

Special Homework

Provide a carefully explained solution to all the questions in the following problem:

Problem. Two matrices A and B are said to be similar, if there exists an invertible matrix (a “change of basis matrix”) C , so that $A = C^{-1}BC$.

(a) Show that if matrices A and B are similar then they have the same eigenvalues.

(b) Show that if matrices A and B are similar and A is diagonalizable then B is also diagonalizable.

(c) Are the matrices

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

similar? Justify your answer.

(d) Determine if any of the following three matrices are similar? In each of the three cases justify your answer.

$$A_1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$