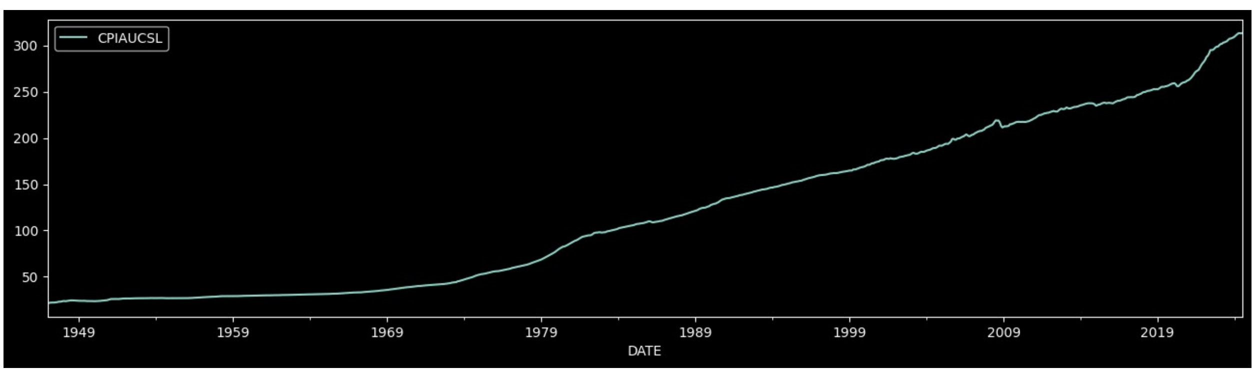
Case Study : Multivariate Time Series

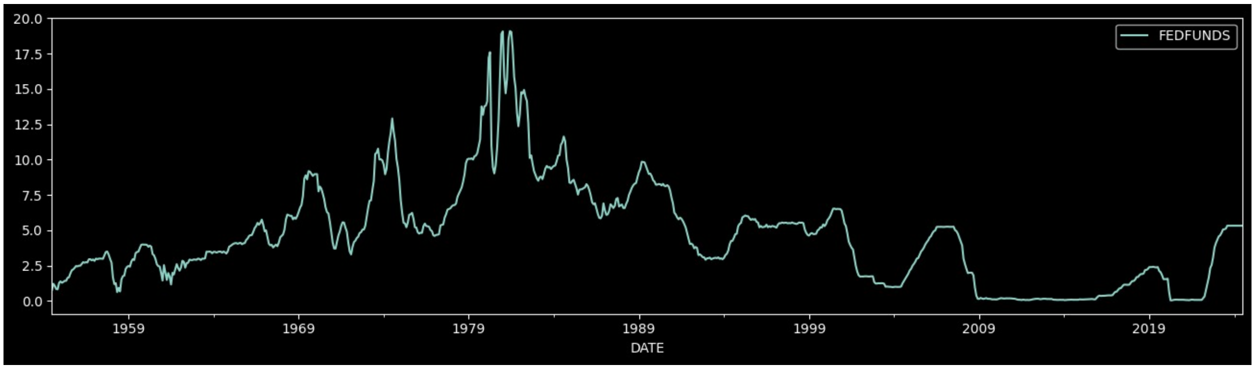
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import yfinance as yf
from statsmodels.tsa.api import VAR

%matplotlib inline

import warnings
warnings.filterwarnings('ignore')

plt.style.use('dark_background')
plt.rcParams['figure.figsize'] = (16, 4)

cpi = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\CPIAUCSL.csv',parse_dates=['DATE'],
                  index_col='DATE')

cpi.plot()
```

```
<Axes: xlabel='DATE'>
```



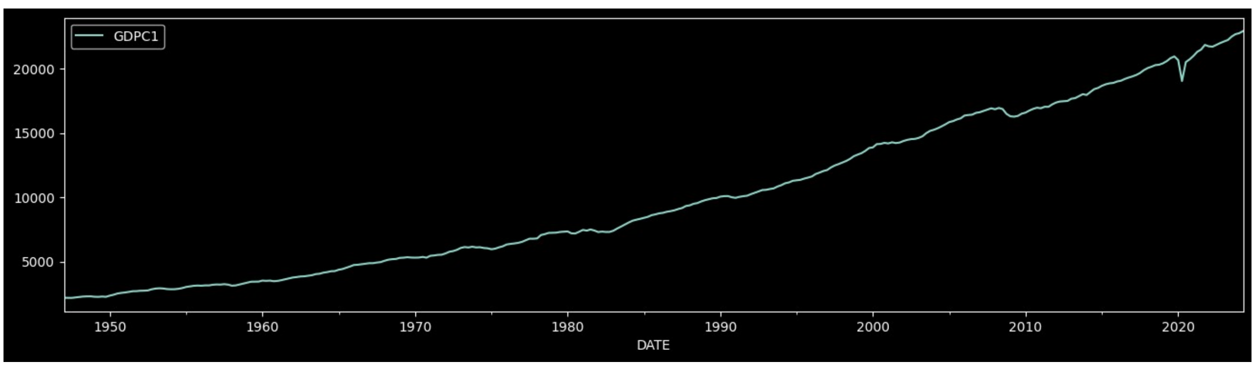

```
fed_funds = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\FEDFUNDS.csv',parse_dates=['DATE'],
                        index_col='DATE')

fed_funds.plot()
```

```
<Axes: xlabel='DATE'>
```



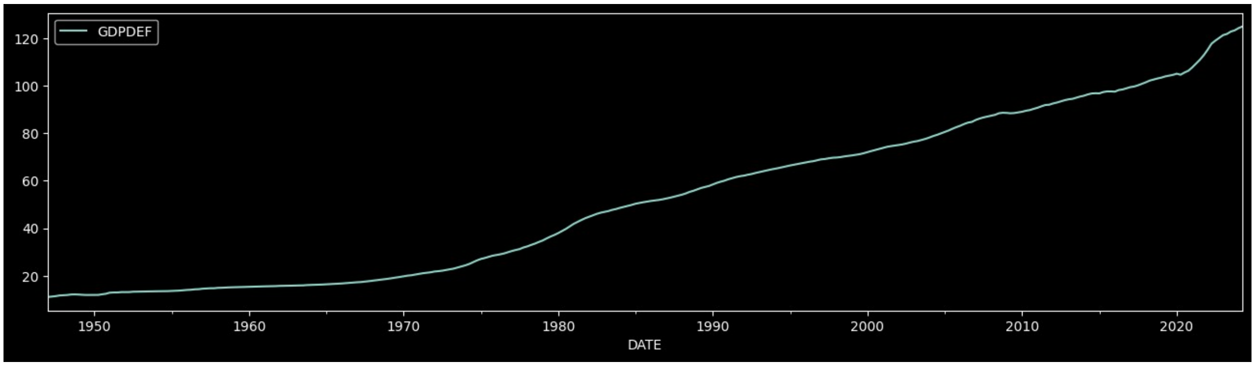

```
gdp = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\GDPC1.csv',parse_dates=['observation_date'],
                  index_col='observation_date')

gdp.index.name = 'DATE'

gdp.plot()
```

```
<Axes: xlabel='DATE'>
```

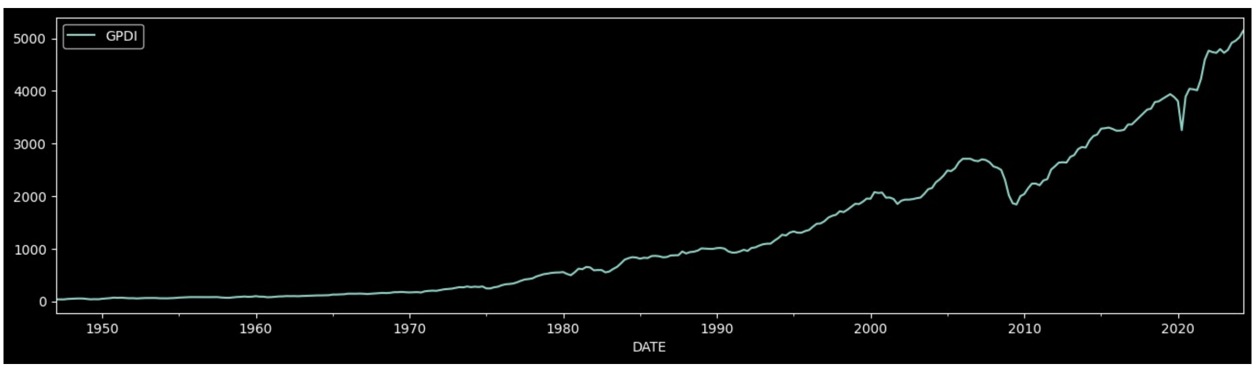



```
gdpdef = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\GDPDEF.csv',parse_dates=['DATE'],
                     index_col='DATE')

gdpdef.plot()
```

```
<Axes: xlabel='DATE'>
```



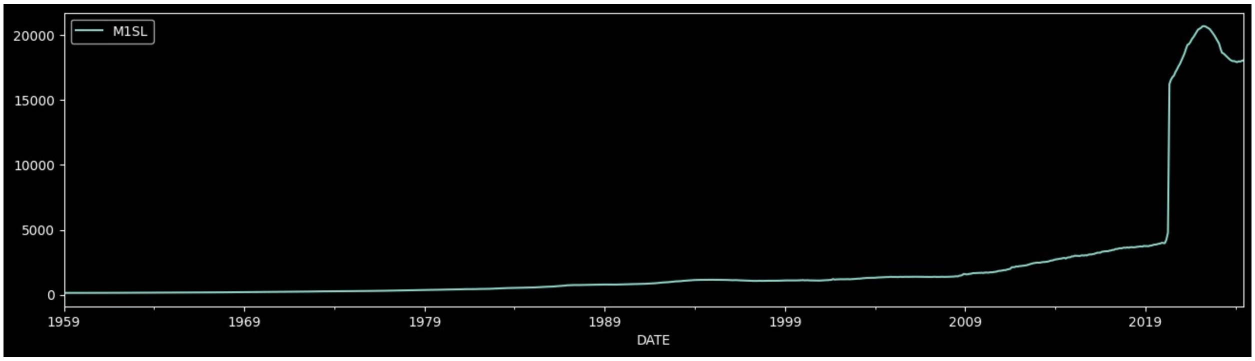

```
gdpi = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\GPDI.csv',parse_dates=['DATE'],
                   index_col='DATE')

gdpi.plot()

