Computer Graphics Assignment 4

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Code will be uploaded to
`https://github.com/Riften/SJTU-Computer-Graphics-2020-Assignments`
after deadline.

1 Question 1

Describe the difference in appearance you would expect between a Phnog illumination model that used $(\bar{N} \cdot \bar{H})^n$ and one that used $(\bar{R} \cdot \bar{V})^n$.

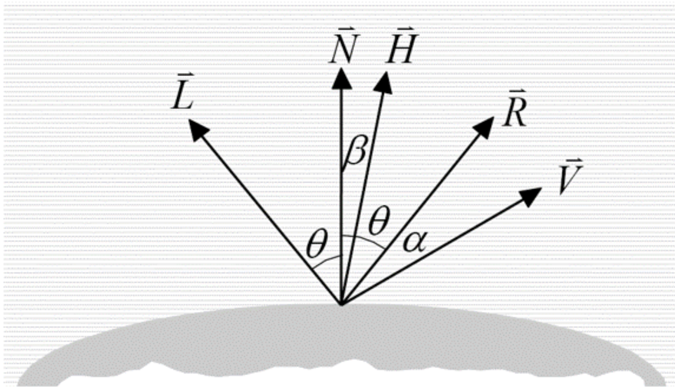


Figure 1.1: Vectors for calculating Phong shading

Solution. The definitions of these vectors are as following

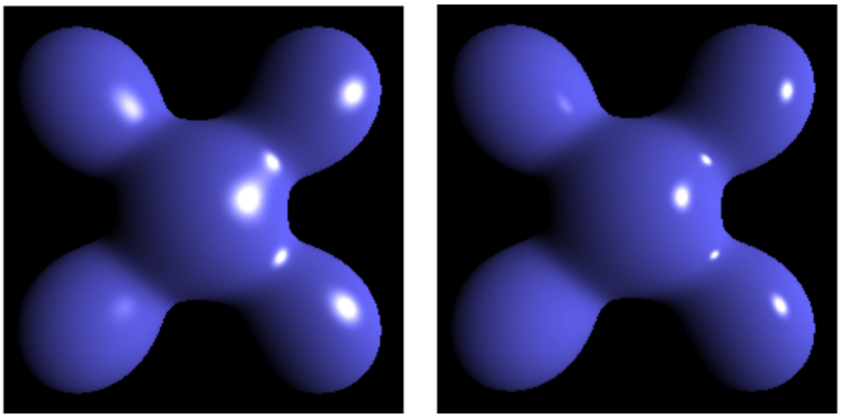
$$\begin{aligned}
&\bar{L} : \text{Direction to the light source.}\\
&\bar{R} : \text{Direction of reflection.}\\
&\bar{V} : \text{Direction to the viewpoint.}\\
&\bar{N} : \text{Surface normal.}\\
&\bar{H} = \frac{\bar{L} + \bar{V}}{||\bar{L} + \bar{V}||}\\
&\quad \text{Halfway vector between the viewer and light-source vectors}
\end{aligned}$$

The intuitive way to calculate specular hightlights is $(\bar{R} \cdot \bar{V})^n$. It gets the maximum when $\bar{R} = \bar{V}$, which is in line with common sence. However, $(\bar{N} \cdot \bar{H})^n$ also gets maximum when $\bar{R} = \bar{V}$. That means the brightest position of the specular hightlights is exactly the same when using these two formulas.

But the size and intensity of hightlights would be different. Note that

$$\bar{R} \cdot \bar{V} = \cos\alpha$$
$$\bar{N} \cdot \bar{H} = \cos\beta$$
$$\beta = \frac{\alpha + 2\theta}{2} - \theta = \frac{\alpha}{2}$$

So there are $\bar{R} \cdot \bar{V} \leq \bar{N} \cdot \bar{H}$, and $\bar{R} \cdot \bar{V} = \bar{N} \cdot \bar{H}$ when $\bar{R} = \bar{V}$. In that case, with the same specular exponent n, Phong illumination model used $(\bar{N} \cdot \bar{H})^n$ would have a larger hightlight area than the one used $(\bar{R} \cdot \bar{V})^n$ as shown in Fig.1.2.

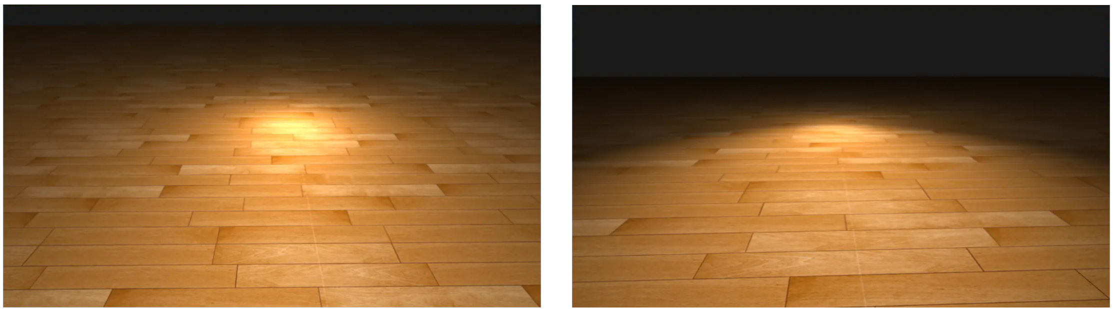


(a) $(\bar{N} \cdot \bar{H})^n$ (b) $(\bar{R} \cdot \bar{V})^n$

Figure 1.2: Phong illumination model used $(\bar{N} \cdot \bar{H})^n$ and $(\bar{R} \cdot \bar{V})^n$[1]

In fact, Phong shading using $(\bar{N} \cdot \bar{H})^n$ to calculate specular hightlights is called **Blinn-Phong Shading**[1].

