Solution to Problem A

Q: A SEQUENCE IS DEFINED BY $A_1 = 1$, $A_{N+1} = A_N + N + 1$.

FIND A CLOSED-FORM EQUATION ~~AND DETERMINE~~ TO CALCULATE THE VALUE OF A_N AND DETERMINE THE VALUE OF A_{100} '

ANS:

$$A_{N+1} = A_N + N + 1$$

$$A_1 = 1$$

$$A_{1+1} = A_1 + 1 + 1 = 1 + 1 + 1$$

$$\Rightarrow A_2 = 3$$

$$A_{2+1} = A_2 + 2 + 1 = 3 + 2 + 1$$

$$\Rightarrow A_3 = 6$$

A

$$\therefore A_{3+1} = A_3 + \cancel{N}\ 3 + 1 = 6 + 3 + 1$$

$$\Rightarrow A_4 = 10$$

$$A_{4+1} = A_4 + 4 + 1 = 10 + 4 + 1$$

$$\Rightarrow A_5 = 15$$

$$\therefore A_1, A_2, A_3, A_4, A_5 = 1, 3, 6, 10, 15$$

2 3 4 5

SO, EVERYTIME WE GO FOR N, It's LIKE, $A_N = 1 + 2 + 3 + \dots + (N-1) + N$

SO THAT'S ~~BINOMIAL N th Triangular Formula~~, TRIANGULAR FORMULA

$$\therefore A_N = \frac{\cancel{N+}\ N \cdot (N+1)}{2}$$

THEREFOR, $A_{100} = \dfrac{100 \times (100+1)}{2} = \dfrac{100 \times 101}{2} = \dfrac{10100}{2} = 5050$

(ANS)

Solution to Problem B

Q: SHOW THAT THERE IS NO $x \in R$ THAT SOLVES THE EQUATION $\sqrt{3+x} + \sqrt{7-x} = 5$.

Ans: $\sqrt{3+x} + \sqrt{7-x} = 5$

$\Rightarrow \sqrt{3+x} = 5 - \sqrt{7-x}$

$\Rightarrow (\sqrt{3+x})^2 = (5-\sqrt{7-x})^2$

$\Rightarrow \cancel{x}\ 3+x = 25 - 2\cdot 5\cdot \sqrt{7-x} + 7 - x$

$\Rightarrow 3+x-25-7+x = \cancel{10}\ -10\sqrt{7-x}$

$\Rightarrow 2x - 29 = -10\sqrt{7-x}$

$\Rightarrow (2x-29)^2 = (-10\sqrt{7-x})^2$

$\Rightarrow 4x^2 - 2\cdot 2x\cdot 29 + 29^2 = 100(7-x)$

$\Rightarrow 4x^2 - 116x + 841 - 700 + 100x = 0$

$\Rightarrow 4x^2 - 16x + 141 = 0$

$\Rightarrow x = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4\cdot 4\cdot 141}}{2\cdot 4}$

$\Rightarrow x = \dfrac{16 \pm \sqrt{256 - 2256}}{8}$

$\Rightarrow x = \dfrac{16 \pm \sqrt{-2000}}{8}$

~~Here~~ $= \dfrac{16 + \sqrt{-2000}}{8}, \dfrac{16 - \sqrt{-2000}}{8}$

Here,

$\sqrt{-2000}$ can't be possible. So, This is an imaginary number.

$\therefore x \notin R$ (Showed)

Solution to ~~Ques~~ Problem D

Q: PROVE THAT, EVERY POSITIVE INTEGER n, THE NUMBER n^4+4 IS A COMPOSITE NUMBER, EXCEPT FOR ONE VALUE OF ~~A.~~ n.

Ans: HERE,

n = POSITIVE INTEGER

n^4+4 = COMPOSITE NUMBER

$$= n^4+4n^2+4-4n^2$$

$$= (n^2)^2+2\cdot n^2\cdot 2+2^2-\cancel{4n^2}(2n)^2$$

$$=(n^2+2)^2-(2n)^2$$

$$=(n+2-2n)(n+2+2n)$$

$$=(n+2n+2)(n-2n+2)$$

LET SUPPOSE

$$A = n^2+2n+2, \quad B = n^2-2n+2$$

$$= n^2+2n+1+1, \quad = n^2-2n+1+1$$

$$= (n+1)^2+1^2, \quad = (n-1)^2+1^2$$

IF $n=1$, $A=(1+1)^2+1$, $B=(1-1)^2+1$

$$= 4+1 \qquad = 0+1$$

$$= 5 \qquad = 1$$

IF $n=2$, $A=(2+1)^2+1$, $B=(2-1)^2+1$

$$= 10 \qquad = 2$$

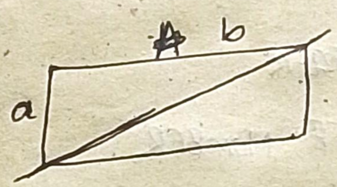
IF $n=3$, $A=(3+1)^2+1$, $B=(3-1)^2+1$

$$= 17 \qquad = 5$$

So, we can see Except $n=1$, Every number consider has more number to devide. (Proved)

Solution to Problem E

Q: A RECTANGLE WITH SIDE LENGTHS ~~A~~ a AND ~~B~~ b IS CUT BY A DIAGONAL AS SHOWN IN THE DRAWING BELOW. FIND THE PERIMETER AND THE AREA OF TWO TRIANGLES FORMED.



ANS: AS SHOWN,

THE RECTANGLE DEVIDED EQUALLY BY A DIAGONAL.

~~CREATED~~ FORMED 2 TRIANGLES.

~~A~~ WE ALL ~~KNOW~~ KNOW THAT

EVERY ANGLES OF A ~~PRORF~~ PROPER RECTANGLE IS 90°.

SO BOTH TRIANGLE ARE FORMED AS RIGHT TRIANGLE.

~~ACC~~ ACCORDING TO PYTHAGORAS,

$$\text{DIAGONAL} = \sqrt{a^2+b^2}$$

LET SUPPOSE, DIAGONAL $= c$

$$\therefore \text{PERIMETER} = a+b+c$$

$$= a+b+\sqrt{a^2+b^2}$$

EACH TRIANGLE'S FORMED AREA $= \frac{1}{2} \times a \times b$

$\therefore$ THE AREA 2 TRIANGLES FORMED $= 2 \times \frac{1}{2} \cdot ab$

$$= ab.$$

(ANS)