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Partial Differential Equations|| Project Report

Image Processing

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1. Abstract:

Partial differential equations and their superiority in physical mechanism analysis from image processing, functional analysis, combining variational methods, differential geometry, projective geometry, and other mathematical tools to the image denoising smoothing, image reconstruction, image segmentation, image restoration, image to enlarge, medical image processing, remote sensing, and other fields are widely used. The digital image can be obtained from the analog image by sampling and quantization, which is made by the digital camera by superimposing a regular grid on the analog image and assigning a value. This mathematical representation of the digital image is the image function u(x,y) that is defined on a two-dimensional image domain; the grid.

2. Introduction:

Digital image processing is one of the most important topics that merge computer science, social applications, and mathematics. Mathematics is the core of digital image processing as it is involved in all of the image processing types. Image processing is developed over time as applied mathematics and technologies are being developed. The partial differential equations made a huge difference in image processing in the past 20 years. The performance of digital image processing is far better with PDEs than traditional methods as PDEs offer new directions in this field which were not available by using the traditional methods like image structure, affine invariant feature extraction, and texture decomposition.

3. Problem Definition:

Image processing is divided into three processes each one has a different aim, they are image compression, restoration, and analysis. In image compression, the image has been represented in the form that has the least size. The second process is restoration in which the image processing is done to get the best version of the image by getting rid of noise, blur, and perturbation. While in the analysis process, the structures in the image are re-shaped to be perfect by neglecting the details.

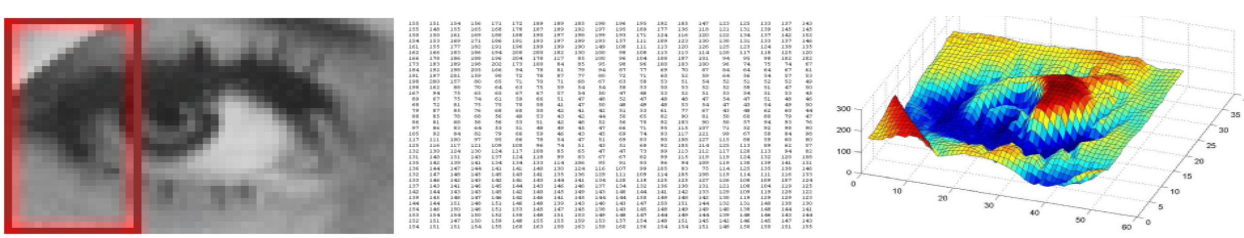
Image analysis processing

All these processes require high precision and accuracy, the accuracy mainly depends on the mathematical process that will be done on the image. PDEs provide high accuracy as it gives the ability to deal with each pixel in the image.

4. Methodology:

- The mathematical representation of a digital image:

The image has a built-in function in any software compiler called "image function" **u(x,y)** which is defined on a two dimensional grid. This function returns a scalar value in the case of a gray value image, or a vector value in the case of a coloured image as values to RGB, Presenting Red, Green and Blue.



- Image Processing Application: Image Denoising:

One of the most interesting applications of image processing is image denoising; and it has many approaches. One of the simplest and easiest approaches is threatening the noise in an image as a highly-oscillatory (high frequency) component of the acquired image data. And one of the most efficient ways to think about denoising an image is smoothing! Aiming to "smooth an image" is to "smooth" away the noise, So let's have one simplet investigated method for smoothing images by applying a linear filter to them: "Gaussian smoothing".

