DIODE MODEL PARAMETER EXTRACTION

PURPOSE

The purpose of this short project is to learn how to extract the parameters most important for modelling the static characteristcs of the diode. The diode simulated in this project is in fact the LED VSLY5940.

INTRODUCTION

We seek to use the data available in the diode datasheet to determine the numerical values of:

1. Series Resistance (ESR)
2. Ideality Factor (η)
3. Saturation Current (I_S)

A regular datasheet does not usually mention the values of the aforementioned parameters. To obtain the necessary values we must take a graphical approach and analyse the data being provided by the Forward Current vs Forward Voltage graphic (see below).

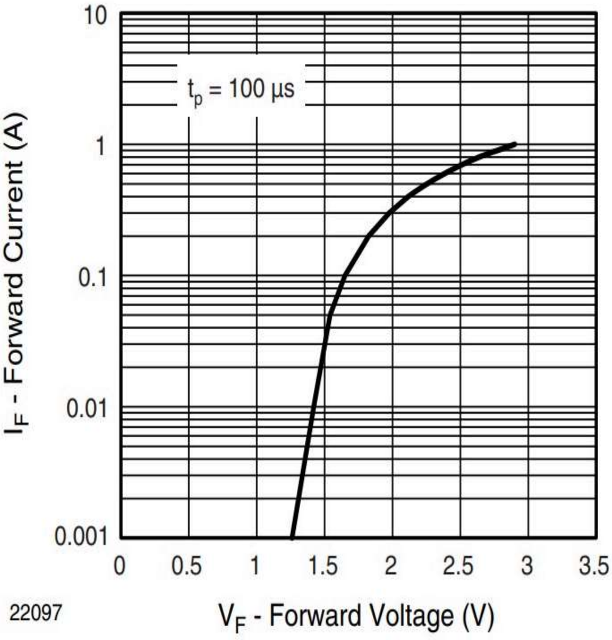
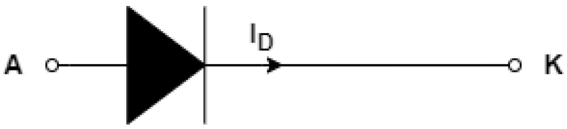


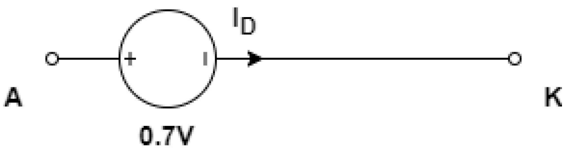
Fig. 4 - Forward Current vs. Forward Voltage

By taking the values of V_F and I_F in three different points spaced reasonably apart (to minimize errors) we will obtain with help from the Shockley Diode Equation $\left(I_D = I_S \cdot exp\left(\frac{V_D}{N \cdot V_{TH}}\right)\right)$ a solvable system of two equations and three unknowns.

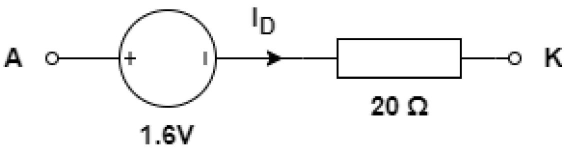
As a brief aside, it is necessary to discuss the various diode models in existance and how they relate to our small project.



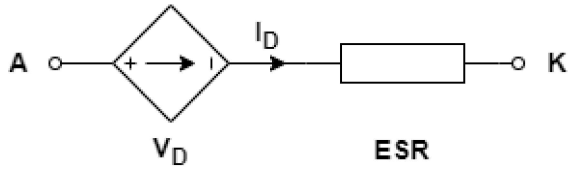
1. LVL 1: The diode has a set voltage drop which is presumed not to vary significantly in most scenarios



3. LVL 2: The diode is replaced by a fixed voltage drop in series with a fixed resistance



3. LVL 3: The diode is replaced by a dependent voltage source in series with a fixed resistance



For our purposes we will use the third diode model, where $V_D = N \cdot V_{TH} \cdot ln\left(\frac{I_D}{I_S}\right)$ is the command function and $V_F = V_D + ESR \cdot I_D$ is the voltage drop experienced by the diode.

