PH3120 – Computational Physics Laboratory 1
CPL105 – Root Finding
Lab Sheet

1. Bisection Method

I. Answer the following questions based on the concepts of bisection method.

(a) What is the Intermediate value theorem?

(b) Explain how the intermediate value theorem is used in bisection method to find roots.

(c) Let's consider that there is a function $y(x)$. If there are two values on x axis, γ and δ, where $\delta > \gamma$, what are the conditions for having a root for $y(x)$ between γ and δ.

(d) If $\beta = \frac{\gamma+\delta}{2}$, how do we determine whether there is a root for $y(\beta)$ using a tolerance?

(e) Discuss some practical considerations and limitations of the bisection method.

II. Develop a Python function with the following signature.

```python
def bisection_method(f, a, b, tol):
    """
    Finds a root of the function 'f' within the interval [a, b] using the bisection method.

    Args:
        f (function): The function for which the root is to be found.
        a (float): The lower bound of the interval.
        b (float): The upper bound of the interval.
        tol (float): The tolerance level for the root approximation.

    Returns:
        float: The approximate root value.

    Raises:
        ValueError: If 'f(a)' and 'f(b)' have the same sign, indicating no root in the interval.
    """
```

III. Apply the above function to solve the following problems.

(a) Find the root of $f(x) = x^2 - \exp(-x) - 2\sin(x)$ within [0,2].

(b) A projectile is launched vertically upward with an initial velocity of v_0=10 m/s and initial height of $h_0 = 5$ m. The acceleration due to gravity is $g = 9.8$ ms^{-2}. Find the time at which the projectile reaches its initial position using the bisection method.

(c) Using the bisection method, approximate the value of the square root of 3 within the interval (1, 2).

(d) A pendulum of length L is oscillating back and forth. The period T of the pendulum is given by the equation $T = 2\pi\sqrt{\frac{L}{g}}$, where g is the acceleration due to gravity (9.8ms^{-2}). Find the length L of the pendulum that corresponds to a period of T= 10 seconds using the bisection method.

(e) An RC circuit is connected to a voltage source. The voltage across the capacitor in the circuit is given by the equation $V_c(t) = V_0 * (1 - \exp(-t / (RC)))$, where $V_c(t)$ is the voltage across the capacitor at time t, V_0 is the initial voltage across the capacitor, R is the resistance, and C is the capacitance. Assume that $V_0 = 5\text{V}$, $R = 10\ \text{M}\Omega$ and $C = 1\mu\text{F}$. Find the time t at which the voltage across the capacitor reaches a certain value $V_c(t) = 2.5$ V using the bisection method.

2. Newton-Raphson Method

I. Answer the following questions based on the concepts of bisection method.

(a) What is the formula used to update the approximation in each iteration of the Newton-Raphson method?

(b) How does the choice of the initial guess affect the convergence of the Newton-Raphson method?

(c) Explain how the derivative of a function is used in the Newton-Raphson method to find the root. You can use a diagram to illustrate this.

(d) Under what conditions does the Newton-Raphson method fail to converge or converge slowly?

II. Develop a Python function with the following signature.

```python
def newton_raphson(f, f_prime, initial_guess, tolerance=1e-6, max_iterations=100):
    """
    Newton-Raphson method for finding the root of a function.

    Parameters:
    f (function): The function for which the root needs to be found.
    f_prime (function): The derivative of the function f.
    initial_guess (float): The initial guess for the root.
    tolerance (float, optional): The desired level of accuracy. Defaults to 1e-6.
    max_iterations (int, optional): The maximum number of iterations. Defaults to 100.

    Returns:
    float: The approximate root of the function f.
    """
```

III. Apply the above function to solve the problems in part 1 III.

3. Python Root Finding Functions

I. Use the fsolve() function in Scipy Python package to solve the problems in part 1 III.

II. Discuss the advantages of the fsolve() function over the two methods you developed in parts 1 and 2 for finding the roots of a function.