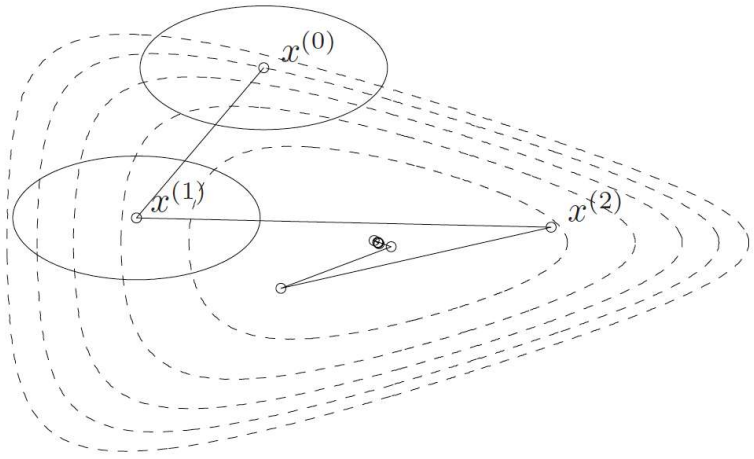
Section 9.4.4

Thursday, February 22, 2024 1:28 PM

$f(x) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1-0.1}$



$P_1 = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$

$P_2 = \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$

Figure 9.11 Steepest descent method with a quadratic norm $\|\cdot\|_{P_1}$. The ellipses are the boundaries of the norm balls $\{x \mid \|x - x^{(k)}\|_{P_1} \leq 1\}$ at $x^{(0)}$ and $x^{(1)}$.

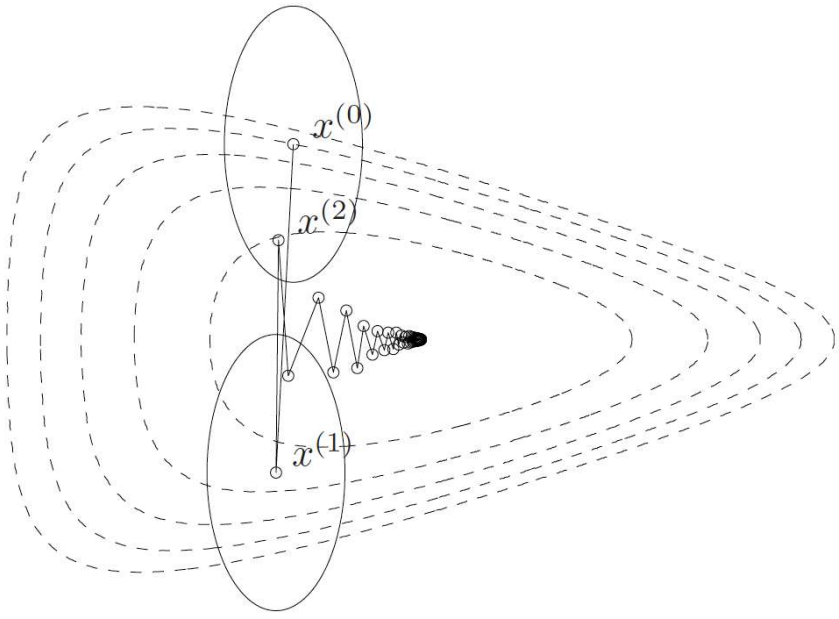


Figure 9.12 Steepest descent method, with quadratic norm $\|\cdot\|_{P_2}$.

