The Tools of Metaphysics and the Metaphysics of Science

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Preface

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Chapter 1

Metaphysical tools and structuralism

1.1 Tools in metaphysics

By "tools in metaphysics" I mean the core concepts used to articulate metaphysical problems and structure metaphysical discourse. They are a lens through which we view metaphysics.

The metaphysical tools of choice change over time, and as they do, the problems of metaphysics are transformed. We view the very same problems (in some sense) through different lenses. In the 1950s and 60s the preferred tools were concepts of meaning and analysis. So when personal identity over time was discussed, for example, the question asked was, what are we *saying* when we re-identify persons over time?[1] In the 1970s through the 1990s, the tools became modal, and the questions of personal identity underwent a corresponding transformation: what conditions governing personal identity hold of metaphysical necessity? Would it be possible to survive the loss of all of one's memories?

The mind-body problem had a similar arc. In the 1950s the goal was to give an analysis of mental concepts; but later the questions became modal, whether, for instance, it would be possible for a world physically like ours to lack consciousness.

Philosophical questions begin life in vague, primordial form. The mind and the body: what's up with that? How are they related? Before real progress

[1]See Strawson (1959), for instance.

can be made, the questions must be made precise, and this is the job of tools of metaphysics. With particular tools in hand, the primordial questions begin to seem, in retrospect, as first attempts to ask what was the proper question all along—say, whether it would be possible for two physically alike bodies to have different mental properties. The proper questions will be viewed as *better* than the questions yielded by rival tools—clearer perhaps, or more precise, substantive, or objective; or better in lacking false presuppositions, or being less susceptible to being confused by misleading natural language, or having a better associated methodology, or being more likely to connect with questions elsewhere in philosophy.

Recently there has been a shift to new tools, which I will call “postmodal”. David Lewis (who had also been a leader in the modal revolution) enriched his conceptual toolkit with the concept of *natural properties and relations*, those elite properties and relations that determine objective similarities, occur in the fundamental laws, and whose distribution modally fixes everything else. In a recent book I have argued for the centrality of a concept that is closely related to Lewis’s notion of naturalness, the concept of structure, or as I’ll put it here, the concept of a *fundamental concept*.[2] Fundamental concepts need not be predicates; we may ask, for instance, whether quantifiers or modal operators are fundamental concepts—whether they help to determine the world’s fundamental structure. Kit Fine (re-)introduced the concept of *essence*, and argued that it should not be understood modally. He pointed out that although it does seem to be an essential feature of the singleton set {Socrates} that it contain Socrates, it does not seem to be an essential feature of Socrates that he be contained in {Socrates}; it is not “part of what Socrates *is*” that he be a member of this set. Thus we cannot define a thing’s essential features, as it had been common to do in the halcyon days of the modal era, to be those features that the thing possesses necessarily, for it is plausible that Socrates possesses the feature of being a member of {Socrates} necessarily. Fine also (re-)introduced a notion of *ground*. One fact grounds another, he says, if the second holds in virtue of the first, if the first explains, in a distinctively metaphysical way, the second. Interest in ground and related concepts over the past ten years or so has been intense.

Friends of the postmodal revolution think that modal conceptual tools need to be supplemented, or perhaps even replaced, by one or more of these

[2] Ultimately I don’t think structure/fundamentality is a matter of concepts or any other entity, abstract or otherwise, but I’ll set aside those scruples here. See Sider (2011, section 6.3).

postmodal concepts.[3] A vague theme has been that modal concepts are too crude for many purposes, in that even after modal questions are settled, there remain important questions that can be raised only by using the postmodal tools. Fine's example of Socrates and {Socrates} illustrates this, as does the often-cited example of the Euthyphro question: even if all hands agree that something is pious if and only if the gods love it, one can still wonder whether the gods love something because it is pious, or whether something is pious because the gods love it. This appears to be a question of ground, of whether piety grounds the gods' love or the gods' love grounds piety.[4] A further theme has been that modal questions are often epiphenomenal, a mere reflection of deeper postmodal phenomena.

The story of a linear progression from conceptual analysis to modality to fundamentality/essence/ground is an oversimplification. For instance, inspired by Quine's "On What there Is", much metaphysical inquiry has centered on ontological questions, questions structured by the central concepts of ontology (for Quineans: the existential and universal quantifiers of first-order logic). From 1980–1990, three of the major works of metaphysics were focused on ontology: Field's *Science without Numbers*, Lewis's *On the Plurality of Worlds*, and van Inwagen's *Material Beings*.[5]

Nevertheless, our primary focus will be on the final transition in the simplified story: from modal to postmodal tools. I'm interested in how the shift to postmodal tools affects first-order metaphysical questions (and also in the reverse direction of influence—what the tools' repercussions for first-order questions can teach us about the tools). The postmodal revolution has been very "meta", about what we're asking when we ask metaphysical questions. But the choice of tools also affects the questions' answers.

1.2 Structuralism

If this book has a single thesis, it is that the choice of metaphysical tools matters to first-order metaphysics, especially when it comes to "structuralist" positions in the metaphysics of science and mathematics.

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[3] See Bennett (2017); Fine (1994*a*,*b*, 2001, 2012); Rosen (2010); Schaffer (2009); Sider (2011).

[4] See Evans (2012).

[5] And indeed, Schaffer's (2009) defense of ground focuses on the limitations of a purely ontological approach to metaphysics more than on the limitations of a purely modal approach.

'Structuralism' is pretty vague, but the idea is that patterns or structure are primary, and the entities or nodes in the pattern are secondary.

The argument for structuralism is often epistemic: our evidence is only for patterns. One could respond with a merely epistemic doctrine: all we know is the pattern; what instantiates the pattern is real but unknown.[6] But structuralists respond metaphysically: the patterns are metaphysically, not just epistemically, primary.

Other arguments are nonepistemic: that mere differences in nodes are distinctions without a difference, or that dispensing with the nodes while keeping the structure yields a simpler picture of the world.

Structuralist positions have been defended in a number of different areas in the metaphysics of science and mathematics (and elsewhere).

According to nomic essentialism, networks of nomic, or lawlike, relations between properties are primary and the properties themselves are secondary. When a law of nature governs a property, this isn't something that just happens to the property. The nature of the property itself is somehow bound up with the laws governing it and other properties.

Why believe such a claim? One putative reason is epistemic. We know the property of charge (for example) through its nomic profile: entities with this property are correlated, by law, with the electromagnetic field, which is in turn correlated with the motions of other particles, depending, in part, on their charges. What do we know of the property of charge *in itself*? Nothing—we know of it only as: "that which is correlated, by law, with such-and-such". So why assume that there *is* anything more to the property than this lawful correlation?

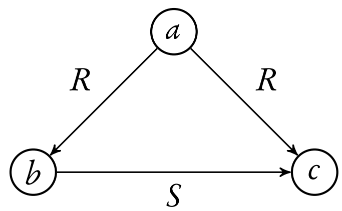
That was nomic essentialism, but there are also the closely related doctrines of dispositional and causal essentialism, according to which the dispositional and causal roles of properties are prior to the properties themselves.

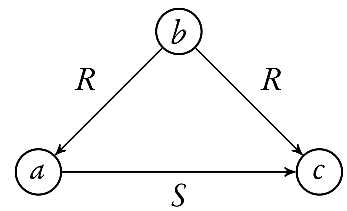
Another form of structuralism pertaining to properties concerns quantitative properties, those that can be measured by numbers. Charge and mass, for instance, come in degrees: one can measure amounts of these properties with real numbers. Now, for any distribution of values for a given quantity across all individuals—an assignment of 2g mass to this thing, of 1g mass to that thing, and so on—there is a network of corresponding relations amongst those individuals: one individual is twice as massive as another; a certain pair of

[6] Examples include what Ladyman (1998) calls epistemic structural realism and Lewis's (2009) "Ramseyan Humility".

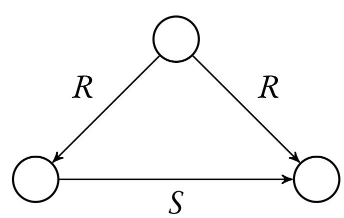
individuals are together exactly as massive as a certain other pair; and so on. A structuralist view of a quantity would say that the network of relations is prior to the individual values for that quantity. As before, this form of structuralism is often supported on epistemic grounds: what we observe is relations of quantity rather than particular values of quantity, as when we use a pan balance to establish that two things are exactly as massive as each other.

Yet another form of structuralism pertains to individuals: the network of qualities—properties and/or relations—had by individuals is primary and the individuals themselves are secondary. And again there is an epistemic argument. We seem to have no way to distinguish the following two arrangements:





Observation tells us only the qualities of individuals, and not which individuals they are; individuals don't have metaphysical nametags. So why suppose that there exists something beyond the qualities, an extra fact of which things have which properties, which can vary independently of the pattern of properties and relations? Why suppose there's a different possible world that is qualitatively just like ours, except that Barack Obama and I have "swapped places", so that I am a 6 feet 1 inch politician born in Hawaii, and he is a 5 feet 9 inch philosopher born in New Haven? Why not suppose instead that the identities of individuals cannot vary independently of the pattern, and indeed that the pattern is all there is:



Structuralisms about individuals have been defended within pure metaphysics—the bundle theory, for example. Recently such a position has been developed in the philosophy of science: structural realism. A related position is also defended in the philosophy of mathematics, only here the chief argument is not that all we *observe* is the pattern—since we don't observe mathematical entities—but rather that the pattern is all that matters to the practice of mathematics. What's distinctive of the natural numbers, for instance, is that they be an "infinite series each of whose members has only finitely many precursors" (Quine, 1960*b*, p. 242). It doesn't matter to mathematics what individuals are *in* this structure; all that matters is that they be so structured. So perhaps all there is to the natural numbers is this structure.

1.3 Modal and postmodal structuralism

All this talk of patterns being "primary", of patterns being "all there is", is extremely vague, and how it is precisified depends on the metaphysical tools one adopts. For instance, using modal concepts one can articulate theses to the effect that nodes and patterns cannot vary independently; and many structuralist positions have in fact been formulated in this way.

One form a modal structuralist thesis can take is this: the pattern cannot vary while the nodes remain constant. Dispositional essentialism, for instance, has usually been articulated as the claim that the very same properties and relations could not have existed while having different dispositional features; the network of dispositional relationships amongst properties and relations cannot vary independently of the identities of those properties and relations. Modal structuralist theses can also take the converse form: nodes cannot vary while the pattern remains constant. Structuralism about quantities can be understood as the claim that any two possible worlds that are alike in their distribution of quantitative relations (relations like being-twice-as-massive-as) are alike simpliciter with respect to quantities; thus doubling everything in mass does not result in a different possible world. Structuralism about individuals can be articulated as anti-haecceitism, the claim that it's impossible for individuals to vary independently of qualitative facts—that is, that there are no two possible worlds that have the same distribution of qualities over individuals, but in which different individuals occupy different qualitative roles; there is no duplicate possible world in which I have swapped places with Barack Obama.

Now, in the case of structuralism about mathematical individuals, modal articulations of the structuralist thesis have rarely been discussed, since facts about mathematical entities are generally taken, even by structuralists, to be necessary. Anyone who accepts this dogma already thinks that it's necessarily true that the number 1 occupies its place in the structure of natural numbers, for instance, and yet it's often thought that some question of structuralism remains open.

From a postmodal point of view, the failure of modal tools to articulate a meaningful thesis of mathematical structuralism is a sign of a deeper problem. *Any* modal thesis is bound to be unsatisfying as a formulation of any form of structuralism, because modality is "insensitive to source", as Fine (1994*a*, p. 9) puts it. A modal structuralist thesis says that independent variation of patterns and nodes is impossible, but says nothing about *why* this is impossible; the impossibility might be due to something that, intuitively, has nothing to do with structuralism. This can be dramatized by an example of Shamik Dasgupta's (2011, p. 118). Suppose that a very surprising "Spinozistic" thesis is in fact true of modal reality, namely that all truths are necessarily true. Then each modal structuralist thesis would automatically be true. Nodes and patterns can't vary independently because nothing can vary at all. But this would not be because of any priority of patterns over nodes; it would be because of the quirky nature of modality.[7] A more satisfying statement of a structuralist position will no doubt *imply* a modal thesis, but that modal thesis would be due to some deeper nonmodal thesis: nodes and patterns can't vary independently because nodes and patterns are tied together in some nonmodal way. Consider antihaecceitism, the claim that individuals cannot vary independently of the qualitative facts. A postmodalist won't take this modal claim as the statement of a structuralist position, but will seek instead some nonmodal formulation, for example the thesis that individuals just are bundles of universals. This thesis *implies* the modal thesis (given plausible principles connecting modality to claims of the form "X just is Y"[8]), but is a distinctively structuralist claim about the nonmodal tie between individuals and qualities.

Postmodalists have a similar attitude to modal formulations of many other metaphysical doctrines, not just structuralism. The modal thesis of mind-body materialism, that there is no mental difference without a physical difference, is all well and good, but to what is it due? What is it about the nature of mind that rules out the possibility of independent variation of the mental? A satisfying materialism would give some answer, such as that there are no fundamental mental properties or relations, and that all fundamental properties and relations are physical.[9]

[7] See also Fine's (1995, p. 271) point about the Tractarian view that all objects exist necessarily.

[8] See Dorr (2016); Rayo (2013) on this sort of language.

[9] It's tempting to regard many of the contortions philosophers of mind underwent to construct a proper modal formulation of materialism as the result of struggling to find a modal

I'm going to assume that modal articulations of all the structuralist positions to be considered in this book are indeed inadequate, and further, that postmodal articulations are needed. Though I won't say much in support of this assumption, it's worth distinguishing some different groups of philosophers who would accept it.

One group thinks that modality is *nonfundamental*. Of course, plenty of important metaphysical claims are not fundamentally true. But if a modal structuralist thesis is not fundamentally true, it must, somehow, be due to some fact or facts about fundamental reality, and a structuralist might prefer to articulate those facts directly. (This group faces a powerful objection: perhaps a modal structuralist thesis is the only way to concisely state the relevant facts about fundamental reality. One rejoinder might be that the relevant facts ought to be concisely stateable in fundamental terms—a kind of metaphysical law—if structuralism is to be the kind of significant thesis about fundamental reality that it aspires to be.)

A second group is a subgroup of the first: those who think that modality is not only nonfundamental, but also metaphysically *superficial*. On my own view, for instance, the necessary truths are just certain truths that we "hold constant" when talking about alternatives to actuality, and the distinction between truths we hold constant in this way and truths that we don't hold constant is more-or-less conventional.[10] Given this approach, if a structuralist thesis aspires to articulate something metaphysically important about reality, it should not do so via the metaphysically superficial language of modality. At best this would be a misleading way to get at an important nonmodal fact, and at worst it would not reflect anything important at all.

A third group thinks that the necessary truths are *minimal*. (Equivalently, they think that the possible truths are plentiful.) Suppose, for example, you think that, with a few exceptions (logical truths, perhaps), no truth is necessary unless it is underwritten by some postmodal claim (such as that individuals just are bundles of universals). You will then be dissatisfied with modal articulations of structuralism, for you will think they can't be true unless underwritten by some appropriate postmodal claim.

Finally, a fourth group thinks that modal structuralist theses may well be true, metaphysically deep, and even fundamental, but nevertheless are *unsuitable*

proxy for a simple idea about what is fundamental.

[10]See Sider (2011, chapter 12). Sidelle (1989) holds a similar view about modality; see also Nolan (2011).

statements of structuralism because they are not supported by structuralist arguments. Suppose, for instance, that one's argument for structuralism about individuals is that permutations of individuals amongst qualitative roles are distinctions without a difference. The modal formulation of structuralism about individuals—antihaecceitism—wouldn't be supported by this argument, since it doesn't imply that permutationally different scenarios aren't different; it just implies that they aren't both possible. Or suppose that one's main argument for structuralism is an epistemic argument that only structuralism can explain our knowledge of the domain in question; one might think that a merely modal formulation of structuralism, even if true, couldn't explain our knowledge.[11]

1.4 The challenge for postmodal structuralism

The demand for nonmodal formulations of metaphysical theses can make a difference: there is no guarantee that a given modal thesis *can* be backed by a suitably attractive nonmodal thesis.

One obstacle is that there may not be any coherent postmodal thesis in the vicinity. Consider structuralism about individuals. As we saw, that has a perfectly coherent modal formulation: antihaecceitism. But suppose we seek an account of the fundamental facts that underlies this pattern: an account of what is ultimately going on that explains why individuals can't vary independently of the qualitative pattern. The problem now is that it's hard to see what that account might be. The structuralist slogan "patterns without nodes" is absurd if taken flat-footedly, since what a pattern *is*, flat-footedly, is a set of facts about nodes; a pattern without nodes makes no more apparent sense than the Cheshire Cat's lingering smile. The modal formulation avoids incoherence by backing away from the flat-footed reading. It doesn't deny the existence of the nodes, only the possibility that they vary independently of the qualitative pattern. But if one is trying to say something distinctive about what is ultimately going on, the concern is that nothing other than the flat-footed, incoherent slogan is left. For instance, it's natural to think that a specification of the fundamental facts—a story about "what is ultimately going on"—just is a specification of which objects fundamentally exist and what their fundamental properties and relations are. If so, there would be no remaining room to state a coherent structuralist thesis; there would be only the incoherent flat-footed transposition

[11] Compare Dasgupta (2011).

of the slogan: "fundamentally, there are no objects, but nevertheless, there is a pattern—a set of facts about objects instantiating properties and relations".

I don't mean to suggest that no response is possible—hence the term 'obstacle'. One might, for instance, deny that fundamental facts must take the specified form. The bundle theory, in fact, denies this, but is inadequate for reasons we'll see in chapter 3.10. The question is whether any account is both coherent and avoids other obstacles.

(Many structuralist views merely prioritize relations over properties, rather than prioritizing relations (or properties and relations) over their relata, and hence don't face this obstacle. For example, meaning holists claim that meaning ultimately consists, not in the possession of semantic properties by individual words or sentences, but rather in a network of semantic relations across all words or sentences. This is a view of which kinds of features are present in the most fundamental semantic facts—relations, not properties—but the existence of the objects possessing those features—words, or sentences, understood in some nonsemantic sense—isn't denied or understood structurally. This all is perfectly straightforward, metaphysically.)

Another obstacle is that there might be a conflict with "postmodal logic". A natural strategy for formulating structuralism appeals to ground: facts about the pattern somehow ground facts about the nodes. And it's natural to take "facts about the pattern" to be existentially quantified facts whose instances are facts about nodes. Thus existential facts would ground their instances. But the usual logic of ground demands the reverse: instances ground existentials. The problem, again, simply doesn't arise if one articulates structuralism in merely modal terms. Ground is a hierarchical notion: facts are arranged in a hierarchy of more or less basic facts according to certain rules; and this additional imposed constraint can conflict with a structuralist thesis.

