Universidad Nacional de Colombia

Facultad de ciencias

Departamento de matemáticas

Contingencias de vida 2025-I

Trabajo de seguros de vida

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Contenido

HOLA

1. Crecimiento aritmético fraccionado

2. Crecimiento aritmético dentro del año fraccionado

3. Demostración

Asumiendo UDD, simplificar en términos de A_x y $(IA)_x$ la prima simple neta de un seguro de vida entero fraccionado a m meses con crecimiento aritmético de r en cada mes de su valor asegurado.

$$\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}\left(1+kr+r\frac{j}{m}\right)v^{k+\frac{j+1}{m}}\,{}_{k+\frac{j}{m}}p_x\frac{1}{m}q_{x+k+\frac{j}{m}} = \frac{i}{i^{(m)}}[A_x+r[(IA)_x-A_x]]+r\frac{i-i^{(m)}}{(i^{(m)})^2}A_x \tag{1}$$

Demostración:
Sabemos que:

$$A_x^{(m)} = \sum_{k=0}^{\infty}\sum_{j=0}^{m-1}v^{k+\frac{j+1}{m}}\,{}_{k+\frac{j}{m}}p_x\frac{1}{m}q_{x+k+\frac{j}{m}} \approx \frac{i}{i^{(m)}}A_x \tag{2}$$

$$\sum_{k=0}^{\infty}(1+kr)\,{}_kA_{x:1}^{(m)} = \frac{i}{i^{(m)}}[A_x+r[(IA)_x-A_x]] \tag{3}$$

Con ${}_kA_{x:1}^{(m)} = \frac{i}{i^{(m)}}v^{k+1}\,{}_kq_x$, entonces:

$$\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}\left(1+kr+r\frac{j}{m}\right)v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{4}$$

$$=\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}(1+kr)v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{5}$$

$$+\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}r\frac{j}{m}v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{6}$$

Analizando las dos sumas por partes:
i) Para la primera suma:

$$\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}(1+kr)v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{7}$$

$$=\sum_{k=0}^{\infty}(1+kr)\sum_{j=0}^{m-1}v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{8}$$

$$=\sum_{k=0}^{\infty}(1+kr)\,{}_{k}A^{(m)}_{x:1} \tag{9}$$

$$=\frac{i}{i^{(m)}}[A_x+r[(IA)_x-A_x]] \tag{10}$$

ii) Para la segunda suma:

$$\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}r\frac{j}{m}v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{11}$$

$$=r\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}\frac{j}{m}v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{12}$$

$$=r\sum_{k=0}^{\infty}v^{k+1}\,{}_{k}p_x\sum_{j=0}^{m-1}\frac{j}{m}v^{\frac{j+1}{m}-1}\ {}_{\frac{j}{m}}p_x\,{}_{\frac{1}{m}}q_{x+k+\frac{j}{m}} \tag{13}$$

$$=r\sum_{k=0}^{\infty}v^{k+1}\,{}_{k}p_x q_{x+k}\sum_{j=0}^{m-1}\frac{j}{m^2}v^{\frac{j+1}{m}-1}\quad \text{(Utilizando DUM)} \tag{14}$$

$$=rA_x\sum_{j=0}^{m-1}\frac{j}{m^2}v^{\frac{j+1}{m}-1} \tag{15}$$

Examinando $v^{\frac{1}{m}}(1+i)\frac{1}{m^2}\sum_{j=0}^{m-1}jv^{\frac{j}{m}}$, observamos que:

$$
\begin{aligned}
v^{\frac{1}{m}}(1+i)\frac{1}{m^2}\sum_{j=0}^{m-1} jv^{\frac{j}{m}} &= \frac{v^{\frac{1}{m}}(1+i)}{m^2}\left[\sum_{j=0}^{m} jv^{\frac{j}{m}} - mv\right] && (16)\\
&= \frac{v^{\frac{1}{m}}(1+i)}{m^2}\sum_{j=0}^{m} jv^{\frac{j}{m}} - \frac{v^{\frac{1}{m}}}{m} && (17)\\
&= v^{\frac{1}{m}}(1+i)(I^{(m)}\ddot{a})_1^{(m)} - \frac{v^{\frac{1}{m}}}{m} && (18)\\
&= v^{\frac{1}{m}}(1+i)\left(\ddot{a}_{x:1}^{(m)} - \frac{v}{i^{(m)}}\right) - \frac{v^{\frac{1}{m}}}{m} && (19)\\
&= v^{\frac{1}{m}}(1+i)\left[\frac{1-v}{d^{(m)}} - \frac{v}{i^{(m)}}\right] - \frac{v^{\frac{1}{m}}}{m} && (20)\\
&= \frac{(1+i)-1-d^{(m)}}{(i^{(m)})^2} - \frac{v^{\frac{1}{m}}}{m} && (21)\\
&= \frac{i-d^{(m)}}{(i^{(m)})^2} - \frac{v^{\frac{1}{m}}}{m} && (22)
\end{aligned}
$$

Como $v^{\frac{1}{m}} = \frac{1}{1+(i^{(m)})/m} = \frac{m}{m+i^{(m)}}$, entonces:

$$
\begin{aligned}
&\frac{m(i-d^{(m)}) - (i^{(m)})^2 v^{\frac{1}{m}}}{(i^{(m)})^2 m} && (23)\\
&= \frac{m(i-d^{(m)}) - (i^{(m)})^2 \frac{m}{m+i^{(m)}}}{(i^{(m)})^2 m} && (24)\\
&= \frac{mi - \frac{i^{(m)}m^2}{m+i^{(m)}} - \frac{(i^{(m)})^2 m}{m+i^{(m)}}}{(i^{(m)})^2 m} && (25)\\
&= \frac{mi - \frac{i^{(m)}m^2 + (i^{(m)})^2 m}{m+i^{(m)}}}{(i^{(m)})^2 m} && (26)\\
&= \frac{m((i-i^{(m)})(m+i^{(m)}))}{(m+i^{(m)})(i^{(m)})^2 m} && (27)\\
&= \frac{i-i^{(m)}}{(i^{(m)})^2} && (28)
\end{aligned}
$$

Por lo tanto:

$$
rA_x v^{\frac{1}{m}}(1+i)\frac{1}{m^2}\sum_{j=0}^{m-1} jv^{\frac{j}{m}} = rA_x\frac{i-i^{(m)}}{(i^{(m)})^2} \tag{29}
$$

Así, juntando i) y ii):

$$
\begin{aligned}
&\sum_{k=0}^{\infty}\sum_{j=0}^{m-1}\left(1+kr+r\frac{j}{m}\right)v^{k+\frac{j+1}{m}}\ {}_{k+\frac{j}{m}}p_x\frac{1}{m}q_{x+k+\frac{j}{m}} && (30)\\
&= \frac{i}{i^{(m)}}[A_x + r[(IA)_x - A_x]] + r\frac{i-i^{(m)}}{(i^{(m)})^2}A_x \quad \blacksquare && (31)
\end{aligned}
$$

4. Crecimiento geométrico

5. Tabla y gráfica de comparación