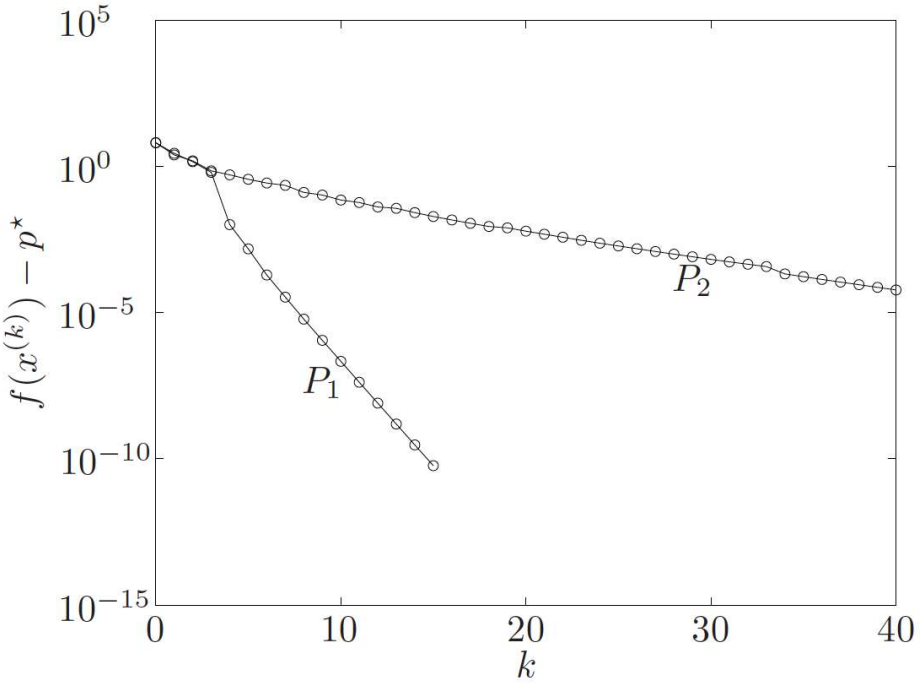


Figure 9.13 Error $f(x^{(k)}) - p^\star$ versus iteration k, for the steepest descent method with the quadratic norm $\|\cdot\|_{P_1}$ and the quadratic norm $\|\cdot\|_{P_2}$. Convergence is rapid for the norm $\|\cdot\|_{P_1}$ and very slow for $\|\cdot\|_{P_2}$.

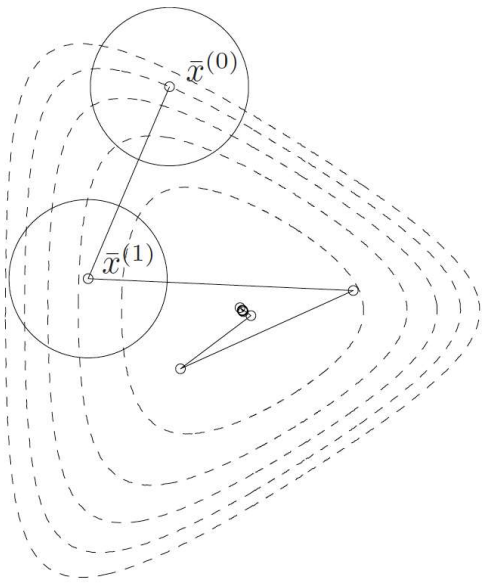


Figure 9.14 The iterates of steepest descent with norm $\|\cdot\|_{P_1}$, after the change of coordinates. This change of coordinates reduces the condition number of the sublevel sets, and so speeds up convergence.

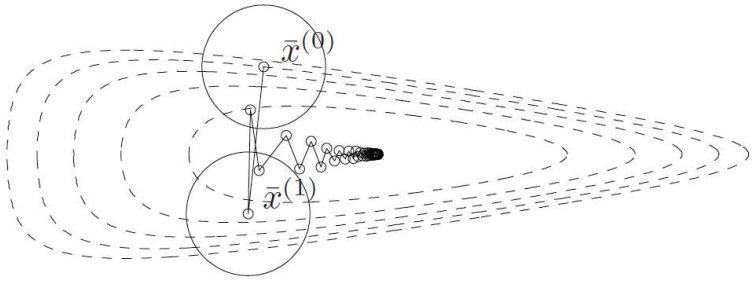


Figure 9.15 The iterates of steepest descent with norm $\|\cdot\|_{P_2}$, after the change of coordinates. This change of coordinates increases the condition number of the sublevel sets, and so slows down convergence.