A third potential obstacle is that even if a modal position can be "translated" into a coherent and consistent postmodal thesis, that thesis might be theoretically unattractive from a distinctively postmodal point of view. For instance, if a postmodal structuralist thesis is a claim that certain concepts are fundamental, it may be that the required concepts to state the structuralist thesis are complex in certain objectionable ways, or cannot be used to state simple laws of nature—complaints that flow from a natural epistemology for fundamentality.

1.5 Essence

These concerns about postmodal structuralism will be discussed in more detail when we examine particular structuralist views. For the remainder of this chapter we will look more closely at various postmodal concepts, beginning with essence.

Fine's example of Socrates and the singleton set of Socrates is the intuitive heart and soul of the contemporary discussion of essence: it is meant to convince us that there is a real distinction between those facts or features that are part of a given thing's nature and those that are not; and it is thought that this distinction cannot be captured in modal terms.

Fine explores various ways to formalize the conception of essence; we can focus on the regimentation $\Box_{x_1,x_2\ldots}A$, which says that A holds in virtue of the natures of entities $x_1, x_2 \ldots$. Thus the true claim that it's of the essence of {Socrates} to have Socrates as a member would be regimented as $\Box_{\{\text{Socrates}\}}\text{Socrates} \in \{\text{Socrates}\}$, and the false claim that it's of the essence of Socrates to be a member of {Socrates} would be regimented as $\Box_{\text{Socrates}}\text{Socrates} \in \{\text{Socrates}\}$.

As we've seen, Fine denies that essence should be defined in terms of necessity. Indeed, Fine accepts the reverse definition: a necessary truth is a truth that holds in virtue of the essences of all things.[12]

[12] Fine (1994*a*, p. 9). Incidentally, I doubt this is right. There are some subject matters where the truth is necessary, whatever that truth happens to be, but where the truth isn't settled by the essences of the entities involved. For example, for a certain sort of realist about set theory, either the continuum hypothesis or its negation is true; and whichever is true is necessarily true. But this doesn't seem to be settled by essences (by the essence of set-membership, say). It's just a fact about what sets happen to exist. (See Sider (2011, p. 267).) Similarly for the principle of universal composition, according to which any plurality has a universal sum: it's necessary if it's true, but its truth doesn't seem to be due to essences. Fine himself might bring in his postulational account of existence (2007) to claim that these truths are essential after all: the idea might be, in the case of set theory, that we can choose which notion of set-membership to adopt, and that sets obeying the laws corresponding to that notion are thereby postulationally introduced, with truths about them holding in virtue of the essence of the chosen notion. But this reply seems unavailable given a more orthodox Platonist conception of mathematical existence.

1.6 Ground

Turning next to ground, we may again begin with Fine's regimentation: one or more facts $F_1, F_2 \ldots$ are said to ground another fact, G:

$$F_1, F_2 \cdots \Rightarrow G$$

There are many subtle details which I'll mostly ignore or elide. I'll move back and forth between speaking of grounding of facts, propositions, and speaking of grounding using a sentence operator; I'll mostly ignore distinctions between full, partial, strict, weak ground, etc.

Philosophers often speak of facts "holding in virtue of", "being grounded in", "depending on", "consisting in", "being explained by", or "being made true by" other facts. As Gideon Rosen (2010) vividly recounts, we have long viewed such talk with suspicion, preferring instead allegedly clear modal and other language, at least when we're trying to be rigorous. But Rosen, Fine (2001, 2012), Jonathan Schaffer (2009), and many others now say that such talk is legitimate after all. It concerns a relation of grounding, which is an irreplaceable conceptual tool in philosophy.

Claims of grounding *imply* modal claims: if P grounds Q then P necessitates Q. But the converse implication doesn't hold: even if it happens to be necessary that Q is true whenever P is true, there may not be the right sort of connection between P and Q so that P grounds Q.

Many of the traditional questions of philosophy, it is said, are really about grounding. The question of moral naturalism, for instance, should really be understood as the question of whether moral facts are grounded in natural facts. It is a distortion to understand the question in modal terms, for instance, as the question of whether moral facts are necessitated by natural facts, since according to many moral nonnaturalists, even though moral facts are "above and beyond" the natural facts, they nevertheless cannot vary independently of the natural facts. (And recall Dasgupta's point about the Spinozistic view that every truth is necessary.)

1.6.1 Ground and levels

There is a familiar "levels" picture of reality, in which facts at "higher" levels rest on facts at "lower" levels, with everything ultimately based on a ground floor of fundamental facts. Perhaps psychological facts are higher than chemical facts, which in turn are higher than physical facts, which are in turn fundamental facts.

The levels picture has always faced the question of how the levels are related. In his classic discussion of theoretical reduction, Ernest Nagel (1961, p. 354) himself mentioned three views of the status of his "coordinating definitions", which connect higher- and lower-level concepts in theoretical reductions. Coordinating definitions can be analytic, Nagel said, they can be stipulated by fiat, or they can be "factual" or "material". None of these three ideas seems correct as an account of the relationship between facts or properties at different levels. The third idea is apparently a relationship of lawful co-variation between metaphysically separate, metaphysically coequal partners; but the levels pictures is meant to articulate some metaphysically "tighter" connection—the lower levels in some sense *constitute* the higher levels. The relationship between thermodynamics and statistical mechanics should not be assimilated to that between the past and the future. The first and second ideas move in the direction of a tighter connection, but go too far: the relationship between the levels is discovered, not stipulated, and isn't a mere matter of meaning. What is wanted is a metaphysical, not semantic relationship, but a tighter one than "material".

When I was in graduate school, a certain view was common of the available options for connecting distinct properties or subject matters, and more generally, for giving a philosophical account of something. The main two were definitions and synthetic necessities. One could say "x is good $=_{\mathrm{df}}$ x causes pleasure", where the "$=_{\mathrm{df}}$" was understood to be, in some sense, a matter of ordinary meaning. Or one could deny that 'good' can be defined (in the $=_{\mathrm{df}}$ sense) but still hold it to be necessarily true that, for instance, anything that causes pleasure is good. A few other options were recognized (though not all of them were regarded as appropriate for philosophical accounts): extensional, nomic, and apriori. These connections can be ordered by the tightness of the connection:[13]

[13] The ordering is oversimplified, in light of, for instance, the contingent apriori (Kripke, 1972). Also, to facilitate comparison with ground, the diagram lists "conditional" connections (except $=_{\mathrm{df}}$); but one could instead consider biconditional connections: $\forall x(Fx \longleftrightarrow Gx)$, $\boxed{\mathbb{N}}\forall x(Fx \longleftrightarrow Gx)$, etc. Correspondingly, one could consider a biconditional groundlike concept $\Leftrightarrow$, intermediate in tightness between modal and apriori equivalence. It would be biconditional in that $A \Leftrightarrow B$ would imply $\Box(A \longleftrightarrow B)$, but nevertheless be asymmetric (as ground is normally held to be): if $A \Leftrightarrow B$ then $B \not\Leftrightarrow A$.

	extensional	$\forall x(Fx \to Gx)$
	nomic	$\boxed{\scriptstyle N}\forall x(Fx \to Gx)$
tighter connections	modal	$\Box\forall x(Fx \to Gx)$
	apriori	$\boxed{\scriptstyle A}\forall x(Fx \to Gx)$
	analytic	$Fx =_{\mathrm{df}} Gx$

But it is natural to think that some further connection, intermediate in tightness between the modal and apriori, must be recognized. The connection between levels is metaphysical, not apriori; but it's tighter than a merely modal connection, for as we saw, domains can be modally connected even when there is no "constitutive connection", as was illustrated most dramatically by the Spinozistic view that all truths are necessary. Also, as Fine especially has emphasized, modal connections are not asymmetric. It's being necessary that all Fs are Gs leaves open that it's also necessary that all Gs are Fs; A supervening on B (in various senses) leaves open that B might also supervene on A. But, Fine argued, the levels picture (not his phrase) demands an asymmetric connection between lower and higher levels. Fine puts all this well, in a discussion of how to understand materialism in the philosophy of mind:

> It will not do, for example, to say that the physical is causally determinative of the mental, since that leaves open the possibility that the mental has a distinct reality over and above that of the physical. Nor will it do to require that there should be an analytic definition of the mental in terms of the physical, since that imposes far too great a burden on the [materialist][14]. Nor is it enough to require that the mental should modally supervene on the physical, since that still leaves open the possibility that the physical is itself ultimately to be understood in terms of the mental.
>
> The history of analytic philosophy is littered with attempts to explain the special way in which one might attempt to "reduce" the reality of one thing to another. But I believe that it is only by embracing the concept of a ground as a metaphysical form of explanation in its own right that

[14] Fine actually says "anti-realist" here, but he uses that phrase idiosyncratically, and my substitution is arguably equivalent.

> one can adequately explain how such a reduction should be understood. For we need a connection as strong as that of metaphysical necessity to exclude the possibility of a "gap" between the one thing and the other; and we need to impose a form of determination upon the modal connection if we are to have any general assurance that the reduction should go in one direction rather than another. Fine (2012, pp. 41–2)

Thus the levels structure is naturally construed in terms of ground; lower-level facts ground higher-level facts.[15]

1.6.2 Wilson's challenge

Jessica Wilson (2014) has argued that ground is in fact useless in philosophy. Consider its putative use in articulating "naturalistic" positions. According to Wilson, the bare claim that the mental, say, is grounded in the natural is neutral over a range of more specific positions involving more specific metaphysical relations such as type identity, token identity, functional realization, part-whole, and so forth. (Wilson calls the generic grounding relation Grounding with a capital G, and calls the specific metaphysical relations grounding-with-a-lowercase-g relations.) She says:[16]

> Hence it is that naturalists almost never rest with the schematically expressed locutions of metaphysical dependence, but rather go on to stake out different positions concerning how, exactly, the normative or other goings-on metaphysically depend on the naturalistic ones.

On Wilson's view, then, grounding (i.e., Grounding) claims have no point; only the more specific claims are of interest.[17]

Someone who viewed ground as metaphysically fundamental, as a sort of super-added metaphysical force, would not agree that grounding claims are neutral over more specific positions; the grounding claim itself would count as another one of those specific positions. This isn't a very attractive view of ground though (we'll discuss it further shortly).

[15] There are open questions here, of how exactly to define the levels in terms of ground, and to what extent the grounding relation really partitions all facts into discrete layers (not that the levels picture really requires this).

[16] P. 546. Bennett (2017) and Kovacs (2017, 2016) make related claims, though Bennett holds that grounding claims (generic building claims, in her terms) have a role to play in metaphysics.

[17] See also Koslicki (2015).

In my view Wilson is importantly right about something. When we attempt to say what is going on, metaphysically speaking, in some domain, we don't stop with a claim of ground. We don't just say that the the mind is grounded in the body and leave it at that (setting aside the super-added force conception of ground). Instead, as Wilson says, we go on to say something more specific about the connection between the mind and the body. As we'll see in subsequent chapters, this sometimes means that ground is unsuitable as a tool for articulating forms of structuralism. But in other contexts what we want is precisely a more neutral claim. We *do* stop with neutral claims of grounding, for instance, when we're stating overarching positions like physicalism or naturalism, or saying what makes more specific positions count as instances of physicalism or naturalism.

The usefulness of overarching positions like physicalism about the mind emerges when we look at their epistemic role. Such sweeping doctrines are epistemically important even if they're in a sense metaphysically superficial—because unspecific. Take the case of consciousness. Physicalists work very hard to try to show that consciousness is somehow a physical (material) phenomenon. They begin by exploring one sort of way to ground consciousness in the physical, but if that doesn't work, they try another way. Why do they stick to this path? It's because they take themselves to have very good evidence that everything is grounded—in one way or another—in the physical. They look at many cases in the history of science, in which various phenomenon that initially seemed not to be physicalistic were all shown to be grounded in the physical, and conclude that these cases provide evidence for a sweeping doctrine of physicalism, to the effect that *all* phenomena are grounded in the physical.

This line of thought essentially uses the general notion of ground, and cannot be reconstructed using any more specific relation, since different specific relations are at issue in different cases in the history of science. Chemical phenomena, many of them anyway, can be *identified* with physical phenomena, whereas biological and geological phenomena seem different, and call for different specific metaphysical relations. Thus the neutrality of ground over more specific metaphysical relations is essential to its epistemic role. (Modality also shares this neutrality.) When we're trying to get to the bottom of things, metaphysically, we go deeper than ground. But in certain epistemic contexts it's important not to do this.[18]

[18]Schaffer (2009) has stressed the value of ground in preserving a role for traditional metaphysical disputes given a Moorean respect for common sense. This is another case in which the

1.6.3 Grounding ground

Are facts about ground themselves grounded? Or are they fundamental, as the "super-added force" view would have it, where a fundamental fact is one that isn't grounded in other facts?[19]

For many grounding facts, such as those connecting levels, there is a powerful argument that they must be nonfundamental. Levels-connecting grounding facts always involve higher-level concepts, because the higher-level fact getting grounded will involve such concepts; and surely no fundamental fact involves a higher-level concept. Any fact of the form "X grounds the fact that I am in pain", for instance, involves the property of being in pain, and hence is surely not a fundamental fact.[20]

The only exception to this sort of argument would be grounding facts involving only fundamental concepts, such as the fact that some particular *e*'s being charged grounds the fact that something is charged. But even these facts seem unlikely candidates to be fundamental; for why posit a new "super-added" force without good reason?

Thus grounding facts themselves have grounds. What grounds? I doubt there is any simple story to be told here, just as, in general, there is no simple story to be told about how nonfundamental facts are grounded. Ground, after all, is itself a high-level notion (assuming we reject the super-added force view), and one of the main reasons to accept ground in the first place is to allow for higher-level facts to depend on lower-level facts in complex ways that may not be accessible to us a priori. But we can make a good guess at the kinds of facts that help to ground grounding facts: patterns in what actually happens, modal facts, facts about the form or constituents of the grounding fact in question, metalinguistic facts, and even (according to some friends of grounding though not me), certain grounding facts involving only fundamental concepts.[21]

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metaphysical neutrality of ground is essential: the Moorean demand is that one's fundamental metaphysics be capable of grounding *in one way or another* common sense, not that it ground common sense in one particular way. (See also Fine's (2012, p. 41) discussion of the importance of ground to the project of "critical" metaphysics.)

[19]This definition is commonly accepted, but controversial. Fine (2001) argues for the need for a concept of *reality*, in addition to that of ground; in his terms there are two concepts in the neighborhood of a fundamental fact: a fact that holds in reality, and a fact that holds in reality and is ungrounded.

[20]See Sider (2011, sections 7.2, 7.3, 8.2.1).

[21]See Sider (2018) for a fuller discussion of the issues in this section.

1.7 Fundamentality

Another postmodal concept that will occupy us is that of fundamentality.[22] Actually there are two concepts here worth distinguishing: that of a *fundamental fact*, and that of a *fundamental concept*. The fundamental facts are (intuitively) those ground-level facts on which everything else rests, whereas the fundamental concepts stand for the most basic elements of fundamental facts, the ultimate "building blocks" of the world.

Lewis's natural properties are akin to fundamental concepts. Lewis says (Lewis, 1986, p. 60):

> Sharing of them makes for qualitative similarity, they carve at the joints, they are intrinsic, they are highly specific, the sets of their instances are ipso facto not entirely miscellaneous, there are only just enough of them to characterise things completely and without redundancy.

But fundamental concepts are not restricted to predicates; one can ask whether logical concepts are fundamental, for instance. Just as Lewis would articulate the view that there is fundamental mass structure by saying that mass properties (or relations) are natural, I would articulate the view that the world has fundamental ontological, or modal, or disjunctive structure by saying that quantifiers, modal operators, or the concept of disjunction are fundamental concepts. The idea is that a concept—whether logical or no—is fundamental if and only if it plays a role in articulating the world's fundamental structure, if and only if it is one of reality's ultimate building blocks.

There are certain structural differences between the various postmodal notions. For instance, ground and fundamental facthood are "factual" (or propositional) whereas fundamental concepthood is "sub-factual" (or subpropositional): it is entire facts that ground and are grounded, or are fundamental facts, whereas it is components of facts—or rather, their corresponding concepts—that are fundamental concepts. Essential claims $\Box_{x_1,x_2\ldots}A$ are partially factual (A) and partially subfactual ($x_1, x_2 \ldots$). Second, ground is comparative, in that each grounding claim involve a pair of facts, whereas both fundamental concepthood and fundamental facthood (on my usage anyway) are absolute: fundamentality is fundamentality simpliciter—absolute fundamentality.[23] Essential claims $\Box_{x_1,x_2\ldots}A$ can be regarded as comparative: the natures of $x_1, x_2 \ldots$

[22] I will be comparatively brief here, but see my Sider (2011), especially chapters 7 and 8.

[23] I'm open to various concepts of relative fundamentality, but the more fundamental (!) concepts on my view are the absolute ones. See Sider (2011, pp. ??).

are said to give rise to A. However, the relevant facts about the natures of $x_1, x_2 \ldots$ aren't specified in the essential claim; indeed, there is no commitment to any such facts being specifiable. (We will return to this.)

Because it is comparative, there is a sense in which ground is a richer notion than either sort of fundamentality. Ground can be used to make assertions about high-level subject matters, and about how high-level matters relate to the lowest level, whereas (absolute) concept-fundamentality concerns only the lowest level. Moreover, as we saw, one can define a fundamental fact as one that has no ground, whereas there is no immediate definition of ground in terms of fact-fundamentality. Thus the concepts of fundamentality cannot "go it alone" for certain philosophical endeavors. But for certain purposes this austerity of fundamentality can be welcome. Focusing exclusively on what is fundamental might be deemed appropriate if what one is giving an account of is itself a fundamental matter.

Let us finally discuss the epistemology of concept fundamentality. Realist epistemology of science generally stresses the super-empirical virtues, notably simplicity of various sorts. The believer in fundamental concepts is ideally placed to make sense of those kinds of simplicity in realist terms. For many kinds of simplicity, a theory's simplicity can be altered when it is "rewritten" using alternate concepts (as when we convert a theory about blue and green to one about grue and bleen (Goodman, 1955, chapter 3)). A skeptic about fundamental concepts will likely regard the rewritten theory as a notational variant of the original, and so will be correspondingly skeptical of the epistemic value of these kinds of simplicity: they are sensitive to notational, nonworldly differences between theories. But a realist about fundamental concepts will not regard the difference as notational if the concepts in question are putatively fundamental: the theories may differ in how well they correspond to the world's fundamental structure. Thus it is open to the realist to regard these sorts of simplicity as being epistemically significant.