- Gaussian Smoothing:

Gaussian smoothing denotes the construction of a smoothed version u of g " which is a grayscale image represented as a real valued mapping g ∈ L 1 (R 2)" by convolving g with a Gaussian kernel, and G_σ denotes the two-dimensional Gaussian of width σ > 0.

$$u(x) = (G_\sigma * g)(x) := \int_{\mathbb{R}^2} G_\sigma(x-y)\, g(y)\, dy,$$

$$G_\sigma(x) := \frac{1}{2\pi\sigma^2}\, e^{-|x|^2/(2\sigma^2)}$$

And as it's known, the Fourier transform is one of the most popular ways to solve this convolution through converting it to simple multiplication in the frequency domain:

$$(\mathcal{F}g)(\omega) := \int_{\mathbb{R}^2} g(x)\, e^{-i\omega x}\, dx.$$

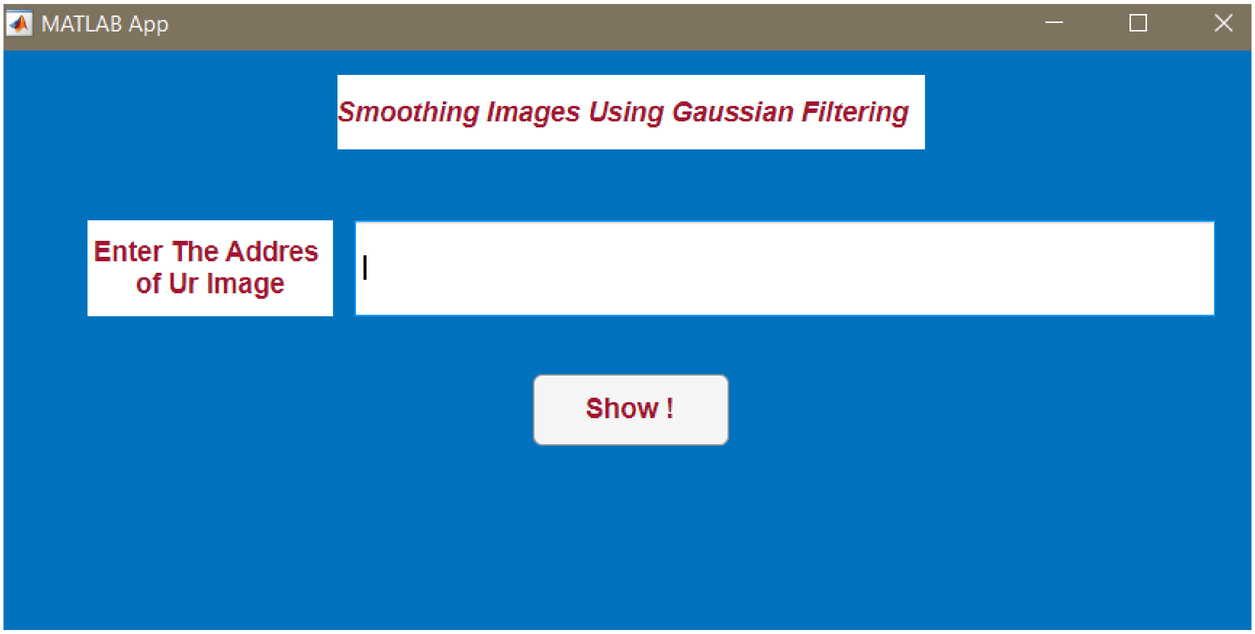
$$\begin{aligned}(\mathcal{F}(G_\sigma * g))(\omega) &= (\mathcal{F}G_\sigma)(\omega)\cdot(\mathcal{F}g)(\omega)\\ &= e^{-|\omega|^2/(2/\sigma^2)}\cdot(\mathcal{F}g)(\omega)\end{aligned}$$

The parameter sigma (σ > 0) determines the size of the eliminated details by the filter: the bigger σ, the smoother the result, the lesser details are kept.

