E_1)}}$

题目中已经给出$P(E_1|H_i)$与$P(H_i)$了,下面来求解$P(E_1)$

根据$\boxed{\text{全概率公式}}$,应该有$\boxed{P(E_1)=\sum_{i=1}^{3}P(E_1|H_i)\times P(H_i)}$

带入会有:$\boxed{P(H_i|E_1)=\dfrac{P(E_1|H_i)\times P(H_i)}{\sum_{i=1}^{3}P(E_1|H_i)\times P(H_i)}}$

带入数值:
$$\begin{cases} P(H_1|E_1)=\dfrac{P(E_1|H_1)\times P(H_1)}{P(E_1|H_1)\times P(H_1)+P(E_1|H_2)\times P(H_2)+P(E_1|H_3)\times P(H_3)} \\ \qquad =\dfrac{0.5\times 0.4}{0.5\times 0.4+0.3\times 0.3+0.5\times 0.3}=\dfrac{5}{11}=0.454545 \\ P(H_2|E_1)=\dfrac{P(E_1|H_2)\times P(H_2)}{P(E_1|H_1)\times P(H_1)+P(E_1|H_2)\times P(H_2)+P(E_1|H_3)\times P(H_3)} \\ \qquad =\dfrac{0.3\times 0.3}{0.5\times 0.4+0.3\times 0.3+0.5\times 0.3}=\dfrac{9}{44}=0.204545 \\ P(H_3|E_1)=\dfrac{P(E_1|H_3)\times P(H_3)}{P(E_1|H_1)\times P(H_1)+P(E_1|H_2)\times P(H_2)+P(E_1|H_3)\times P(H_3)} \\ \qquad =\dfrac{0.5\times 0.3}{0.5\times 0.4+0.3\times 0.3+0.5\times 0.3}=\dfrac{15}{44}=0.340909 \end{cases}$$

观察结果,可知E_1对结论H_1的影响较大,对H_3的影响其次,对H_2的影响较小

(2):已知E_1,E_2同时出现时$P(H_1|E_1E_2),P(H_2|E_1E_2),P(H_3|E_1E_2)$的概率值,试说明$E_1$ E_2同时出现出现对结论H_1,H_2,H_3的影响;

解:

对于$P(H_i|E_1E_2)$,有贝叶斯公式:$\boxed{P(H_i|E_1E_2)\times P(E_1E_2)=P(E_1E_2|H_i)P(H_i)}$

于是有:$\boxed{P(H_i|E_1E_2)=\dfrac{P(E_1E_2|H_i)P(H_i)}{P(E_1E_2)}}$

其中有:$\boxed{P(E_1E_2|H_i)=P(E_1|H_i)\times P(E_2|H_i)}$

以及有:$\boxed{P(E_1E_2)=P(E_1E_2|H_i)P(H_i)+P(E_1E_2|\neg H_i)P(\neg H_i)=\sum_{i=1}^{3}(P(E_1|H_i)\times P(E_2|H_i)\times P(H_i))}$

带入有:$\boxed{P(H_i|E_1E_2)=\dfrac{P(E_1|H_i)\times P(E_2|H_i)\times P(H_i)}{\sum_{i=1}^{3}(P(E_1|H_i)\times P(E_2|H_i)\times P(H_i))}}$

带入数据,有:

