ECE 509 (Spring'25): Homework #5

75 points

Problem 1 (10 points): Recall that Newton's method with a full Newton step (i.e., step size is 1) is given by the iteration:

$$x^{(k+1)} = x^{(k)} - (\nabla^2 f(x^{(k)}))^{-1} \nabla f(x^{(k)}).$$

Consider the quadratic function:

$$f(x) = \frac{1}{2} x^\top P x + q^\top x + r,$$

where P is a positive definite matrix.

1. Show that applying one step of full Newton's method from an arbitrary starting point $x^{(0)}$ finds the exact minimizer of $f(x)$.
2. Can you discuss why this might be happening? Think about the derivation of Newton's method in Class #10.

Problem 2 (10 points): Complete Exercise 9.10 from Boyd and Vandenberghe. Submit well-commented code along with a clear explanation of your observations, drawing on the discussions from Class #11.

Problem 3 (20 points): Complete Exercise 9.30 from Boyd and Vandenberghe. As in Homework #4 for gradient descent, structure your implementation of Newton's method in a modular way, allowing it to take different input parameters, as you will be reusing it throughout the course.

In part (b), disregard the instruction related to "Look for quadratic convergence" and anything beyond that, as we will cover this concept later in the course in the context of convex optimization. However, compare the convergence speed of gradient descent and Newton's method for this problem, and provide comments on their relative performance.

Submit your well-commented code, all output results including numerical values and plots, and a detailed answer to the comparison question.

Problem 4 (5 points): Complete Exercise 2.1 from Boyd and Vandenberghe.

Problem 5 (10 points): Complete Exercise 2.2 from Boyd and Vandenberghe.

Problem 6 (20 points): Complete Exercise 3.1 from Boyd and Vandenberghe.