One such sort, call it ideological parsimony, concerns the number and nature of undefined concepts: fewer and "simpler" concepts are better. Another sort concerns laws: a theory is better when it contains powerful yet simple laws, where the simplicity of a law corresponds to something about its syntax when stated using the theory's undefined concepts. To be sure, there are difficult questions about how to measure either sort of simplicity: simplicity of fundamental concepts is not just a matter of counting the theory's fundamental concepts, nor is simplicity of laws just a matter of measuring the length of their statements. But however these questions are answered, realism about fundamental concepts (or something a lot like it) is needed to regard these sorts of simplicity as corresponding to something worldly.

Conversely, if one is a realist about fundamental concepts, it's very natural, for a scientific realist anyway, to think that parsimony and simple-yet-powerful laws are epistemically important. For the realist about fundamental concepts believes in worldly distinctions corresponding to differences in these kinds of simplicity; and they seem like an exact match for the intuitive basis of realist thinking about theory choice, which is that the world is a priori likely to be simple.

Frank Arntzenius's (2012) book *Space, Time, and Stuff* is an example of a recent inquiry into the metaphysics of science that gives pride of place to the simplicity and strength of laws. Arntzenius writes that:

> …our knowledge of the structure of the world derives from one basic idea: the idea that the laws of the world are simple in terms of the fundamental objects and predicates. In particular, what we can know and do know about the way things could have been—what we can know and do know about the metaphysical, and physical, possibilities—derives from our knowledge of what the fundamental objects and predicates are, and what the fundamental laws are in which they figure. I argue that it is bad epistemology to infer what the fundamental objects, predicates, and laws are on the basis of intuitions as to what is, and what is not, possible. (p. 1)

Notice how distinctively postmodal this epistemology is. Modal beliefs—about fundamental reality anyway—are epistemically downstream from nonmodal beliefs about the way reality is, and these nonmodal beliefs should in large part be determined by considerations involving laws (and ideological parsimony, in my view).

I don't pretend that pairing a simplicity-centered epistemology of fundamental reality with concept-fundamentality-centered metaphysics is inevitable, only that the pairing is natural. It's an interesting question what pairing is most natural for a ground-centered metaphysics. As it happens, the friends of ground have tended to prefer a Moorean/modalist epistemology of fundamental reality: look for fundamental grounds that mesh with intuitive modal beliefs and ordinary nonmodal beliefs. (Each of these sorts of beliefs constrains ground given commonly accepted principles: the former via the principle that $A \Rightarrow B$ implies $\Box(A \rightarrow B)$, the latter by the principle that any truth must be grounded in fundamental truths.) But perhaps those writers just happen to be partial to this epistemology for independent reasons.

Chapter 2

Nomic Essentialism

We turn now to the relation between scientific properties and the laws of nature, but with an eye to our more general topic: how metaphysical inquiry is sensitive to one's chosen "metaphysical tools".

Are scientific properties "independent" of the laws of nature in which they figure? "Quidditists" say yes. "Nomic essentialists" say no: properties are bound up with laws.

Actually, the literature has mostly discussed the relation of properties to causation, dispositions, and powers, rather than to laws.[1] But I'm going to talk about laws since I want to explore the issue as it arises at the fundamental level, and laws, I think, are more likely to be fundamental than causation or dispositions or powers.

Arguments in favor of nomic essentialism often involve "swapping of nomic roles". A property's nomic role is the set of laws of nature that govern it. If properties are "independent" of the laws, as quidditists think, then it should be possible for some property to obey different laws from what it actually obeys—to have a different nomic role—or even to have a pair of properties swap their roles in the laws, so that, e.g., charge obeys the kinds of laws that mass actually does, while mass obeys the kinds of laws that charge actually obeys. Nomic essentialists then object to this consequence in various ways. Some object that if role-swapping were possible, then we couldn't know certain things that we obviously do know, such as that I am more massive than a mouse. For a world in which charge and mass properties have swapped roles would

[1] E.g. Shoemaker (1980); Swoyer (1982); Bird (2007); (Cartwright?); but see Hawthorne (2001).

appear exactly the same to us, but I might not be more massive than a mouse in that world. Others object that a world in which charge and mass have swapped roles is a "distinction without a difference".

For the record, I don't myself find either argument convincing. I don't find the epistemic argument convincing for Rae Langton (2004) and Jonathan Schaffer's (2005) reasons: in the ordinary sense of knowledge, we have knowledge of which properties play which roles despite the possibility of role-swapping, and it is harmless to admit that we don't know these things in extraordinary senses of knowledge. And the metaphysical argument seems either to rely on an observational criterion for counting as a difference (the possibilities would "look the same"), in which case it should obviously be rejected, or else is nothing more than the bald assertion that there is in fact no fundamental difference between role-swapping worlds.

But our focus here will be on how nomic essentialism should be understood, not on whether it is true. The question of whether properties are "independent" of the laws of nature needs to be made precise, and how this is done depends on one's metaphysical tools. The issue has mostly been approached from a modal perspective, with the central questions being whether a property could have obeyed different laws, or whether there are possible worlds in which properties swap nomic roles. But it's natural to be dissatisfied with a merely modal formulation. There ought to be some deeper postmodal claim from which this modal claim would flow. Such a claim will likely have modal implications, so addressing the modal issues remains important, but the deeper claim is the one of ultimate interest, or so many postmodalists will insist.

In fact, nomic essentialism is hard to formulate postmodally; that is the main thesis of this chapter. Quidditism, on the other hand, is perfectly straightforward from a postmodal point of view, and is therefore congenial. We will also begin to encounter the obstacles confronting "structuralist" views in general.

2.1 Nomic essentialism and ground

First let's look for a ground-theoretic formulation of nomic essentialism. It's natural to look for some thesis to the effect that scientific properties are grounded in laws, for that would amount to privileging the pattern of lawlike relations amongst properties over the individual properties themselves. And it's natural to assume that such a thesis would say that whenever a property is instantiated, this fact is grounded in facts about laws. Thus whenever an object *a* has a property P, the nomic essentialist would accept a claim of the following form:

$$[\text{something about the laws}] \Rightarrow a \text{ has } P$$

But what about the laws, exactly? It must connect to the object a. And laws say nothing directly about particulars like a, only about their properties. Moreover, the relevant property for a will surely be P. Won't this involve a circularity of ground?[2]

To be concrete, suppose the view has this form:

$$\exists p(\mathscr{L}(p) \wedge a \text{ has } p) \Rightarrow a \text{ has } P$$

"a has P because a has some property with nomic role $\mathscr{L}$"

$\mathscr{L}(p)$ is the description of the nomic role played by p—perhaps in relationship to other properties. (The other properties might be mentioned by name, or they might be "ramsified out", in which case $\mathscr{L}(p)$ would say that there exist other properties $q, r \ldots$ that are connected by law to p in a certain way.)

Next make some standard assumptions about the logic of ground. To make these assumptions we need a distinction between "partial" and "full" ground (2012). Full ground ($\Rightarrow$) is the notion of ground we've been discussing so far. A partial ground is a "part" of a full ground: one fact partially grounds ($\rightsquigarrow$) another fact if the first, perhaps together with other facts, fully grounds the second. The assumptions are then these:

For all y, if $A(y)$ then: $A(y) \Rightarrow \exists x A(x)$
("existentials are fully grounded in their instances")

If $A \wedge B$ then: $A \rightsquigarrow (A \wedge B)$, and $B \rightsquigarrow (A \wedge B)$
("conjunctions are partly grounded in their conjuncts")

If $A \Rightarrow B$ then $A \rightsquigarrow B$
("full ground implies partial ground")

$\rightsquigarrow$ is transitive and irreflexive

[2] The argument is surely a cousin of the "everyone doing everyone else's washing" argument [references], as well as the argument in section IV of Whittle (2008).

The problem is then as follows. We've supposed that a has P fully because $\exists p(\mathscr{L}(p) \wedge a \text{ has } p)$. A true instance of this existential is $\mathscr{L}(P) \wedge a$ has P; and so the existential holds fully and hence partially because of the instance (existentials…); but the instance holds partially because a has P (conjuncts…), which given transitivity violates irreflexivity.

Shamik Dasgupta once suggested to me that perhaps the core insight of "structuralist" positions is precisely that existentials are *not* grounded in their instances. This isn't his considered view; indeed in subsequent work he has argued that it's analytic that existentials are grounded in their instances.[3] But in any case it's an interesting suggestion, to which we will return.

2.2 Other grounding claims: existence, identity

Given the previous section, a nomic essentialist might back away from the claim that facts about the *instantiation* of properties are grounded in the laws, and say instead that some other facts about properties are grounded in the laws.

They might, for instance, say that it's the *existence* of a property, and not facts about its instantiation, that is grounded in the laws. Perhaps facts about the property's instantiation are fundamental, on this view, having nothing to do with the laws. The laws enable the property to get its foot in the door of being; but once it's in, it no longer needs them to be instantiated (though of course its instantiations are then guided by the laws, in a sense dependent on the particular view of lawhood adopted by the nomic essentialist).

But what kind of fact about the laws will ground a property P's existence? It surely can't be the fact $\mathscr{L}(P)$, the fact that P itself plays a certain role in the laws of nature, for that fact "presupposes" P's existence. How could P's possession of *any* feature ground its own existence? Nor can it be the fact that $\exists p \mathscr{L}(p)$, the fact that some property plays that role, since this fact is grounded in its instance $\mathscr{L}(P)$ which presupposes P's existence. The intuitive problem here is this: the idea was to ground the existence of P in "the laws", but the relevant facts about the laws involve P, or are grounded in facts that involve P, and hence cannot ground P's existence.

Instead of saying that the laws ground facts about property instantiations or existence, the nomic essentialist might instead say that they ground facts about property identitites. Nomic essentialists in fact do say things like this:

[3] See Dasgupta (2009, p. 50), though he's less committed to this in Dasgupta (2015).

"the *identity* of a property involves the laws".[4] So perhaps the nomic essentialist could claim:

$$\mathscr{L}(P) \Rightarrow P = P$$

But for one thing, I doubt identity facts have grounds.[5] For another, even if they do, surely the reason for an object's identity with itself should be the same in every case, regardless of the object involved, in which case a ground of $P = P$ should be of a sort that could apply to any object whatseover, such as the fact that P exists, and not a fact like $\mathscr{L}(P)$. And anyway, the nomic essentialist idea that properties are tied to their nomic roles seems distant from the idea that facts about the laws ground such facts as $P = P$. To verify this impression, consider the modal implication of the claim that $\mathscr{L}(P) \Rightarrow P = P$: $\Box(\mathscr{L}(P) \to P = P)$. (I assume the usual view that ground implies necessitation.) This modal claim does nothing to block the possibility of properties "swapping" nomic roles. Where $\mathscr{L}$ and $\mathscr{L}'$ are the nomic roles of properties P and P', respectively, so that $\mathscr{L}(P)$ and $\mathscr{L}'(P')$, for all the modal claim says there might be a possible world in which $\mathscr{L}(P')$ and $\mathscr{L}'(P)$—the modal claim's meager implication about this world is just that $P = P$. The original ground-theoretic proposal, in contrast, does block role-swapping. That proposal was that $\exists p(\mathscr{L}(p) \wedge a \text{ has } p) \Rightarrow a \text{ has } P$, whose modal implication is that $\Box(\exists p(\mathscr{L}(p) \wedge a \text{ has } p) \to a \text{ has } P)$. Thus in a putative world in which P swaps nomic roles with P', a and other objects that instantiate the property that has P's actual role—namely, $\mathscr{L}$—would still be instantiating P.

It wouldn't help to change the grounding claim to:

$$\exists p \mathscr{L}(p) \Rightarrow P = P$$

The corresponding modal claim is then $\Box(\exists p \mathscr{L}(p) \to P = P)$, which still allows the role-swapping world in which $\mathscr{L}(P')$ and $\mathscr{L}'(P)$; its meager implication about this world is again just that $P = P$ (via the implication of $\exists p \mathscr{L}(p)$ by $\mathscr{L}(P')$).

A more promising suggestion would be to interpret the claim about how P's identity is grounded as involving an irreducibly generic or general concept of grounding, of the sort recently advocated by a number of writers.[6] The idea can be implemented by attaching the grounding operator to sentences with

[4]See, for example, Bird (2016, p. 4); Mumford (2004, p. 151).

[5]Though see Burgess (2012); Fine (2015); Shumener (2015).

[6]Fine, Glazier, Wilsch; thanks to Sarah Moss for helpful suggestions here.

free variables; in these terms the most recent proposal could be amended to:

$$\mathscr{L}(p) \Rightarrow p = P$$

(with p a variable). Thus instead of specifying the ground of the particular fact that P is identical to itself, we specify the ground of the *kind* of fact (or property) of being a fact of identity with P. The usual view that ground implies necessitation has its analog for general grounding: if $A(x) \Rightarrow B(x)$ then $\Box\forall x(A(x) \rightarrow B(x))$; thus the general-grounding claim implies that any property satisfying $\mathscr{L}$ must be P itself, which does preclude the possibility of swapping nomic roles.

There are some complaints about this proposal that I myself am inclined to make, but which may have limited dialectical force. First, the involvement of the identity relation in the proposed claim of general grounding seems extraneous to the proper concerns of nomic essentialists. Those concerns involve charge, mass, and so forth, and not the identity relation at all. Second, the same reason for doubting that identity facts have grounds is also a reason to doubt that identity-with-a-given-thing has a ground—identity just seems basic. Third, it seems intuitively odd that laws play the limited grounding role of securing identities of properties but no role at all in the instantiations of those properties. I do have a further complaint which is considerably more forceful (I think), but discussion of it must await the next section.

2.3 Nomic essentialism and essence

Given some of the roadblocks of the previous section, it is natural to ask whether the key nomic essentialist claim might be better understood as involving Kit Fine's notion of essence. Fine often glosses the claim that an object is essentially thus-and-so by saying that being thus-and-so is part of the thing's "identity". So perhaps the talk of a property's "identity" involving the laws should be understood, not as a grounding claim about some fact involving the identity relation, but rather as a Finean essentialist claim:

$$\Box_P \mathscr{L}(P) \qquad \text{("}\mathscr{L}(P)\text{ holds in virtue of the nature of }P\text{")}$$

Notice a difference between the essence-theoretic formulation of nomic essentialism and formulations we'd been considering previously. The previous formulations all embrace a laws-to-properties "direction of influence", in one way or another. The initial formulation considered, for example, claimed that particular property instantiations of the form *a* has P hold in virtue of facts about laws. But the essence-theoretic formulation embraces the reverse, saying that the laws hold in virtue of the natures of properties.

This interpretation of nomic essentialism does not bring in the extraneous subject matter of the identity relation—a plus, I think. Also, the modal claim implied by this reading of nomic essentialism (following Fine in assuming that essence implies necessity), namely that $\Box\mathscr{L}(P)$, is certainly close to the concerns of nomic essentialists. It rules out the role-swapping world considered earlier: since $\mathscr{L}(P)$ holds in actuality, it must given the essentialist formulation hold in every possible world, in which case the putative world in which $\mathscr{L}'(P)$ and $\mathscr{L}(P')$ is impossible (assuming the roles $\mathscr{L}$ and $\mathscr{L}'$ to be incompatible). It also agrees at an intuitive level with many of the things that nomic essentialists say. As Alexander Bird (2007, p. 2) puts it, "…laws are not thrust upon properties, irrespective, as it were, of what those properties are. Rather the laws spring from within the properties themselves."

I suspect the essence-theoretic proposal will be the most popular I'll consider. But in my view it is very unsatisfying, even if true. It says that "something flows from the essence of P", without saying *how* that something flows. We aren't given a picture of the "innards" of P from which we can just "read off" the claim. What do the fundamental facts about P and other properties (their "innards") look like, and how do these facts give rise to laws? My criticism of the essentialist formulation isn't that it's false, but rather that it's incomplete, since it doesn't begin to address these questions.

The gloss on a Finean essentialist claim $\Box_{x_1,x_2\ldots}A$ is that "A holds in virtue of the natures of $x_1, x_2\ldots$". But it isn't as if one has been given an account of these things, "natures" of $x_1, x_2\ldots$, or an account of now natures give rise to the truth of statements. Ultimately, natures are given no more explicit articulation than: "are such as to give rise to certain essential truths".[7] For certain purposes this

[7] Compare Fine (1995, p. 273):

> Although the form of words 'it is true in virtue of the identity of x' might appear to suggest an analysis of the operator into the notions of the identity of an object and of a proposition being true in virtue of the identity of an object, I do not wish to suggest such an analysis. The notation should be taken to indicate an unanalyzed relation between an object and a proposition. Thus we should understand the identity or being of the object in terms of the propositions rendered true by its identity rather than the other way round.

is just fine. Some important metaphysical questions are perhaps answered by saying which facts flow from things' natures, and by identifying the particular objects giving rise to those facts, but without saying anything more about those objects' natures. But nomic essentialism is surely meant to be a claim about fundamental reality, about the ultimate natures of properties and laws. So an articulation of nomic essentialism shouldn't just say: "the natures of properties—whatever those are, exactly—guarantee that their laws hold".

This criticism is in the spirit of Wilson's (2014) critique of ground, her claim that grounding claims are useless because they're neutral over more specific claims. I argued in chapter 1 that this neutrality of ground doesn't undermine their use for certain purposes, but the statement of nomic essentialism is one of those cases where I think that a point like Wilson's is right.

Is it appropriate to demand a more informative account of the nature scientific properties that underwrites the essentialist claim? Sometimes such an informative account is possible. Consider, for example, the fact that the ancestor relation is essentially transitive. This essentialist claim is explained by an informative statement of the nature of the ancestor relation: for x to be an ancestor of y is for x and y to be the first and last, respectively, in a finite series of objects, each of which is a parent of the previous; this definition implies that being-an-ancestor is transitive (since if x and y are connected by such a series and y and z are as well, then x and z are connected by a series consisting of the first two series concatenated). But, it may be replied, in other cases we can't give a definition, or any other such account of the "innards" of a property or relation, and we can say only that a claim holds in virtue of that property or relation's nature. Perhaps it's essential to the concept of disjunction that it obeys the rule of disjunction introduction, but this is presumably not underwritten by any definition of, or any other more fundamental account of, disjunction; all we can say is that disjunction is essentially such as to obey that rule. And perhaps the nomic essentialist will say that the case of scientific properties and their laws is similar.

