Robust Bayesian Inference for Set-Identified Models

Contents

Section 1

quick overview of Bayesian statistics

Bayes rule

$$P(\text{Hypothesis}|\text{Data}) = \frac{P(\text{Data}|\text{Hypothesis})P(\text{Hypothesis})}{P(\text{data})}$$

- Prior: Our Hyptothesis (= parameters) before seeing the data, set by the researcher

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- Evidence: (Usually unobserved) normalising constant

Example model, Bayesian linear regression

Given we chose the model

$$y = \theta_0 + \theta_1 x + \epsilon \quad y, x, \theta_0, \theta_1 \in \mathbb{R} \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$

We set relatively uninformative priors

$$\theta = \begin{pmatrix} \theta_0 \\ \theta_1 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix} \right) = \mathcal{N}(\mu_{prior}, \Sigma_{prior}), \quad \sigma^2 \sim \text{Inv-Gamr}$$

The likelihood defined by the model given that we have k samples follows the distribution

$$y|\beta_0, \beta_1 \sim \mathcal{N}_k(X\beta, diag(\sigma^2)), \quad X = \begin{pmatrix} 1 & x_1 \\ ... & ... \\ 1 & x_k \end{pmatrix}$$

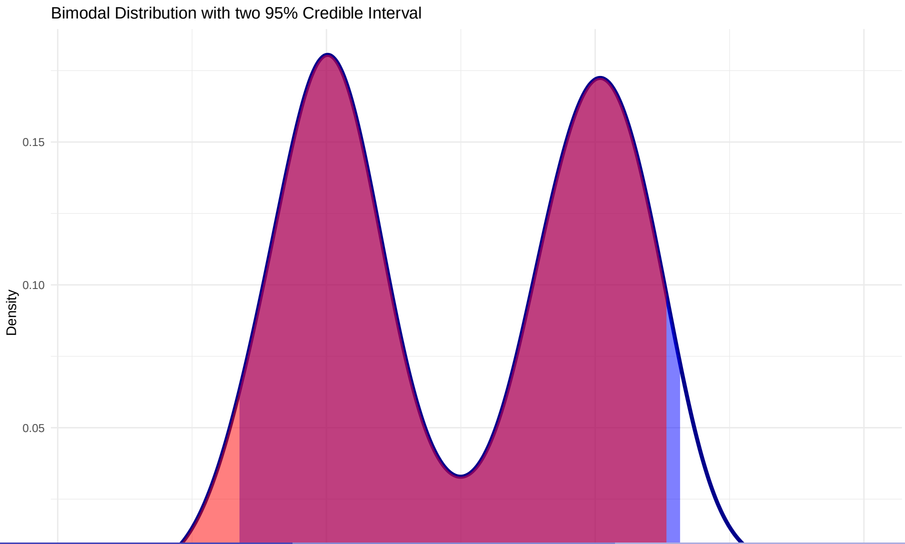
Therefore the posterior is

$$P((\theta_0, \theta_1)^T | y) \propto \mathcal{N}\left(\Sigma_{posterior} \frac{1}{\sigma^2} X^T y, (\Sigma_{prior}^{-1} + \frac{1}{\sigma^2} X^T X)^{-1} \right)$$

!!!! Note the above is false, do not bother with the full distribution

Inference: credible intervals

Say, given any sample $Y = (y_1, ..., y_n)$ the posterior $ P(| y) $ looks approximately like



