Macroeconomics I
Problem Set 4

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Instructions

Please form groups of four persons per group. Give your group a group name that remains the same throughout the course.

In terms of grading, the weight on the average grade across all problem sets for the final grade will be 33%, whereas the final exam will receive weight 67%.

For all problem sets, we want you to compile a PDF document which clearly answers each question (with the help of tables and figures if needed). This also implies that, for any computational exercises we may have in the problem sets, you do not need to send code. Your report should be sent to Ludovic.Roussel@eui.eu.

Deadline: January 9, 2026 (11:59 pm)

1 The Stochastic Neoclassical Growth Model (60 points)

Consider the social planner's solution of the neoclassical growth model in discrete time.

The aggregate resource constraint of the economy is

$$C_t + K_{t+1} = \zeta_t \Lambda_t F(K_t, L_t) + (1-\delta)K_t$$

where $L_t = N_t$ are aggregate hours worked and N_t is the total population and we assume that $N_t = (1+n)^t$. $\Lambda_t = (1+g)^t$ is an exogenous technology level. ζ_t is an aggregate real business cycle shock which we assume to be i.i.d. distributed. We assume a Cobb–Douglas technology and you can interpret productivity as labor-augmenting (or rewritte it as such). The objective is

$$U = \max_{\{c_t, \tilde{k}_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\mathbb{E}_0$ is the expectations operator and $c_t = \frac{C_t}{N_t}$ is per capita consumption and $\tilde{k}_t = \frac{K_t}{\Lambda_t N_t}$ is the capital stock per efficiency unit of labor. Expectations are taken with respect to the i.i.d. distributed aggregate productivity shocks ζ_t.

Throughout, we assume the familiar CRRA utility function

$$u(c) = \frac{c^{1-\theta}}{1-\theta}$$

- Derive the detrended version of the economy.
- Reformulate the problem as a dynamic programming problem and show that it is a contraction.
- Derive the Euler equation of consumption.
- Show that, in the detrended economy, for the specific case of logarithmic utility ($\theta = 1$) and full depreciation ($\delta = 1$), the consumption and savings policy functions take the form

$$\tilde{c} = \zeta\Gamma\tilde{k}^{\alpha},$$
$$G\tilde{k}' = \zeta(1-\Gamma)\tilde{k}^{\alpha},$$

  where $G = (1+g)(1+n)$ is the growth factor of efficiency units. Derive an analytical expression for Γ.
- Argue why the saving rate is not affected by ζ and thus also not by the variance of ζ (and any higher-order risk moments)? Interpret your finding.
- Use the Euler equation to show that for $\theta > 1, \delta = 1$ there is precautionary savings, whereas for $\theta \in (0,1)$ there is not. Briefly interpret this finding.
- With $\theta = 1, \delta = 1$, how does an increase of risk affect life-time utility? Provide an analytical expression and an interpretation of your finding. Measure the variance of per capita consumption in quarterly data and compute the CEV of setting the variance to zero.
- Which additional effects are at work for $\theta > 1$?

2 Numerical Solution of the Stochastic Neoclassical Growth Model (30 points)

- Compute the numerical solution of the neoclassical growth model using dynamic programming techniques for your estimate of δ from the previous problem set and $\theta = 4$. Assume a quarterly frequency of the model to calibrate β, otherwise base the calibration on the previous problem sets. Discretely approximate ζ by assuming that it can only take on two values $\zeta \in \{\zeta^l, \zeta^h\}$.
- To measure ζ_t^d (the fluctuation component of aggregate output) apply a Hodrick Prescott filter to real data on per capita output.
  - Estimate an AR(1) regression $\zeta_{t+1}^d = \rho\zeta_t^d + \epsilon_{t+1}$ and estimate $\hat{\rho}, \hat{\sigma}_\epsilon$.
  - Show that the unconditional variance of ζ_t^d is $\hat{\sigma}_\zeta^2 = \frac{\hat{\sigma}^2}{1-\hat{\rho}^2}$.

- For the remainder of the analysis, calibrate the model with the unconditional variance $\hat{\sigma}_{\zeta}^2$. Use random number generator to simulate the model for $T = 10,000$ periods. Compute the variance of per consumption, output and investment as well as the covariances and the autocorrelations of these variables. How do these moments change if you set $T = 1,000$ or $T = 100,000$? Interpret this finding.

3 Bonus: The Welfare Costs of Fluctuations (10 points)

- Compute the welfare costs of reducing the fluctuations to zero. Decompose the total effect as the sum of two effects by computing the welfare costs holding constant the consumption and savings policy functions for the high variance economy. Interpret this decomposition.
- Download the respective data on per capita consumption, output and investment. Apply HP filters. Compare the simulated moments from the model with these moments and interpret your findings.

4 Literature (10 points)

Provide summaries of Imrohoroglu (1989) and Christiano and Eichenbaum (1990).