Section 9.3.2

Monday, March 10, 2025 10:47 PM

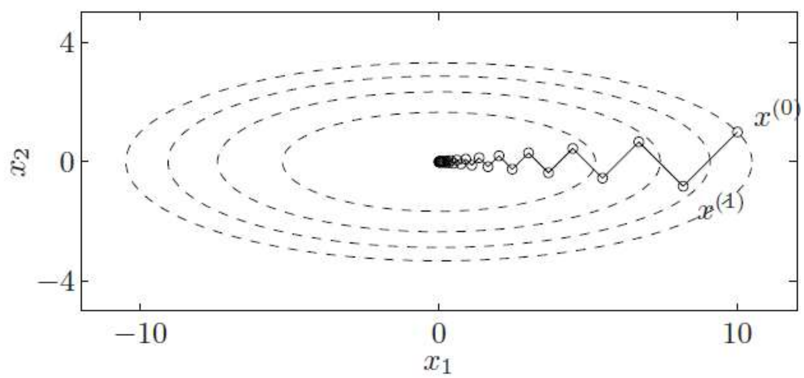

Figure 9.2 Some contour lines of the function $f(x) = (1/2)(x_1^2 + 10x_2^2)$. The condition number of the sublevel sets, which are ellipsoids, is exactly 10. The figure shows the iterates of the gradient method with exact line search, started at $x^{(0)} = (10, 1)$.

$f(x)$ with $x \in \mathbb{R}^2$; $f(x) = e^{x_1+3x_2-0.1} + e^{x_1-3x_2-0.1} + e^{-x_1-0.1}$

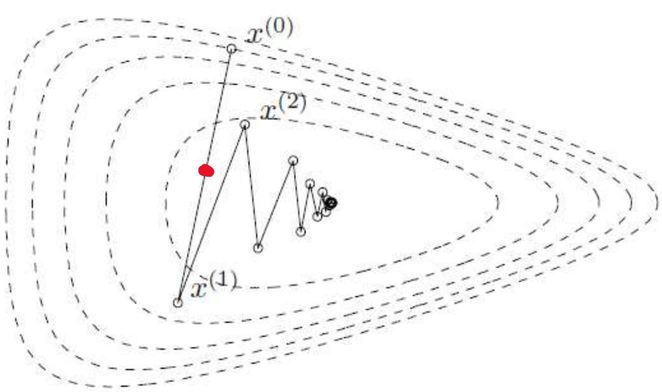

Figure 9.3 Iterates of the gradient method with backtracking line search, for the problem in $\mathbf{R}^2$ with objective f given in (9.20). The dashed curves are level curves of f, and the small circles are the iterates of the gradient method. The solid lines, which connect successive iterates, show the scaled steps $t^{(k)}\Delta x^{(k)}$.

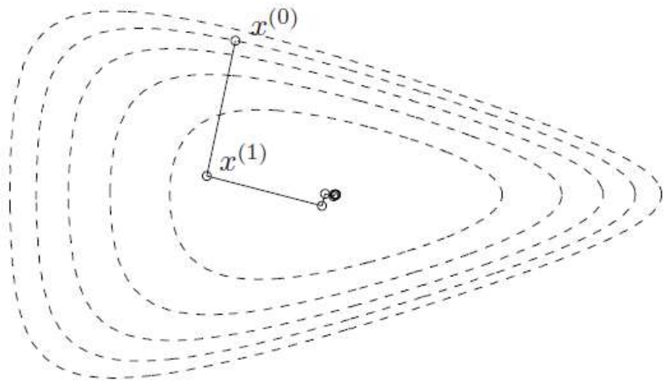

Figure 9.5 Iterates of the gradient method with exact line search for the problem in $\mathbf{R}^2$ with objective f given in (9.20).

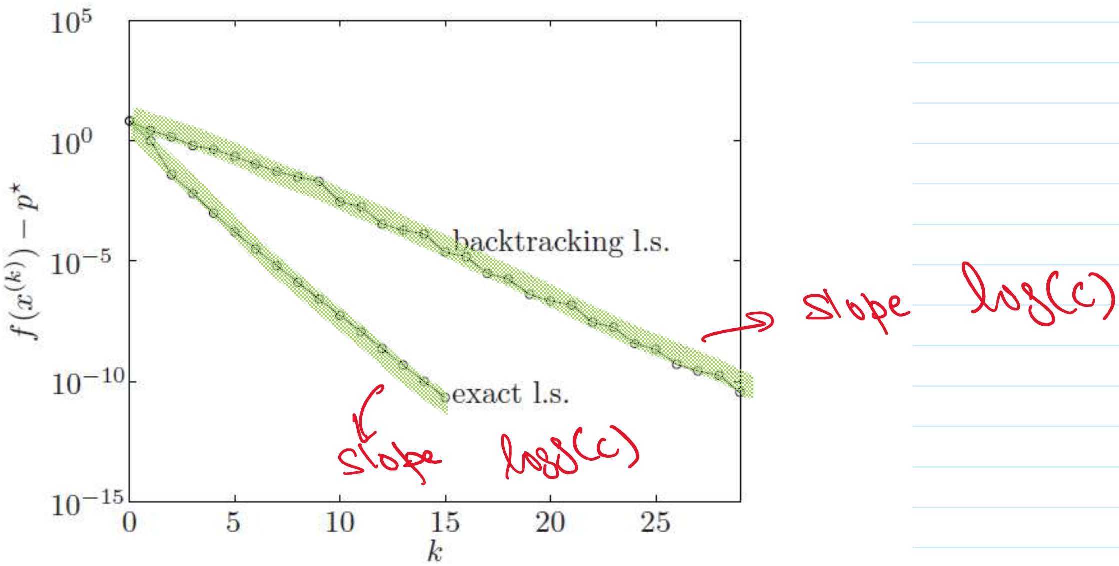

Figure 9.4 Error $f(x^{(k)}) - p^\star$ versus iteration k of the gradient method with backtracking and exact line search, for the problem in $\mathbf{R}^2$ with objective f given in (9.20). The plot shows nearly linear convergence, with the error reduced approximately by the factor 0.4 in each iteration of the gradient method with backtracking line search, and by the factor 0.2 in each iteration of the gradient method with exact line search.

