Why do we call a diode a square-law device

Purpose

When searching about AM (Amplitude Modulation) mixers and demodulators / envelope detectors a frequently encountered electronic component is the humble diode, often assumed to be a square-law device.

Why do we call the diode, with its clearly exponential IV characteristic, a square-law device? That is the question that I seek to answer in this doc.

Theoretical

DC square law derivation

The IV characteristic of all diodes is described by the Shockley equation: $I_D = I_S \cdot \left(e^{\frac{V_D}{\eta V_T}} - 1\right)$. The equation at hand does not in any way, shape or form resemble a quadratic equation, so what gives?

It's important to note that the diode (as opposed to FET transistors) acts as a square-law device for only very small voltages/ RF signals. Being a square law device means that the diode's I_D will be directly proportional to the square of V_D.

To better see the quadratic aspect of the IV curve we must employ a powerful mathematical apparatus: The Taylor Series. The Taylor series of e^x centered around zero is:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Which we then expand and find to be of the form shown below:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

For the very small RF signals we will encounter only the first and second order terms will be taken into account, such that a reasonable approximation for the exponential function around zero will be: $e^x \approx 1 + x + \frac{1}{2}x^2$.

Now, back to our dear Shockley and his equation:

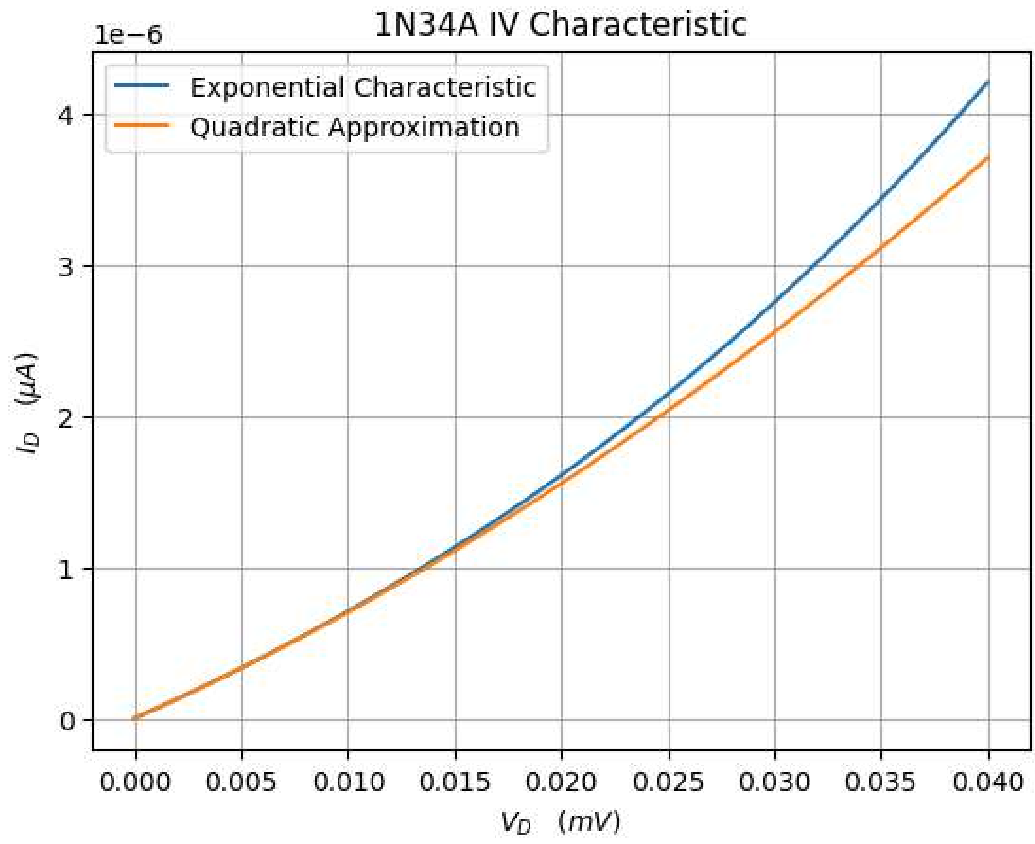
$$\begin{aligned} I_D &= I_S \cdot \left(e^{\frac{V_D}{\eta V_T}} - 1\right) \\ &= I_S \cdot \left[1 + \frac{V_D}{\eta V_T} + \frac{1}{2} \cdot \left(\frac{V_D}{\eta V_T}\right)^2 - 1\right] \\ &= I_S \cdot \left[\frac{V_D}{\eta V_T} + \frac{1}{2} \cdot \left(\frac{V_D}{\eta V_T}\right)^2\right] \end{aligned}$$

Looking at the final form of the equation from above we can clearly see the quadratic component of the current.