<Axes: xlabel='DATE'>
```




```
m1

    observation_date     M1SL
0         1959-01-01    138.9
1         1959-02-01    139.4
2         1959-03-01    139.7
3         1959-04-01    139.7
4         1959-05-01    140.7
..               ...      ...
781       2024-02-01  17930.2
782       2024-03-01  17990.1
783       2024-04-01  17978.9
784       2024-05-01  18022.8
785       2024-06-01  18063.4

[786 rows x 2 columns]

m1 = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\M1SL.csv',parse_dates=['observation_date'],
```

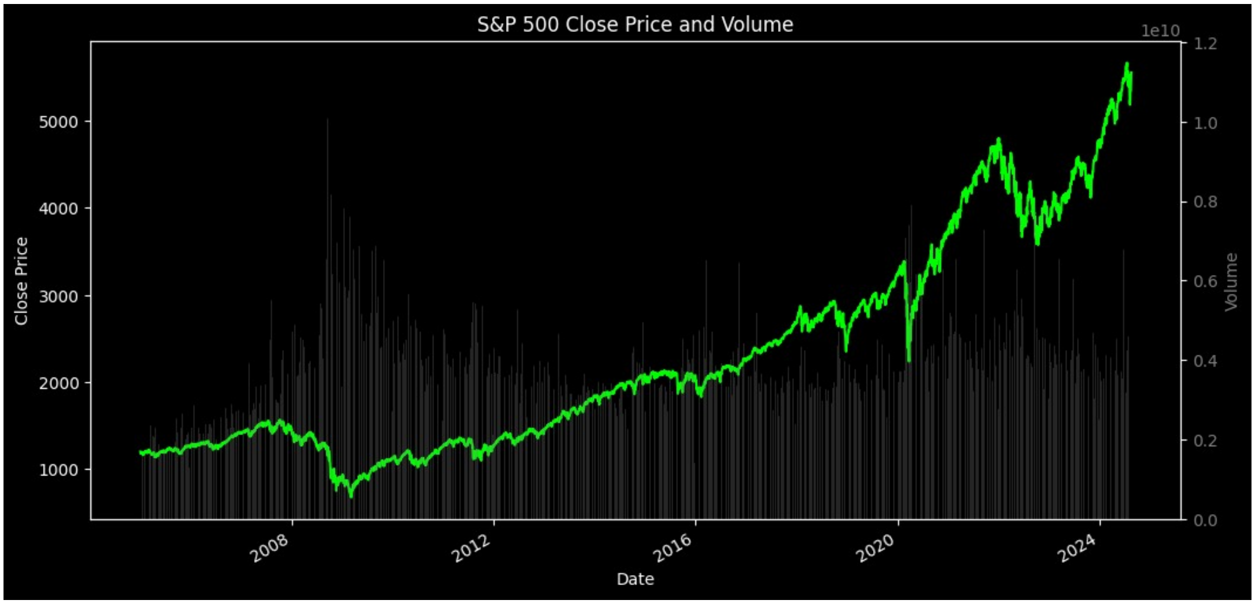
```
                 index_col='observation_date')

m1.rename_axis('DATE',inplace=True)

m1.plot()
```

```
<Axes: xlabel='DATE'>
```

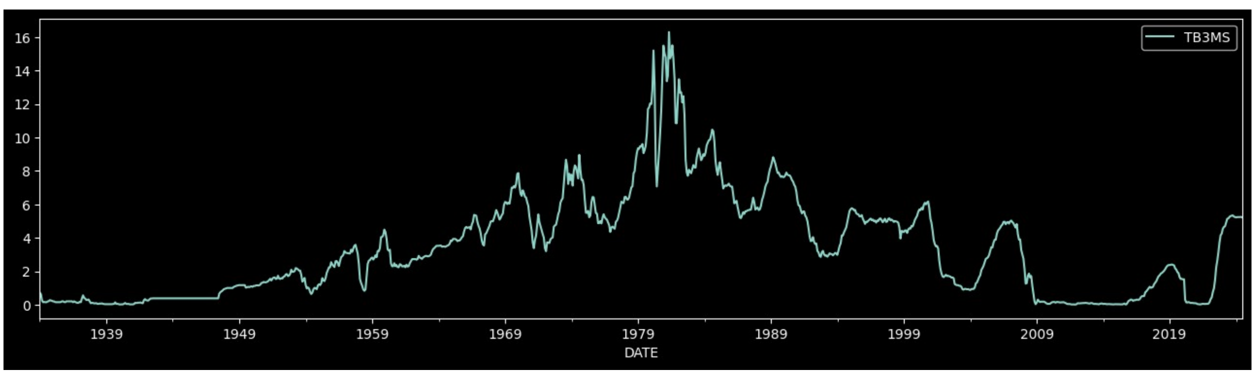



```
sp500 = yf.download('^GSPC',start='2005-01-01')

# Create a figure and axes
fig, ax1 = plt.subplots(figsize=(12, 6))

# Plot the price data as a line plot on the primary y-axis
ax1.plot(sp500.index, sp500['Close'], color='lime', label='Close
Price')
ax1.set_xlabel('Date')
ax1.set_ylabel('Close Price',)
ax1.tick_params(axis='y',)

# Create a secondary y-axis for the volume data
ax2 = ax1.twinx()
ax2.bar(sp500.index, sp500['Volume'], color='grey', alpha=0.3,
label='Volume')
ax2.set_ylabel('Volume', color='grey')
ax2.tick_params(axis='y', labelcolor='grey')

# Set title and format x-axis labels
plt.title('S&P 500 Close Price and Volume')
fig.autofmt_xdate()  # Rotate date labels for better readability

# Show plot
plt.show()
```

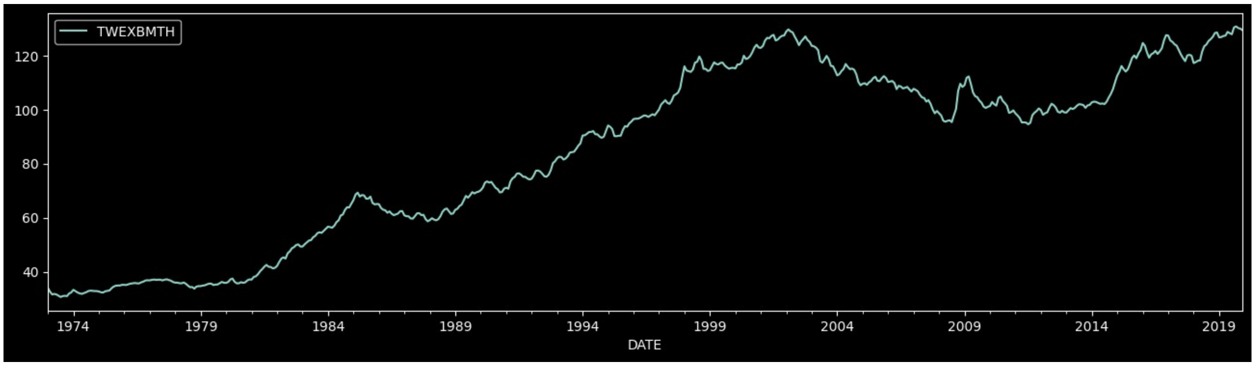
```
[*********************100%%***********************]  1 of 1 completed
```



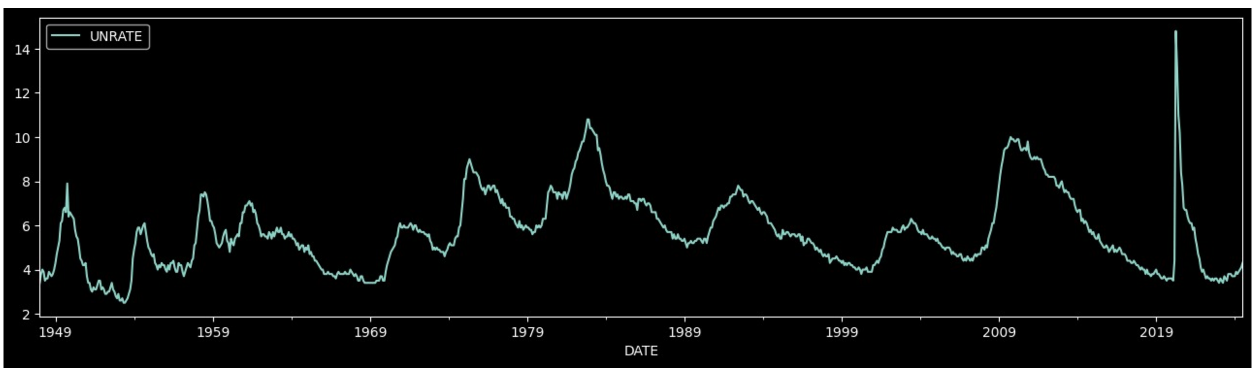

```
tb3ms = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\TB3MS.csv',parse_dates=['DATE'],
                    index_col='DATE')

tb3ms.plot()

<Axes: xlabel='DATE'>
```




```
TWEXBMTH = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector autoregression\data\TWEXBMTH.csv',parse_dates=['DATE'],
                    index_col='DATE')

TWEXBMTH.plot()