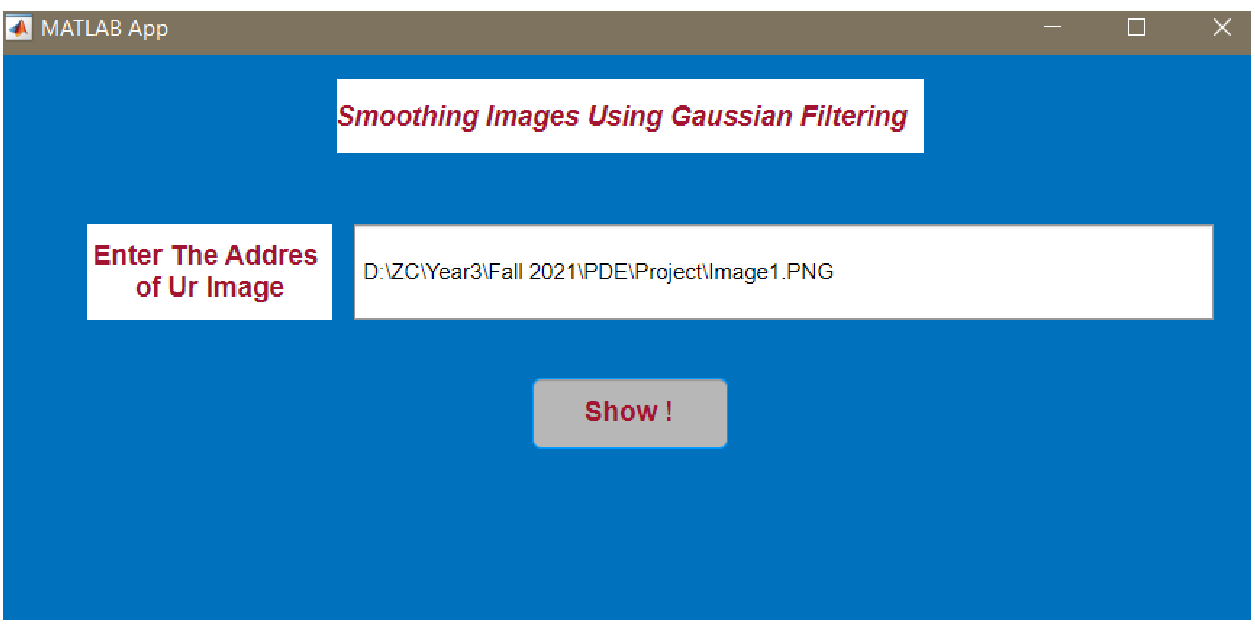
5. Software Application:

Smoothing Images Using Gaussian Filtering:

- First, Our code gets the original image as an input from user:



- Then, Once user clicks the button "Show !", The original image Vs the filtered one will be shown.



- Results:

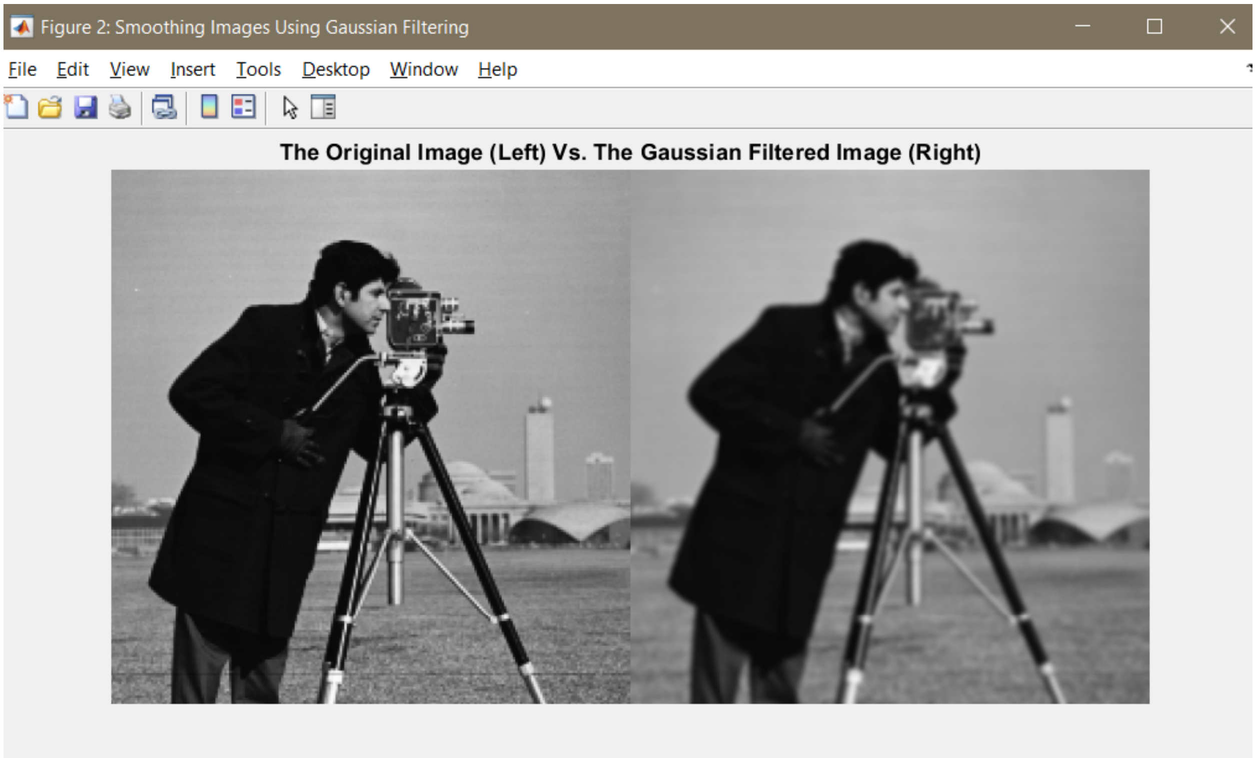
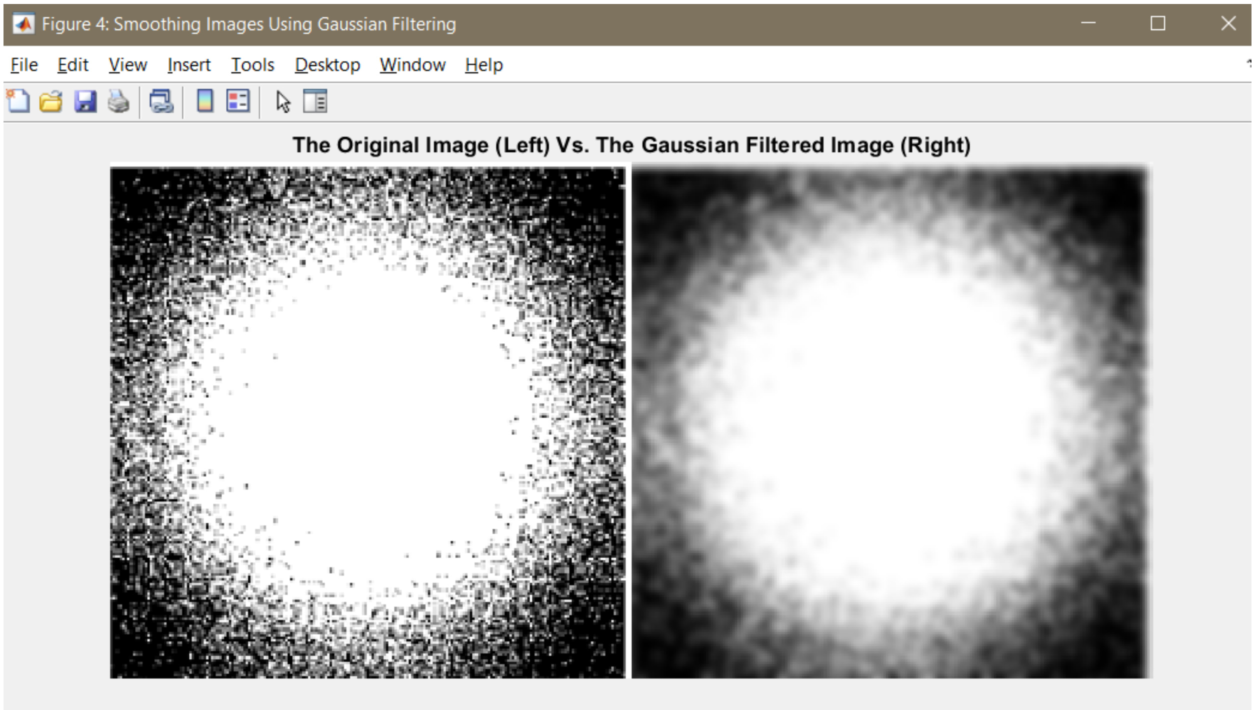
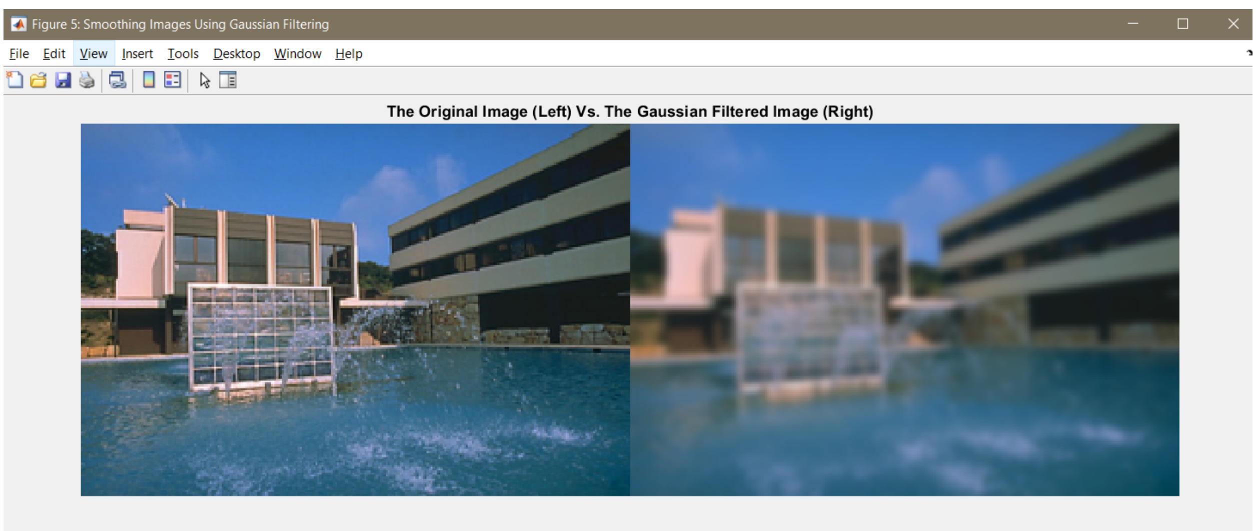


Figure 2: Smoothing Images Using Gaussian Filtering
File Edit View Insert Tools Desktop Window Help
The Original Image (Left) Vs. The Gaussian Filtered Image (Right)


Figure 3: Smoothing Images Using Gaussian Filtering
File Edit View Insert Tools Desktop Window Help
The Original Image (Left) Vs. The Gaussian Filtered Image (Right)





Our Application:
https://drive.google.com/drive/folders/1I1KmlCexI2CMemAPPgYTVZ-bK_AL4EK1?usp=sharing

6. Conclusion:

In conclusion: Partial differential equations in the field of image processing have their various applications from: image denoising smoothing, image reconstruction, image segmentation, image restoration, image to enlarge, medical image processing. And the application of Gaussian Filtering to smooth away the noise is one of the great applications uptil now.

7. References:

[1] Guichard, F. & Moisan, Lionel & Morel, Jean-Michel. (2002). A review of P.D.E. models in image processing and image analysis. Journal De Physique Iv - J PHYS IV. 12. 137-154. 10.1051/jp42002006.

[2] Schonlieb ,B.(2013/14) Image Processing – Variational and PDE Methods. Mathematical Tripos Part III.