EXTRACTING THE PARAMETERS:

Known values: The thermal voltage V_{TH} is 26mV as the test temperature is presumed to be 300K.

$V_F \quad [V]$	$I_F \quad [mA]$
1.25	1
1.5	30
2.875	1000

According to the diode model in use the voltage drop across the diode is $V_F = ESR \cdot I_D + N \cdot V_{TH} \cdot ln\left(\frac{I_D}{I_S}\right)$ (1).

We write (1) for each of the three voltages in the table above:

$$V_{F1} = ESR \cdot I_{D1} + N \cdot V_{TH} \cdot ln\left(\frac{I_{D1}}{I_S}\right) \ (1.1)$$

$$V_{F2} = ESR \cdot I_{D2} + N \cdot V_{TH} \cdot ln\left(\frac{I_{D2}}{I_S}\right) \ (1.2)$$

$$V_{F3} = ESR \cdot I_{D3} + N \cdot V_{TH} \cdot ln\left(\frac{I_{D3}}{I_S}\right) \ (1.3)$$

To simplify the equations we introduce several coefficients:

$$V_{F1} - V_{F2} = A_1 \cdot ESR + B_1 \cdot N$$
$$V_{F1} - V_{F3} = A_2 \cdot ESR + B_2 \cdot N$$

where: $A_1 = I_{D1} - I_{D2}$; $B1 = V_{TH} \cdot ln\left(\frac{I_{D1}}{I_{D2}}\right)$ for the first equation and $A_2 = I_{D1} - I_{D3}$; $B2 = V_{TH} \cdot ln\left(\frac{I_{D1}}{I_{D3}}\right)$ for the second equation

$$V_{F1} - V_{F2} = A_1 \cdot ESR + B_1 \cdot N \quad || \quad \cdot(-B_2)$$
$$V_{F1} - V_{F3} = A_2 \cdot ESR + B_2 \cdot N \quad || \quad \cdot(B_1)$$

$$B_1 \cdot (V_{F1} - V_{F3}) - B_2 \cdot (V_{F1} - V_{F2}) = ESR \cdot (A_2 B_1 - A_1 B_2)$$

$$ESR = \frac{B_1 \cdot (V_{F1} - V_{F3}) - B_2 \cdot (V_{F1} - V_{F2})}{A_2 B_1 - A_1 B_2}$$

We do the same thing to find N.

$$N = \frac{A_1 \cdot (V_{F1} - V_{F3}) - A_2 \cdot (V_{F1} - V_{F2})}{B_2 A_1 - A_2 B_1}$$

And to find I_S we simply introduce the newly found values in the Shockley equation.