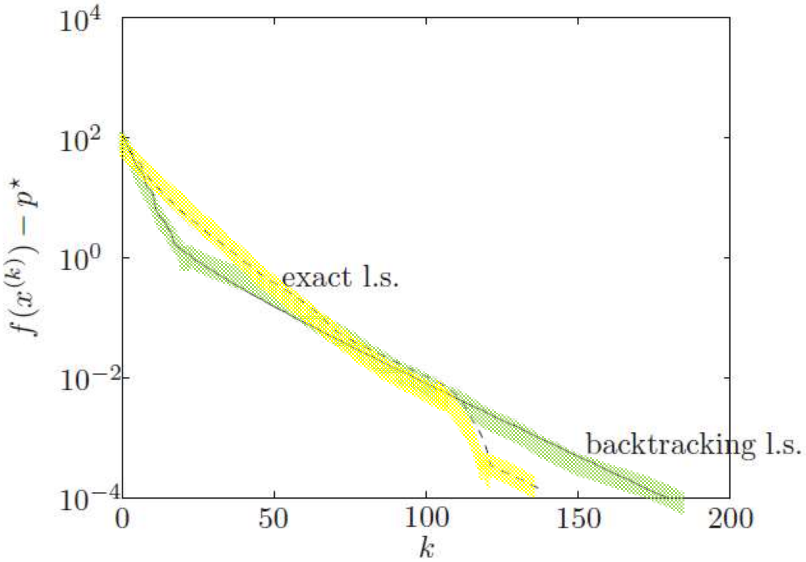

Figure 9.6 Error $f(x^{(k)}) - p^\star$ versus iteration k for the gradient method with backtracking and exact line search, for a problem in $\mathbf{R}^{100}$.

$f(x) : x \in \mathbb{R}^{100}$; $f(x) = c^T x - \sum_{i=1}^{m} \log(b_i - a_i^T x)$ with $m = 500$

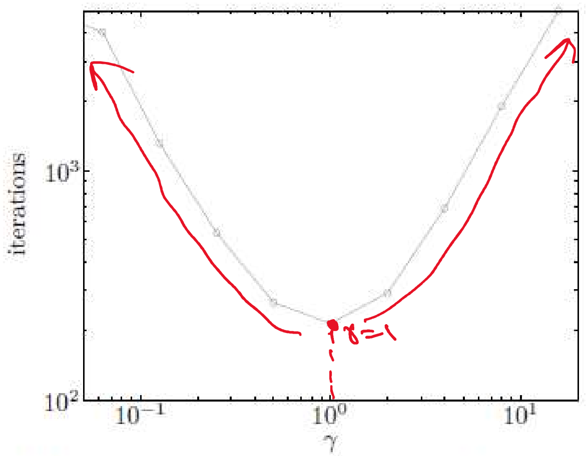

Figure 9.7 Number of iterations of the gradient method applied to problem (9.22). The vertical axis shows the number of iterations required to obtain $\bar{f}(\bar{x}^{(k)}) - \bar{p}^\star < 10^{-5}$. The horizontal axis shows γ, which is a parameter that controls the amount of diagonal scaling. We use a backtracking line search with $\alpha = 0.3$, $\beta = 0.7$.

$$f(x) = c^T \Lambda x - \sum_{i=1}^{m} \log(b_i - a_i^T \Lambda x)$$

$$\Lambda = \operatorname{diag}(1, \gamma^{1/n}, \gamma^{2/n}, \ldots, \gamma^{(n-1)/n})$$

$$\gamma > 0$$

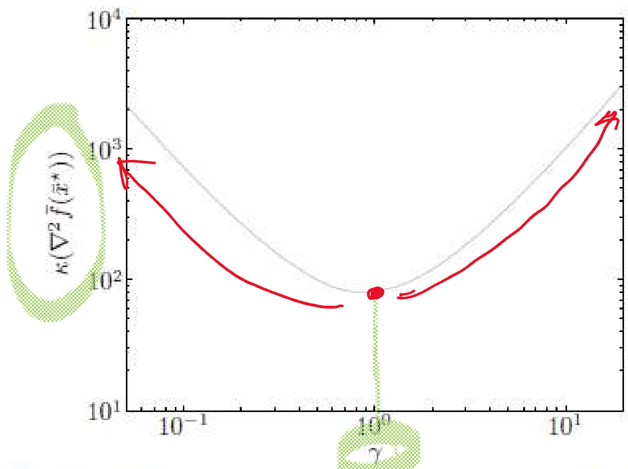

Figure 9.8 Condition number of the Hessian of the function at its minimum, as a function of γ. By comparing this plot with the one in figure 9.7, we see that the condition number has a very strong influence on convergence rate.