Ridwan Sharkar   CS486   HW1   9/24/2024

1a) $Y^T = \begin{pmatrix} Y_{11} & Y_{21} & Y_{31} \\ Y_{12} & Y_{22} & Y_{32} \end{pmatrix}$

$YX^T = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \\ Y_{31} & Y_{32} \end{pmatrix} \begin{pmatrix} X_{11} & X_{21} & X_{31} \\ X_{12} & X_{22} & X_{32} \end{pmatrix}$

$$\begin{bmatrix} (Y_{11}X_{11}) + (Y_{12}X_{12}) & (Y_{11}X_{21}) + (Y_{12}X_{22}) & (Y_{11}X_{31}) + (Y_{12}X_{32}) \\ (Y_{21}X_{11}) + (Y_{22}X_{12}) & (Y_{21}X_{21}) + (Y_{22}X_{22}) & (Y_{21}X_{31}) + (Y_{22}X_{32}) \\ (Y_{31}X_{11}) + (Y_{32}X_{12}) & (Y_{31}X_{21}) + (Y_{32}X_{22}) & (Y_{31}X_{31}) + (Y_{32}X_{32}) \end{bmatrix}$$

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1b) $(Y_{*1}X_{*1}^T) + (Y_{*2}X_{*2}^T) =$

↓

$[Y_{11}][X_{11} \; X_{21} \; X_{3}] = [Y_{11}X_{11} \; Y_{11}X_{21} \; Y_{11}X_{31}] \; [Y_{21}] [Y_{21}X_{11} \; Y_{21}X_{21} \; Y_{21}X_{31}] [Y_{31}X_{11} \; Y_{31}X_{21} \; Y_{31}X_{31}]$

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$= [Y_{12}X_{12} \; Y_{12}X_{22} \; Y_{12}X_{32}] \; [Y_{22}] [Y_{22}X_{12} \; Y_{22}X_{22} \; Y_{22}X_{32}] [Y_{32}X_{12} \; Y_{32}X_{22} \; Y_{32}X_{32}]$

Adding A+B results in the exact same matrix in Part a) ☆

1c)   $A_{i,j}$ is $(m \times n)$ matrix  & $B_{j,i}$ is $(n \times m)$ matrix:
   • trace of square matrix is the sum of its diagonal elements; so

$tr(AB) = \sum_{i=1}^{n}(AB)_{ii} = \sum_{i=1}^{m}\sum_{k=1}^{n} A_{ik}B_{ki}$ $\Big|$ $tr(BA) = \sum_{j=1}^{n}(BA)_{jj} = \sum_{j=1}^{n}\sum_{i=1}^{m} B_{ji}A_{ij}$

(relabel) match indices ↙ to show!

$tr(BA) = \sum_{k=1}^{n}\sum_{i=1}^{m} B_{ki}A_{ik}$
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Multiplication of real #s here is commutative,
SO:
$tr(AB) = tr(BA)$

**3b) Now suppose title A has 1600 impressions, title B has 1200 impressions, and title C has 1500 impressions but the clicks/views are still 60, 31, and 41 respectively. Compare the new p-value with that of part a – is it larger,**
**smaller, or about the same? Why is this the case? Give an intuitive justification.**

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OLD:
A: 60/160 = 0.375
B: 31/120 = 0.258
C: 41/150 = 0.273
NEW:
A: 60/1600 = 0.0375
B: 31/1200 = 0.0258
C: 41/1500 = 0.0273

With many more samples, the new CTRs should yield a smaller p-value with more statistical significance, despite the relative differences between each other being the same.