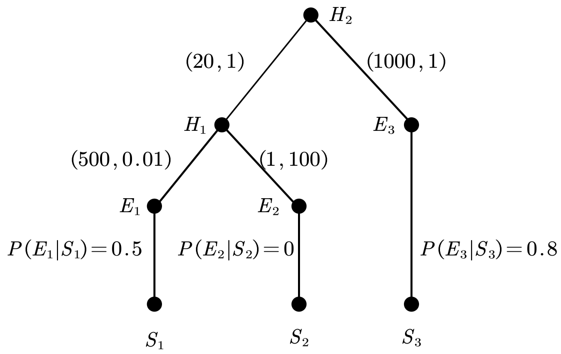
$$
\begin{cases}
P(H_1|E_1E_2)=\dfrac{P(E_1|H_1)P(E_2|H_1)P(H_1)}{P(E_1|H_1)P(E_2|H_1)P(H_1)+P(E_1|H_2)P(E_2|H_2)P(H_2)+P(E_1|H_3)P(E_2|H_3)P(H_3)} \\
\qquad =\dfrac{0.5\times0.7\times0.4}{0.5\times0.7\times0.4+0.3\times0.9\times0.3+0.5\times0.1\times0.3}=\dfrac{35}{59}=0.59322 \\
P(H_2|E_1E_2)=\dfrac{P(E_1|H_2)P(E_2|H_2)P(H_2)}{P(E_1|H_1)P(E_2|H_1)P(H_1)+P(E_1|H_2)P(E_2|H_2)P(H_2)+P(E_1|H_3)P(E_2|H_3)P(H_3)} \\
\qquad =\dfrac{0.3\times0.9\times0.3}{0.5\times0.7\times0.4+0.3\times0.9\times0.3+0.5\times0.1\times0.3}=\dfrac{81}{236}=0.34322 \\
P(H_2|E_1E_2)=\dfrac{P(E_1|H_2)P(E_2|H_2)P(H_2)}{P(E_1|H_1)P(E_2|H_1)P(H_1)+P(E_1|H_2)P(E_2|H_2)P(H_2)+P(E_1|H_3)P(E_2|H_3)P(H_3)} \\
\qquad =\dfrac{0.5\times0.1\times0.3}{0.5\times0.7\times0.4+0.3\times0.9\times0.3+0.5\times0.1\times0.3}=\dfrac{15}{236}=0.0635593
\end{cases}
$$

可见E_1E_2同时出现会使得H_1,H_2成立的概率提升,使得E_3成立的概率大幅下降

2. 设有如下推理规则:

$$
\begin{array}{llllll}
R_1: & \textbf{IF} & E_1 & \textbf{THEN} & (500,0.01) & H_1 \\
R_2: & \textbf{IF} & E_2 & \textbf{THEN} & (1,100) & H_1 \\
R_3: & \textbf{IF} & E_3 & \textbf{THEN} & (1000,1) & H_2 \\
R_4: & \textbf{IF} & H_1 & \textbf{THEN} & (20,1) & H_2
\end{array}
$$

且已知$P(H_1)=0.1,P(H_2)=0.1,P(H_3)=0.1$,初始证据的概率为$P(E_1|S_1)=0.5,P(E_2|S_2)=0,P(E_3|S_3)=0.8$。用主观贝叶斯方法求$H_2$的后验概率$P(H_2|S_1,S_2,S_3)$。



解:

先推导一个数学结论,这在利用LS,LN计算时会方便些☺

方程组:$\begin{cases}\dfrac{x}{y}=a\\ \dfrac{1-x}{1-y}=b\end{cases}\rightarrow x=ay\rightarrow\dfrac{1-ay}{1-y}=b\rightarrow 1-ay=b-by\rightarrow(b-a)y=b-1\rightarrow\boxed{y=\dfrac{b-1}{b-a}\rightarrow x=a\dfrac{b-1}{b-a}}$

做出如图所示的推理网络,便可采用自下向上的方式算出$\boxed{O(H_2|S_1,S_2,S_3)}$,从而得到$\boxed{P(H_2|S_1S_2S_3)}$

1.求出$O(H_1|S_1)$,(利用$P(H_1|S_1)$)

求出$P(H_1|S_1)$,在这一条分支上,H_1的证据是E_1,且$P(E_1|S_1)=0.5$可以用EH公式来计算,而EH公式中要用到$P(E_1)$:

1.1.利用LS公式求出$P(E_1)$,$P(H_1|E_1)$等

LS公式:$\boxed{LS=\dfrac{P(E_1|H_1)}{P(E_1|\neg H_1)}}=500$,且又有公式$\boxed{O(H_1|E_1)=LS\times O(H_1)}$,题目中给到了$\boxed{P(H_1)=0.1}$,

不难做出转化:$\boxed{O(H_1)=\dfrac{P(H_1)}{1-P(H_1)}}=\dfrac{0.1}{1-0.1}=\dfrac{1}{9}=0.111111$,