Perhaps so; but insofar as the essentialist claim gives no informative account of scientific properties and their relationship to laws—other than "the property's nature is such as to guarantee those laws holding"—it draws nearer to the merely modal formulation of nomic essentialism, according to which it's necessary that if a property exists then it obeys its actual laws. Pointing to the properties as the source of the necessity does head off certain concerns about the modal formulation, such as that the necessity could be due to a general feature of modality (the "Spinozistic view", for instance) having nothing to do with the properties. But the account still remains distant from the postmodal ideal, of a satisfying account of the structure of actuality giving rise to the modal claim.

Indeed, if a certain sort of reductionism about essence is true, the account wouldn't differ from quidditism! Suppose a thing's essence just consists in certain sorts of important facts about it. Which facts? Well, specifying them will be the task of the reductionist, but on a rather deflationary approach, the specification might be somewhat conventional. And in the case of a property, one of the important facts might be held to be the property's nomic role. Thus the nomic essentialist view would amount merely to the claim that a property's nomic role is one of the specified important facts about that property—which is something that a quidditist could accept.

At the opposite end of the spectrum from conventionalist reductionism about essence is the view that essence is metaphysically basic. This would be like the "further force" view of ground mentioned in chapter 1. Given that view, the essentialist articulation of nomic essentialism would at least amount to a statement about fundamental reality. Now, I don't myself like this view for essence any more than for ground. But even given the view's truth, the articulation of nomic essentialism that it enables remains unsatisfying. We may again compare it to a merely modal articulation, given the view that modality is metaphysically basic. In each case, a demand for an account of the underlying nature of properties and laws still seems appropriate. A metaphysically basic modal or essentialist connection between laws and properties would be "external" to laws and properties, whereas one would have hoped for an "internal" account, an account of the "innards" of one or the other explaining the distinctive sense in which the two are bound up together.

I have been complaining that "$\mathscr{L}(P)$ holds in virtue of the essence of P" is insufficiently metaphysically revealing to count as the statement of nomic essentialism. I'd make the same complaint about ground-theoretic variants of that formulation, such as that the truth of $\mathscr{L}(P)$ is grounded in the existence of P (or in P, if as Jonathan Schaffer (2009) thinks, particular entities can ground). *How* does the existence of a property ground facts about its nomic role? What are the relevant features of P by virtue of which the putative grounding claim holds?

And, to complete an argument from the previous section, I would make the same complaint about the claim of generic grounding that playing role $\mathscr{L}$ grounds being identical to P, i.e., $\mathscr{L}(p) \Rightarrow p = P$. Like the formulations considered in the present section, this doesn't on its own yield any distinctive general account of fundamental reality, nor does any such theory with which it might be supplemented come to mind.

Claims of generic essence don't on their own constitute a distinctive general account of fundamental reality. What they can do is allow us to "place" nonfundamental matters within a given account of (more) fundamental reality. Suppose it given, as a fundamental fact, that there exists a certain sum of subatomic particles with feature T_1; and suppose the generic grounding claim that T_1 grounds tablehood: $T_1 x \Rightarrow Tx$. We can then say that the sum of particles is a table, and indeed, that it's a table because it's a T_1. We have "placed" the feature tablehood on the pre-existing fundamental grid of particles and their sums, by means of the generic grounding claim. But in order to do so, we needed that grid in the first place.

Can the generic grounding claim $\mathscr{L}(p) \Rightarrow p = P$ be used in this way to place scientific properties P within a pre-existing fundamental grid that, somehow, does justice to the nomic essentialist vision? We're back where we started: what is that fundamental grid? What *is* the nomic essentialist vision of fundamental reality? Also, this particular generic grounding claim concerns an *identity property*, the property of being-identical-to-P. So the kinds of grids in which it can "place" this property must contain a property possessing the feature $\mathscr{L}(p)$. This property will be the very property P! So it isn't as if the generic essentialist claim will enable some fundamental account to dispense with facts about particular scientific properties. At best it would seem to be a kind of add-on to what would otherwise appear to be the quidditist's grid: fundamental facts about the possession of properties by particular entities—*a* has P, b bears R to c, etc.—plus facts about laws, such as the fact that $\mathscr{L}(P)$. It's hard to view the addition of the generic grounding claim to this grid as an advance on the merely modal formulation of nomic essentialism. The initial grid contains nothing distinctive of nomic essentialism, and no hint of why swapping of nomic roles should be impossible. Its impossibility is guaranteed only when the generic grounding claim is added.

This last comparison generalizes: the complaint I've been making throughout this section is akin to the standard postmodal complaint about modal formulations of metaphysical theses. The postmodal complaint that modal theses are insufficiently revealing of "metaphysical structure" is based on the hankering for a more explanatorily satisfying account than mere modal formulations offer. What we have seen is that even certain postmodal formulations leave us with the same hankering.

2.4 Nomic essentialism and fundamentality

Let's now think about how the issue of nomic essentialism would look when framed in terms of concepts of fundamentality—that of fundamental properties, and of fundamental facts—rather than ground or essence.

The intuitive opposition is between those who accept, and those who deny, that properties can be understood independently of laws. This might suggest characterizing quidditism as the view that properties like mass and charge are fundamental properties, and that singular facts involving their instantiation are fundamental facts. (And perhaps adding that certain facts about laws are also fundamental, if this is what the quidditist thinks about laws.) Nomic essentialists, we might then think, deny some part of this picture, since the picture implies that facts about the possession of properties—facts that don't involve lawhood or causation—are fundamental facts. But how exactly?

Could the nomic essentialist simply say, for instance, that mass and charge are not fundamental properties? I don't think so. Simply downgrading the status of mass, charge, and other scientific properties to the level of grue and bleen just doesn't seem right. Also, it omits any distinctive claim about the connection of properties and laws. Also, the view surely[8] must hold that laws are fundamental *facts*; but surely any property that occurs in a fundamental fact must be a fundamental *property*.[9]

Perhaps the part of the quidditist picture that nomic essentialists should reject is instead the claim that singular facts about the instantiation of properties like mass and charge are fundamental facts, on the grounds that such facts speak of scientific properties in isolation from the laws. But of course, such facts can't simply be rejected, since they're essential to the description of the world. (The laws aren't all there is to the world; there is also the matter of which occurrences that are permitted by the laws actually happen! Even in a deterministic world, at least one time slice is needed to characterize the rest of the world.) So some fundamental facts must be introduced that ground (or replace) these singular facts; and these fundamental facts must somehow vindicate the core nomic essentialist intuition that properties can't be "understood independently of" laws.

[8]Caveat: as noted earlier, the literature is dominated by those who make claims about dispositions or causation or powers, not laws, and some of these writers think that facts about laws are not fundamental, but rather emerge from fundamental facts about causation or dispositions or powers.

[9]See my principle of "purity" from Sider (2011).

These fundamental facts might be construed as being relational in some way, so that a complete fundamental fact of x's having a certain mass cannot be formed without somehow bringing in lawhood as well. (Analogy: comparativism about mass implies that you can't "understand my mass independently of the masses of other things", since there simply is no fundamental fact about my mass alone.) Here's a simple version of this thought. Whereas according to the quidditist the fundamental particular matters of facts look like this:

$$a \text{ has } P$$

for the nomic essentialist, these facts are not fundamental; rather, the facts that are fundamental look like this:

$$\exists p(\mathscr{L}(p) \wedge a \text{ has } p)$$

where $\mathscr{L}(p)$ specifies P's role in the laws. But now we're back in familiar territory. This claim is incompatible with the plausible principle that if an existentially quantified statement is a fundamental fact, then so are all of its instances—the fundamentality-theoretic analog of the principle that existentials are grounded in their instances.

2.5 Ungrounded or fundamental existentials?

Several of my arguments have rested on principles about existential quantification: the principle that existentials are grounded in their instances, and the principle that if an existential fact is fundamental, so are its instances (or, more strongly: existential facts are never fundamental).

Although the argument at the beginning of section 2.1 used the full-strength principle that existentials are always grounded in their instances, it could get by with the weaker principle that existentials can never be ungrounded, since the nomic essentialist considered there presumably takes $\exists p(\mathscr{L}(p) \wedge a \text{ has } p)$ to be the ultimate account of a's having P and hence takes it to be ungrounded. And this weaker principle avoids certain objections one might make to the stronger principle. One might be open to strange patterns of grounding at nonfundamental levels while remaining conservative about the most fundamental level. Fine (1994*b*) talks about recriprocal essence: the essence of Holmes is to be assisted by Watson, and the essence of Watson is to be assisted by Holmes. This arguably leads to cyclic grounding: Holmes's existence is grounded in

Watson's, and Watson's in Holmes's. Or (to continue with fiction) if the Holmes stories say that someone stole Holmes's boots without specifying any thief in particular, a sort of fictional realist might argue that it's true that someone stole Holmes's boots but that this existential sentence isn't grounded in any instance. Examples like these don't tend to show that existentials can be fundamentally true or ungrounded.[10]

So what can be said in favor of the existentials principle? Shamik Dasgupta has stressed its intuitive appeal:

> ...we have been brought up to understand that quantifiers range over a domain *of individuals*. So our natural understanding of [existential facts] is that they hold in virtue of facts about individuals... (2009, p. 50)

One might worry that the latter claim doesn't follow from the former: why should the existential quantifier's ranging over a domain of individuals imply that existential statements hold in virtue of their instances? Still, the intuitive basis for the latter claim is undeniable. As Dasgupta says elsewhere:

> Against quantifier generalism [the view that existential facts are fundamental], one might argue that it is analytic, or perhaps essential, of the existential quantifier that existential facts hold in virtue of their instances. (2015)

But what can be said to someone who just denies these thoughts?

One might argue that existentials are analogous to disjunctions, and thus behave analogously with respect to ground. So, since disjunctions are grounded in their true disjuncts, existentials are grounded in their true instances. (An analogous argument may be given for fundamentality.) The argument is somewhat compelling, but the question then becomes how to justify the disjunctions principle. The disjunctions principle and the existentials principle both seem intuitively obvious, but hard to justify on independent grounds.

[10]On my own way of thinking about grounding, the facts about grounding in these cases would be unremarkable. In each case we can identify the sort of metaphysical semantics for a language with a new quantifier; and the metaphysical semantics is holistic in a certain way, directly providing metaphysical truth conditions for pairs of sentences in the case of reciprocal essence, and for existential sentences in the case of the second, rather than via instances. See Sider (2011, section 7.7) and Sider (2008*b*).

2.5.1 The Tractarian and the semi-Tractarian

The existentials principle (and the disjunctions principle) could be defended by appeal to the "Tractarian" view that the fundamental facts consist exclusively of atomic propositions. (Although the Tractarian view doesn't imply any particular story about how existentials (or disjunctions) are grounded, it makes the standard story natural to adopt.) The Tractarian view is certainly intuitively satisfying, but it is hard to uphold: surely some negations are fundamentally true. What would ground the fact that I'm not 6 feet tall? The fact that I'm five-nine, perhaps? But other examples aren't so easily handled. Suppose an object completely lacks any determinate of some fundamental determinable property—suppose an object completely lacks charge, for instance (where this is not the same as having zero charge). It's hard to see what nonnegative fact would ground this. (Even if we could find a necessitating positive fact, it's a stronger claim to say that the positive fact is a *ground*.)

One might support the existentials principle with a related but weaker "semi-Tractarian" view: that the fundamental facts consist of atomic sentences and their negations. (Again, although this doesn't imply any particular claim about how existentials are grounded, the standard story seems likely once fundamental truth is ruled out.) The semi-Tractarian view is more defensible than the Tractarian view (though less satisfying). But it too is hard to uphold, for familiar reasons. Suppose that everything is F. This fact must presumably have a ground, given the Tractarian view. And the only available ground, given the view, would seem to be the plurality consisting of all atomic facts Fa.[11] But grounds must necessitate, and it would seem that each member of this plurality could hold even if not everything is F: some extra object could have existed and failed to be F.

The idea that everything exists necessarily has recently surged (Linsky and Zalta, 1994, 1996; Williamson, 1998, 2002, 2013). But although this view undercuts the argument of the previous paragraph, it doesn't clearly rescue the semi-Tractarian. The mere fact that it's impossible for there to exist further objects doesn't make the instances of $\forall x Fx$ look any more like a (full) *ground* of that fact, since, intuitively, there is no *ground-theoretic* basis for their being the only instances, only a modal basis. (The necessity of mathematics undermines modal arguments against the absurd view that all mathematical facts are grounded in my existence, but the falsity of that view is nevertheless manifest.)

[11] Perhaps this is clearer for some choices of F than others.

What the semi-Tractarian view needs is not merely the modal doctrine of necessitism, but rather some ground-theoretic version. Here is one such: facts about which objects exist are not "apt for being grounded", in Dasgupta's (2014*b*) sense. (This notion of Dasgupta's is a kind of ground-theoretic version of Fine's (2005) notion of unworldliness.) I myself, though, doubt that any facts are not apt for being grounded.

2.5.2 Grounding-qua

Perhaps there is another way out for the semi-Tractarian. Recall a twenty-five year old dispute between David Lewis and David Armstrong. In many writings Armstrong (1980, 1989, 1997) insisted that every truth must have a truthmaker (in arguments for universals, states of affairs, totality facts, and so forth). Lewis objected that this truthmaker principle is "an over-reaction to something right and important and under-appreciated" (1992, p. 218), and should be replaced with the principle that truth supervenes on being: two possible worlds with the same individuals and distribution of natural properties and relations over those individuals are alike in every way. What both the truthmaker principle and supervenience-on-being are reactions to, in Lewis's view, was a perceived flaw in positions like the acceptance of brute counterfactual or tensed facts, namely, the acceptance of truths that lack a basis in facts about things. But moving beyond that dialectic, there is a parallel issue concerning fundamentality: how to formulate a "completeness principle". The fundamental facts must surely be held to be complete in some sense, to provide a basis for everything else; but in what sense exactly? One idea is that every fact must be grounded in some fundamental facts. This is the position parallel to Armstrong's truthmaker principle, and the one that causes trouble for the semi-Tractarian. But one might instead adopt a completeness principle that is analogous to supervenience-on-being, to the effect that differences in facts must be due to differences in fundamental facts. Given an appropriate principle of this sort, the fact that everything is F would not need a ground in atomic facts and their negations; all that would be required is that differences in whether everything is F would need to be due to differences in atomic facts and their negations. The idea behind this weakened principle is that the fundamental facts don't need to *say* that they are complete; they just need to *be* complete.

There is an obstacle to this approach. "Differences in facts must be due to differences in fundamental facts" is vague, and the obvious precisification, namely "possible worlds that share the same fundamental facts share all facts" is modal, and shares in the failings of other modal attempts to say what should rather be said in fundamentality- or ground-theoretic terms. For instance, it does not meaningfully constrain the relationship between necessary truths and the fundamental facts: for all it is concerned, a Platonist who held that mathematical truths are necessary would be free to refrain from accepting any fundamental mathematical truths at all.

Nevertheless, there still seems to be something to the underlying Lewisian idea. A fundamental account of reality that includes a certain roster of individuals doesn't also need a further fact saying that there are no additional individuals; a fundamental account of reality that includes a certain roster of facts doesn't also need a further fact saying that there are no additional facts.[12] The roster on its own is ground-theoretically complete simply by being the complete roster. The question, though, is how to articulate this thought.

Here is one way. In addition to orthodox grounding claims $A_1, \cdots \Rightarrow B$, in which the grounding statements $A_1 \ldots$ are said to ground B without regard for what kinds of statements they are, one might invoke a class of "grounding-qua" statements, in which the grounding statements are said to ground qua satisfying a certain condition C. Grounding-qua statements must be understood as sui generis, in that they must not be defined as meaning that $A_1 \ldots$, together with the further statement that they satisfy the given condition, ground B in the orthodox sense. The further statement is not part of the ground of B; rather, it is in light of the further statement that $A_1, \ldots$ ground B.

To formalize grounding-qua, it's easiest if we take $\Rightarrow$ to be a predicate of facts rather than a sentence operator, and speak of relations between facts rather than conditions on statements. Where R is a relation over facts and g and $f_1 \ldots$ are facts, grounding-qua statements can take the form $f_1 \cdots \Rightarrow_R g$: "$f_1 \ldots$ qua standing in R ground g". The orthodox notion of ground can now be understood as a special case of grounding-qua: $f_1 \cdots \Rightarrow g$ iff $f_1 \cdots \Rightarrow_{R_\top} g$, where $R_\top$ is the trivial relation that holds amongst any relata whatsoever. Factivity for grounding-qua amounts to this: $f_1 \cdots \Rightarrow_R g$ only if $f_1 \ldots$ and g all hold and $f_1 \ldots$ stand in R. The principle connecting grounding-qua to necessity would be this: $f_1 \cdots \Rightarrow_R g$ only if $\Box((f_1 \text{ holds} \wedge \ldots \wedge f_1 \ldots \text{ stand in } R) \rightarrow g \text{ holds})$.

Given this setup, the semi-Tractarian view may be defended. The complete-

[12] Indeed, the second requirement would lead to awkward results. If the fundamental roster $F_1, \ldots$ must contain a fact to the effect that there are no fundamental facts other than $F_1, \ldots$, then for some i, $F_i =$ the fact that there exist no fundamental facts other $F_1, \ldots, F_i, \ldots$; facts must therefore be in a sense not well-founded.

ness principle for ground should be taken to say that every nonfundamental fact is grounded in some fundamental facts *qua* standing in some relation or other. Using $[A]$ to denote the fact that A, return now to the putatively nonfundamental fact that $[\forall x F x]$. The new completeness principle can be satisfied by claiming that $[\forall x F x]$ is grounded by its instances *qua all and only its instances*, i.e., that $[Fa_1], \cdots \Rightarrow_{R_F} [\forall x F x]$, where $a_1, \ldots$ are all the individuals and R_F is the relation that holds amongst some facts if they are all and only the facts consisting of the attribution of F-ness to some individual or other. Unlike the claim that $[\forall x F x]$ is grounded (in the orthodox sense) in its instances, this claim does not have the questionable modal implication that $[\forall x F x]$ holds in every world in which its instances $[Fa_1]\ldots$ hold. Its modal implication (via the connecting principle of the previous paragraph) is merely that it holds in any possible world in which those instances hold and in which they are all and only its instances.

