CALCULUS TRICKS AND TRAPS

Schrödinger's Cat

- Is it true that if a function has a vertical asymptote, its derivative has a vertical asymptote at the same place?
- Is it true that if a function is continuous on the interval (a, b) and its graph is a smooth curve (no sharp corners) on that interval, then the function is differentiable at any point on (a, b)?
- Is it true that if a function is not monotone, then it does not have an inverse function?
- If a function f is continuous for all real x and $\lim_{n\to\infty} f(n) = A$ for natural numbers n, then $\lim_{x\to\infty} f(x) = A$. True or false?
- If a function is defined on $\mathbb{R}^2$, it cannot be continuous at only one point. Discuss.
- Discuss the following statement: If $f(x, y)$ approaches zero, as a point approaches the origin along any algebraic curve $y = cx^{m/n}$, where $c \in \mathbb{R}$ and $m, n \in \mathbb{N}$ ($x \geq 0$ in case the exponential fraction is in lowest terms and n is even), then the limit of $f(x, y)$ is zero at the origin.