带入有$O(H_1|E_1)=LS\times O(H_1)=500\times\dfrac{1}{9}=55.5556$

再利用一步公式:$\boxed{P(X)=\dfrac{O(X)}{1+O(X)}}$令$x=H_1|E_1$,有$\boxed{P(H_1|E_1)}=\dfrac{O(H_1|E_1)}{1+O(H_1|E_1)}=\dfrac{\frac{500}{9}}{1+\frac{500}{9}}=\boxed{\dfrac{500}{509}}=0.982318$

再根据$\begin{cases}LS_1=\dfrac{P(E_1|H_1)}{P(E_1|\neg H_1)}=500\\ LN_1=\dfrac{1-P(E_1|H_1)}{1-P(E_1\neg H_1)}=0.01\end{cases}$

$\rightarrow\boxed{P(E_1|H_1)}=LS_1\dfrac{LN_1-1}{LN_1-LS_1}=500\times\dfrac{0.01-1}{0.01-500}=\boxed{\dfrac{49500}{49999}}=0.99002$

利用一次贝叶斯公式:$\boxed{P(E_1|H_1)\times P(H_1)=P(H_1|E_1)\times P(E_1)}$

$\rightarrow\boxed{P(E_1)}=\dfrac{P(E_1|H_1)\times P(H_1)}{P(H_1|E_1)}=\dfrac{\frac{49500}{49999}\times 0.1}{\frac{500}{509}}=\boxed{\dfrac{53091}{499990}}=0.100784$

1.2.$\boxed{P(E_1)=0.10<P(E_1|S_1)}$,那么利用$EH$公式后半部分进行计算得到$P(H_1|S_1)$

EH公式:$\boxed{P(H_1|S_1)=P(H_1)+\dfrac{P(H_1|E_1)-P(H_1)}{1-P(E_1)}\times[P(E_1|S_1)-P(E_1)]}[P(E_1)\leqslant P(E_1|S_1)\leqslant 1]$

把$\boxed{\begin{cases}P(H_1)=0.1\\ P(H_1|E_1)=\dfrac{500}{509}\\ P(E_1|S_1)=0.5\\ P(E_1)=\dfrac{53091}{499990}\end{cases}}$带入上式:

$\rightarrow\boxed{P(H_1|S_1)}=0.1+\dfrac{\frac{500}{509}-0.1}{1-\frac{53091}{499990}}\times\left[0.5-\dfrac{53091}{499990}\right]=\boxed{\dfrac{222353491}{454943182}}=0.48875$

1.3.再次利用$O(X)=\dfrac{P(X)}{1-P(X)}$得到$O(H_1|S_1)$

令$x=H_1|S_1$,会有$\boxed{O(H_1|S_1)}=\dfrac{P(H_1|S_1)}{1-P(H_1|S_1)}=\dfrac{\frac{222353491}{454943182}}{1-\frac{222353491}{454943182}}=\boxed{\dfrac{222353491}{232589691}}=0.95599$

2.求出$O(H_1|S_2)$,(利用$P(H_1|S_2)$) [此时H_1的证据是E_2了]

2.1.利用EH公式求出$P(H_1|S_2)$

我们理应使用EH公式,但观察题目条件,发现$P(E_2|S_2)=0$,故可以直接使用Duda的公式计算:

当$P(E_2|S_2)=0$时,有$\begin{cases} P(\neg E_2|S_2)=1 \\ \boxed{P(H_1|S_2)=P(H_1|\neg E_2)} \end{cases}$

2.2.利用$\boxed{\text{修改的贝叶斯公式}}$,求出$O(H_1|\neg E_2)$,进一步得到$P(H_1|\neg E_2)$

根据公式$O(H_1|\neg E_2)=LN_2\times O(H_1)$,其中根据上一问提供的$\boxed{O(H_1)=\dfrac{1}{9}}$,可以得到:

$\boxed{O(H_1|\neg E_2)}=100\times\dfrac{1}{9}=\boxed{\dfrac{100}{9}}=11.1111$

进一步根据公式:$\boxed{P(H_1|\neg E_2)}=\dfrac{O(H_1|\neg E_2)}{1+O(H_1|\neg E_2)}=\dfrac{\frac{100}{9}}{1+\frac{100}{9}}=\boxed{\dfrac{100}{109}}=0.917431$

2.3.得到$O(H_1|S_2)$

那么再次根据$\boxed{P(H_1|S_2)=P(H_1|\neg E_2)}$会有$P(H_1|S_2)=\dfrac{100}{109}$

根据公式$\boxed{O(H_1|S_2)}=\dfrac{P(H_1|S_2)}{1-P(H_1|S_2)}=\dfrac{\frac{100}{109}}{1-\frac{100}{109}}=\boxed{\dfrac{100}{9}}$(好$ba$绕回去了...)