<Axes: xlabel='DATE'>
```

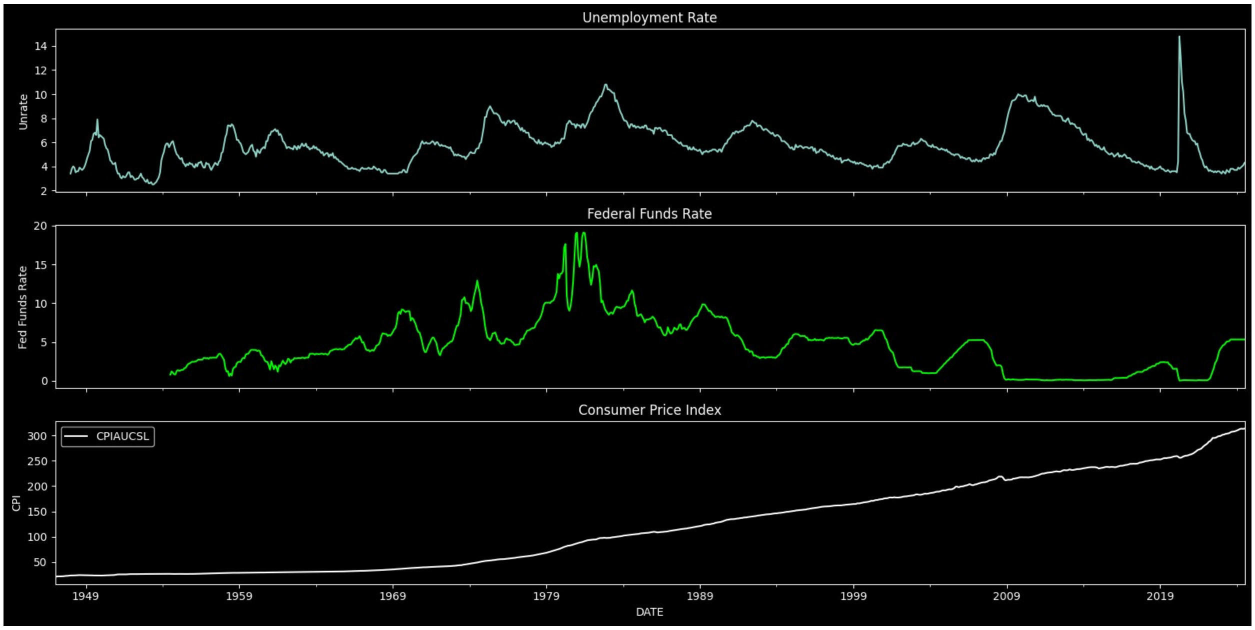



```
unrate = pd.read_csv(r'C:\Users\wassim\Documents\Projects\Vector
autoregression\data\UNRATE.csv',parse_dates=['DATE'],
                      index_col='DATE')

unrate.plot()

<Axes: xlabel='DATE'>
```




```
# Ordinary and Partial Autocorrelations of Reduced Set

merged =
pd.merge(unrate,fed_funds,right_index=True,left_index=True,how='outer'
)
ymat0 =
pd.merge(merged,cpi,right_index=True,left_index=True,how='outer')

fig, axes = plt.subplots(3,1,sharex=True,figsize=(16,8))

ymat0['UNRATE'].plot(ax=axes[0])
axes[0].set_title('Unemployment Rate')
axes[0].set_ylabel('Unrate')

ymat0['FEDFUNDS'].plot(ax=axes[1],color='lime')
axes[1].set_title('Federal Funds Rate')
axes[1].set_ylabel('Fed Funds Rate')

ymat0['CPIAUCSL'].plot(ax=axes[2],color='white')
```

```
axes[2].set_title('Consumer Price Index')
axes[2].set_ylabel('CPI')

plt.xlabel('DATE')
plt.legend()
plt.tight_layout()
```




```
# Extract window from 1960 to 2000

ymat00 = ymat0.loc['1960-01-01':'2000-12-31',:]

ymat00.shape
```

```
(492, 3)
```

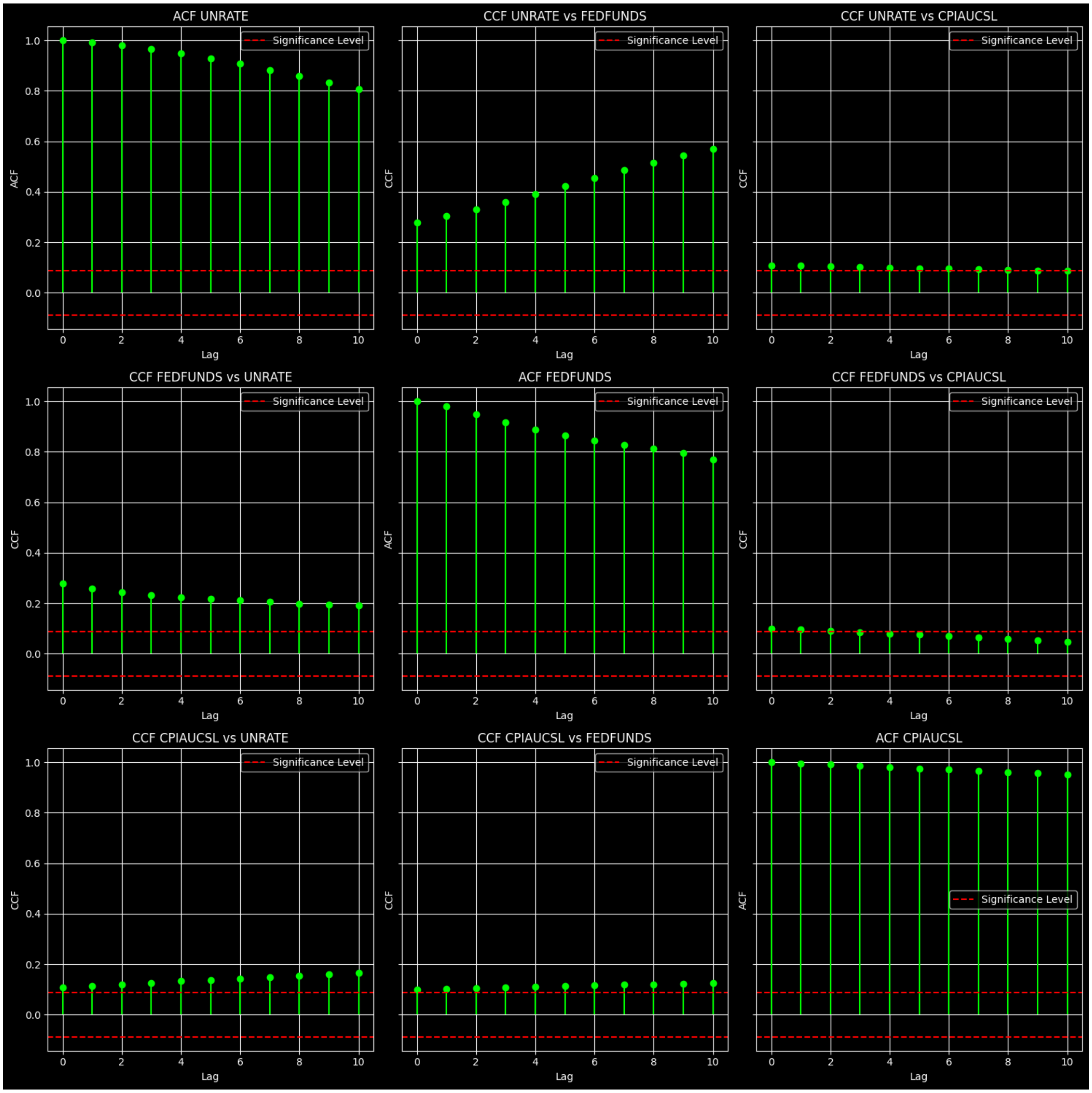
```
from statsmodels.tsa.stattools import ccf,acf,pacf

help(ccf)
```

```
Help on function ccf in module statsmodels.tsa.stattools:

ccf(x, y, adjusted=True, fft=True, *, nlags=None, alpha=None)
    The cross-correlation function.

    Parameters
    ----------
    x, y : array_like
        The time series data to use in the calculation.
    adjusted : bool
        If True, then denominators for cross-correlation are n-k,
otherwise n.
```

```
    fft : bool, default True
        If True, use FFT convolution.  This method should be preferred
        for long time series.
    nlags : int, optional
        Number of lags to return cross-correlations for. If not
provided,
        the number of lags equals len(x).
    alpha : float, optional
        If a number is given, the confidence intervals for the given
level are
        returned. For instance if alpha=.05, 95 % confidence intervals
are
        returned where the standard deviation is computed according to
        1/sqrt(len(x)).

    Returns
    -------
    ndarray
        The cross-correlation function of x and y: the element at
index k
        is the correlation between {x[k], x[k+1], ..., x[n]} and
{y[0], y[1], ..., y[m-k]},
        where n and m are the lengths of x and y, respectively.
    confint : ndarray, optional
        Confidence intervals for the CCF at lags 0, 1, ..., nlags-1
using the level given by
        alpha and the standard deviation calculated as 1/sqrt(len(x))
[1]. Shape (nlags, 2).
        Returned if alpha is not None.

    Notes
    -----
    If adjusted is True, the denominator for the cross-correlation is
adjusted.

    References
    ----------
    .. [1] Brockwell and Davis, 2016. Introduction to Time Series and
       Forecasting, 3rd edition, p. 242.


def plot_acf_ccf(data, lags=10, confidence_interval=1.96):
    """
    Plots ACF and CCF for each pair of columns in the provided
DataFrame.

