Ejercicios de Integración

3. $$\int_a^b f(x)dx \approx \int_a^b P_2(x)\,dx$$

1. Interpolación Polinomial

$$x_0 = a \qquad x_1 = \frac{a+b}{2} = x_m \qquad x_2 = b$$

$$P_2(x) = f(a)L_0(x) + f(x_m)l_1(x) + f(b)l_2(x)$$

$$l_0(x) = \frac{(x-x_m)(x-b)}{(a-x_m)(a-b)}$$

$$l_1(x) = \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}$$

$$L_2(x) = \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$$

2. $$P_2(x) = f(a)\frac{(x-x_m)(x-b)}{(a-x_m)(a-b)} + f(x_m)\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} + f(b)\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$$

3. Aproximación de la Integral

$$\left.\begin{aligned}
&\int_a^b f(a)\,l_0(x)dx = f(a)\cdot\frac{1}{6}(b-a)\\
&\int_a^b f(x_m)\,L_1(x)\,dx = f(x_m)\frac{2}{3}(b-a)\\
&\int_a^b f(b)\,l_2(x)\,dx = f(b)\cdot\frac{1}{6}(b-a)
\end{aligned}\right\}$$

$$f(a)\cdot\frac{1}{6}(b-a) + f(x_m)\frac{2}{3}(b-a) + f(b)\frac{1}{6}(b-a)$$

$$(b-a)\left[f(a)\frac{1}{6} + f(x_m)\frac{2}{3} + f(b)\frac{1}{6}\right]$$

Si $h = \frac{b-a}{2} \rightarrow 2h\left[\frac{1}{6}f(a) + \frac{2}{3}f(x_m) + \frac{1}{6}f(b)\right] = \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right)$

Conclusión $$\int_a^b f(x)dx \approx \int_a^b P_2(x)dx \approx \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right)$$

21

a.

$$\int_{-1}^{1} P_n(x) P_m(x)\,dx = \begin{cases} 0 & \text{si } n \neq m \\ \frac{2}{2n+1} & \text{si } n = m \end{cases}$$

$$f(x) = \sum_{n=0}^{N} c_n P_n(x)$$

$$\int_{-1}^{1} f(x) P_m(x)\,dx = \int_{-1}^{1} \sum_{n=0}^{N} c_n P_n(x) P_m(x)\,dx$$

$$\int_{-1}^{1} f(x) P_m(x)\,dx = \sum_{n=0}^{N} c_n \int_{-1}^{1} P_n(x) P_m(x)\,dx$$

Si $n \neq m$

$$\int_{-1}^{1} P_n(x) P_m(x)\,dx = 0$$

Si $n = m$

$$\int_{-1}^{1} P_n(x) P_n(x)\,dx = \frac{2}{2n+1}$$

$$\int_{-1}^{1} f(x) P_m(x)\,dx = C_m \cdot \frac{2}{2m+1}$$

$$\boxed{C_m = \frac{2m+1}{2} \int_{-1}^{1} f(x) P_m(x)\,dx}$$