3.计算$O(H_1|S_1S_2)$

根据所有观察下的后验概率公式:$\boxed{O(H_1|S_1S_2)=\dfrac{O(H_1|S_1)}{O(H_1)}\times\dfrac{O(H_1|S_2)}{O(H_1)}\times O(H_1)}$

带入有:$O(H_1|S_1S_2)=\dfrac{\frac{222353491}{232589691}}{\frac{1}{9}}\times\dfrac{\frac{100}{9}}{\frac{1}{9}}\times\dfrac{1}{9}=\boxed{\dfrac{22235349100}{232589691}}=95.599$

4.根据推理网络,下面应该求$\boxed{O(E_3|S_3)}$,利用$P(E_3|S_3)=0.8$

根据公式,有$O(E_3|S_3)=\dfrac{P(E_3|S_3)}{1-P(E_3|S_3)}=\dfrac{0.8}{1-0.8}=4$

5.求出$O(H_2|S_1S_2)$,[需要注意此时H_1是H_2的证据(根据$\boldsymbol{R_3}$)]

5.1.计算$P(H_1|S_1S_2)$

既然证据是H_1,那么根据EH公式,比较$\boxed{P(H_1)}$,与$\boxed{P(H_1|S_1S_2)}$

那么$\boxed{P(H_1|S_1S_2)}=\dfrac{O(H_1|S_1S_2)}{1+O(H_1|S_1S_2)}=\dfrac{\frac{22235349100}{232589691}}{1+\frac{22235349100}{232589691}}=\dfrac{22235349100}{22467938791}=0.989648$

显然的,$P(H_1)<P(H_1|S_1S_2)$,需要根据EH公式后一半来计算

$$\boxed{P(H_2|S_1S_2)=P(H_2)+\frac{P(H_2|H_1)-P(H_2)}{1-P(H_1)}\times[P(H_1|S_1S_2)-P(H_1)]}$$

5.2.计算暂时不知道的$P(H_2|H_1)$

根据修改的贝叶斯公式$\boxed{O(H|E)=LS\times O(H)}$,这里$E$是$H_1$,$H$是$H_2$,带入有:

$\boxed{O(H_2|H_1)=LS_4\times O(H_2)}$,其中$O(H_2)=\frac{P(H_2)}{1-P(H_2)}=\frac{0.1}{1-0.1}=\frac{1}{9}=0.111111$

有$\boxed{O(H_2|H_1)}=20\times\frac{1}{9}=\boxed{\frac{20}{9}}=2.22222$

根据公式$\boxed{P(H_2|H_1)}=\frac{O(H_2|H_1)}{1+O(H_2|H_1)}=\frac{\frac{20}{9}}{1+\frac{20}{9}}=\boxed{\frac{20}{29}}=0.689655$

5.3.数据$\begin{cases}P(H_1|S_1S_1)=\frac{22235349100}{22467938791}=0.989648\\P(H_1)=P(H_2)=0.1\\P(H_2|H_1)=\frac{20}{29}\end{cases}$ 带入EH公式:

$$P(H_2|S_1S_2)=P(H_2)+\frac{P(H_2|H_1)-P(H_2)}{1-P(H_1)}\times[P(H_1|S_1S_2)-P(H_1)]$$

$$\boxed{P(H_2|S_1S_2)}=0.1+\frac{\frac{20}{29}-0.1}{1-0.1}\times\left[\frac{22235349100}{22467938791}-0.1\right]=\boxed{\frac{444939571691}{651570224939}}=0.682873$$