    Parameters:
    - data: pd.DataFrame containing the time series data.
    - lags: Number of lags to consider for ACF and CCF.
    - confidence_interval: The confidence interval for significance
```

```
level lines.
    """

    num_vars = len(data.columns)
    fig, axes = plt.subplots(nrows=num_vars, ncols=num_vars,
figsize=(15, 15), sharey=True)

    for i, col1 in enumerate(data.columns):
        for j, col2 in enumerate(data.columns):
            ax = axes[i, j]
            if i != j:

                ccf_values = ccf(data[col1], data[col2],
adjusted=False)[:lags+1]

                ax.stem(range(lags+1), ccf_values, linefmt='lime',
markerfmt='o', basefmt=' ')
                ax.set_title(f'CCF {col1} vs {col2}')
                ax.set_xlabel('Lag')
                ax.set_ylabel('CCF')
                ax.grid(True)

                threshold = confidence_interval /
(len(data[col1])**0.5)
                ax.axhline(y=threshold, color='red', linestyle='--',
label='Significance Level')
                ax.axhline(y=-threshold, color='red', linestyle='--')
                ax.legend()
            else:

                acf_values = acf(data[col1], nlags=lags)

                ax.stem(range(lags+1), acf_values[:lags+1],
linefmt='lime', markerfmt='o', basefmt=' ')
                ax.set_title(f'ACF {col1}')
                ax.set_xlabel('Lag')
                ax.set_ylabel('ACF')
                ax.grid(True)

                threshold = confidence_interval /
(len(data[col1])**0.5)
                ax.axhline(y=threshold, color='red', linestyle='--',
label='Significance Level')
                ax.axhline(y=-threshold, color='red', linestyle='--')
                ax.legend()


    plt.tight_layout()
    plt.show()
```

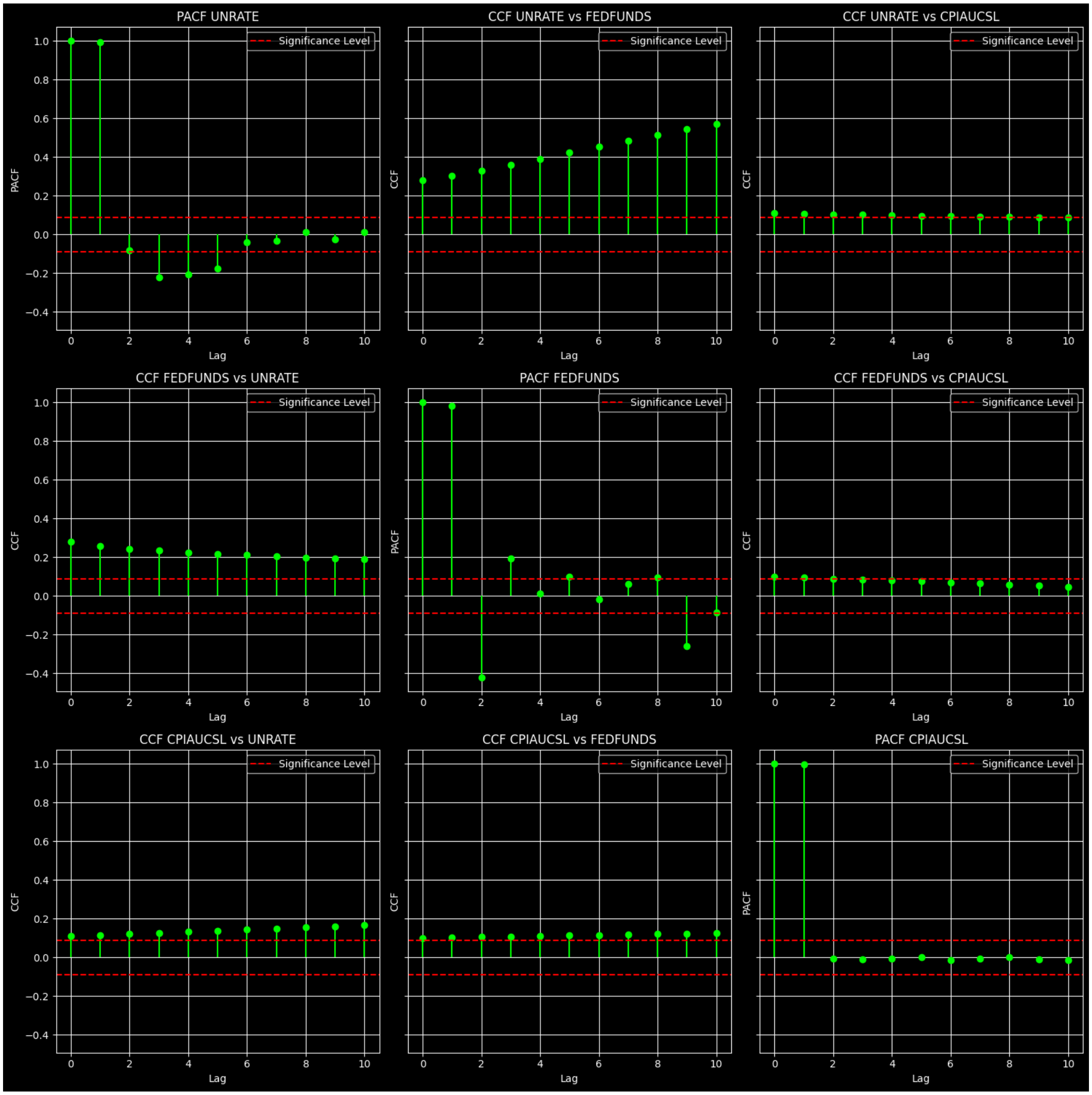
```python
plot_acf_ccf(ymat00, lags=10, confidence_interval=1.96)
```




```python
def plot_pacf_ccf(data, lags=10, confidence_interval=1.96):
    """
    Plots PACF and CCF for each pair of columns in the provided
DataFrame.

    Parameters:
    - data: pd.DataFrame containing the time series data.
    - lags: Number of lags to consider for PACF and CCF.
    - confidence_interval: The confidence interval for significance
```

```
level lines.
    """



    num_vars = len(data.columns)
    fig, axes = plt.subplots(nrows=num_vars, ncols=num_vars,
figsize=(15, 15), sharey=True)

    for i, col1 in enumerate(data.columns):
        for j, col2 in enumerate(data.columns):
            ax = axes[i, j]
            if i == j:

                pacf_values = pacf(data[col1], nlags=lags)

                ax.stem(range(lags+1), pacf_values[:lags+1],
linefmt='lime', markerfmt='o', basefmt=' ')
                ax.set_title(f'PACF {col1}')
                ax.set_xlabel('Lag')
                ax.set_ylabel('PACF')
                ax.grid(True)

                threshold = confidence_interval /
(len(data[col1])**0.5)
                ax.axhline(y=threshold, color='red', linestyle='--',
label='Significance Level')
                ax.axhline(y=-threshold, color='red', linestyle='--')
                ax.legend()
            else:

                ccf_values = ccf(data[col1], data[col2],
adjusted=False)[:lags+1]

                ax.stem(range(lags+1), ccf_values, linefmt='lime',
markerfmt='o', basefmt=' ')
                ax.set_title(f'CCF {col1} vs {col2}')
                ax.set_xlabel('Lag')
                ax.set_ylabel('CCF')
                ax.grid(True)

                threshold = confidence_interval /
(len(data[col1])**0.5)
                ax.axhline(y=threshold, color='red', linestyle='--',
label='Significance Level')
                ax.axhline(y=-threshold, color='red', linestyle='--')
                ax.legend()


    plt.tight_layout()
    plt.show()
```

```python
plot_pacf_ccf(ymat00, lags=10, confidence_interval=1.96)
```



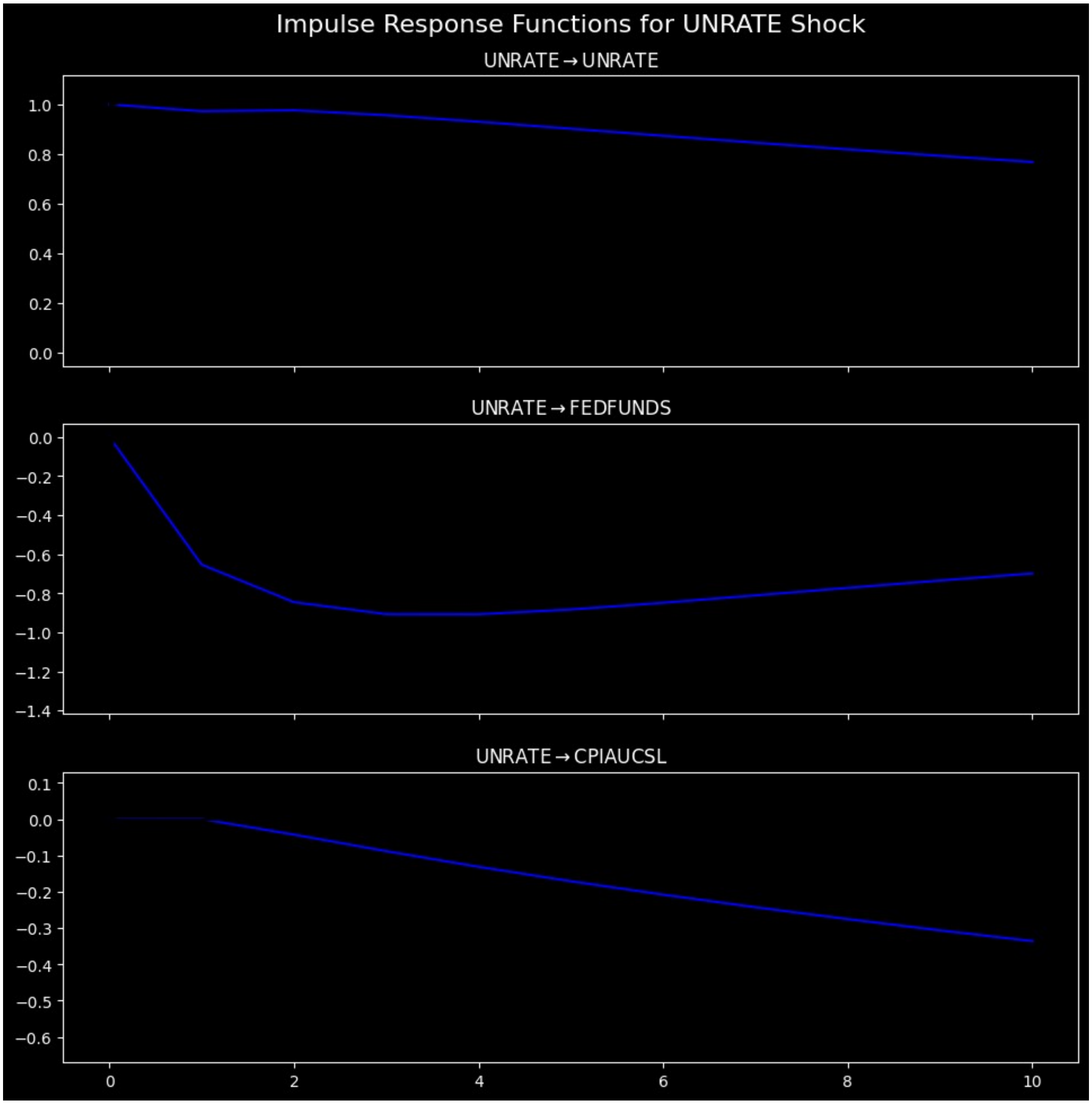