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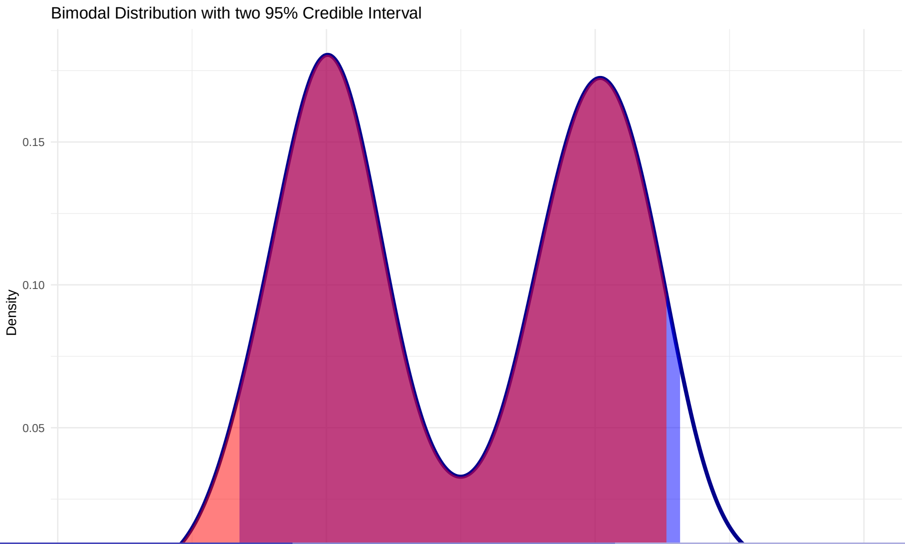
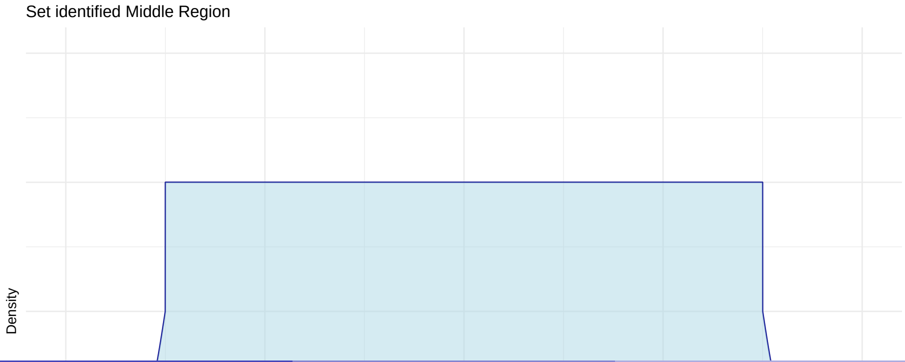


Set identification

- A parameter set Θ is said to be (point) identified if $\Theta = \{\theta\}$



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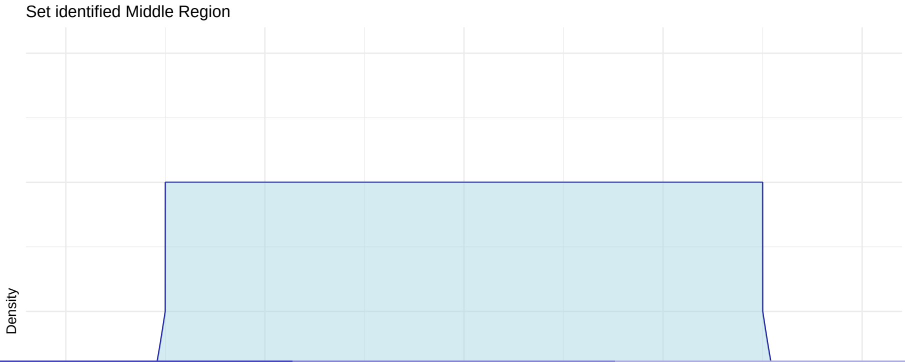

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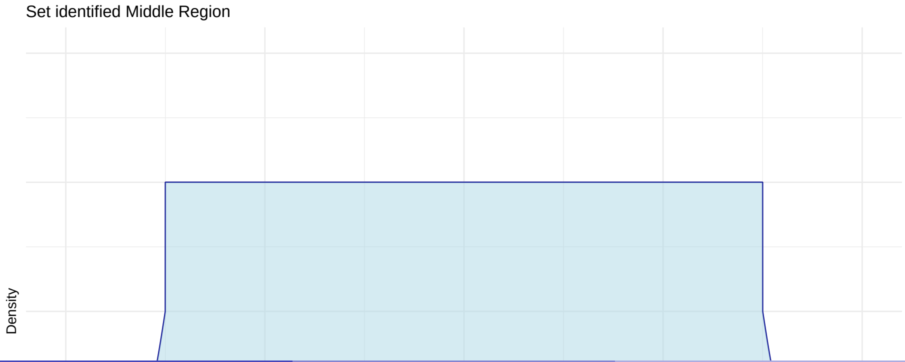

