Problem 1.
(a) Are the three functions

$$\sin^2 t \ , \quad \cos^2 t \ , \quad \text{and } 1$$

linearly dependent or independent? Provide an argument!
(b) Are the three functions

$$\sin^2 t \ , \quad \cos^2 t \ , \quad \text{and } \sin t$$

linearly dependent or independent? Provide an argument!

Problem 2.
Calculate the determinant of the $(n+2) \times (n+2)$ matrix

$$\begin{bmatrix} 1 & 2 & 0 & & & & 0 \\ 2 & 1 & 0 & . & . & & 0 \\ & & 1 & & & & \\ & & & 2 & & & \\ 0 & . & & . & . & & 0 \\ & & & . & & & \\ . & . & . & 0 & & 0 & n \end{bmatrix} ,$$

i.e., the symmetric matrix with $a_{11} = a_{22} = 1$, $a_{12} = a_{21} = 2$, $a_{i+2\ i+2} = i$ for $1 \le i \le n$, and $a_{ij} = 0$ otherwise.

Problem 3. With $\mathbf{v}(t) = (v_1(t), v_2(t))^T$ solve the initial value problem

$$\frac{d}{dt}\mathbf{v} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \mathbf{v} \ , \qquad \mathbf{v}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ .$$

Problem 4. Find a simple formula for

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^N ,$$

valid for any integer N.

Problem 5. Find the QR decomposition of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} .$$

Problem 6. Let A denote the synmetric matrix

$$A = \begin{bmatrix} 1 & \sqrt{6} \\ \sqrt{6} & 2 \end{bmatrix} .$$

(a) Find the characteristic polynomial for A.
(b) calculate the expression

$$A^2 - 3A - 4I \ .$$

can you explain your result?