```python
# Vector Autoregressive (VAR) Model for Reduced Set

model = VAR(ymat00)

lag_order = model.select_order(maxlags=12)

print(lag_order.summary())
print('Best orders :',lag_order.selected_orders)
```

```
 VAR Order Selection (* highlights the minimums)
==================================================
       AIC         BIC         FPE         HQIC
--------------------------------------------------
0        10.85       10.87   5.130e+04       10.86
1       -7.763      -7.659   0.0004252      -7.722
2       -8.090     -7.908*   0.0003065      -8.018
3       -8.162      -7.901   0.0002854      -8.059
4       -8.234      -7.895   0.0002655      -8.101
5       -8.265      -7.848   0.0002573     -8.101*
6       -8.260      -7.764   0.0002587      -8.065
7       -8.266      -7.693   0.0002570      -8.041
8       -8.309      -7.657   0.0002464      -8.052
9       -8.317      -7.586   0.0002445      -8.030
10      -8.334      -7.525   0.0002403      -8.016
11      -8.323      -7.436   0.0002432      -7.974
12     -8.345*      -7.380  0.0002378*      -7.966
--------------------------------------------------
Best orders : {'aic': 12, 'bic': 2, 'hqic': 5, 'fpe': 12}

#Fit the VAR model based on Bayes Information criterion (BIC)

var_model = model.fit(2)

print(var_model.summary())

  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 18, Aug, 2024
Time:                     22:50:21
--------------------------------------------------------------------
No. of Equations:         3.00000    BIC:                   -7.87807
Nobs:                     490.000    HQIC:                  -7.98724
Log likelihood:          -90.6701    FPE:                0.000316613
AIC:                     -8.05783    Det(Omega_mle):     0.000303423
--------------------------------------------------------------------
Results for equation UNRATE
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const               0.023873         0.035582            0.671
0.502
L1.UNRATE           0.972402         0.045932           21.170
0.000
L1.FEDFUNDS        -0.029274         0.013634           -2.147
```

```
0.032
L1.CPIAUCSL         0.017452         0.041142            0.424
0.671
L2.UNRATE           0.011561         0.045568            0.254
0.800
L2.FEDFUNDS         0.043466         0.013726            3.167
0.002
L2.CPIAUCSL        -0.017783         0.041210           -0.432
0.666
======================================================================
========

Results for equation FEDFUNDS
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const               0.207075         0.110980            1.866
0.062
L1.UNRATE          -0.653295         0.143260           -4.560
0.000
L1.FEDFUNDS         1.310460         0.042525           30.816
0.000
L1.CPIAUCSL         0.202530         0.128320            1.578
0.114
L2.UNRATE           0.645737         0.142126            4.543
0.000
L2.FEDFUNDS        -0.336354         0.042812           -7.856
0.000
L2.CPIAUCSL        -0.203102         0.128532           -1.580
0.114
======================================================================
========

Results for equation CPIAUCSL
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const              -0.065469         0.036647           -1.786
0.074
L1.UNRATE           0.000410         0.047307            0.009
0.993
L1.FEDFUNDS         0.064809         0.014042            4.615
0.000
```

```
L1.CPIAUCSL          1.372401         0.042373           32.388
0.000
L2.UNRATE           -0.001120         0.046932           -0.024
0.981
L2.FEDFUNDS         -0.039541         0.014137           -2.797
0.005
L2.CPIAUCSL         -0.371371         0.042443           -8.750
0.000
======================================================================
========

Correlation matrix of residuals
              UNRATE  FEDFUNDS  CPIAUCSL
UNRATE      1.000000 -0.191066 -0.040362
FEDFUNDS   -0.191066  1.000000  0.082336
CPIAUCSL   -0.040362  0.082336  1.000000
```

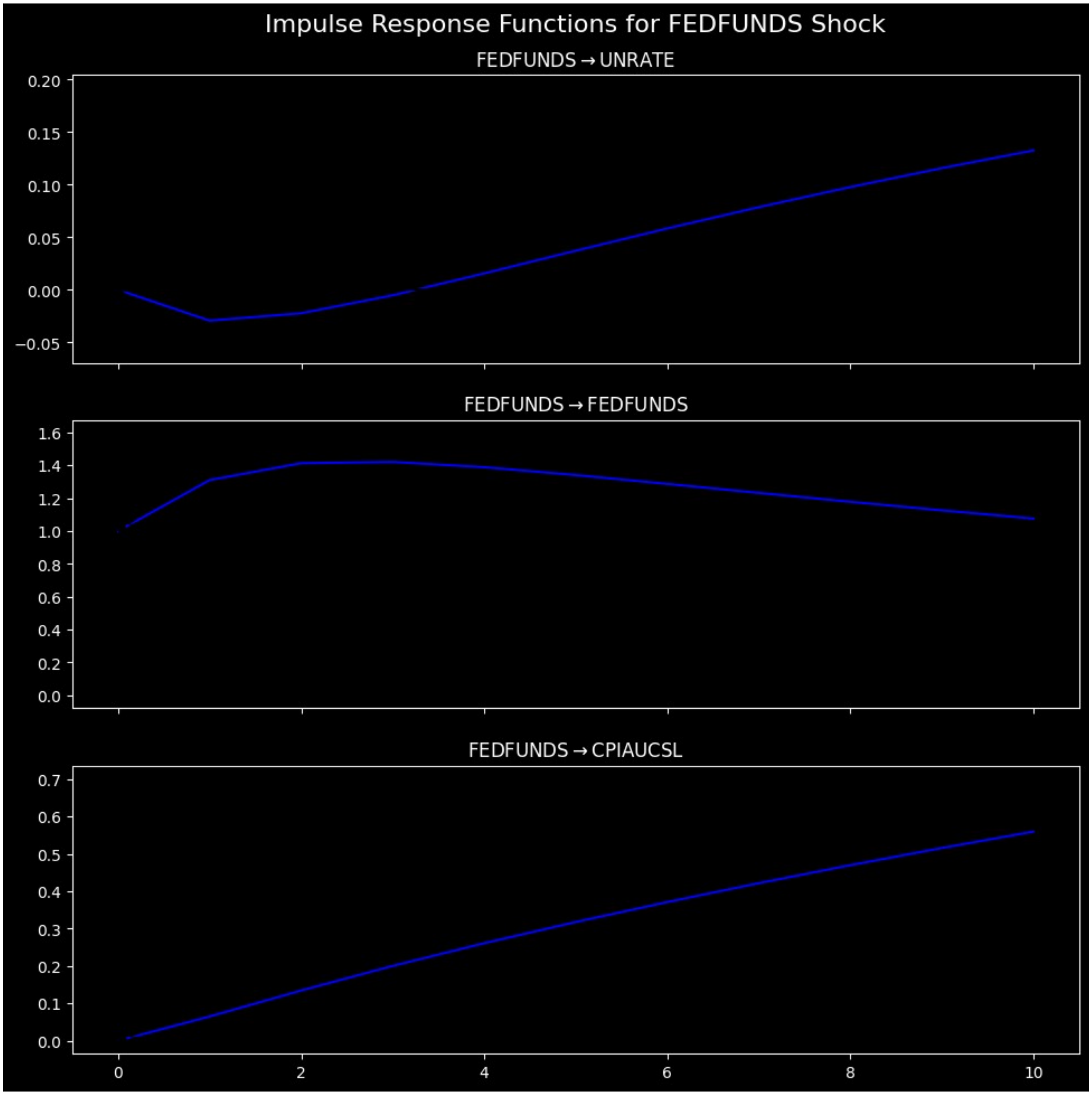
```
# Impulse Responde Function for a Fitted VAR(p) Model

irf = var_model.irf(10)

fig = irf.plot(impulse='UNRATE')
fig.suptitle('Impulse Response Functions for UNRATE Shock',
fontsize=16)

plt.show()
```



- When unemployment rises:
  - the Federal Funds rate is projected to decline (consistent with Federal Reserve Policy)
  - the CPI decreases (lower employment results in lesspressure to increase consumer prices)

