Tarea 2 Interpolación

Ejercicios de derivación 8

a) Polinomio de Interpolación

$$P_2(x) = L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2)$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

Son las bases de Langrange.

Si $x_1 = x_0 + h$ y $x_2 = x_0 + 2h$

$$l_0(x) = \frac{(x-(x_0+h))(x-(x_0+2h))}{(x_0-(x_0+h))(x_0-(x_0+2h))} = \frac{(x-x_0-h)(x-x_0-2h)}{-h(-2h)} = \frac{(x-x_0-h)(x-x_0-2h)}{2h^2}$$

$$L_1(x) = \frac{(x-x_0)(x-(x_0+2h))}{((x_0+h)-x_0)((x_0+h)-(x_0+2h))} = \frac{(x-x_0)(x-x_0-2h)}{h(-h)} = \frac{-(x-x_0)(x-x_0-2h)}{h^2}$$

$$L_2(x) = \frac{(x-x_0)(x-(x_0+h))}{((x_0+2h)-x_0)((x_0+2h)-(x_0+h))} = \frac{(x-x_0)(x-x_0-h)}{2h(h)} = \frac{(x-x_0)(x-x_0-h)}{2h^2}$$

Entonces:

$$P_2(x) = f(x_0)\frac{(x-x_0-h)(x-x_0-2h)}{2h^2} - f(x_1)\cdot\frac{(x-x_0)(x-x_0-2h)}{h^2} + f(x_2)\cdot\frac{(x-x_0)(x-x_0-h)}{2h^2}$$

b) · Derivemos los terminos por separado

$$T_1(x) = f(x_0)\,\frac{(x-x_0-h)(x-x_0-2h)}{2h^2}$$

$$T_1'(x) = f(x_0)\,\frac{(x-x_0-2h)+(x-x_0-h)}{2h^2}$$

$$T_1'(x) = f(x_0)\,\frac{2(x-x_0)-3h}{2h^2} = \frac{-3f(x_0)}{2h}$$

$$T_2(x) = -f(x_1)\,\frac{(x-x_0)(x-x_0-2h)}{h^2}$$

$$T_2'(x) = -f(x_1)\,\frac{(x-x_0-2h)+(x-x_0)}{h^2}$$

$$T_2'(x) = -f(x_1)\,\frac{2(x-x_0)-2h}{h^2} = \frac{2f(x_1)}{h}$$

$$T_3(x) = f(x_2)\,\frac{(x-x_0-h)+(x-x_0-h)}{2h^2}$$

$$T_3'(x) = f(x_2)\,\frac{(x-x_0-h)+(x-x_0)}{2h^2}$$

$$T_3'(x) = f(x_2)\,\frac{2(x-x_0)-h}{2h^2} = \frac{-f(x_2)}{2h}$$

Luego

$$P_2'(x_0) = \frac{-3f(x_0)}{2h} + \frac{2f(x_1)}{h} + \frac{-f(x_2)}{2h}$$

$$P_2'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_1) - f(x_2)\right)$$

$$f'(x_0) \approx P_2'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_1) - f(x_2)\right)$$

Entonces si $x_1 = x_0 + h$ y $x_2 = x_0 + 2h$

$$f'(x) = \frac{1}{2h}\left(-3f(x) + 4f(x+h) - f(x+2h)\right)$$

e) Derivada de $f(x) = \sqrt{\tan(x)}$

$$(\tan(x))^{1/2} = \frac{1}{2}(\tan(x))^{-1/2}\cdot \sec^2(x) = \frac{1}{2\sqrt{\tan(x)}}\sec^2(x)$$