CMBslow

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Introduction

In the mid 20th century, the scientific community was divided between supporters of the Big Bang model and the rival Steady State Model. This changed abruptly when in 1964 the radio astronomers Arno Penzias and Robert Wilson serendipitously discovered an isotropic background of microwave radiation, called the cosmic microwave background (CMB). The CMB is highly isotropic exquisitely well fitted by a blackbody spectrum of temperature [1]

$$T_0 = 2.7255 \pm 0.0006\,\mathrm{K}.$$

Within the Steady State Model, the isotropy and closeness of the spectrum to that of a black body are hard to explain. The Big Bang Model on the other hand predicts the CMB, which helped it become favoured over the Steady State Model.

At the time of last scattering, at which a typical CMB photon underwent its last scattering with an electron, the presence of small inhomogeneities in the matter distribution lead to angular fluctuations of the CMB temperature that we observe today. Since these fluctuations $\frac{\delta T}{T}$ are defined on a sphere (in our case, the celestial sphere), we can express them in terms of spherical harmonics

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\theta,\phi).$$

The multipole moments of this decomposition are then given by

$$C_\ell = \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2.$$

Motivation

The multipole moments are sensitive to changes in cosmological parameters. This can be analyzed with the software `CAMB` [2], which is a helpful yet complicated tool with a high-dimensional parameter space. However, one can derive elementary analytical expressions that explicitly describe the dependence of the CMB on cosmological parameters. The goal of this project was to implement the framework provided by Reference [3] in Python to explore the influence of parameters on the CMB power spectrum, e.g. in an animation by sweeping values for h.

Derivation of analytic expressions

In this section we summarize key assumptions and steps taken to get to the analytical expressions because the full derivation is lengthy.

Given the observationally most favoured case of a flat universe and assuming that only the radiation and the baryonic matter can be approximated by a single fluid before recombination (*tight-coupling approximation*), one can show that in the conformal Newtonian coordinate system the temperature fluctuations system for the present moment η_0 at point $\mathbf{x}_0$ in the direction $\mathbf{n}$ are given by [3, 4]

$$\frac{\delta T}{T}(\eta_0,\mathbf{x}_0,\mathbf{n}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(\left(\Phi + \frac{\delta}{4}\right)_{\mathbf{k}} - \frac{3\delta'_{\mathbf{k}}}{4k^2}\frac{\partial}{\partial\eta_0} \right)_{\eta_r} e^{i\mathbf{k}(\mathbf{x}_0 + \mathbf{n}(\eta_r - \eta_0))},$$

where $\Phi \ll 1$ is the gravitational potential, $\delta \ll 1$ denotes a fluctuation in the radiation energy density, η_0 is the present time, and η_r is the recombination moment. The first term represents both the Sachs-Wolfe effect and the result from the initial inhomogeneities in the radiation energy density $\epsilon_\gamma(1+\delta)$. The second term is called the *Doppler contribution* because it is related to the velocities of the baryon-radiation fluid at radiation.