```
irf = var_model.irf(10)

fig = irf.plot(impulse='FEDFUNDS')
fig.suptitle('Impulse Response Functions for FEDFUNDS Shock',
fontsize=16)
```

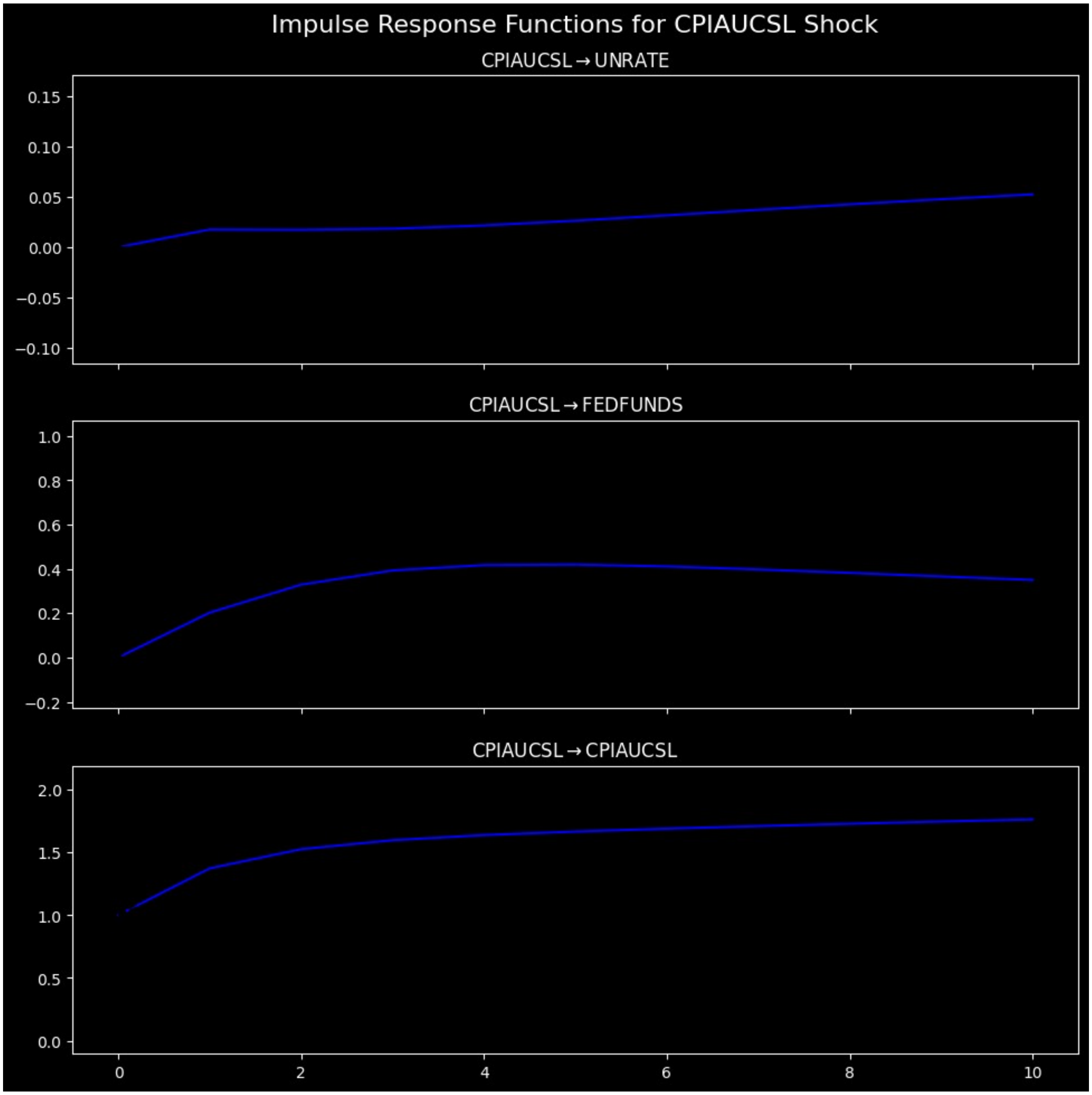
```
plt.show()
```



- When the Fed Funds rate increases:
  - The Unemployment rate tends to increase; so reducing the Fed Funds rate would tend to reduce unemployment
  - The CPI increases; increases in the Fed Funds rate are associated with increase in CPI over future quarters

```
irf = var_model.irf(10)
```

```
fig = irf.plot(impulse='CPIAUCSL')
fig.suptitle('Impulse Response Functions for CPIAUCSL Shock',
fontsize=16)

plt.show()
```



- When the CPI increases:
  - The Federal Funds rate tends to increase over subsequent quarters.
  - This is consistent with Federal Reserve policy of raising interest rates to control for inflation.

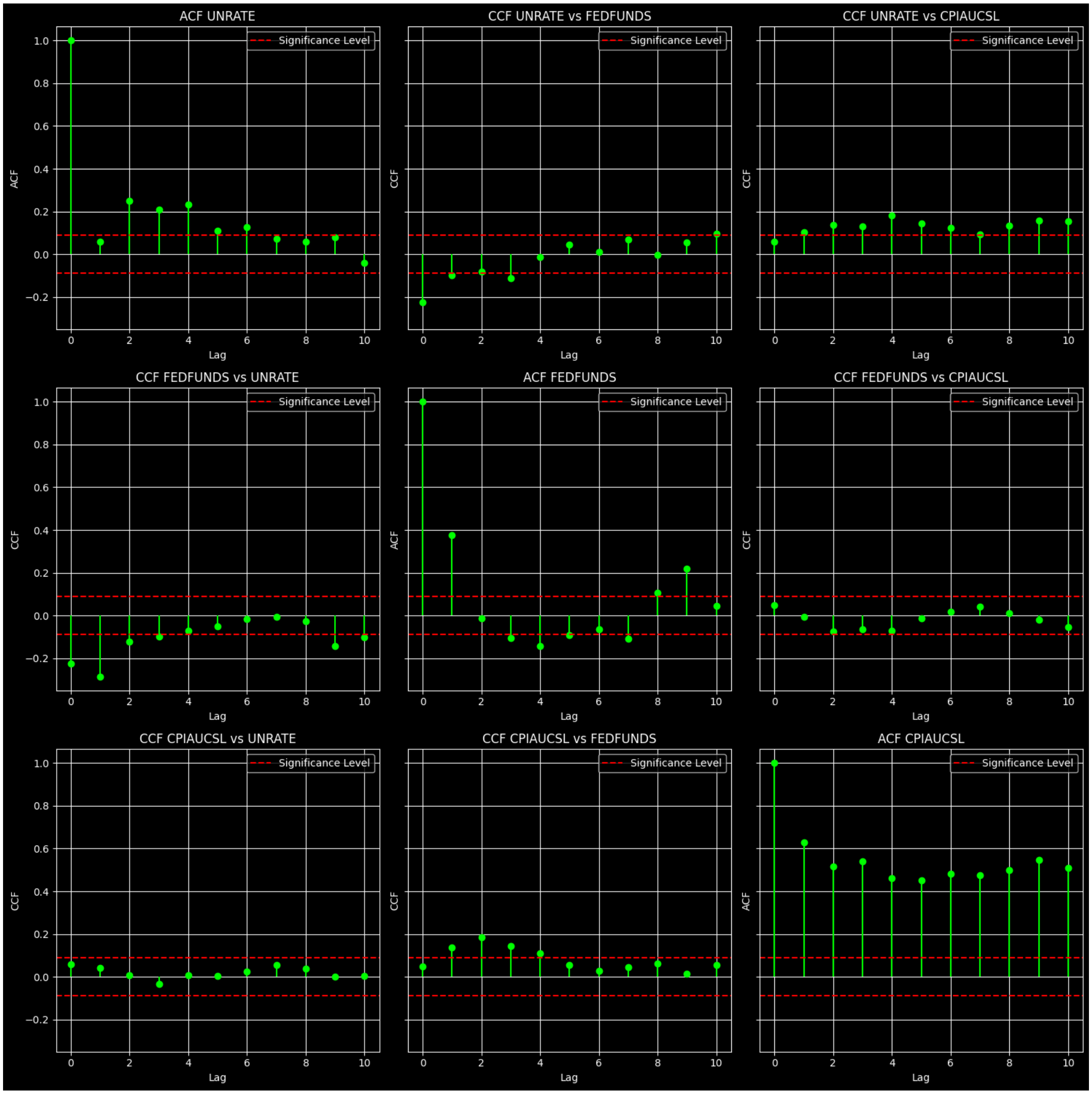
```python
# Ordinary and Partial Autocorrelations of Differenced Series

diff_ymat00 = ymat00.diff().dropna()

diff_ymat00
```

```
            UNRATE  FEDFUNDS  CPIAUCSL
DATE
1960-02-01    -0.4     -0.02      0.04
1960-03-01     0.6     -0.13      0.00
1960-04-01    -0.2      0.08      0.13
1960-05-01    -0.1     -0.07      0.03
1960-06-01     0.3     -0.53      0.04
...            ...       ...       ...
2000-08-01     0.1     -0.04      0.00
2000-09-01    -0.2      0.02      0.90
2000-10-01     0.0     -0.01      0.30
2000-11-01     0.0      0.00      0.30
2000-12-01     0.0     -0.11      0.40

