PH3120 – Computational Physics Laboratory 1
CPL104 – Vectors and Matrices
Section 1 & 2

1. Vectors

I. Answer the following questions based on the concepts of vectors. Support your answers with relevant examples.

(a) What is the difference between a vector and a scalar quantity?

(b) How do you determine the magnitude of a vector?

(c) How do you calculate the dot product of two vectors?

(d) How do you calculate the cross product of two vectors?

(e) How do you find the angle between two vectors using the dot product?

(f) How do you find the unit vector of a given vector?

II. Develop a Python program using the NumPy package to solve the following problems based on vectors. Solve the problems using analytical methods and verify the accuracy of the solution.

(a) Three-point charges, Q1 = +2 μC, Q2 = -4 μC, and Q3 = +6 μC, are positioned at (1, 0), (0, 2), and (-2, -1) meters, respectively.

i. Calculate the electric field at the origin caused by each charge individually and find the total electric field as a vector at the origin.

ii. Find the magnitude and the direction of the total electric field at the origin.

iii. Find the unit vector of the total electric field.

(b) A rotating object has an angular velocity of (0, 0, 5) rad/s and a moment of inertia tensor given by:

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Calculate the angular momentum of the object.

(c) A particle with a charge of +2 μC and a velocity of (3, 4, 0) m/s enters a magnetic field of strength (0, 0.2, 0.3) T. Calculate the magnetic force acting on the particle.

2. Matrices

I. Answer the following questions based on the concepts of matrices.

(a) What are the properties of matrix addition and matrix multiplication?

(b) Can you explain the concept of matrix transposition and its effect on the matrix dimensions?

(c) What is the inverse of a matrix? Under what conditions does a matrix have an inverse?

(d) What is a square matrix? How does it differ from a rectangular matrix?

(e) Define a diagonal matrix and provide an example.

(f) What is an identity matrix? What are its properties and uses in matrix computations?

(g) Define the rank of a matrix. How is it related to the linear independence of its columns or rows?

(h) Can the rank of a matrix change under matrix operations like addition, multiplication, or inversion? Explain.

II. Develop a Python program using the NumPy package for the following.

(a) Given two matrices A and B, write a Python function to compute the matrix sum A + B.

(b) Write a Python function to calculate the determinant of a given square matrix.

(c) Given a matrix A, write a Python function to find the transpose of A.

(d) Given a matrix A, write a Python function to check if A is symmetric. A matrix is symmetric if it is equal to its transpose.

(e) Write a Python function to multiply two matrices A and B. The function should handle matrices of compatible sizes for multiplication.

III. Solve the following mathematical problems using the above functions. Solve the problems using analytical methods and verify the accuracy of the solution.

(a) $A = XX^T + YY^T$, where X and Y are defined as follows.

$X = \begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}, Y = \begin{pmatrix}\sin(\theta)\\ -\cos(\theta)\end{pmatrix}$, where $\theta = \pi/6$

(b) Is the following matrix symmetric?

$$C = \begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & -3 \\ -3 & -3 & -3 \end{pmatrix}$$

(c) $U = \exp(i\alpha)\begin{pmatrix}\exp(-i\beta/2) & 0\\ 0 & \exp(\mathrm{i}\beta/2)\end{pmatrix}\begin{pmatrix}\cos(\frac{\gamma}{2}) & -\sin(\frac{\gamma}{2})\\ \sin(\frac{\gamma}{2}) & \cos(\frac{\gamma}{2})\end{pmatrix}\begin{pmatrix}\exp(-i\delta/2) & 0\\ 0 & \exp(-\mathrm{i}\delta/2)\end{pmatrix}$

Here, $\alpha = \pi/3, \beta = \delta = \pi/4, \gamma = 3\pi/2$. Note that here you need to work with complex numbers. You need to learn how to construct a complex number using NumPy package.

3. Eigenvalues and Eigenvectors

I. Answer the following questions based on the concepts of the eigenvalues and eigenvectors.

(d) Define eigenvalues and eigenvectors of a matrix.

(e) Explain the concept of the characteristic equation and how it is used to find eigenvalues.

(f) Describe the geometric interpretation of eigenvalues and eigenvectors in terms of matrix transformations.

II. Develop a Python function to find the Eigenvalues and Eigenvectors of a given matrix.

III. Use the above function to solve the following problems. Solve the problem using analytical methods and verify the accuracy of the solution.

(a) Consider a homogeneous sphere of radius 2 cm and mass 10 kg rotating about an axis passing through its center. Determine the eigenvalues and eigenvectors of the moment of inertia tensor for this rotating sphere using the function you created in part 2 II.

(b) Consider a quantum harmonic oscillator with a Hamiltonian given by:

$$H_{ij} = \frac{\hbar\omega}{2}(i+j)\sqrt{2}$$

where ħ is the reduced Planck's constant, ω is the angular frequency, i is the index of raw ($i = 1,2,3$) and j is the index of column ($j = 1,2,3$). Write down the Hamiltonian as a 3×3 matrix. Determine the eigenvalues and eigenvectors of the Hamiltonian matrix by assuming that $\hbar\omega = 1$.

4. System of Linear Equations (SLE)

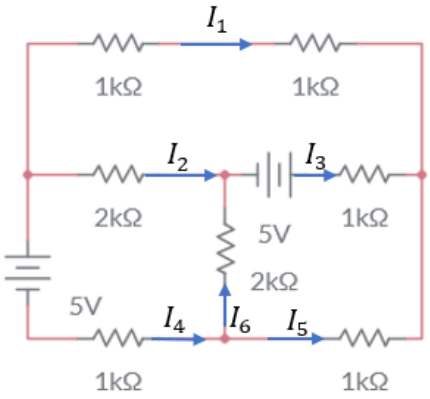
I. Answer the following questions based on the concepts of SLE. Support your answers with relevant examples.

(a) How do you represent a SLE using matrix notation?

(b) How do you use the determinant of the matrix of the SLE to determine whether there is a solution or not?

(c) How do you find the solutions of a SLE using matrix inversion.

II. Solve the following problem using the concepts of SLE.



(a) Using Kirchhoff's laws, construct a SLE to find the currents I_1, I_2, I_3, I_4, I_5 and I_6.

(b) Write a Python program to solve the above SLE using the properties of matrices. Solve the problem using analytical methods and verify the accuracy of the solution.