Error Control Codes

Course Code: COE 3206 Course Title: Computer Networks

Dept. of Computer Science
Faculty of Science and Technology

Instructed by: Dr. Nazib Abdun Nasir

Lecture Outline

1. Linear block code

Linear Block Code

Generator Matrix

Linear Block Code: A code in which addition of any two codewords gives another codeword [2].

Message, M: k bits long

Redundant bits, Q: q bits long

Codeword length, N: $k+q$ bits long

Generator matrix, $G = [P_{k\times q} I_k]$

For $k = 3$ and q= 3,

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{3\times 3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, it is a $(n, k) = (6, 3)$ block code

Linear Block Code....

Codeword calculation

The codeword for the message [0 1 1] is

$$C = M \times G$$

$$C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$C = [\underbrace{1 \quad 1 \quad 0}_{Q} \quad \underbrace{0 \quad 1 \quad 1}_{M}]$$

Modulo-2 summation

$$0 \times 1 \oplus 0 \times 1 \oplus 1 \times 1 = 1$$
$$0 \times 1 \oplus 1 \times 1 \oplus 1 \times 0 = 1$$
$$0 \times 0 \oplus 1 \times 1 \oplus 1 \times 1 = 0$$
$$0 \times 1 \oplus 1 \times 0 \oplus 1 \times 0 = 0$$
$$0 \times 0 \oplus 1 \times 1 \oplus 1 \times 0 = 1$$
$$0 \times 0 \oplus 1 \times 0 \oplus 1 \times 1 = 1$$

Linear Block Code….

Error-detection

Receiving end

Parity check matrix,

$$H = [I_q \;\; P^T_{k\times q}]$$

$$H = [I_3 \;\; P^T_{3\times 3}]$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$P_{3\times 3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^T_{3\times 3} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$P^T_{3\times 3}$ is the transpose of $P_{3\times 3}$

Linear Block Code....

Error-detection....

Suppose that there is no error in the received sequence.

Hence the received sequence, r, is the same as the transmit sequence, C.

$$r = C$$

$$r = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Syndrome, $s = rH^T$

$$s = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$s = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

The all-zero syndrome indicates a correct reception !

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Linear Block Code....

Error-detection....

Suppose that there is an error in the received sequence.
The second bit (from left side) has altered from 1 to 0

$$r = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]$$

Syndrome, $s = rH^T$

$$s = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$s = [0 \quad 1 \quad 0]$$

The non-zero syndrome indicates an erroneous reception !

Linear Block Code….

Error-correction

How to correct the error?

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

1. Syndrome , $s = [0 \quad 1 \quad 0]$
2. Locate the syndrome in H^T
3. It is in second row
4. So, the second element in the received sequence, $r = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1]$ is erroneous.
4. Alter the second bit from 0 to 1.
5. So the correct received sequence is [1 1 0 0 1 1].

Note: The given generator matrix enables correction of at most 1 bits.

It is possible to correct more bits , but it requires quite a lot work! No Free Lunch!

Homework

- Consider a (7, 4) code whose generator matrix is given by

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find all the codewords of the code

(b) Find the parity-check matrix

(c) Find the syndrome for the received vector [1 1 0 1 0 1 0]. Is it a valid codeword?

References

[1] W. Stallings, *Data and Computer Communication*, 10th ed., Pearson Education, Inc., 2014, USA, pp. 194 - 196.

[2] B. Sklar, *Digital Communications*, 2nd ed., Prentice Hall. 2017, USA, pp. 328 - 345.

Recommended Books

1. **Data Communications and Networking**, *B. A. Forouzan*, McGraw-Hill, Inc., Fourth Edition, 2007, USA.
2. **Computer Networking: A Top-Down Approach**, *J. F., Kurose, K. W. Ross*, Pearson Education, Inc., Sixth Edition, USA.
3. **Official Cert Guide CCNA 200-301 , vol. 1**, *W. Odom*, Cisco Press, First Edition, 2019, USA.
4. **CCNA Routing and Switching**, *T. Lammle*, John Wily & Sons, Second Edition, 2016, USA.
5. **TCP/IP Protocol Suite**, *B. A. Forouzan*, McGraw-Hill, Inc., Fourth Edition, 2009, USA.
6. **Data and Computer Communication**, *W. Stallings*, Pearson Education, Inc., Tenth Education, 2013, USA.