Bioinformatics I
WS 15/16
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1	2	3	4	$\sum$ (7)

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Assignment 5

(Handed in 16. November 2015)

Theoretical Assignment - *Comparison with at most l mismatches*

General we are now only looking at the worst case scenario, which means uniform distributed mistmatches in an alignment. Also we only look at $k >= 1$, as $k = 0$ is in our application not a useful result. Assume two sequences of length t with l mismatches.
Than both sequences contain l tuples of length $L = \lfloor \frac{t}{l+1} \rfloor$ and one tuple which has a length of maximal $\lfloor \frac{t}{l+1} \rfloor$.

So both sequences share l or $l+1$ k-tuples, if $k = \lfloor \frac{t}{l+1} \rfloor$, and for each $k \leq \lfloor \frac{t}{l+1} \rfloor$ they share $(l+1) * \lfloor \frac{L}{k} \rfloor$ k-tuples.

Theoretical Assignment - *Linear programming by hand*

The feasible region of this linear program is shown in figure 1, where the red line is constraint 1, green line is constraint 2, constraint 3 is drawn as a blue line and the yellow line is constraint 4.

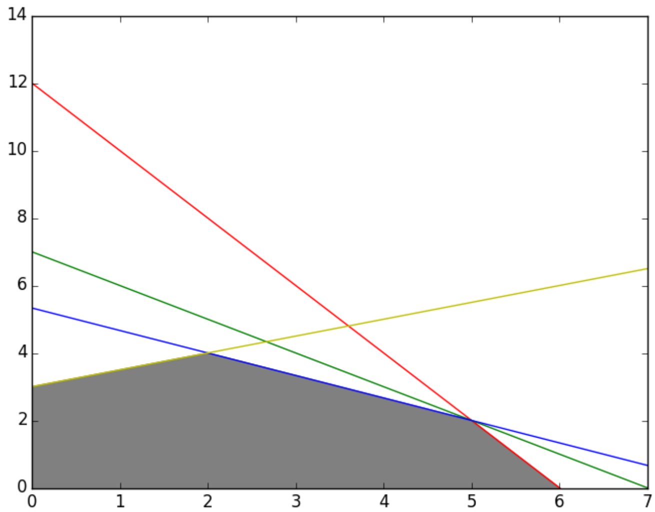


Figure 1: feasible region of linear program (grey area)

solution if t=s=1

Generally we only used integers in this task. The constraint for the algorithm are as following if t and s value is 1 now:

$$max.1 * x_1 + 1 * x_2 \quad (1)$$
$$s.t.2 * x_1 + x_2 \leq 12 \quad (2)$$
$$2 * x_1 + 3 * x_2 \leq 7 \quad (3)$$
$$-1 * x_1 + 2 * x_2 \leq 16 \quad (4)$$
$$x_1, x_2 \geq 0 \quad (5)$$

the next step we started to compute each line of the tuple. Starting with constraint 3 under the aspect of constraint 5. All possible tuples respectively to both constraints from the assignment are:
$temp1 = \{(0,7); (1,6); (2,5); (3,4); (4,3); (5,2); (6,1); (7,0)\}$

the constraint 2 was applied to temp1:
$temp2 = \{(0,7); (1,6); (2,5); (3,4); (4,3); (5,2); \}$

the next constraint (constraint 4) was applied:
$temp3 = \{(5,2)\}$

but not least the last constraint (4) is checked and the remaining tuple from the last step is valid:
$r = \{(5,2)\}$

the final result for the Algorithm with s=t=1 is the tuple: (5,2). Solving the formula with this numbers leads us to the following maximal value: $1 * x_1 + 1 * x_2 = 5 + 2 = 7$

How to make unsolvable?

To get a not solvable algorithm for example the following constraint could be added:

$$x_1 + x_2 \geq 7 \quad (6)$$

a new constraint just needs to be a contradiction to another constraint.

How to get infinite solutions with t and s?

Getting infinite solutions, requires to describe with the max function of a linear algorithm a line in the feasible region and not just one point. Therefore we decided to use constraint 3 and wanted to generate a max function along constraint (3). For the modelling of the max function we used two points from constraint (3) $(P_1(2,4); P_2(5,2))$ to determine $s = \frac{2}{3}$ and $t = 1$ in the following equations.

$$
\begin{aligned}
s * x1_{P_1} + t * x2_{P_1} &= s * x1_{P_2} + t * x2_{P_2} \\
\Leftrightarrow s * 2 + t * 4 &= s * 5 + t * 2 \\
\Leftrightarrow 2t &= 3s \\
\Leftrightarrow s &= \frac{2}{3} t \\
\rightarrow t &= 1
\end{aligned}
$$

Now we controlled the resuld by inserting the extreme data points P_1 and P_2.

$$
\begin{aligned}
t &= 1 \\
\frac{2}{3} * t * x_1 + t * x_2 & \\
\Leftrightarrow \frac{2}{3} * x_1 + x_2 &
\end{aligned}
$$

$$
\frac{2}{3} * 2 + 4 = \frac{2}{3} * 5 + 2 = 5 + \frac{1}{3}
$$

Not serious ;) The equation influence by t and s is:

$$
max : s * x_1 + t * x_2
$$

way to get an infinite solution could be to generate a endless loop with values for s and t like:

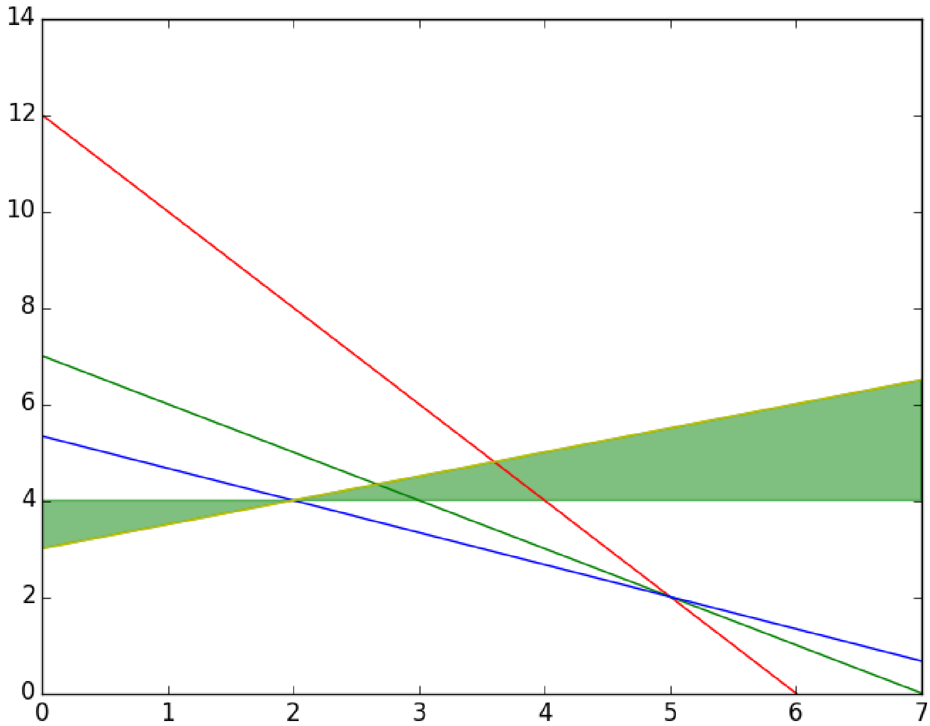
$$
\begin{aligned}
s &= s * x_1 \\
t &= t * x_2 \\
& or \\
s &= t * x_2 \\
t &= s * x_1
\end{aligned}
$$

how to get the points?

For the first point $P_1 = (0, 3)$ we have to set $s = -1$ and $t = 1$. That result in a maximal value for (0,3) because we cannot set x_2 arbitrary big due to constraint 4. And if we assign a bigger value to x_1, would decrease the result. Also we would violate constraint 2, if x_1 is to big.

P2=(2,4)

To achive that P(2,4) is the only maximal value, we need to use the borders used in figure . The lower one is described by the maximal constraint (4) (equation 8)and the higher one is described by the horizontal line at $x2 = 4$. Next we can define relations, for the variables m_2 and c_2 of the searched linear equation. These are shown in equation 11 and .



$$
\begin{aligned}
-1x_1 + 2x_2 &= 6 && (7)\\
\Leftrightarrow x_2 &= \frac{1}{2}x_1 + 3 && (8)\\
= x_2 &= m_1x_1 + c_1 && (9)\\
& && (10)\\
m_1 &> m_2 > 0 && (11)\\
c_1 &< c_2 < 4 && (12)\\
& && (13)
\end{aligned}
$$

So any linear equation, which lies in the green area of figure , is a possible solution. We will now just choose one and define $c_2 = 3.5 = \frac{7}{2}$ according to relation . From this now we can deduce $m_2 = \frac{1}{4}$ as shown in Equation 15 so that P(2,4) is the only intersection with the feasible region.

$$
\begin{aligned}
4 &= m_2 * 2 + 3.5 && (14)\\
m_2 &= \frac{4-3.5}{2} = \frac{1}{4} && (15)\\
& && (16)
\end{aligned}
$$

The last step is now to get s and t, this is achieved by forming the found linear equation back to the desired form (Equation 19). The complete result is for $s = \frac{-1}{14}$ and for $t = \frac{2}{7}$.

$$
\begin{align}
x_2 &= \frac{1}{4} * x_1 + \frac{7}{2} \tag{17}\\
\Leftrightarrow -\frac{1}{4}x_1 + x_2 &= \frac{7}{2} \tag{18}\\
\Leftrightarrow -\frac{1}{14}x_1 + \frac{2}{7}x_2 &= 1 \tag{19}\\
\Rightarrow s = -\frac{1}{14}; t &= \frac{2}{7} \tag{20}\\
\tag{21}
\end{align}
$$

P3=(1,3)

To make $P_4 = (6, 0)$ the maximum we set $s = 1$ and $t = -1$. We cannot set x_1 to a bigger value than 6 without violating constraint 1.

Theoretical Assignment - *Bonus: Carillo-Lipman bound*