To be sure, there is a real question of the legitimacy of grounding-qua. If some facts standing in a relation is relevant to their grounding something, then, one might object, the fact that they stand in the relation simply must be counted as part of the ground. The objection certainly carries force in certain cases. It would be absurd to defend the idea that conjunctions are grounded solely in their left conjuncts by saying that $[A \wedge B]$ is grounded-qua-$[B]$'s-holding in $[A]$. In this case, anyway, $[B]$ must be counted as part of the ground, and cannot be "moved" from that position into the condition on the grounding. More generally, the "import" principle that $f_1, f_2, \cdots \Rightarrow g$ implies $f_2, \cdots \Rightarrow_{\text{being such that } f_1 \text{ holds}} g$ must be rejected. (Even more generally, $f_1, f_2, \cdots \Rightarrow_R g$ does not imply $f_2, \cdots \Rightarrow_{\text{being such that } f_1 \text{ holds and } f_1, f_2 \ldots \text{ stand in } R}\, g$.) What the defender of grounding-qua must say is that although facts that one might have expected to be grounds cannot in general be regarded as mere conditions on ground, in certain special cases they can be, such as the case of the totality condition on the instances of a universal generalization.

If grounding-qua is accepted, perhaps even the full-on Tractarian view could be defended. First consider negative facts $[{\sim}Fa]$ where $[Fa]$ would be a fundamental fact if it held. In such cases one might claim that $[{\sim}Fa]$ is grounded in the totality of fundamental atomic facts about a, qua the totality of fundamental facts about a. This proposal passes the modal test, anyway: where $[F_1 a]\ldots$ are, in fact, all and only the fundamental atomic facts about a, it's impossible for them to continue to be all and only the fundamental facts about a while $[{\sim}Fa]$ fails to hold, since then $[Fa]$ would hold, and so would be fundamental by hypothesis, but would be distinct from each of $[F_1 a]\ldots$. So at least in these cases, grounds for negative facts can be found; and perhaps these, together with universally quantified facts (which have already been discussed) suffice to ground all other negative facts.

2.5.3 Metaphysical indeterminacy and the disjunction principle

Setting aside attempts to argue directly for the existentials principle, let's get a deeper sense of what denying it would involve. The principle, recall, is this schema:

$$\text{For all } y\text{, if } A(y) \text{ then: } \exists x A(x) \text{ because } A(y)$$

If it is false then (fixing some particular formula A) for some y, $A(y)$ is true but nevertheless doesn't ground $\exists x A(x)$. How could that be? The instance is true, and hence the existential is as well; why isn't the true instance enough to ground the existential?

The answer must be that the existential is "already" true, in a way that somehow preempts the instance, either by being fundamentally true or by having an independent ground that preempts the instance. Which is of course what we expected: denying the principle amounts to accepting a metaphysics on which existentials are "directly made true", rather than being made true via their instances.

The analogous disjunctions schema is:

$$\text{If } A \text{ then: } A \vee B \text{ because } A\text{, and } B \vee A \text{ because } A$$

So to resist it in any particular case, one must deny some "$A \vee B$ because A" (without loss of generality) while supposing A (and hence $A \vee B$). Why isn't the disjunct enough to ground the disjunction? The answer must again be that the disjunction is "already" true in a way that preempts the disjunct, either by being fundamentally true or by having an independent ground that preempts the disjunct. Again this is what we expected: denying the principle amounts to accepting a metaphysics on which disjunctions are directly made true by reality, rather than inheriting their truth via their disjuncts.

Such a metaphysics, in the case of disjunctions, might be one that embraces "metaphysical indeterminacy". Now, in cases where it's borderline whether A or B, some approaches to indeterminacy recommend taking the intuitionist's attitude toward the disjunction $A \vee B$: refrain from accepting it while also refraining from accepting its negation.[13] This approach is not the one needed here, since denying the disjunctions principle requires accepting the disjunction (while denying that it's grounded in its true disjunct or disjuncts). The denier of the disjunctions principle needs instead an approach like that of Elizabeth Barnes and Robbie Williams (2011), who accept disjunctions $A \vee B$ in borderline cases. According to them, one expresses the indeterminacy, not by refraining from saying "either A or B", but rather by saying "either A or B, but it's indeterminate which", where "it's indeterminate which" means that it's not determinate that A and it's not determinate that B.

It is by no means inevitable that Barnes and Williams would reject the disjunction principle.[14] They could say that the disjunction *is* grounded in its true disjunct, but that it's indeterminate *which* disjunct that is.[15] That is, either: $A \vee B$ because A, or $A \vee B$ because B; but it's not determinate that $A \vee B$ because A, and it's not determinate that $A \vee B$ because B. This position is not only open to them, but is also a good fit with part of their program. Their view is attractive in large part because it is logically conservative: it enables acceptance of classical logic along with classical principles about truth like the T-schema. Accepting the "classical" principle that disjunctions are grounded in their true disjuncts (though it might be indeterminate which one that is) is in the spirit of their view.

Barnes and Williams could, then, accept the disjunctions principle. But let us focus on what it would look like for them to reject the principle. Barnes suggested an interesting reason to me: that in a borderline case, $A \vee B$ is grounded, not by its true disjunct, but rather by the fact that A and B "are the only possibilities".[16] They might be the only possibilities because of their logical forms,

[13]This outlook on indeterminacy in general is associated with Crispin Wright (2001), though Wright does not himself present his view as one about "metaphysical" indeterminacy.

[14]Thanks to Barnes for discussion of these issues.

[15]Compare Cameron's (2011, section 6) analogous claim about truthmaking.

[16]Two other potential reasons come to mind, but should be disavowed. One is that grounds must be determinate: A because B only determinately B. This reason is unmotivated: why couldn't the mere truth of B be enough to ground the mere truth of A? Also, if there can be indeterminacy at the fundamental level, implausible results about the grounding status of higher-level facts would result. For example, fundamental but indeterminate A couldn't even ground $A \wedge A$, in which case $A \wedge A$ presumably wouldn't have any ground at all. The other bad reason is that an indeterminate truth can never ground a determinate one. This is unmotivated (and worryingly close to notorious false relatives, such as that a contingent truth can never imply a necessary truth (Routley and Routley, 1969)). The increase in determinacy in passing from A to $A \vee B$ needn't be due solely to the "ground-theoretic force" of the indeterminate A.

as in the case of $A \vee {\sim} A$. Or they might be the only possibilities because of further facts about the situation, as when a color patch is borderline pink/red: the relevant facts are the precise shade of the patch and the fact that pink and red are exhaustive possibilities for a certain range of precise shades.

One might doubt—I do doubt—the suggested role of modality in making nonmodal facts true. There is a general question of whether to say such things as that $1+1=2$ because it's necessary that $1+1=2$; and saying that $A \vee B$ is true because A and B are the only possibilities (in the situation) is akin to answering yes to this general question. This general question is a hard one, which I won't try to answer here.

But set that objection aside. What is important about Barnes's suggestion is that it is of a direct ground of the disjunction that doesn't run via the disjuncts. We have again confirmed (what we already knew) that the right model for the denier of the disjunction and existentials principle is to adopt a "holistic" metaphysics in which certain logically complex statements, disjunctions and existentials, have direct grounds that do not proceed via their constituent sentences. In chapter 3.10 we will look at this sort of metaphysics in depth.

But even if Barnes has given us an attractive model of a disjunction being grounded other than via its true disjuncts, it still gives us no attractive model of how a disjunct might be fundamentally true—and thus, no attractive precedent for the main view criticized in section 2.1.

So where are we left with the attempt to justify the claim that existentials are grounded in their instances? I've put forward a number of considerations, but none of them strike me as especially decisive.

2.6 The nature of the problem

We've struggled to articulate nomic essentialism using ground and fundamentality. The problems are due to the "hierarchical" demands of the concepts of ground and fundamentality. They evaporate when the issue is formulated with nonhierarchical tools such as modality: "it's impossible for (e.g.) charge to exist (or things to be charged) without charge obeying certain laws". But surely, this modal fact ought to follow from some deeper, nonmodal fact about the natures of properties. The modal formulation just hides a genuine problem

It could instead be due to the logical form of the grounded statement $A \vee B$ and the nature of the disjuncts. (Consider, for example, the logical form of the disjunction in the special case of $A \vee {\sim} A$.)

by considering the issue at a superficial level. Thus we have an instance of our larger theme, that "structuralist" views are more difficult to articulate in postmodal terms.

2.7 The replacement strategy and resemblance nominalism

In our dealings with problematic entities, it can be liberating to *replace* them with something else altogether, something from which one can recover whatever was of value in our practice of talking about them. Russell (1905), for instance, famously replaced Meinong's ontology of nonexistent entities with his metaphysics of quantification and his semantics of descriptions.

In the case of structuralism, such a replacement theory would completely eliminate reference to the entities comprising the structure in question—at the fundamental level, anyway[17]—and would account for the facts in question using new vocabulary that somehow gets at the structural facts directly. There's no guarantee that a suitable replacement theory exists; part of what's so interesting about structuralism is that replacement theories are so hard to find. But such a replacement theory is desirable if it's attainable, and so it's worth asking whether it's attainable in the case of nomic essentialism.

In fact there is an available replacement theory in this case, which is metaphysically tamer than anything we've been considering so far. Consider a form of resemblance nominalism, which does away with properties altogether, and instead makes use of a primitive plural predicate $R(X)$, read "the Xs resemble one another perfectly in some one fundamental respect".[18] Now, resemblance nominalism is usually a Humean view whereas nomic essentialists tend to take laws more seriously, but one could add a primitive sentence operator "it is a law that" to the mix. Thus the fundamental facts, on this view, are given by all the truths that can be expressed in the language of plural quantification plus the predicate R plus the sentence operator "it is a law that".

What nomic essentialists really want is for it to just not make sense to talk about permuting properties amongst nomic roles. (The main argument for nomic essentialist is the thought that such permutations are distinctions

[17] This would help if the entities would be less problematic if confined to the nonfundamental level.

[18] See Rodriguez-Pereyra (2002) for a defense of resemblance nominalism.

without a difference.) But the form of resemblance nominalism just mentioned gives them this: permuting makes no sense because there are no properties to permute! (Moreover, given this view, a property-theoretic sentence S in ordinary thought will be admissible insofar as its R-theoretic uphshots are true; e.g. 'some property P is instantiated by a, b, and c' will be admissible iff $R(X)$ for some Xs containing a, b, and c; but given this approach, permutations of properties will not affect the set of admissible property-theoretic sentences.)

I doubt, though, that many nomic essentialists will accept this olive branch.[19]

[19] Here is a related olive branch that the nomic essentialist will undoubtedly reject. One could be a quidditist about spatiotemporal relations, but then, as Ned Hall (2015, section 5.2) describes, provide a reduction of other scientific properties that is like Lewis's best-system reduction of chance. Like resemblance nominalism, this view rules out the possibility of permutations of nomic roles (excepting spatiotemporal relations). But see section 3.5.

Chapter 3

Quantities

3.1 The problem of quantity

Quantitative properties are those that come in degrees. We use numbers to talk about quantitative properties like mass, charge, and distance—both to talk about the properties of individual things ("this thing is 5 kg") and also to state laws governing quantitative properties ("$F = ma$").

What is the right metaphysics of quantity? What do quantitative theories tell us about the ultimate nature of the world? These questions are especially pressing given the ubiquity of quantitative properties in physics.

Interpreted in the most straightforward and flat-footed way, quantitative theories contain relational predicates relating concrete objects to numbers. Thus to say that a is 5 kilograms is to attribute the two place predicate 'mass-in-kilograms(x, y)' to a and the real number 5. Or, if we're happy to reify properties and relations, it is to say that a bears the mass-in-kilograms relation to the real number 5. And to say that $F = ma$ (neglecting the directionality of force and acceleration) is to say something like this: "for any object, x, the number to which x bears the force-in-newtons relation is the product of the numbers to which x bears the mass-in-kilograms and acceleration-in-meters-per-square-second relations".

(Even though the facts of mass recognized by the flat-footed account involve abstract entities, those facts are capable of empirical confirmation since they are correlated with observable facts. If x bears the mass-in-kg relation to 5 and y bears it to 4 then x will be more massive than y, which can be observed by placing x and y on a pan balance. The confirmation is admittedly indirect, but no one open to the kinds of metaphysical questions we've been entertaining can insist on direct confirmation of each bit of a foundational theory.)

Does the flat-footed interpretation yield an adequate metaphysics? Your answer will turn on which metaphysical tools you accept.

Suppose the only metaphysical tools you accept are ontological; the only metaphysical questions you recognize concern which entities exist. You might then be happy with the flat-footed interpretation. You might question whether real numbers (or relations) really exist. But otherwise the flat-footed account is fine.

Even if you also accept modal tools, the account still seems to be the basis for an acceptable metaphysics of quantity. For instance, in modal terms one could recognize necessary connections between numerical predicates corresponding to different scales:

> Necessarily, for any object x and real number y, mass-in-kilograms(x, y) iff mass-in-grams$(x, 1000y)$

One could also recognize necessary connections between the numerical predicates and further, nonnumerical predicates:

> Necessarily, for any objects x, x' and real numbers y, y' where mass-in-kilograms(x, y) and mass-in-kilograms(x', y'), $y > y'$ iff x is more massive than y

Viewed through a modal-cum-ontological lens, the flat-footed account is still looking fine.

But suppose you accept richer metaphysical tools. Suppose in particular that you accept concept fundamentality, so that a fundamental theory must specify which concepts are fundamental. You then face the question of which quantity-theoretic concepts are fundamental.

The flat-footed interpretation suggests an answer: that relational predicates like 'mass-in-kilograms(x, y)' express fundamental concepts, or, reifying relations, that mass-in-kilograms is a fundamental relation.

One might object that mass is *physical* and thus couldn't involve abstract entities. But who made that rule?[1]

A more serious problem arises when we ask *which* relations to numbers (or which relational predicates) are fundamental. In particular, which units are

[1]See Sider (2013*a*, pp. 287–8) on this sort of issue.

involved? In the case of mass, is the fundamental relation mass-in-kilograms? mass-in-grams? mass-in-some-other-unit?

It would surely be intolerably arbitrary to say that one of these relations, mass-in-kilograms, say, is fundamental to the exclusion of all the others. But the only alternative, apparently, is to say that *all* of the relations are fundamental, which would amount to accepting massive redundancy in the fundamental properties and relations.[2]

The problem here, notice, simply doesn't arise, if one is only concerned with ontological or modal issues. It arises when one accepts the demand to provide an account of the fundamental nature of mass, by saying which concepts (or relations) of mass are fundamental. (It also arises if one's metaphysical tool of choice is ground or fact-fundamentality rather than concept fundamentality.[3] For one still faces the question of which mass relation is involved in the fundamental or ungrounded facts of mass.)

There are some hard general questions about fundamentality that intrude here. Is some redundancy in the fundamental properties tolerable? That seems hard to avoid: how to choose between, for instance, parthood and overlap in the theory of parthood? But then, how much is tolerable? And the problem of redundancy becomes only harder if, like me, you think fundamentality can be applied to logical notions such as existential and universal quantification, or conjunction, disjunction, and negation. We will return to these issues in chapter 5. For now I continue to assume that massive redundancy in the fundamental concepts is to be avoided, as is arbitrariness, and hence that the flat-footed interpretation does not yield a viable metaphysics of quantity.

3.2 Simple absolutism

We need a different idea about what the fundamental mass properties and relations (or concepts) are, one that avoids the problems of arbitrariness generated by privileging a single unit. Here is one such idea:

Simple Absolutism the "determinate masses" are the only fundamental mass properties or relations

[2]See Field (1980, 1985) on all this, though his concern is defending nominalism rather than fundamentality.

[3]Provided that ground and fact-fundamentality are understood as being hyperintensional, as they normally are.

By the determinate masses, I mean properties like *having exactly this mass*, or *having exactly that mass*. Such properties can be *named* by mentioning numbers and a unit—"having exactly 5 kg mass"—but the numbers aren't "built into" the properties; the property 5kg is the very same property as the property 5000g.

Simple absolutism avoids the problems of the "pythagorean" views of the previous section. But it goes too far in the opposite direction, away from numbers. The representation of mass using numbers is essential to science. Something fundamental must justify this procedure, and simple absolutism does not say what that is. We'll make this argument more precise soon.

Simple absolutism also comes in a nominalistic version, according to which the only fundamental mass predicates are the continuum many monadic predicates that the platonist would regard as standing for the determinate absolute masses. Of course there are questions about the status of these continuum many predicates, which we'll discuss below.

3.3 Representation theorems

Much of the literature on quantity has been in the philosophy of science, where metaphysical concerns are not always central. But the main theories there have close metaphysical cousins, and these cousins can avoid the difficulties faced by the flat-footed account and simple absolutism.

One such cousin may be called "comparativism".[4] According to the comparativist account, the fundamental quantitative concepts are comparative, or relational. Two examples of such concepts, in the case of mass, are these:

- $x \succeq y$: individual x is at least as massive as individual y
- $Cxyz$: x and y's combined masses equal z's

One comparativist account of mass can then be formulated as follows:

Comparativism The only fundamental mass predicates are $\succeq$ and C

The displayed glosses on the predicates $\succeq$ and C in the previous paragraph are not meant to be definitions of those predicates in terms of an underlying numerical scale: $x \succeq y$ is not defined as meaning that the real number that is x's mass in grams, say, is greater than or equal to the real number that is

[4]Here I'll state a fundamentality-theoretic version, but the label has more general application, including for example Dasgupta's (2013) ground-theoretic approach.

y's mass in grams. That would apparently return us to the problems of the flat-footed account. The predicate $\succeq$ isn't defined in this way, or in any other way for that matter; it is said to be fundamental, after all. To be sure, numerical scales might facilitate our understanding what the comparativist has in mind by these predicates. Given a practice of measuring mass by numbers, an object x will be assigned at least as large a number as an object y iff $x \succeq y$, and an object z will be assigned the sum of the numbers assigned to objects x and y iff $Cxyz$ (this is true regardless of the numerical scale, i.e., regardless of the chosen units for mass); and since we're familiar with numerical scales, this fact might help us grasp what the comparativist means by $\succeq$ and C. But as we'll see, the comparativist grounds the practice of measuring mass by numbers in comparative relations like $\succeq$ and C, rather than the other way around.

To dispel lingering worries about the comparativist's claim that predicates like $\succeq$ and C are not defined in terms of numerical scales, it may help to note that there are physical processes that can be used to directly test whether these predicates apply, without knowledge of a numerical scale. A good test for whether $x \succeq y$ is to put x and y on opposite ends of a pan balance and see whether y's side fails to move downward; and a good test for whether $Cxyz$ is to put x and y on one side and z on the other and see whether they balance.[5]

The comparativist's fundamental predicates, then, do not presuppose an underlying numerical scale. Neither do the simple absolutist's fundamental properties. But the choice of relational predicates enables the comparativist to recognize a richer structure for quantities than the simple absolutist could, which allows her to "justify" the use of numbers to measure quantities.

