# ECE 509 (Spring'25): Homework #7

70 points

**Problem 1 (5 points):** Complete Exercise 2.5 from Boyd and Vandenberghe.

**Problem 2 (5 points):** Complete Exercises 2.8(b) and 2.8(c) from Boyd and Vandenberghe. Ignore the second sentence in the main problem statement starting with "If possible, express $S$ in the form $S = \{x \mid Ax \preceq b, Fx = g\}$."

**Problem 3 (12 points):** Complete Exercises 2.12(a), 2.12(b), and 2.12(c) from Boyd and Vandenberghe.

**Problem 4 (5 points):** Complete Exercises 2.15(a) and 2.15(b) from Boyd and Vandenberghe.

**Problem 5 (8 points):** Complete Exercises 3.16(b) and 3.16(c) from Boyd and Vandenberghe, but change the question statement to: "For each of the following functions, determine whether it is convex, concave, or neither."

**Problem 6 (5 points):** Prove that the sublevel set of a convex function is always convex.

**Problem 7 (5 points):** Let $f$ be convex. Prove that $f(Ax + b)$ is also convex; that is, show that the composition of a convex function with an affine map is convex.

**Problem 8 (5 points):** Complete Exercise 4.1 from Boyd and Vandenberghe, but only the part: "Make a sketch of the feasible set."

**Problem 9 (5 points):** Complete Exercise 4.3 from Boyd and Vandenberghe.

**Problem 10 (15 points):** Complete Exercise 4.1 from Boyd and Vandenberghe using CVXPY. You should implement and solve the problem using Python. Submit both your code and its output. You may use a Jupyter notebook or a standalone Python script.

The following code snippet gives a basic CVXPY setup to solve a linear program. Use this as a starting point and modify it according to the objective and constraints of the problem.

```
import cvxpy as cvx

# Define variables
x1 = cvx.Variable()
x2 = cvx.Variable()

# Define constraints (inequalities)
constraints = [
    2*x1 + x2 >= 1,
    x1 + 3*x2 >= 1,
    x1 >= 0,
    x2 >= 0
]

# Define the objective
objective = cvx.Minimize(x1 + x2)

# Formulate and solve the problem
```

```
prob = cvx.Problem(objective, constraints)
prob.solve()

# Output the result
print("Optimal value:", prob.value)
print("Optimal x1:", x1.value)
print("Optimal x2:", x2.value)
```

**Note:** To include **equality constraints** in CVXPY, simply use the `==` operator. For example, to enforce $x + y = 5$, you would write:

```
x = cvx.Variable()
y = cvx.Variable()
constraints = [x + y == 5]
```

This can be combined with inequality constraints in the same list. CVXPY will handle both types of constraints when solving the optimization problem.