[491 rows x 3 columns]
```

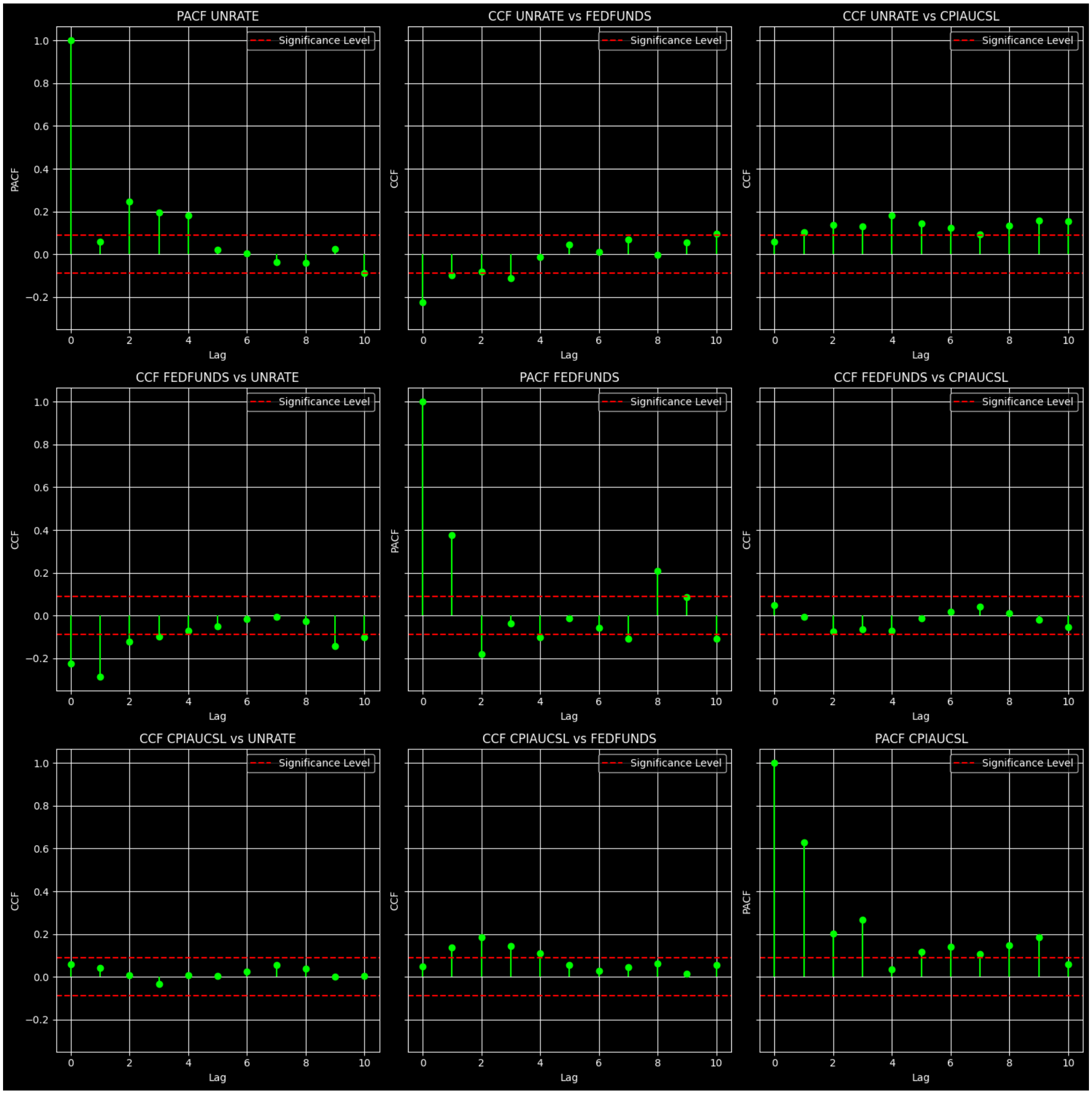
```python
help(plot_acf_ccf)
```

```
Help on function plot_acf_ccf in module __main__:

plot_acf_ccf(data, lags=10, confidence_interval=1.96)
    Plots ACF and CCF for each pair of columns in the provided
DataFrame.

    Parameters:
    - data: pd.DataFrame containing the time series data.
    - lags: Number of lags to consider for ACF and CCF.
    - confidence_interval: The confidence interval for significance
level lines.
```

```python
plot_acf_ccf(diff_ymat00,10,1.96)
```




```
plot_pacf_ccf(diff_ymat00,10,1.96)
```



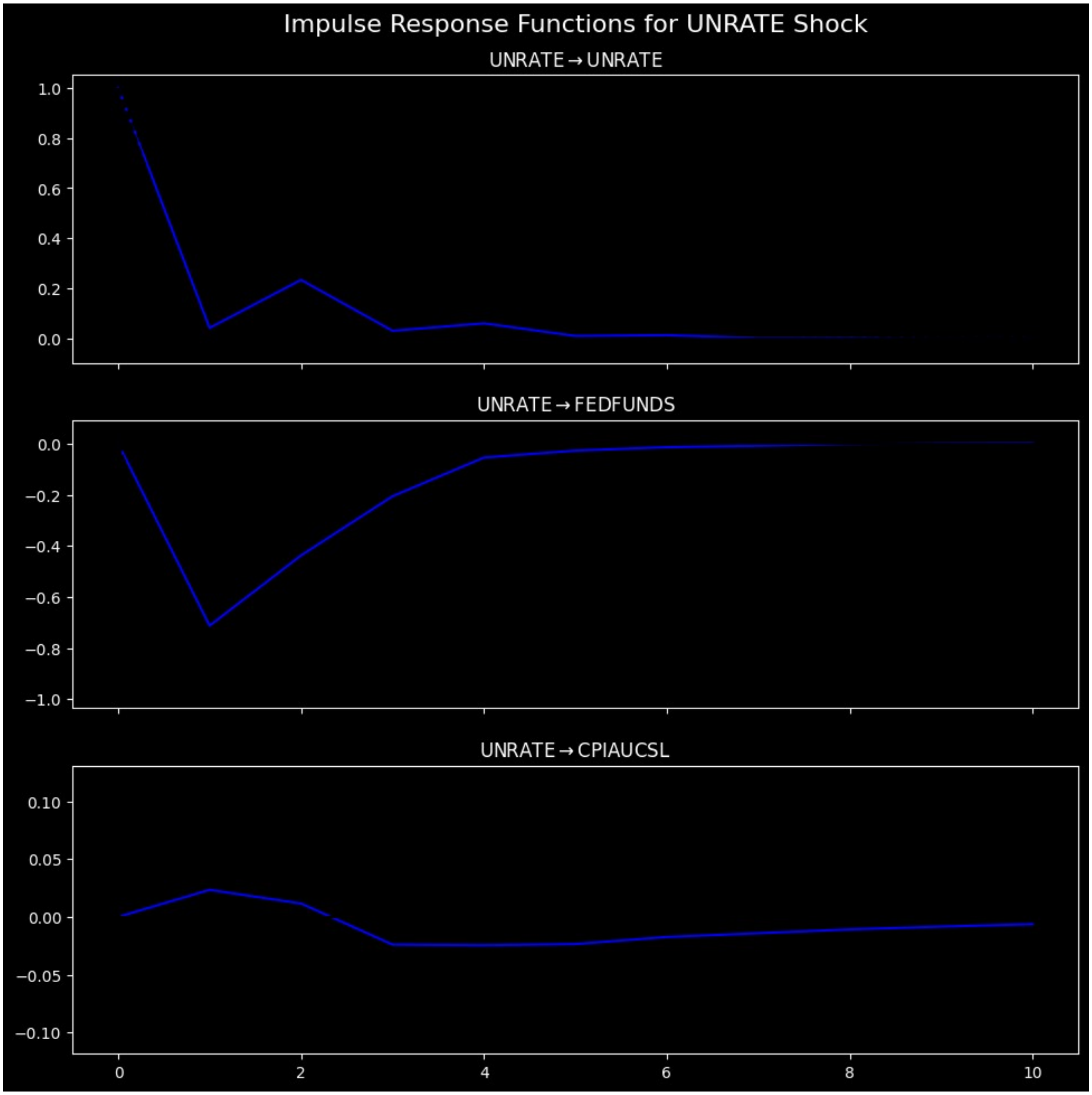

```
#Vector Autoregressive (VAR) Model with Differenced Series

model2 = VAR(diff_ymat00)

lag_order2 = model2.select_order(maxlags=12)

print(lag_order2.summary())
print('Best orders: ',lag_order2.selected_orders)
```

```
 VAR Order Selection (* highlights the minimums)
==================================================
       AIC         BIC         FPE         HQIC
--------------------------------------------------
```

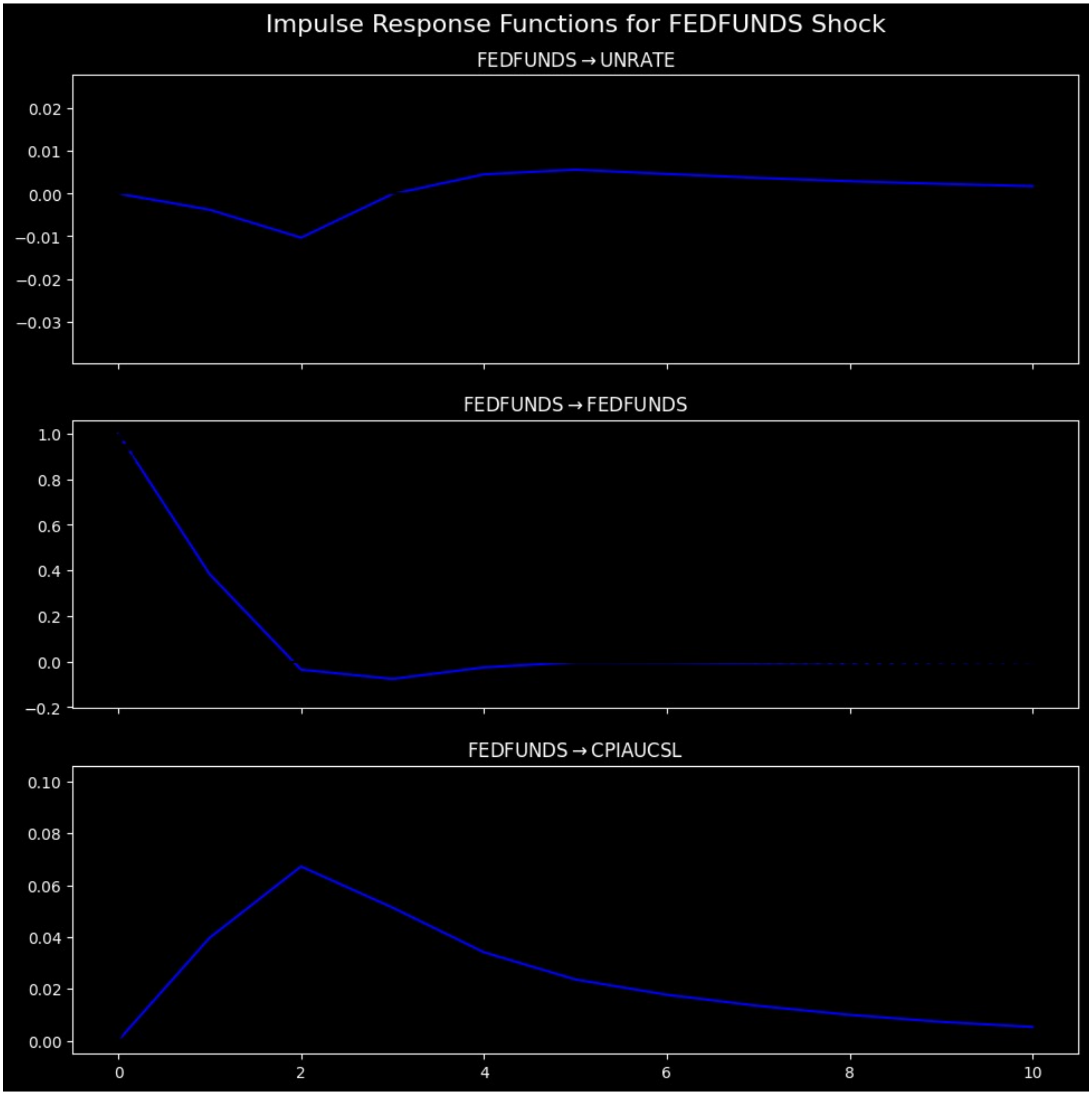
```
0        -7.198      -7.171   0.0007484      -7.187
1        -7.884      -7.779   0.0003768      -7.843
2        -8.004      -7.821   0.0003342      -7.932
3        -8.108     -7.847*   0.0003011     -8.005*
4        -8.138      -7.798   0.0002922      -8.005
5        -8.139      -7.721   0.0002920      -7.975
6        -8.140      -7.644   0.0002917      -7.945
7        -8.161      -7.586   0.0002856      -7.935
8        -8.199      -7.546   0.0002749      -7.943
9        -8.244      -7.513   0.0002629      -7.957
10       -8.232      -7.422   0.0002662      -7.913
11       -8.242      -7.354   0.0002635      -7.893
12      -8.265*      -7.299  0.0002575*      -7.885
--------------------------------------------------
Best orders:  {'aic': 12, 'bic': 3, 'hqic': 3, 'fpe': 12}