If we supply the diode with a very small signal and apply some low pass filtering we will obtain a current proportional to the square of the input signal.

Do note that the approximation presented here is valid only for a narrow interval. From 0 to $\approx \frac{\eta}{2} \cdot V_T$ the approximation will fit the exponential characteristic in a satisfactory manner.

Neglecting the impact of the series resistance of the diode we observe in the image from below the degree to which the quadratic approximation fits the exponential characteristic of an older germanium diode (1N34A):



1N34A IV Characteristic

Increasing the value of the ideality factor will shift the curve to the right, enlarging the spread of values for which the quadratic approximation remains valid.

AC square law derivation

If we desire to use diodes in RF/ small signal detection roles, we of course must understand the AC small signal behaviour of the diode. In this chapter I attempt to obtain both the usual AC dynamic resistance and the a clear square-law-like expression for diode current and voltage.

First we explain the notation used when dealing with this sort of analysis:

$$\begin{cases} v_D(t) = V_D + v_d(t) \\ i_D(t) = I_D + v_d(t) \end{cases}$$

$v_D(t)$ and $i_D(t)$ are the instantaneous current and voltage experienced by the diode at any one moment in time. V_D and I_D are the DC components, whilst $v_d(t)$ and $i_d(t)$ are the AC components.

An important aspect that we must discuss is how to define what a small signal is. When it comes to the linear approximations needed for the small signal model of the diode, a small signal $v_d(t)$ is defined such that $\frac{v_d(t)}{\eta V_T} \ll 1$. In more practical terms, at most 10-15 mV and at best upto 5mV.

$$i_D(t) = I_S \cdot \left(e^{\frac{V_D+v_d}{\eta V_T}} - 1\right)$$

$$= I_S \cdot \left(e^{\frac{V_D}{\eta V_T}} \cdot e^{\frac{v_d}{\eta V_T}} - 1\right)$$

$$= I_S \cdot e^{\frac{V_D}{\eta V_T}} \cdot e^{\frac{v_d}{\eta V_T}} - I_S$$

$$= (I_D + I_S) \cdot e^{\frac{v_d}{\eta V_T}} - I_S$$

$$= I_D \cdot e^{\frac{v_d}{\eta V_T}} + I_S \cdot \left(e^{\frac{v_d}{\eta V_T}} - 1\right)$$

$$= (I_D + I_S) \cdot e^{\frac{v_d}{\eta V_T}} - I_S$$

At this point we must employ the use of the Taylor series expansion of the function e^x. The formulas used here are valid only in the vicinity of $x \approx 0$, so, at first, we'll take $|v_d(t)|$ to be no bigger than $\frac{\eta}{2} \cdot V_T$.

$$i_D = \left(1 + \frac{v_d}{\eta V_T} + \frac{1}{2} \cdot \left[\frac{v_d}{\eta V_T}\right]^2\right) \cdot (I_D + I_S) - I_S$$

$$= (I_D + I_S) \cdot \left(\frac{v_d}{\eta V_T} + \frac{1}{2} \cdot \left[\frac{v_d}{\eta V_T}\right]^2\right) + I_D$$

This is the complete form of the quadratic approximation of the diode equation for small AC signals.

Note that for our detector diode applications in the simplest of circuits there is no DC biasing of the diode, such that I_D is zero and i_D takes a simpler form.

If we are taking about the diode in AC small signal applications (where there is a DC bias), I_D is not zero, but the value of $\frac{v_d}{\eta V_T}$ needs to be as small as possible. In this case the quadratic term of the series expansion can be ignored.

$$i_D = I_D + (I_D + I_S) \cdot \frac{v_d}{\eta V_T}$$

$$i_d = (I_D + I_S) \cdot \frac{v_d}{\eta V_T}$$

$$i_d = g_d \cdot v_d$$

$$g_d = \frac{I_D + I_S}{\eta V_T}$$

$$r_d = \frac{1}{g_d} = \frac{\eta V_T}{I_D + I_S}$$

r_d is the dynamic AC resistance of the diode, so called dynamic because it depends on the bias point of the diode.

The full AC small signal model of the diode is more complex, taking into account the junction capacitance (C_j) and the diffusion capacitance (C_d) of the diode. I will explore this on a later date.

Plugging in an AM signal and filtering the output for unwanted harmonics we obtain a clear square law relationship .

For future exploration

I am currently interested in building a detector diode or a diode demodulator, but have thus far failed in building a suitable circuit. In university we are currently learning about microwaves, and I had the pleasure to play with such a detector.

Resources

- https://inst.eecs.berkeley.edu/~ee105/fa07/labs/Miniproject.pdf
- https://xdevs.com/doc/HP/pub/Pratt_Diode_detectors.pdf
- https://electronics.stackexchange.com/questions/301404/modulation-scheme-for-simple-signal-generator?rq=1