Given certain assumptions about how $\succeq$ and C behave, one can prove "representation" and "uniqueness" theorems:

Representation theorem There exists a function f which assigns real numbers to individuals, subject to the constraints i) $f(x) \geq f(y)$ iff $x \succeq y$ and ii) $f(x)+f(y)=f(z)$ iff $Cxyz$

Uniqueness theorem Any two functions g and h obeying constraints i) and ii) in the representation theorem are scalar multiples—i.e., for some positive real number k, $g(x)=kh(x)$ for all individuals x

Similar theorems may be proven for other quantities, provided the comparativist's relations for those quantities—relations analogous to C and $\succeq$—obey appropriate assumptions.

[5] These are defeasible: the balance may be defective, the objects may overlap, etc.

These theorems establish the existence and uniqueness of certain functions, which we may call "representation functions" in general, and "mass functions", "charge functions", and so on in particular cases. These functions assign numbers to concrete objects that are correlated with the Comparativist's fundamental relations for the quantity in question. For example, mass functions are correlated with the as-or-more-massive relation $\succeq$: a mass function assigns at least as high a number to one individual as to another iff the first individual is at least as massive as the second.

I complained above that simple absolutism cannot "justify" the practice of using numbers to measure quantities. Comparativism can justify this practice, given the theorems, in the following sense. Statements about mass using numbers may be understood as concerning some chosen mass function, whose values are systematically correlated with the comparativist's nonnumeric fundamental mass facts and numbers. Numerical talk about mass is just a way of coding up statements about $\succeq$ and C. The representation theorem guarantees that some mass function or other exists, and the uniqueness theorem shows that the range of mass functions corresponds exactly to the (arbitrary) choice of a unit of measure. Simple absolutism can't justify the use of numbers in this sense, since it includes no fundamental relations of the sort needed for the representation and uniqueness theorems.

The move to Comparativism from the flat-footed mass-in-kilograms theory can be regarded as fitting the structuralist template. The flat-footed view of the fundamental mass facts is of an array of numerical values distributed over individuals. But in this array, too much significance is accorded to the individual nodes, the individual numbers assigned (this was the arbitrariness objection). All that matters to physics is the relations between the numbers—specifically, their ratios. (One could also make the familiar structuralist epistemic complaint that we can have no knowledge of the particular numerical values.) So we should choose a metaphysics on which only the structure of the array is accorded fundamental significance. Comparativism is such a metaphysics.

Comparativism is another instance of the "replacement" strategy for dealing with problematic entities (section 2.7). The problematic entities were the individual mass values, and they were simply jettisoned, and replaced by the fundamental predicates $\succeq$ and C. (Simple absolutism also jettisoned the numbers, but jettisoned too much since insufficient fundamental structure remained to justify the use of numbers to measure quantities.) This particular instance of structuralism is thus unproblematic so far—no metaphysical funny business like ungrounded existential sentences.

3.4 Laws and fundamentality

I've claimed that comparativism can, while simple absolutism cannot, "justify" the use of numbers to measure mass, because only the former supplies fundamental concepts sufficient to prove a representation theorem. These claims need to be sharpened.

Let's begin with the nominalistic form of simple absolutism, according to which the fundamental concepts of mass consist of continuum-many determinate mass predicates. The usual argument against this sort of approach is that a language with continuum many primitive predicates would not be learnable by humans. But I don't think such arguments get to the heart of the matter. In the spirit of Joseph Melia (1995), a nominalist might reply that *the* best theory may not be *our* best theory; the best theory may be inaccessible to us. Or she may reply in the spirit of Stephen Yablo (2000): we have ways to get at the contents of this best theory even without continuum many predicates, via metaphorical contents of sentences in the language of mathematics.

But how would the nominalist simple absolutist account for *laws* involving mass? Someone sympathetic to Melia might reply thus: even though *we* can't formulate general laws using just the monadic predicates, the contents of those laws are out there, so to speak. Beings speaking an infinitary language could say "if a particle has exactly *this* mass and experiences exactly *that* force, then it will undergo exactly *such-and-such* acceleration; but if it has exactly this other mass and experiences exactly that other force, then it will undergo exactly thus-and-so acceleration; and if…". So there is no reason to accept any further account of the metaphysics of mass beyond the continuum of monadic mass predicates.

This might seem alright at first, until one reflects that this line could be pushed much further, to avoid all commitment to fundamental predicates or properties of charge, mass, force, and so on. Reference to any given mass, for instance, would be replaced by reference to the type of spatiotemporal trajectory distinctive of an object with that mass. The type of trajectory will need to be disjunctive and relational (involving relations to the trajectories of other objects), but this is no obstacle if laws like those in the previous paragraph are countenanced. Instead of predicting the acceleration of particles as functions of their masses and the forces acting on them, Newton's second law would instead "predict" each particle's acceleration as a function of its trajectory: "if an object has precisely this sort of trajectory (vis-a-vis all the other trajectories of all other particles in the universe), it will undergo precisely such-and-such acceleration; but if …". It's only our finitude, so the argument would go, that requires us to speak of mass and charge.[6]

What blocks this line of thought, in my view, is the simplicity-centric epistemology of fundamentality discussed in section 1.7. According to that view, it's reasonable to regard concepts as being fundamental to the extent that they figure into simple and strong laws. This speaks against the Melian defense of nominalistic simple absolutism: with only the monadic mass predicates, the only laws of mass we could formulate would be the complex infinitary and disjunctive ones. It also argues against the attempt to dispense with charge and mass altogether: if we had no predicates other than spatiotemporal ones, we couldn't formulate simple laws of motion. And it also argues against non-nominalistic simple absolutism: using only predicates for the continuum of determinate mass properties, we couldn't formulate simple laws of mass.

Thus our vague assertion that simple absolutism cannot "justify" the use of numbers to measure quantities is now clarified. More fully: the use of numbers to measure quantities is not merely a convenient shorthand, needed only because of our finite nature. Numerical representation is also intertwined with the statement of simple laws. The statement that the acceleration on an object is directly proportional to the net force on that object and inversely proportional to its mass is essentially tied to those structural features of mass, force, and acceleration in virtue of which talk of proportionality is well-defined; and these structural features are missing from the simple absolutist account. To state simple and strong laws, where simplicity is measured by reference to the fundamental concepts occurring in the law, one must recognize fundamental concepts tied to these structural features. Comparativism, on the other hand, recognizes those structural features in its fundamental concepts $\succeq$ and C. (Below we will look more closely at exactly what laws comparativism allows.)

3.5 Interlude: ramsifying fundamental properties away

Before resuming our discussion of quantities, let's look more closely at the possibility of dispensing with fundamental properties like mass. My objection to the Melian strategy was that it doesn't enable simple laws. But a variant strategy

[6] Distinguish this method for eliminating the properties from Valia Allori's; Allori appealed to a robust conception of nonqualitative laws to pick up the slack.

might appear to reinstate them. Instead of replacing reference to mass by disjunctive reference to the trajectories of massive particles, one could instead "ramsify mass out": replace physical theories that make reference to mass with theories that say that there exists some property or other that plays the mass-role. Simplify (for the moment) by pretending that 'mass' isn't quantitative, and is a simple monadic predicate. Then, where $\mathscr{L}(\text{mass})$ is the conjunction of the laws normally thought to govern the fundamental predicate 'mass', the proposal is to instead regard the law as being $\exists p \mathscr{L}(p)$—which isn't infinitary and disjunctive.

(The pretense could be eliminated in various ways, depending on one's approach to quantities. The laws recognized by a comparativist, for instance, will take the form $\mathscr{L}(\succeq, C)$, and could be replaced with $\exists R^2 \exists R^3 \mathscr{L}(R^2, R^3)$. But let's continue pretending that 'mass' is a monadic predicate.)

If the existentially quantified variable p in the ramsey sentence $\exists p \mathscr{L}(p)$ were restricted to "sparse" properties, then the only candidate value for that variable would be the property of mass itself, the very property we were trying to avoid, and no advance would have been made. (Unless we regressed back to the view that existentials need not be grounded by their instances.) But the idea instead is to not restrict the variable in this way. The variable p is to range over properties in the abundant sense, which can be arbitrarily disjunctive, or even "nonqualitative", corresponding to sets with no defining condition. Such a law therefore says, in effect, that there is a division of all things such that things on one side of the divide behave as massive objects are normally supposed to behave.

This approach has an inherent limitation. One might wish to use the approach to eliminate more than one fundamental predicate. To do so, one would simultaneously eliminate all the targeted predicates $F_1, F_2, \ldots$, by replacing the conjunction of the usual laws governing those predicates, $\mathscr{L}(F_1, F_2, \ldots)$, with the ramsey sentence $\exists p_1 \exists p_2 \ldots \mathscr{L}(p_1, p_2 \ldots)$. But there is a danger that this ramsey sentence will be too weak a statement to be a law. This will occur if *all* the predicates in $\mathscr{L}(F_1, F_2, \ldots)$ are ramsified out, for then the ramsey sentence will be guaranteed to be true provided $\mathscr{L}(F_1, F_2, \ldots)$ was consistent (and provided there exist enough objects). Thus defenders of this strategy do not try to eliminate absolutely all fundamental concepts from physics. In particular, they normally exempt spatiotemporal concepts from ramsification.

There has been some recent interest in this kind of approach, especially as applied to the wave function in quantum mechanics.[7] It has normally been paired with the best-system approach to laws, according to which there is nothing more to being a law than being part of the simplest and strongest deductive system.[8] Probably this is because its defenders have modeled their approach after David Lewis's (1994) reduction of chance. But the approach isn't essentially tied to the best-system view of laws; all that's needed is that laws can be existential in form. Even a primitivist about laws could admit this, as far as I can see.

I find it strange that the recent discussions of this approach have not replied to the powerful critique in Cian Dorr's "Of Numbers and Electrons". Dorr writes:

> If we want to weaken a theory so as to eliminate its commitment to some sort of hidden structure, we can often do so by replacing the vocabulary which purports to characterize this structure with variables of an appropriate sort bound by initial existential quantifiers. Philosophers who are suspicious of particular putative bits of hidden structure keep on rediscovering this fact, and announcing that they have shown how to eliminate the structure in question. But once we have realized the complete generality of the trick, we should not be impressed by their achievements. Here are some more examples. (p. 160)

The examples Dorr goes on to mention include eliminating fundamental kinds of particles, eliminating spatiotemporal structure above a certain level (say, topological) by quantification over coordinate systems, and (alas) my own defense of four-dimensionalism against the problem of the rotating disk.

To Dorr's list I would add two others. First there is the recently popular proposal to "ramsify out" the wavefunction in quantum mechanics.[9] Second,

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[7] Notes

[8] Notes.

[9] This view takes particle trajectories to be physically fundamental, and ramsifies out wave function values on configuration space. [Fix; look up….] Here is an additional concern about this instance of the ramsification approach: since configuration space is not taken to itself be physically fundamental, this introduces another degree of freedom that moves the ramsified laws one step closer to triviality. Even if one is happy with the approach in the case of, say, eliminating particle-kinds—happy, that is, with laws saying in effect that particles behave as if they come in a small number of different kinds which have distinctive sorts of trajectories—one may not be happy with a second layer of "as if", with laws saying that particles behave as if there is a fundamental space with the geometry of configuration space and as if there is a field on that space, which field constrains trajectories in ordinary three-dimensional space. To see why

there is the way that reductionists about time's arrow regard the "past hypothesis"

the second layer of "as if" raises a concern, compare ramsifying out the electromagnetic field in classical electromagnetism (while not ramsifying out anything else—not trajectories or the masses and charges of particles). The usual laws $\mathscr{L}(F)$ in part concern a particular physically distinguished function F assigning values for the electromagnetic field to points of space at times; these laws constrain the relationships between this field, particle charges, masses, and trajectories. Ramifying out the field F means replacing $\mathscr{L}(F)$ with this statement:

> there exists some function or other f from pairs of points of space and times to field values such that $\mathscr{L}(f)$.

This replacement law isn't trivially true. But if the existentially quantified function could live, not in physical space, but rather in another space whose relation to physical space was unconstrained, the "law" would be closer to triviality. Consider a possible world, w, that obeys $\mathscr{L}(F)$, let t_0 be some time, and now construct a second possible world v in which all charges, masses, and field values remain the same, and in which particle positions are the same at times up until t_0, but in which each particle's position is, at every time $t > t_0$, shifted in some fixed direction D by a distance proportional to $t - t_0$ (abusing notation) from the position it had in w. The laws no longer hold in v, neither in their ordinary $\mathscr{L}(F)$ form nor in their ramsified form. [right?] But now construct the following "shifted" space $\mathscr{S}'$, whose geometry is the same as v's physically real space $\mathscr{S}$, but in which particles are said to be "located" at points that are shifted relative to their physical locations, where the shifts "undo" the shifts used to construct v. Thus at times up until t_0 each particle is "located" in $\mathscr{S}'$ at the point it occupies in $\mathscr{S}$, but at times $t > t_0$, each particle is located at a point that is shifted in the reverse of direction D by a distance proportional to $t - t_0$ from its location in $\mathscr{S}$. At every time, each particle is "located" in $\mathscr{S}'$ exactly where it is located in the original world w, which means that the following statement is true in v:

> there exists a function f from pairs of points of space and time in $\mathscr{S}'$ to field values such that: $\mathscr{L}(f)$ is true of $\mathscr{S}'$

If this is an acceptable law then we must say that v is a law-governed world. (Not just a world in which the laws change at time t_0; the single quoted law holds at all times.) Clearly we could play this trick for more drastically shifted worlds v.

So now, consider the analogy with wavefunction ramsification. In each case, the ramsified-out field lives in a space—$\mathscr{S}'$ in one case, configuration space in the other—that isn't physically fundamental, but rather is a constructed space with a certain specified correlation between "location" in that space and real, physical location. The moral is that if ramsified-out fields are allowed to live in any space whatsoever, then the existence of ramsified laws threatens to be trivial [a general result can surely be proven here]. The wavefunction ramsifier will likely reply that configuration space is, unlike the shifted space $\mathscr{S}'$, defined simply, and without reference to special cases, from physical space. That's true, but I still cling to a "bad-company" concern: if configuration space is not fundamental (as it isn't according to the view in question), why think it physically significant that particles behave as if there is a distinguished field on that space, when behaving as if there is a distinguished field on a space-in-general is not physically significant?

that the universe began in a low-entropy initial microstate.[10] On the face of it this putative law appeals to a physically distinguished direction of time, since it specifies something about how the universe *began*. Faced with this concern, reductionists about time's arrow have reconstrued the law in existential form: *one temporal end of the universe* is in a low-entropy microstate.[11]

The ramsifier may be tempted to dig in, and accept that each such instance of the strategy is acceptable.

Next bit unfinished:

- There's a danger of ramsifying too much, short of trivial truth. In classical physics, suppose one claimed that the only fundamental structure was the following: there exist points of time and points of space and particles; there is a fundamental relation of occupation that particles bear to pairs of points of space and time (so that particle trajectories are fundamental); space and time have only topological fundamental structure. Thus there is no fundamental metric, no fundamental electromagnetic field, and no fundamental charges or masses. The laws are existential in form.: particle trajectories behave, by law, as if there exists a metric on space and on time, as if there is an electromagnetic field on space over time, and as if there are particle charges and masses, which all constrain the trajectories as classical physics says. These claims is vulnerable to the following objection. Even if the existential sentences aren't so weak as to be trivially true (provided the domain is large enough), they may be weak to a lesser extent that still damages the view. Suppose these existential laws don't constrain trajectories any more than, say, to continuous trajectories. Then they may not be the laws—or anyway, we may not be justified in thinking they're the laws—because the more simple intrinsic statement "objects move continuously" seems a more likely candidate to be a law.

  Also, the weakening of the law might, for example, make it indeterministic.

- Combinations of acceptable instances of the strategy might, in aggregate, be unacceptable. Ramsifying out just one of two aspects while leaving the other unramsified might keep the laws reasonably strong; but ramsifying both out could make the whole thing too weak.

[10]Notes.

[11]Should Lewis's account of chance also be considered an instance (and hence subject to Dorr's critique)?

- Versions of the strategy, as applied to the structure of space—as in Fraassen (1970) for example—might have relied on stuff about observation to cut down on trivialization. For example, one might say that the metric of space is given by any assignment of coordinates + a distance function that interacts with the distinguished topology of physical space appropriately, and assigns equal distances to certain distinguished measuring rods (or something like that). But when the strategy is applied at the fundamental level, trivialization can't be cut down in that way.

- In chapter 2 we saw that resemblance nominalism gives nomic essentialists the main thing they want: it blocks swapping of nomic roles. But now we can see a problem for resemblance nominalism: its conception of laws will need to be of the existentially quantified variety that Dorr criticizes. The resemblance nominalist can't state a law for charge, say, by name. The law must instead be that there exists some resemblance class that behaves in such-and-such a way.

Dorr also includes a further instance of the ramsification strategy: ramsifying-out all unobservable facts!

> We need only define up some notion of what it is for a model to 'accurately represent the observable facts': then our new theory can simply say that the old theory [which posited unobservable facts] is true in some model that accurately represents the observable facts. (p. 162)

If the ramsifier needed to admit this instance of her strategy, this would be a real blow. However, our question here (unlike Dorr's) is that of which concepts are fundamental, and thus our ramsifier pursues her strategy only when the predicates that remain unramsified are fundamental. Since the concepts used to articulate "the observable facts" would need to be nonfundamental, our ramsifier can give a principled reason for denying that her approach leads to eliminating all unobservables: "the old theory is true in some model that accurately represents the observable facts" needn't be regarded as a law because it contains nonfundamental vocabulary.

Still, even though the death blow can be avoided, Dorr's objection to the ramsification strategy strikes me as serious. The ramsifiers owe him a reply.

3.6 Comparativism and existence assumptions

Back now to quantities, beginning with a brief recap. The flat-footed view that the fundamental concepts of quantity are relations to numbers was seen to either arbitrarily privilege a unit or else require massive redundancy. Simple absolutism avoided these problems, but could not "justify" the use of numbers in measuring quantities since it delivers no fundamental concepts appropriate for representation and uniqueness theorems and hence cannot deliver simple and strong laws based on its fundamental concepts. Comparativism, on the other hand, does deliver fundamental concepts appropriate for representation and uniqueness theorems.

However—moving forward now—comparativism has a commonly recognized problem. In order to prove the representation and uniqueness theorems, it is necessary to make certain strong assumptions about the existence of objects standing in relations like C and $\succeq$, assumptions according to which there exist infinitely many objects. Here are two assumptions of this type, in the case of mass:

Existence of sums For any x and y there is some z such that $Cxyz$

Density If $x \succ y$ then for some z, $x \succ z \succ y$

(where "$x \succ y$" means that $x \succeq y$ but $y \not\succeq x$—i.e., that x is more massive than y.) The problem is that the existence assumptions may be false. Perhaps the objects possessing mass are finite in number, or perhaps the range of possessed mass values contains "gaps".

