**Problem 1.**
Let $\mathbb{I}(\mathbf{x}) = \mathbf{x}$ be the identity mapping $\mathbb{R}^3 \to \mathbb{R}^3$. Let $B_1$ denote the canonical basis $\{[1\ 0\ 0]^T, [0\ 1\ 0]^T, [0\ 0\ 1]^T\}$ and let $B_2$ denote the basis $\{[1\ 1\ 1]^T, [1\ 0\ 1]^T, [0\ 0\ 2]^T\}$.

(a) Find the matrix representation of $\mathbf{x} \to \mathbb{I}(\mathbf{x})$ when using basis $B_2$ in preimage and basis $B_1$ in the image.

(b) Find the matrix representation of $\mathbf{x} \to \mathbb{I}(\mathbf{x})$ when using basis $B_1$ in preimage and basis $B_2$ in the image.

(c) What is the relation between the two matrices you found in (a) and (b)?

Let $A$ denote the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix},$$

and let $L : \mathbb{R}^3 \to \mathbb{R}^3$ denote the linear mapping $L(\mathbf{x}) = A\mathbf{x}$.

(d) Find the matrix representation of $L$ in the basis $B_1$ (for both image and preimage).

(e) Find the matrix representation of $L$ in the basis $B_2$ (for both image and preimage) and express it in terms of matrix multiplication of $A$ by the two matrices you found in (a) and (b).

Problem **2.**
Let $B$ denote the $3 \times 4$ matrix

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix},$$

(a) Find a decomposition $B = LU$ where $L$ is a lower triangular $3 \times 3$ matrix and $U$ is a $3 \times 4$ matrix in **reduced** row echelon form.

(b) What is the rank of the matrix $B$ above.

(c) Find $R(B)$, the range of $B$ and $N(B)$, its nullspace.

Problem **3.**
Let $L$ denote the operator

$$\mathcal{P}_3 \ni p \to Lp(t) = \frac{d}{dt}\left[(1-t^2)\frac{d}{dt}p(t)\right].$$

(a) Show that $L$ is a linear operator from $\mathcal{P}_3$ to $\mathcal{P}_3$ (polynomials of degree $\leq 3$).

(b) Show that $\{1, t, t^2, t^3\}$ is a basis for $\mathcal{P}_3$, and find the matrix $A$, that represents $L$ in that basis.

(c) Show that $\{1, t, t^2 - \frac{1}{3}, t^3 - \frac{3}{5}t\}$ is a basis for $\mathcal{P}_3$ and find the matrix $B$, that represents $L$ in that basis.

(d) Find the coordinate transformation matrix $C$ which relates $A$ to $B$ by the change basis formula $B = C^{-1}AC$.