Another appearance difference between Blinn-Phong and Phong shading model occurs when $\alpha > \pi/2$. In that case, $\bar{R} \cdot \bar{V} < 0$ and the specular hightlight would be 0 for Phong shading model. That would generate a sharp border of hightlights when specular exponent n is small as shown in Fig.1.3. In that case, Blinn-Phong is more similar with real sence.



(a) Blinn Phong (b) Phong

Figure 1.3: Difference between Blinn Phong and Phong shading when $n = 1$.[2]

□

2 Question 2

a. Prove that $\alpha = 2\beta$ when all vectors of Fig.1.1 are coplanar.

Solution. That has been shown in **Question 1**.

When $\bar{V}$ is lower than $\bar{R}$, there is

$$\beta = \frac{\alpha + 2\theta}{2} - \theta = \frac{\alpha}{2}$$

When $\bar{V}$ is between $\bar{L}$ and $\bar{R}$, there is

$$\beta = \theta - \frac{2\theta - \alpha}{2} = \frac{\alpha}{2}$$

When $\bar{V}$ is lower than $\bar{L}$, there is

$$\beta = \theta + \frac{\alpha - 2\theta}{2} = \frac{\alpha}{2}$$

So that $\alpha = 2\beta$ when all vectors are coplanar. $\square$

b. Prove that this relationship is not true in general.

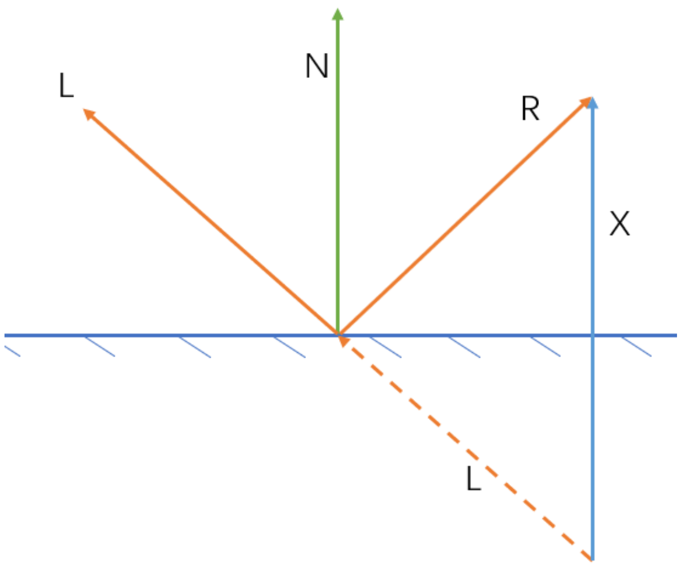


Figure 2.1: Calculate reflected vector

Solution. The basic idea is that $\bar{V}$ can be seen as the reflected vector of $\bar{L}$ about $\bar{H}$. First, there is a simple method to calculate reflected vector. As shown in Fig.2.1, there is

$$\begin{aligned}\bar{X} &= 2(\bar{L} \cdot \bar{N})\bar{N} \\ \bar{R} &= \bar{X} - \bar{L} \\ &= 2(\bar{L} \cdot \bar{N})\bar{N} - \bar{L}\end{aligned}$$

If we see $\bar{V}$ as the reflected vector of $\bar{L}$ about $\bar{H}$, there is

$$\bar{V} = 2(\bar{L} \cdot \bar{H})\bar{H} - \bar{L}$$

So that

$$\begin{aligned}\bar{R} \cdot \bar{V} &= 2(\bar{L} \cdot \bar{H})(\bar{H} \cdot \bar{R}) - \bar{L} \cdot \bar{R} && (2.1) \\ &= 4(\bar{L} \cdot \bar{N})(\bar{L} \cdot \bar{H})(\bar{N} \cdot \bar{H}) - 2(\bar{L} \cdot \bar{N})^2 - 2(\bar{L} \cdot \bar{H})^2 + \bar{L}^2 && (2.2)\end{aligned}$$

Define

$$\begin{aligned}a &= -2 \\ b &= 4(\bar{L} \cdot \bar{N})(\bar{N} \cdot \bar{H}) \\ c &= \bar{L}^2 - 2(\bar{L} \cdot \bar{N})^2 \\ x &= \bar{L} \cdot \bar{H}\end{aligned}$$

Then equation 2.2 would be

$$\bar{R} \cdot \bar{V} = \cos\alpha = ax^2 + bx + c \tag{2.3}$$

Assume that $\bar{H}'$ is a halfway vector that is not coplanar with $\{L, N, H\}$. It can be seen from a coplanar vector $\bar{H}$ rotating around $\bar{N}$. Note that when $\bar{H}$ rotating around $\bar{N}$, $(\bar{N} \cdot \bar{H})$ will remain unchanged.

In that case, when $\bar{H}$ is rotating around $\bar{N}$, both $\{a, b, c\}$ would be unchanged constant. Only x is changing unless $\bar{L} = \bar{N}$.

So when $\bar{H}$ is rotating around $\bar{N}$, β will remain unchanged, while α is changed according to equation 2.3. So $\alpha \neq 2\beta$ when $\bar{V}$ is not conplanar with $\{L, N, H\}$.

$\square$

References

[1] "Blinn–phong reflection model wikipedia," https://en.wikipedia.org/wiki/Blinn%E2%80%93Phong_reflection_model.

[2] "Blog: Blinn–phong," https://www.jianshu.com/p/2294cce3db2c.