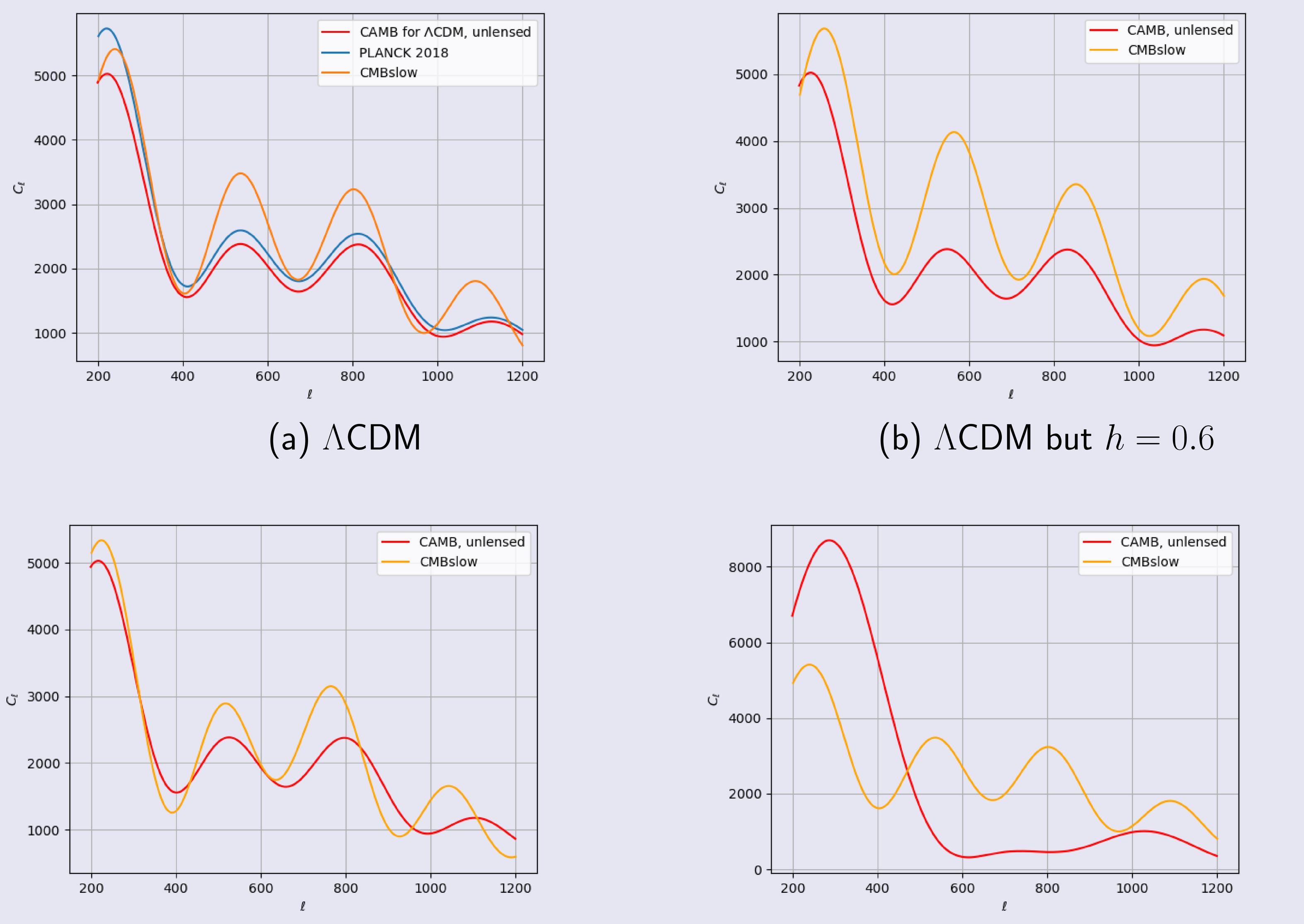
Plugging this result into the formula for C_ℓ, we find that the multipoles are made up of five contributions, three of which are non-oscillating integrals (denoted by N_i), the others are oscillating integrals (O_i)

$$C_\ell \propto \frac{1}{l^2}(N_1 + N_2 + N_3 + O_1 + O_2).$$

The next step is to express each contribution in terms of the relevant cosmological parameters, which in our case are the density parameters Ω_b and Ω_m and the (dimensionless) Hubble constant h ("little h"). Then, with good accuracy the oscillating integrals can be approximated by employing the saddle-point technique, whereas the N-terms can be evaluated in terms of hypergeometric functions. Subsequently, to keep the solution tractable, these are fitted numerically.

Comparison with `CAMB` and Planck 2018 data

For the ΛCDM model (the standard model of cosmology), we plotted the power spectrum of the CMB as calculated by `CAMB` and our version of CMBslow together with the 2018 data of the Planck satellite [5] below. Note that in order to obtain this result, we had to correct several functions from Reference [3] because they contained errors. For details, see Reference [6].



(a) ΛCDM

(b) ΛCDM but $h = 0.6$

(c) ΛCDM but $h = 0.75$

(d) ΛCDM but $\Omega_b = 0.3$

As can be seen in Fig. 1a, `CAMB` predicts the shape and location of the peaks well and merely underestimates the C_ℓ slightly, whereas CMBslow becomes increasingly inaccurate in predicting position and amplitude of the peaks for higher ℓ. In Figs. 1b,c h has been varied, which in general increased deviations even further. Fig. 1d shows that a variation of Ω_b changes the CMBslow result only slightly, although `CAMB` predicts a completely different curve.

Conclusion

The result of this project is a set of Python functions that computes the power spectrum of the CMB given merely the three cosmological parameters Ω_b, Ω_m and h. It is to be understood as a simple, qualitative tool that allows the user to understand the influence of these parameters on the spectrum intuitively in a wide range of parameters around the standard model of cosmology.

Our implementation qualitatively produces the same curve as the state-of-the-art software `CAMB`, though fails to correctly predict the location and amplitude of peaks with accuracy. This is partly due to errors in the original paper [3] that could not be corrected completely within the limited scope of this project. The author has been contacted about this issue and we will continue to improve the code beyond the frame of this project.

The code is publicly available online in a GitHub repository [6].

References

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