# We will chose again the bayesian information criterion

var_model2 = model2.fit(2)

print(var_model2.summary())

  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Mon, 19, Aug, 2024
Time:                     00:01:33
--------------------------------------------------------------------
No. of Equations:         3.00000    BIC:                   -7.82286
Nobs:                     489.000    HQIC:                  -7.93218
Log likelihood:          -103.874    FPE:                0.000334494
AIC:                     -8.00290    Det(Omega_mle):     0.000320531
--------------------------------------------------------------------
Results for equation UNRATE
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const              -0.032066         0.012989           -2.469
0.014
L1.UNRATE           0.041119         0.044488            0.924
0.355
L1.FEDFUNDS        -0.003781         0.014573           -0.259
0.795
L1.CPIAUCSL         0.033265         0.040306            0.825
0.409
L2.UNRATE           0.227982         0.045285            5.034
```

```
0.000
L2.FEDFUNDS        -0.010052         0.014180           -0.709
0.478
L2.CPIAUCSL         0.067336         0.040050            1.681
0.093
======================================================================
========

Results for equation FEDFUNDS
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const               0.013074         0.040563            0.322
0.747
L1.UNRATE          -0.712756         0.138930           -5.130
0.000
L1.FEDFUNDS         0.383472         0.045509            8.426
0.000
L1.CPIAUCSL         0.146516         0.125870            1.164
0.244
L2.UNRATE          -0.138538         0.141419           -0.980
0.327
L2.FEDFUNDS        -0.191170         0.044281           -4.317
0.000
L2.CPIAUCSL        -0.182392         0.125071           -1.458
0.145
======================================================================
========

Results for equation CPIAUCSL
======================================================================
========
                 coefficient       std. error           t-stat
prob
----------------------------------------------------------------------
--------
const               0.093276         0.014291            6.527
0.000
L1.UNRATE           0.023367         0.048947            0.477
0.633
L1.FEDFUNDS         0.039709         0.016033            2.477
0.013
L1.CPIAUCSL         0.477476         0.044345           10.767
0.000
L2.UNRATE           0.027755         0.049823            0.557
0.577
```

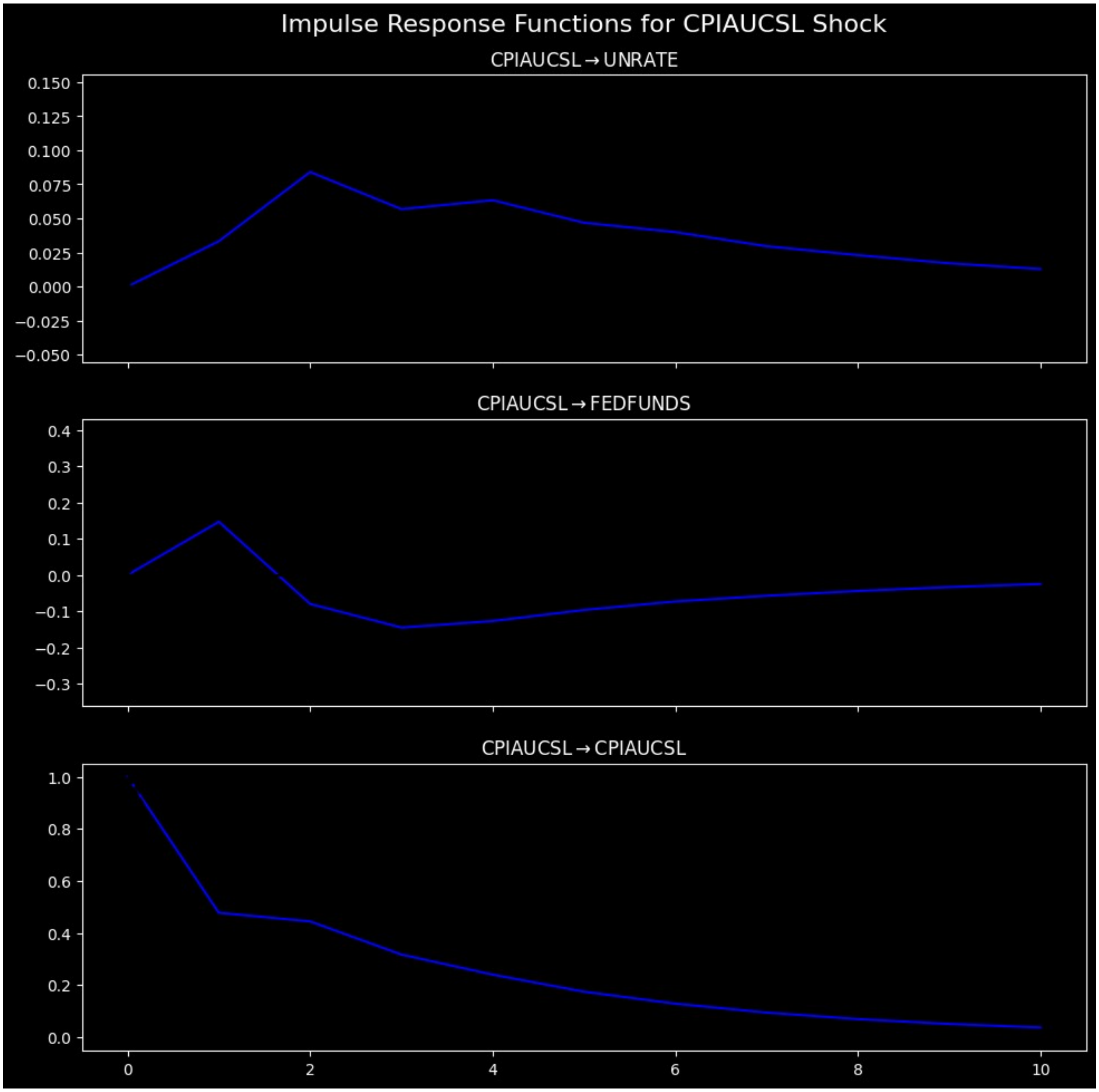
```
L2.FEDFUNDS          0.033114         0.015601            2.123
0.034
L2.CPIAUCSL          0.209762         0.044064            4.760
0.000
======================================================================
========

Correlation matrix of residuals
              UNRATE  FEDFUNDS  CPIAUCSL
UNRATE      1.000000 -0.203665 -0.014787
FEDFUNDS   -0.203665  1.000000  0.060508
CPIAUCSL   -0.014787  0.060508  1.000000




# Impulse Responde Function for a VAR(p) fitted of Differenced Series

irf = var_model2.irf(10)

fig = irf.plot(impulse='UNRATE')
fig.suptitle('Impulse Response Functions for UNRATE Shock',
fontsize=16)

plt.show()
```




```
irf = var_model2.irf(10)

fig = irf.plot(impulse='FEDFUNDS')
fig.suptitle('Impulse Response Functions for FEDFUNDS Shock',
fontsize=16)

plt.show()
```




```
irf = var_model2.irf(10)

fig = irf.plot(impulse='CPIAUCSL')
fig.suptitle('Impulse Response Functions for CPIAUCSL Shock',
fontsize=16)

plt.show()
```



- Interpreting the impulse response functions for the VAR model of the differenced series, we note:
  - When unemployment increases, the Fed Funds rate tends to decrease over subsequent quarters, consistent with Federal Reserve policies (i.e., stimulating economic growth and employment with lower interest rates).
  - When the Fed Funds rate increases, there is a modest increase in inflation(CPIA). This is consistent with the Fed raising rates to control inflation which tends to persist for several quarters (note the high 3-rd quarter lag partial autocorrelation in CPIAUCSL).
  - When in ation (CPIAUCSL) increases, unemployment tends to rise modestly, and the Fed Funds rate tends to increase.