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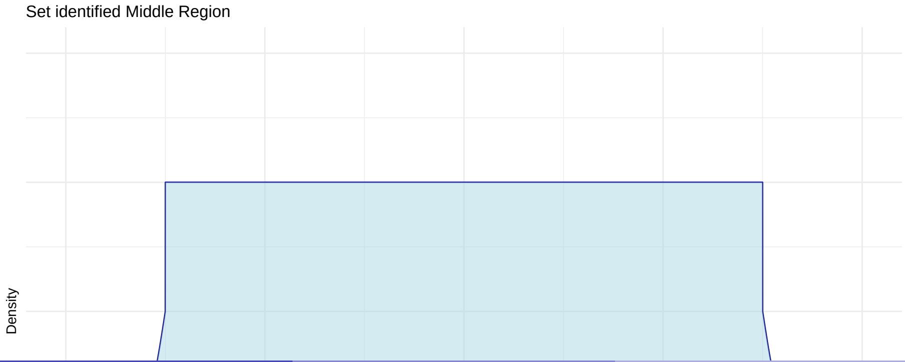
- Note this is for any draw of the data and the priors must be equal. Not necessarily the posteriors



Section 2

The paper

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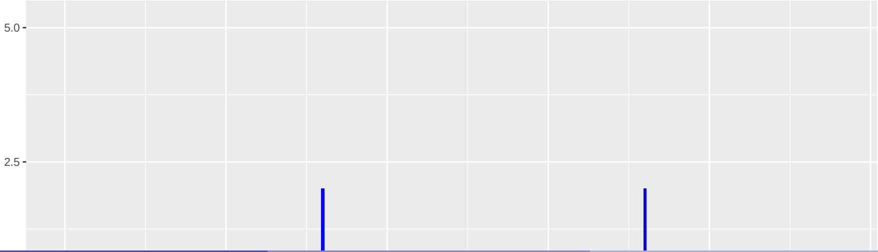

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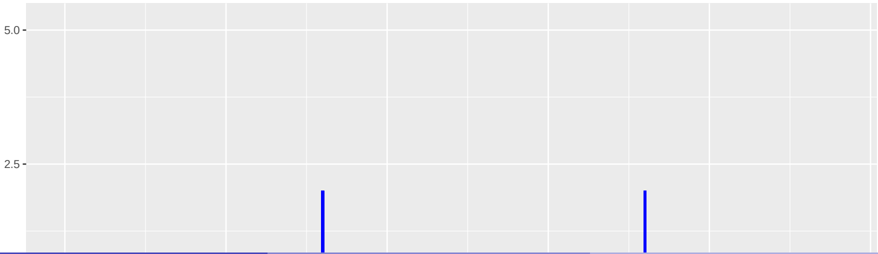
- Suppose we only observe a reduced for parameter $\phi = g(\theta)$, where $g : \Theta \to \Phi, \quad g(\theta) = g(\theta^{'})$ iff $p(Y|\theta) = p(Y|\theta^{'}) \forall Y \in \mathbf{Y}$. Note in set identification g is not injective.



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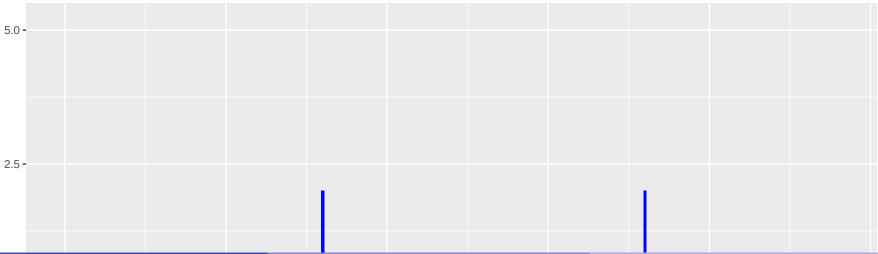

