ECE 509 (Spring'25): Homework #8

45 points

Problem 1 (6 points): Let $a_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $a_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $a_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^2$. Let $C = \text{cone}\{a_1, a_2\}$ and $D = \text{cone}\{a_1, a_2, a_3\}$, where

$$\text{cone}\{v_1, \ldots, v_k\} = \left\{ \sum_{i=1}^{k} \theta_i v_i \mid \theta_i \geq 0 \right\}.$$

Sketch the sets C and D in $\mathbb{R}^2$. Label the generating vectors clearly.

Problem 2 (5 points): Complete Exercise 2.16 from Boyd and Vandenberghe.

Problem 3 (5 points): Complete Exercise 2.6(a) from *Additional Exercises for Convex Optimization* by Boyd and Vandenberghe.

Problem 4 (6 points): Suppose $f : \mathbb{R} \to \mathbb{R}$ is increasing and convex on its domain (a, b). Let g denote its inverse, i.e., the function with domain $(f(a), f(b))$ such that $g(f(x)) = x$ for all $x \in (a, b)$. Use the definition of concavity based on the hypograph (i.e., the set $\{(x, t) \mid x \in \text{dom } g,\ t \leq g(x)\}$ is convex) to prove that g is a concave function.

Problem 5 (8 points): Complete Exercise 3.18(a) from Boyd and Vandenberghe.

Problem 6 (5 points): Complete Exercise 3.21(a) from Boyd and Vandenberghe.

Problem 7 (5 points): Complete Exercise 3.22(a) from Boyd and Vandenberghe.

Problem 8 (5 points): Let $f(x)$ and $g(x)$ be convex, positive, and both nondecreasing (or both nonincreasing) functions on an interval $I \subseteq \mathbb{R}$. Define $h(x) = f(x)g(x)$. Use Jensen's inequality to prove that $h(x)$ is convex on I.