5.4.根据$P(H_2|S_1S_2)$得到$O(H_2|S_1S_2)$

那么容易知道$\boxed{O(H_2|S_1S_2)}=\frac{P(H_2|S_1S_2)}{1-P(H_2|S_1S_2)}=\frac{\frac{444939571691}{651570224939}}{1-\frac{444939571691}{651570224939}}=\boxed{\frac{444939571691}{206630653248}}=2.15331$

6.计算$O(H_2|S_3)$[利用$P(H_2|S_3)$],根据推理网络,这里H_2对应的证据是E_3

6.1肯定还需要利用EH公式,那么一定要先求出$\boxed{P(E_3)}$

可以利用贝叶斯公式$P(E_3|H_2)\times P(H_2)=P(H_2|E_3)\times P(E_3)$求

根据修改的贝叶斯公式:$O(H_2|E_3)=LS_3\times O(H_2)=1000\times\frac{1}{9}=\frac{1000}{9}=111.111$

$$\boxed{P(H_2|E_3)}=\frac{O(H_2|E_3)}{1+O(H_2|E_3)}=\frac{\frac{1000}{9}}{1+\frac{1000}{9}}=\boxed{\frac{1000}{1009}}=0.99108$$

根据LS,LN公式:$\begin{cases}LS_3=\frac{P(E_3|H_2)}{P(E_3|\neg H_2)}=1000\\LN_3=\frac{1-P(E_3|H_2)}{1-P(E_3|\neg H_2)}=1\end{cases}$,有$\boxed{P(E_3|H_2)}=LS_3\times\frac{LN_3-1}{LN_3-LS_3}=\boxed{0}$

那么带入会发现$\boxed{P(E_3)=0}$

6.2.使用EH公式计算

由于$\boxed{P(E_3)=0<P(E_3|S_3)=0.8}$,采用$EH$公式第二部分计算,带入数据$\begin{cases}P(H_2)=0.1\\P(E_3|S_3)=0.8\\P(E_3)=0\\P(H_2|E_3)=\frac{1000}{1009}\end{cases}$

$$P(H_2|S_3)=P(H_2)+\frac{P(H_2|E_3)-P(H_2)}{1-P(E_3)}\times(P(E_3|S_3)-P(E_3))$$

$$\boxed{P(H_2|S_3)}=0.1+\frac{\frac{1000}{1009}-0.1}{1-0}\times(0.8-0)=\boxed{\frac{41009}{50450}}=0.812864$$

6.3.使用公式得到$O(H_2|S_3)$

$$\boxed{O(H_2|S_3)}=\frac{P(H_2|S_3)}{1-P(H_2|S_3)}=\frac{\frac{41009}{50450}}{1-\frac{41009}{50450}}=\boxed{\frac{41009}{9441}}=4.34371$$

7.最终合成出$O(H_2|S_1S_2S_3)$,得到$P(H_2|S_1S_2S_3)$

7.1.得到$O(H_2|S_1S_2S_3)$

根据所有观察下H_2的后验概率:$\boxed{O(H_2|S_1S_2S_3)=\frac{O(H_2|S_1S_2)}{O(H_2)}\times\frac{O(H_2|S_3)}{O(H_2)}\times O(H_2)}$

把数据:$\begin{cases} O(H_2|S_1S_2)=\frac{444939571691}{206630653248}=2.15331 \\ O(H_2)=\frac{1}{9} \\ O(H_2|S_3)=\frac{41009}{9441} \end{cases}$ 带入

$$O(H_2|S_1S_2S_3)=\frac{\frac{444939571691}{206630653248}}{\frac{1}{9}}\times\frac{\frac{41009}{9441}}{\frac{1}{9}}\times\frac{1}{9}=\frac{18246526895476219}{216755555257152}=84.1802$$

7.2.得到$P(H_2|S_1S_2S_3)$

根据公式:$\boxed{P(H_2|S_1S_2S_3)}=\frac{O(H_2|S_1S_2S_3)}{1+O(H_2|S_1S_2S_3)}=\frac{\frac{18246526895476219}{216755555257152}}{1+\frac{18246526895476219}{216755555257152}}=\boxed{\frac{18246526895476219}{18463282450733371}}=0.98826$