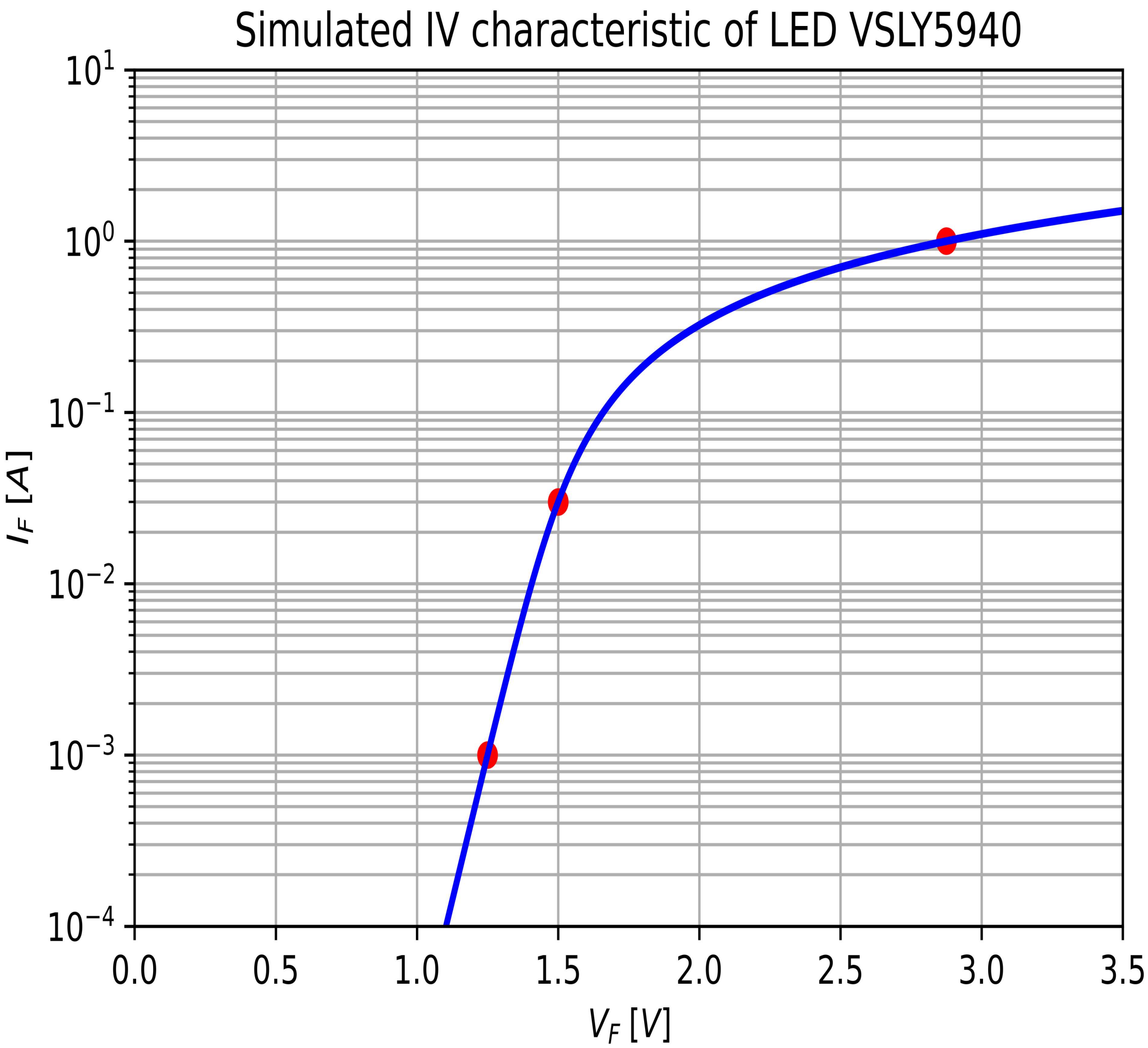
$$I_S = I_{D1} \cdot exp\left(-\frac{V_{F1} - ESR \cdot I_{D1}}{N \cdot V_{TH}}\right)$$

The code:

```
A1 = Id1-Id2
B1 = Vth * np.log(Id1/Id2)

A2 = Id1-Id3
B2 = Vth * np.log(Id1/Id3)


'''
Ideality factor (N)
'''
N = (A1 * (V1-V3) - A2 * (V1-V2)) / (A1*B2 - A2*B1)


'''
Internal series resistance (Rs)
'''
Rs = (B1 * (V1-V3) - B2 * (V1-V2)) / (A2*B1 - A1*B2)


'''
Saturation current
'''
Isat = Id1 * np.exp(-(V1-Rs*Id1)/(N*Vth))
```

For the values provided in the table above we obtain:
$ESR = 1.1884\ \Omega, \quad N = 2.4467, \quad I_S = 2.7641\ pA$

And with these values we obtain the following static characteristic, clearly very close to the original datasheet one:



Testing and matching the new diode model

Now that we've found the necessary parameters, we input known voltages/ currents and gauge the correctness of the output and to make things more interesting we'll use the Lambert W function.