One might defend the existence assumptions in certain cases. In *Science without Numbers*, Hartry Field defends a comparativist approach to mass by construing mass as a scalar field on points of space. On this approach Density is a natural assumption to make, since it is guaranteed to be true provided the field varies continuously; and his approach relies only on Density, not the Existence of sums.[12] However, it may be objected that gaps in the mass field should not be ruled out, nor should treating mass as a field be mandatory (Arntzenius and Dorr, 2011, p. 227).

Because of the problem of existence assumptions, Brent Mundy (1987) argued that Comparativism should be replaced by a view which I'll call "mixed absolutism":

[12] See pp. 72–3 and note 41.

Mixed absolutism the determinate masses, plus two higher-order "structuring relations" over the determinate masses, $\geqslant$ and $*$, are the only fundamental mass properties or relations

where the structuring relations—"second-order" counterparts of $\succeq$ and C—may be glossed as follows:

- $p \geqslant q$: p is at least as "large" as q
- $*(p,q,r)$: p and q "sum to" r

(As before, the glosses aren't definitions; $\geqslant$ and $*$ are fundamental, and not defined in terms of an underlying numerical scale.) The view is "absolutist" because it retains the monadic determinate masses. But it is "mixed" because, unlike Simple Absolutism, it also includes the fundamental structuring relations $*$ and $\geqslant$, which enables representation and uniqueness theorems (and can therefore "justify the use of numbers" to measure quantities in the same sense in which Comparativism can.[13]) The functions in these theorems now assign numbers to the determinate mass properties rather than massive objects. The versions of Density and Existence of Sums needed for the theorems say that the set of determinate mass properties, rather than the set of massive objects, is dense and closed under sums, which are apparently more plausible assumptions (provided one is happy with an ontology of properties in the first place!).

Arntzenius and Dorr (2011) defend a related view. They too apply structuring relations (like $\geqslant$ and $*$) to entities other than familiar concreta, which are hypothesized to exist even when there are gaps in which quantities are possessed and hence which can safely be assumed to obey the existence assumptions. But for them, the hypothesized additional entities are not properties, but rather are points in substantival "quality spaces". In the case of mass, for instance, they posit the existence of a "mass space" consisting of points structured by the likes of $\geqslant$ and $*$, as well as a distinguished relation of occupation between massive objects and points in this space. (They de-emphasize the importance of whether points in this space should be considered properties in the traditional sense,

[13] Incidentally, Mixed Absolutism brings out another contrast between the modal and postmodal approaches to metaphysics. Despite the fact that the higher-order relations $\succeq$ and C presumably hold necessarily whenever they hold, and so are supervenient on anything, from a postmodal point of view—one emphasizing concept-fundamentality anyway—they must still be recognized as fundamental since they are needed for simple and strong laws. (See Eddon (2013).) This is another illustration of the shortcoming of modal tools like supervenience as "measures of metaphysical commitment".

and stress the analogies between the reasons for positing their quality spaces and the reasons for positing the points of familiar space (pp. 229–30)—rightly in each case, in my view.)

Instead of pursuing further the question of whether the existence assumptions are really legitimate, I would like to turn instead to the question of what exactly is threatened, if the existence assumptions fail.

Of the theorems, it is the uniqueness theorem that is threatened (the representation theorem will hold provided the actual structure of objects is embeddable within a structure in which the existence assumptions hold). If the existence assumptions don't hold then not all representation functions will be scalar transformations of one another. But what problem, exactly, does this create? In a world like our own, in which there are a great many massive objects, the various mass functions won't stray *too* far from being scalar multiples (this will become clearer below when we explore the comparativist account of ratios). So it might be thought that the failure of the uniqueness theorem wouldn't affect scientific practice much.

The real problem with the failure of uniqueness, in my view, involves laws. But getting to the deepest problems here will take a bit.

Suppose there exist representation functions for the various fundamental quantities—e.g., a force function F, mass function M, and acceleration function A (idealizing away, for simplicity, certain complexities). These functions might stand in a numerical relationship like those in familiar laws. For example, it might be that for any x, $F(x) = M(x)A(x)$. But this numerical relationship could not be considered a law, since it has no significance outside the actual world, and thus would, for example, be irrelevant to counterfactuals. The functions F, M, and A are just assignments of values to objects, and have no physical significance respect to other possible worlds in which different objects exist, or in which the actual objects stand in different patterns of the comparativist's fundamental relations.

Statements that do have physical significance in other worlds can be constructed by quantifying over representation functions, rather than naming particular ones. Consider, for example, this existential form of Newton's second law:

∃-Newton There exist representation functions for force, mass, and acceleration, f, m, and a, such that for any object x, $f(x) = m(x)a(x)$

This statement has physical significance in other worlds, because it is correlated, via the definitions of representation functions for force, mass, and acceleration, with the facts about the comparativist relations for those quantities within those other worlds. Thus it might be claimed to be a law.

Now, although the *truth* of a statement like ∃-Newton isn't threatened by the failure of the existence assumptions, its uniqueness is. In addition to a set of functions obeying the equation $f(x) = m(x)a(x)$, there may also exist another set of functions that doesn't quite obey this equation, or even obeys other equations. (This is consistent with both sets of functions obeying the constraints in the representation theorems.) Thus in addition to ∃-Newton, there may also be other truths of the form

(∃) There exist representation functions for force, mass, and acceleration, f, m, and a, such that for any object x, $A(f, m, a, x)$

but in which A specifies mathematical relationships other than that in Newton's second law. Is *this* failure of uniqueness a problem?

It might be regarded as undermining determinism. But first a more basic threat to determinism from comparativism must be addressed. One standard definition of determinism is the following: a world is deterministic iff any description at any time of that world plus the laws of that world necessitates any description at any later time of that world. What is the description of a world "at a time"? There are controversies here having to do with the at-at theory of motion, but let's simplify the discussion by pretending that the at-at theory is false, so that velocities, accelerations, etc., are not constituted by positions at neighboring times. Then we could define the description of a world at a time as a complete description of the intrinsic features of the world at that time. But now a more fundamental threat to determinism from comparativism emerges: the comparativist description of a world at a time contains very little information, because it omits quantitative relations between objects at that time and objects at other times; only quantitative relations between things at that time will be included. What one would ordinarily think of as the intrinsic facts about a thing at a time are, for comparativists, constituted by (if they can be recognized at all) the quantitative relations of that object to all other objects, including objects at other times. This on its own makes it unclear clear whether the proposed formulation of determinism can hold.

A natural strategy for defining determinism in a comparativist-friendly way is to use representation functions to construct descriptions of worlds at times. The definition of a representation function requires the function to encode quantitative relations amongst all objects, across all times; thus a description of a time using representation functions will encode more information than just the quantitative relations amongst objects at that time. Here is a definition of determinism based on this thought:[14]

Comparativist determinism A world is deterministic iff for any representation functions $f_1, \ldots$ at that world, a complete description of any time using $f_1 \ldots$ plus the world's laws necessitates any complete description using $f_1 \ldots$ of that world at any future time

(Note, though, that this introduces a striking sort of nonlocality. We ordinarily think of Newton's second law as having implications for a localized region of spacetime: it tells us what an object is doing to do, solely as a function of what is going on in that object's immediate vicinity. This isn't true given the current comparativist understanding, since the current understanding of a particle's instantaneous state is given by representation functions, which are constrained by the relations amongst things at arbitrary spatiotemporal distances from that particle.)

Now we're in a position to see how determinism might be threatened by the failure of the uniqueness theorems. If the uniqueness theorems hold, then the laws might take the form exemplified by this stronger form of Newton's second law:

∀-Newton There exist representation functions for force, mass, and acceleration; and for *any* representation functions for those quantities, f, m, and a, there exists some real number k such that for any object x, $f(x) = km(x)a(x)$

Laws of this form have a fighting chance of satisfying comparative determinism. But if the uniqueness theorems fail, then there won't be laws of this form. There will merely be laws of the form exemplified by ∃-Newton. Such laws won't satisfy comparativist determinism; ∃-Newton tells us merely that *some* representation functions satisfy Newton's second law, and tells us nothing about other representation functions; and comparativist determinism requires complete descriptions under *every* representation function to lawfully necessitate future descriptions.

(It wouldn't be right to weaken the statement of determinism to say merely that under some representation functions, past descriptions lawfully necessitate

[14]Is there a concern that this is too weak, since if the laws are allowed to contain many statements of the form (∃), the lawful necessitation of future states by past states under different representation functions might appeal to quite different statements of the form (∃)?

future descriptions. Numerical descriptions under one particular set of representation functions aren't particularly physically distinguished if the uniqueness theorems don't hold. Such a weakened statement might hold even in cases that intuitively contain indeterministic laws.)

This concern is valid, but I'm not sure it goes to the heart of the problem that would be caused by the failure of the uniqueness theorems, since the following comparativist response has some plausibility. Suppose the uniqueness theorems fail, so that there are multiple sets of representation functions standing in different numerical relationships—multiple statements of the form (∃). Provided the world is reasonably complex, only one of these numerical relationships will be simple—the relationship in ∃-Newton in the preceding example, say. Other truths of the form (∃) can be excluded as laws on that basis alone, and deterministic predictions of the future can be based solely on the representation functions standing in a simple numerical relationship.

This thought could be implemented by restricting the quantification over representation functions in the definition of comparativist determinism to "simple" representation functions, where these are understood as those that satisfy simple numerical laws, and by understanding there to exist (in sufficiently complex worlds) laws of the following sort:

∀S-Newton There exist simple representation functions for force, mass, and acceleration; and for any simple representation functions for those quantities, f, m, and a, there exists some real number k such that for any object x, $f(x) = km(x)a(x)$

In effect this approach insists that descriptions of worlds at times are more closely tied to the laws than one might have thought: they are descriptions in terms of representation functions that stand in simple lawful numerical relationships.

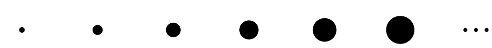
3.7 Existence assumptions and intrinsic laws

The putative laws considered in the previous section—statements like ∃-Newton, ∀-Newton, and ∀S-Newton—in essence say that the pattern in which $\succeq$ and C and other comparativist predicates hold is such as to constrain numerical functions in certain ways, but don't say directly what it is about the pattern that does the constraining. In Field's (1980) terminology, they characterize the fundamental predicates "extrinsically", only by their relationships to representation functions. "Intrinsic" laws, on the other hand, would be simple statements that directly concern the comparative predicates (which would, given representation and uniqueness theorems, have extrinsic statements as consequences).

Field argued in *Science without Numbers* that intrinsic explanations—explanations appealing to intrinsic laws—are superior to extrinsic ones, and sought to find intrinsic laws backing extrinsic statements like ∃-Newton. Arntzenius and Dorr (2011) agree with the need for intrinsic laws, and call the search for them the "hard problem" of quantity. In section 3.9 we will discuss the insistence on intrinsic laws in more detail. It has great initial appeal—especially for a realist about natural kinds.

One very natural view—and my own view until recently—is that the real problem caused by the failure of the existence assumptions would be that there wouldn't exist intrinsic laws. So insofar as the existence assumptions are illegitimate, and it is right to insist on intrinsic laws, comparativism is to be rejected.

I don't have a general argument that comparativist intrinsic laws can't exist without existence assumptions, but it is clear that this is true of the best developed attempt to give such intrinsic laws, namely Field's. Field's method uses quantification over "standard sequences" as described in Krantz et al. (1971). A standard sequence for a quantity Q—mass, say—is a sequence of massive objects that is "evenly-spaced" in the sense that the difference in mass between adjacent objects in the sequence is constant:[15]



$1q \quad 2q \quad 3q \quad 4q \quad 5q \quad 6q$

Think of a standard sequence for quantity Q—a "Q sequence"—as a "grid" one can lay down on all objects. Most objects do not have exactly the same Q value as a point of the grid (only those whose Q values are integer multiples of the grid's unit—the Q value of its first member—do), but they can nevertheless be represented as points on the grid with accuracy that increases as the grid's resolution increases—i.e., as the size of its unit decreases. By quantifying over Q-sequences of increasingly high resolution Field develops intrinsic correlates of statements about, for example, ratios between real-valued quantities. And

[15] I'll require standard sequences to be infinite, for simplicity's sake; but this isn't actually necessary.

the problem is that if Density fails then this strategy fails since there won't exist arbitrarily fine-grained grids.

In more detail: let Q_1 and Q_2 be two quantities that are ratio scales[16] (ratios of values of these quantities are significant; mass is an example), with corresponding fundamental relations $\succeq_{q_1}$, C_{q_1}, $\succeq_{q_2}$, and C_{q_2}; and let q_1 and q_2 be representation functions for Q_1 and Q_2, respectively. Suppose we want an intrinsic statement of this:

$$\frac{q_1(x)}{q_1(v)} > \frac{q_2(u)}{q_2(y)} \tag{*}$$

where x, y, u, and v are any objects. Here is a method, in the spirit of Field's. We'll be using mereology; and for simplicity, let's assume atomism and also that all objects possessing Q_1 and Q_2 are mereological atoms. (This assumption could be avoided, but it's harmless in Field's case anyway, since the quantities in question are taken to be fields defined at points of space.) First a definition, for any ratio scale quantity with corresponding $\succeq$ and C:

> S is a Q *sequence* $=_{\text{df}}$ S has an atomic part, s_1, such that for every atomic part x of S, there is some atomic part y of S such that: a) Cs_1xy, and b) for any atomic part z of S, if $y \succeq z \succeq x$ then $z = x$ or $z = y$

A Q sequence is a sum of atoms $s_1, s_2, s_3 \ldots$ where $q(s_k) = kq(s_1)$ (for any representation function q for Q), and thus can be pictured as in the diagram above. An intrinsic correlate of (*) can then be given:

(**) There exists a Q_1 sequence S_1 and a Q_2 sequence S_2 such that i) x is an atomic part of S_1; ii) y is an atomic part of S_2; iii) there are exactly as many atomic parts of S_1 that are $\preceq_{q_1} x$ as there are atomic parts of S_2 that are $\preceq_{q_2} u$; and iv) there are fewer atomic parts of S_2 that are $\preceq_{q_2} y$ than there are atomic parts of S_1 that are $\preceq_{q_1} v$

(Note the use of the generalized quantifiers 'there are exactly as many Fs as Gs' and 'there are fewer Fs than Gs'; see Field (1980, chapter 9) for discussion.) (**) is an intrinsic correlate of (*) in the sense that—and this can be proven—for any representation functions q_1 and q_2 for Q_1 and Q_2, (**) holds iff (*) holds

[16] Field's method also applies to other quantities, e.g. quantities where it is ratios of differences that are significant.

under q_1 and q_2.[17]

But the proof depends on Density. As mentioned earlier, we can think of Q sequences as grids for measuring Q values; the proof relies on the ability to choose grids of arbitrarily high resolution. If Density fails then the grids won't be guaranteed to exist, and as a result, even if the ratio of x's Q_1 to v's "ought" to exceed the ratio of u's Q_2 to y's, (**) might nevertheless be false: there might not exist the appropriate sequences S_1 and S_2. This then threatens the existence of intrinsic laws concerning ratios (or products) of quantities. *[This isn't quite right as things are set up. I've defined standard sequences in such a way that they need to be infinite and have arbitrarily massive objects; thus the existence of* any *standard sequences relies on Existence of sums.]*

To illustrate, consider Newton's second law $F = ma$. To keep things simple, pretend that acceleration is a primitive scalar quantity taking only positive

[17] Proof sketch of this fact (following Field's note 48, using without comment consequences of standard axioms on ratio scale quantities): let q_1 and q_2 be representation functions for Q_1 and Q_2, and suppose first that there exist S_1 and S_2 as described in (**). Let a and b be the "first" (i.e., Q_1-least and Q_2-least) atomic parts of S_1 and S_2 respectively, and let x and y be the k^{th} and l^{th} atomic parts of S_1 and S_2, respectively. $q_1(x) = kq_1(a)$; but given (iv) in (**) we have $q_1(v) > lq_1(a)$; hence $\frac{q_1(x)}{q_1(v)} < \frac{k}{l}$. Similarly, $q_2(y) = lq_2(b)$; but given (iii) in (**), $q_2(u) \geq kq_2(b)$; and so $\frac{k}{l} \leq \frac{q_2(u)}{q_2(y)}$. Thus $\frac{q_1(x)}{q_1(v)} < \frac{q_2(u)}{q_2(y)}$; we have shown that (*) holds if (**) does. For the other direction, we assume (*) and show how to choose S_1 and S_2 as in (**). Some notation: where S is a Q sequence and o is any object (not necessarily an atomic part of S), let $S(o)$ be the number of atomic parts of S that are $\preceq_q o$. Notice that $S(o)q(a) \leq q(o) < (S(o)+1)q(a)$, where a is the first atomic part of S. Now, we can choose Q_1 and Q_2 sequences S_1 and S_2 that contain x and y as atomic parts, respectively, and where $S_2(u) = S_1(x)$. Such sequences satisfy i)–iii) in (**); it remains to show that they can also be chosen to satisfy iv). Letting a and b be the first atomic parts of any such S_1 and S_2, we have a) $q_1(x) = S_1(x)q_1(a)$, b) $q_2(y) = S_2(y)q_2(b)$, and c) $q_1(v) < (S_1(v)+1)q_1(a)$. Also $q_2(u) < (S_2(u)+1)q_2(b)$; but $S_2(u) = S_1(x)$, so d) $q_2(u) < (S_1(x)+1)q_2(b)$. (a)–(d) yield $\frac{q_2(u)q_1(v)}{q_1(x)q_2(y)} < \frac{(S_1(x)+1)(S_1(v)+1)}{S_1(x)S_2(y)}$; or, rewriting:

$$\frac{q_2(u)q_1(v)}{q_1(x)q_2(y)} < \left(1 + \frac{1}{S_1(x)}\right)\left(\frac{S_1(v)}{S_2(y)} + \frac{1}{S_2(y)}\right)$$

By choosing increasingly closely spaced sequences S_1 and S_2 we can make both $S_1(x)$ and $S_2(y)$ arbitrarily large, and so can get the right hand side of the displayed formula arbitrarily close to $\frac{S_1(v)}{S_2(y)}$. But (*) tells us that $\frac{q_2(u)q_1(v)}{q_1(x)q_2(y)} > 1$. S_1 and S_2 can therefore be chosen so that $\frac{S_1(v)}{S_2(y)} > 1$ (the difference δ between the right hand side of the displayed formula and $\frac{S_1(v)}{S_2(y)}$ can be chosen to be less than the difference ϵ between $\frac{q_2(u)q_1(v)}{q_1(x)q_2(y)}$ and 1), and hence so that $S_1(v) > S_2(y)$.

values (so that it's a ratio scale), and pretend that all objects undergo exactly the same net force, so that the law says that the product of mass and acceleration for any object is the same as for any other object:

$$m(x)a(x) = m(y)a(y) \qquad \text{(for any } x, y\text{)}$$

or, equivalently:

$$\frac{m(x)}{m(y)} = \frac{a(y)}{a(x)}$$

or, equivalently:

$$\frac{m(x)}{m(y)} \not> \frac{a(y)}{a(x)} \text{ and } \frac{a(y)}{a(x)} \not> \frac{m(x)}{m(y)} t$$

The Fieldian intrinsic correlate is then:

I-Newton For any objects x and y: there do *not* exist a mass sequence S_1 and an acceleration sequence S_2 such that i) x is an atomic part of S_1; ii) x is an atomic part of S_2; iii) there are exactly as many atomic parts of S_1 that are $\preceq_m x$ as there are atomic parts of S_2 that are $\preceq_a y$; and iv) there are fewer atomic parts of S_2 that are $\preceq_a x$ than there are atomic parts of S_1 that are $\preceq_m y$; and there do *not* exist an acceleration sequence S_1 and a mass sequence S_2 such that i) y is an atomic part of S_1; ii) y is an atomic part of S_2; iii) there are exactly as many atomic parts of S_1 that are $\preceq_a y$ as there are atomic parts of S_2 that are $\preceq_m$ than x; and iv) there are fewer atomic parts of S_2 that are $\preceq_m y$ than there are atomic parts of S_1 that are $\preceq_a x$

This statement can be true simply because of the nonexistence of appropriate mass or acceleration sequences. Why is that a problem? If I-Newton can so easily be true when the existence assumptions don't hold, that means that it doesn't have the consequences we would expect it to have in those circumstances—in particular, it won't have the same consequences about C_m, C_a, $\succeq_m$, and $\succeq_a$ that, say, the extrinsic statement ∃-Newton has. For example, suppose $C_m xxy$. Then y's mass is twice that of x for any mass function, and so ∃-Newton implies that y's acceleration is half x's for some acceleration function, and so $C_a yyx$. But I-Newton doesn't have this consequence without assuming Density, since we can construct models in which I-Newton holds simply because of the absence of mass and acceleration sequences.[18] It therefore cannot be regarded as a law (even though it's true) in such circumstances: a law must be an appropriately strong statement.

