This is predominantly a computational homework

Problem 1.
Write a program (in Matlab, C++ or Python) that, given an $N \times N$ matrix A as input, finds the LU decomposition of A. In writing this program you do not have to take into account pivoting (row interchanges). In other words: you may assume that the row reductions never generate a zero on the diagonal.

Problem 2.
For $N = 4$, $N = 10$ and $N = 20$ run the program you wrote on the **symmetric** matrix A, whose entries are given by

$$\begin{cases} A[i,i] = 6 \quad i = 1, 2 \ldots , N \quad , \\ A[i,i+1] = 1.25 \quad i = 1, 2 \ldots , N-1 \quad , \\ A[i,i+2] = 1.25 \quad i = 1, 2 \ldots , N-2 \quad , \\ A[i,j] = 0 \quad \text{for } |i-j| \geq 3 \quad . \end{cases} \tag{1}$$

Display the full matrices L and U only for $N = 10$.

Problem 3.
(a) Given the LU decomposition of a square matrix A, how can you easily calculate the determinant of A?
(b) Add a few lines to the program you wrote in Problem 1 so that it now also computes the determinant of A. Run the new program on the matrices from Problem 2. Do not display the full matrices L and U, but only the computed determinants of A.

Problem 4.
Let B be the $N \times N$ matrix with entries

$$B[i,j] = \frac{1}{i+j+1} \text{ for } 1 \leq i,j \leq N \ .$$

Run your program (including the determinant computation) on B for $N = 3, 4, 5$ and 10. How does your program perform if you take $N = 20$? Display only the full matrices for $N = 10$. Display the determinants for $N = 3, 4, 5, 10$ and 20 (if available).