Given that we know $ESR,\ N,\ I_S$ find the corresponding currents for $V_F = 2V$ and $V_F = 2.5V$.

$$V_F - ESR\ I_D - \eta\ V_{TH}\ ln\left(\frac{I_D}{I_{SAT}}\right) = 0$$

$$exp\left(\frac{V_F}{\eta\ V_{TH}}\right) = \frac{I_D}{I_{SAT}} \cdot exp\left(\frac{ESR\ I_D}{\eta\ V_{TH}}\right)$$

$$\frac{ESR\ I_{SAT}}{\eta\ V_{TH}} \cdot exp\left(\frac{V_F}{\eta\ V_{TH}}\right) = \frac{ESR\ I_D}{\eta\ V_{TH}} \cdot exp\left(\frac{ESR\ I_D}{\eta\ V_{TH}}\right)$$

We set $w = \frac{ESR\ I_D}{\eta\ V_{TH}}$

$$w\ e^w = \frac{ESR\ I_{SAT}}{\eta\ V_{TH}} \cdot exp\left(\frac{V_F}{\eta\ V_{TH}}\right)$$

$$w = LambertW\left(\frac{ESR\,I_{SAT}}{\eta\,V_{TH}} \cdot exp\left(\frac{V_F}{\eta\,V_{TH}}\right)\right)$$

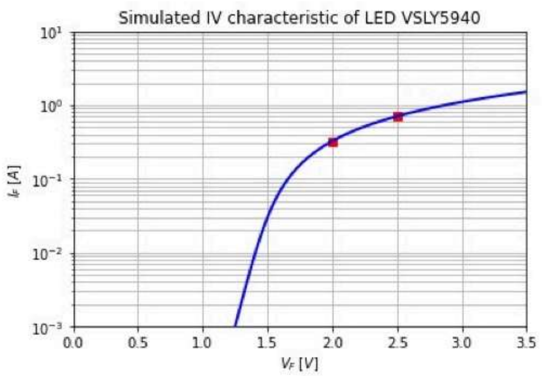
$$\frac{ESR\,I_D}{\eta\,V_{TH}} = LambertW\left(\frac{ESR\,I_{SAT}}{\eta\,V_{TH}} \cdot exp\left(\frac{V_F}{\eta\,V_{TH}}\right)\right)$$

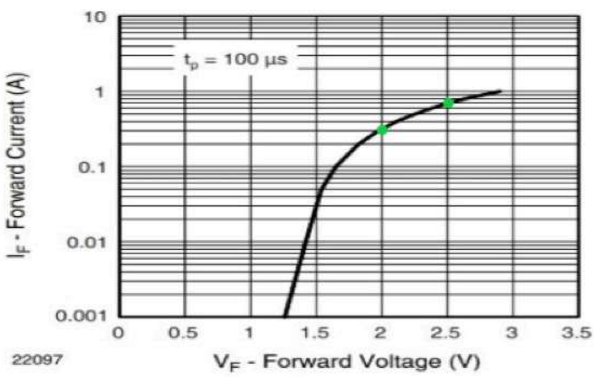
$$I_D = \frac{\eta\,V_{TH}}{ESR} \cdot LambertW\left(\frac{ESR\,I_{SAT}}{\eta\,V_{TH}} \cdot exp\left(\frac{V_F}{\eta\,V_{TH}}\right)\right)$$

The code for doing the math:

```
VFt = np.array([2,2.5])
Idt = np.real(N * Vth / Rs * lambertw((ESR*Isat)/(N*Vth)*np.exp(VFt/(N*Vth))))
#Idt[0]=0.323859  A
#Idt[1]=0.703234  A
```

We obtain $I = 0.323859$ for $V_F = 2V$ and $I_D = 0.703234$ for $V_F = 2.5V$, which are very close to the values presented in the datsheet, as you can see below:





Bibliography and Notes

This project is based on this post on electronics.stackexchange.

Other great resources:

1. https://ltwiki.org/files/SPICEdiodeModel.pdf
2. https://electronics.stackexchange.com/questions/480311/basic-diode-question-about-voltage-drop
3. https://web.ece.ucsb.edu/Faculty/rodwell/Classes/ece2c/labs/CurveFittinginExcel.pdf
4. Semiconductor device modeling with SPICE by Massobrio, Giuseppe

All custom plots are done in matplotlib.

All schematics if not otherwise specified are done in draw.io.