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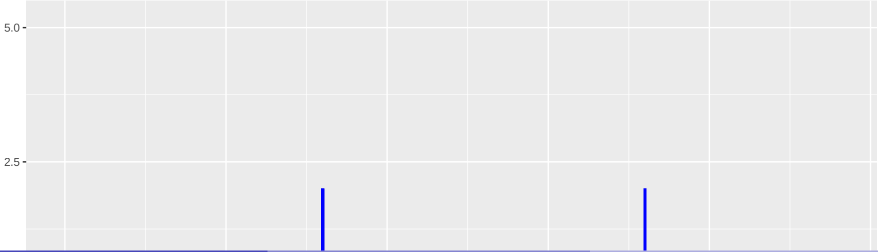

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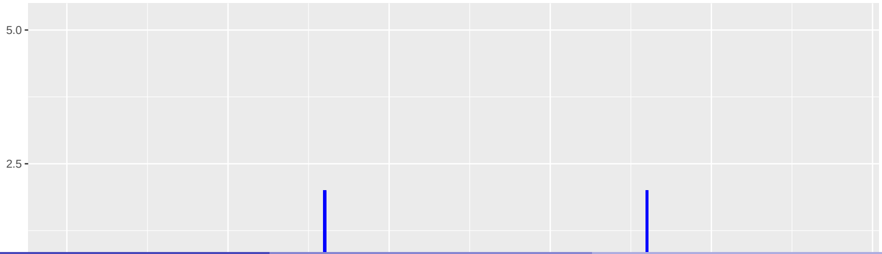

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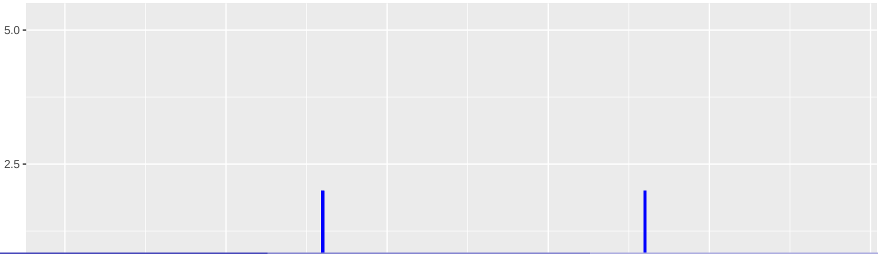

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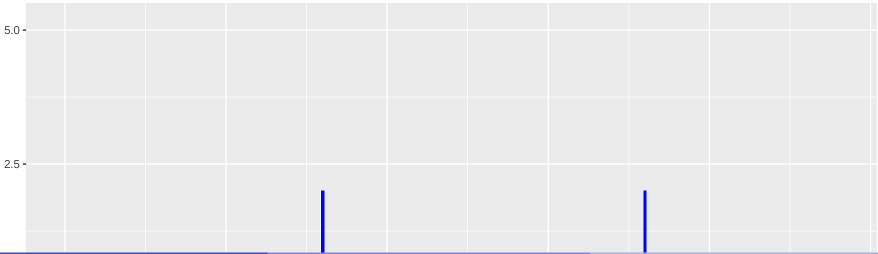

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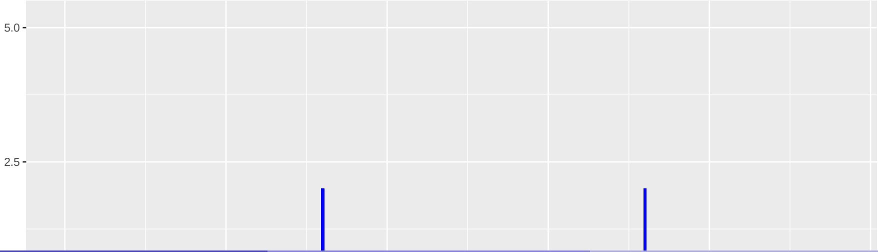

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- Note that $\forall Q \in \mathcal{O}(2)$, we have that $\Sigma = (A^T Q Q^T A)^{-1}$, the model is set identified

Breaking down the priors in set identified models

Multiple priors, posterior lower and upper probabilities

Set of posterior means

Robust credible regions

Diagnostic tools

Asymptotic properties 1

Asymp ic properties 2

Thank you

Section 3

Graveyard