3.8 Intrinsic laws and Mundy

Brent Mundy (1989) ingeniously attempted to avoid problematic existence assumptions in the comparativist theory of quantity by combining and enhancing the standard predicates $\succeq$ and C into a single primitive multigrade predicate "$a_1,\ldots,a_n \succeq b_1,\ldots,b_m$" (for any finite numbers of arguments n,m) meaning, intuitively, that $a_1,\ldots,a_n$ together have a sum total of the quantity in question that is at least as great as the sum total of the quantity possessed by $b_1,\ldots,b_m$. The main virtue of this approach can be seen as follows. Suppose we want to say that the ratio of x to y for the quantity in question is at least as great as some particular fraction $\frac{n}{m}$. Given standard comparativism we can do this by "dividing" up x and y's portion of the quantity:

> There exist $x_1, x_2, \ldots x_{n-1}$ and $y_1, y_2, \ldots, y_{m-1}$ such that i) $Cx_1x_1x_2$, $Cx_1x_2x_3$, …, $Cx_1x_{n-1}x$; ii) $Cy_1y_1y_2$, $Cy_1y_2y_3$, …, $Cy_1y_{m-1}y$; and iii) $x_1 \succeq y_1$

[18] For example, let x and y be the sole massive and accelerated elements in the domain. (The model must also be a model of mereology; so let x and y be mereological atoms; and let the model also contain another object to be the fusion of x and y, which is not in the field of the mass and acceleration relations). Let the only case of either C_m or C_a be C_mxxy (thus C_ayyx does not hold); let $y \succ_m x$ and $y \succ_a x$; let $\succeq_m$ and $\succeq_a$ be reflexive. Other than existence assumptions, the usual axioms for extensive systems hold for both mass and acceleration. (E.g., convert the axioms for an extensive structure with no essential maximum in Krantz et al. (1971, vol. 1, p. 84) to the present notation by defining $Cxyz$ to mean their "$(x,y)\in B$ and $z \sim x \circ y$", and then remove the existence assumptions by deleting axiom 4 and weakening 2 and 3 to:

(2′) If $Cabx$ and $Cxcy$, and *if* $Cbcz$ and $Cazw$, then $y \succeq w$ "associativity"

(3′) If $Cacx$ then $Ccax$ "commutativity"

(3″) If $Cacx$ and $a \succeq b$, and *if* $Cbcy$, then $x \succeq y$ "monotonicity"

The resulting axioms then hold in the model, for $\succeq_m$ and C_m as well as for $\succeq_a$ and C_a.) I-Newton also holds since mass and acceleration sequences must be infinite. Now, Q sequences could have been defined to allow finitude (Field does this); but I-Newton still holds in the model thus reinterpreted (as is easy to verify). (Note that the only mass sequences are $x+y$, x, and y, and that the latter two are the only acceleration sequences.)

$$\underbrace{\bullet \quad \bullet \quad \dots \quad \bullet \quad \bullet \quad x}_{n \text{ evenly spaced things}} \qquad \underbrace{\bullet \quad \bullet \quad \dots \quad \bullet \quad \bullet \quad y}_{m \text{ evenly spaced things}}$$

thus relying on Density, or else else by copying x and y:

> There exist $x_1, x_2, \dots x_{m-1}$ and $y_1, y_2, \dots, y_{n-1}$ such that i) $Cxxx_1$, Cxx_1, x_2, …, $Cxx_{m-2}x_{m-1}$; ii) $Cyyy_1$, Cyy_1y_2, …, $Cyy_{m-2}y_{m-1}$; and iii) $x_{m-1} \succeq y_{n-1}$
>
> $$\underbrace{x \quad \bullet \quad \bullet \quad \dots \quad \bullet \quad \bullet}_{m \text{ evenly spaced things}} \qquad \underbrace{y \quad \bullet \quad \bullet \quad \dots \quad \bullet \quad \bullet}_{n \text{ evenly spaced things}}$$

thus relying on the Existence of sums. But on Mundy's multigrade approach, we needn't rely on any existence assumptions; we can just say:

$$\underbrace{x, \dots, x}_{m \text{ occurrences}} \succeq \underbrace{y, \dots, y}_{n \text{ occurrences}}$$

As a result, as Mundy shows, if one uses his multigrade predicate, existence assumptions (like Density and Existence of sums) aren't needed to prove representation and uniqueness theorems.

But when it comes to stating intrinsic laws (which Mundy doesn't discuss), the need for the existence assumptions reappears. Mundy's representation and uniqueness theorems tell us that the totality of facts stateable in his language fix a unique—up to scalar transformation—numerical representation; they don't tell us that a simple law stated in terms of the numerical representation has a single, simple corresponding sentence in his language. Mundy's method for saying "the mass-ratio between x and y exceeds $\frac{n}{m}$" as described in the previous paragraph is for *fixed* n and m. But in order to construct intrinsic correlates of statements about ratios between real-valued quantities, one needs such comparisons when n and m are variables, as in "for any integers n and m, if the mass-ratio of x to y is greater than or equal to $\frac{n}{m}$ then the mass-ratio of z to w is greater than or equal to $\frac{n}{m}$". Field's method for doing this quantifies over standard sequences and thus requires existence assumptions. In essence it is a modification of the method of division to the case where n and m are variable; a standard sequence is a single entity corresponding to the sequence of values of the existential quantifiers in the method of division. Mundy's method cannot be modified in this way, for without making existence assumptions we have no entities corresponding to his sequences of occurrences of a single variable.

3.9 Intrinsicality of laws

I have said that the main problem that would be caused by the failure of the existence assumptions would be the inability to state intrinsic laws. But why think that laws should be intrinsic?

As we saw in section 1.7, once the notion of fundamental concepts is assumed, it is very natural to take a general bias in favor of simplicity in theory choice to require, at least in part, a bias in favor of simple laws as formulated using the fundamental concepts. But this assumption doesn't itself require intrinsic laws, since even extrinsic laws might be regarded as being appropriately simple constraints on fundamental concepts like the comparativist predicates.[19]

Field himself said three main things against extrinsic laws. He complained about formulations of laws that "appeal to extraneous, causally irrelevant entities" (Field, 1980, p. 43), and he complained about arbitrariness. Let's take these individually.

Field expands on the charge of causal irrelevance:

> If, as at first blush appears to be the case, we need to invoke some real numbers like 6.67×10^{-11} (the gravitational constant in $m^3/kg^{-1}/s^{-2}$) in our explanation of why the moon follows the path that it does, it isn't because we think that that real number plays a role as a cause of the moon's moving that way. (p. 43)

But in what sense would fundamental extrinsic laws render real numbers "causally relevant"? It wouldn't imply, for example, that we can see or touch real numbers, or that purely numeric facts about real numbers (such as that $3 = 2+1$) cause or are caused by physical facts such as that I am sitting, am in a certain location, am more massive than my cat, etc. In what sense would recognizing fundamental extrinsic laws require us to grant the real number 6.67×10^{-11} "a role as a cause of" the moon's motion? It wouldn't imply that the fact that the number exists, or the fact that it has certain mathematical properties, causes the moon to move as it does, or that different possibilities for the moon's motion correspond to different possibilities for the purely mathematical facts about this and other real numbers.[20]

[19]Fundamental predicates sufficient to reconstruct the relevant bit of mathematics would need to be recognized.

[20]It might be objected that the existence of 6.67×10^{-11} *is* a cause of the moon's moving as it does, given the extrinsic law, since if the number hadn't existed then the extrinsic law wouldn't have been true, in which case the moon would have moved differently. But why should the comparativist admit the inference to the final claim?

As far as I can see, the complaint about causal irrelevance amounts to nothing more than insisting that numbers aren't involved in fundamental laws. But who made that rule? One worries that this is just a prejudice.

Consider next the claim that a fundamental law should involve no "extraneous" entities. We have, it would seem, a conception of the proper subject of certain laws, of the kinds of entities those laws really concern. Other entities are extraneous, and the laws should not be interpreted as naming or quantifying over such entities. Real numbers and functions, for instance, seem not to be part of the proper subject of Newton's second law; the law should concern no entities other than bodies in motion.

This is a more promising complaint, but it's still far from clear. What exactly makes an entity "extraneous"? Even Field's preferred laws appear to involve entities that are in some sense extraneous—entities that seem intuitively not to be involved in the facts the law governs. This was pointed out by Melia (1998, section 2). The Fieldian intrinsic correlate of the claim that x and y share the same product of mass and acceleration quantifies over mass and acceleration sequences containing x and y. Such sequences contain arbitrarily many massive and accelerated objects, which objects can be located anywhere whatsoever. Facts about such mass sequences are, intuitively, as extraneous to a law governing the mass and acceleration of x and y as are facts about real numbers. To be sure, mass sequences are *composed* of entities that are themselves the *kinds* of entities that are indeed part of the proper subject of laws of motion. But the standard sequences themselves seem extraneous. They're being used to "code up" what one would have thought was a direct relation between x and y. So now one worries that Comparativism cannot supply intrinsic laws after all. Even given Field's method for avoiding relying on the extrinsicality of laws that quantify over real numbers, we still have the extrinsicality of quantifying over standard sequences.

Actually, there is a further (though related) complaint one can make about Field's allegedly intrinsic laws: they are highly nonlocal. In two ways, actually. First, I-Newton concerns two objects at a time. Second, there is what we already discussed: the quantification over mass and acceleration sequences, where the parts of these sequences can be at arbitrarily large spatial distances from the given pair of objects. So little remains of the idea that to tell what an object is going to do, you only need to look in the immediate spatiotemporal vicinity of that object.

(Notice, by the way, the convergence between the discussion of quantity in the philosophy of science, on the one hand, and pure metaphysics on the other.

Metaphysicians have objected that comparativism turns mass properties into relational rather than intrinsic properties.[21] It is satisfying to see how a brute appeal to "intuition" can be replaced with an appeal to a constraint on laws (though appeal to intuition seems to return with the prohibition of extraneous entities). And notice again how important lawhood is for figuring out what is fundamental.)

Are intuitively "extraneous" entities in laws avoidable by anyone? Even a mixed absolutist will still need to quantify over standard sequences, when attempting to construct intrinsic laws concerning ratios. These will no longer be sequences of, e.g., massive particles, but rather will be sequences of properties (or sequences of points in quality spaces, for Arntzenius and Dorr). But isn't it still true that arbitrary sequences of such properties are, intuitively, "extraneous" to a law governing the mass and acceleration of a given particle? Why does it help that the members of the sequences are properties (or points in quality spaces) rather than concrete things?

In addition to complaining about causally irrelevant and extraneous entities, Field also says that "one of the things that gives plausibility to the idea that extrinsic explanations are unsatisfactory if taken as *ultimate* explanation is that the functions invoked in many extrinsic explanations are so arbitrary" (p. 45). For example, laws concerning one particular set of representation functions involve arbitrary choices of unit for each of the quantities involved.

There's a problem with interpreting Field's complaint about extrinsic laws as a complaint about arbitrariness: extrinsic laws like ∃-Newton, which quantify over representation functions rather than invoking one in particular, seem not to turn on any arbitrary decisions and hence avoid the complaint. For that matter,

[21] Metaphysicians have also focused on modal considerations: comparativism precludes the possibility of, e.g., everything doubling in mass. Reliance on the "intuition" that this is genuinely possible seems to me to suffer from the same problem as relying on the intuition that mass properties are intrinsic: in each case the intuition is nothing but a belief based on an internalized commonsensical proto-theory of mass that has no independent justification. We ordinarily think of quantities as fundamentally intrinsic, and we then apply combinatorial reasoning to yield the possibility of doubling. One could attempt to give a stronger modal argument by arguing that comparativism allows doubling *one* thing's mass (or anyway allows the comparativist equivalent of this) and then arguing that no reasonable conception of the statespace can allow this but not allow the doubling of everything's mass. But this argument is no good, since a perfectly reasonable conception of the statespace can be given in "native" terms: the statespace consists of all the possibilities, given in some combinatorial way, for the comparativist's fundamental relations. What is possible in other terms can then be "read off" from this.

the complaint would also be avoided by a view that involves real numbers even more deeply in the metaphysics of quantity. Like the flat-footed approach to quantities with which we began our discussion, this view characterizes facts about quantities using relations to real numbers, but it avoids privileging a unit by assigning *ratios* of quantities to pairs of objects rather than values of quantities to individual objects. The fundamental concept of mass, on this approach, is a (functional) three-place predicate $Mxyr$ applied to two massive things x and y and the real number r that is the ratio of x's mass to y's mass; mass representation functions can then be defined as functions m such that $\frac{m(x)}{m(y)} = r$ iff $Mxyr$. This privileges no unit since mass ratios are invariant across units.[22] If the extrinsicality complaint were simply a complaint about arbitrariness, then this metaphysics should be exempt.

Perhaps the arbitrariness complaint can still be made against each of these views, if real numbers are regarded as being constructed from sets. There are many ways this construction can go. The usual strategy is to construct integers from sets, rational numbers as equivalence classes of pairs of integers, and real numbers as sets of rationals. But at each stage there are arbitrary-seeming choices about how exactly to carry out the construction. If the fundamental mass concept is a relation to real numbers under one chosen construction, this would seem objectionably arbitrary; it's as arbitrary to privilege a method for constructing real numbers from sets as it is to privilege a unit of measure.[23] So the fundamental facts, for the real-numbers-as-ratios approach, involve an arbitrary element. Moreover, there is also an element of arbitrariness in what to count as laws of the ∃-Newton form, given the arbitrariness in how to define real numbers (and functions, and other such mathematical concepts) in fundamental terms. These concerns could be avoided by taking real numbers to be sui generis, but at the cost of inflationary philosophy of mathematics.

But it isn't clear that Field's own approach is exempt from *this* sort of arbitrariness. Field's intrinsic laws depend on a certain method for nominalizing statements about ratios (via standard sequences), and it's hard to believe that this method is the only one that would do the trick. This arbitrariness seems on a par with the arbitrariness in laws of the form ∃-Newton corresponding to the arbitrariness in how to construct real numbers from sets. (The arbitrariness in the ratios view is perhaps deeper, since it infects the fundamental facts, not

[22] Thanks to Earl Conee for discussion here; and see Mundy (1988) on ratio spaces.

[23] See Sider (1996) for more on this sort of argument. Note that the argument is essentially fundamentality-theoretic.

just the laws.)

My overall concern, zooming out: what Field is recoiling from, when he complains about extrinsic laws, once we set aside the complaint about causal relevance, is some combination of arbitrariness and artificiality in the use of numbers to code up a constraint on $\succeq$ and C. That complaint really does mesh with the fundamentality approach. The intuitive idea of the fundamentality approach's epistemology is that the laws ought to look attractive as laws when you view them as they fundamentally are—when you look at their formulation in a completely fundamental language. And what I'm worried about is that we're now being led to the conclusion that the laws just won't look good when viewed in that way. Because of parsimony—a separate constraint on our epistemology—we are drawn to a minimal basis. But that inevitably means that any statement of powerful quantitative laws is bound to involve some artificiality and/or arbitrariness.

If there is no escape from artificiality and arbitrariness, how should we react? Should we say that intrinsicality comes in degrees and that good laws are as intrinsic as possible? Should we try to make a distinction between problematic and unproblematic ways of being extraneous? Or should we conclude—Cthulu forbid—that it was a mistake from the start to regard laws as the key to identifying the fundamental properties and relations?

3.10 Fundamentality versus ground

I want to close by noting a further respect in which the first order debate is sensitive to the question of tools. I have framed the issue in terms of fundamental properties and relations, which made the search for intrinsic laws central. Shamik Dasgupta's (2013; 2014*a*) important recent defense of comparativism, on the other hand, is framed using the metaphysical tool of ground, rather than of concept-fundamentality. He formulates comparativism as the claim that all facts about quantities are grounded in comparative quantitative facts. Thus "absolute" facts about quantities, such as that I am 165 pounds and that New York City is 100 miles from Philadelphia are grounded in comparative facts such as that New York City is farther from Philadelphia than San Francisco is from Los Angeles, that I am more than twice as massive as my daughter, and so forth. When it comes to developing his account, Dasgupta's energies are focused on subtleties about these ground-theoretic claims. For instance, he argues that absolute facts about quantities are not grounded individually;

rather, the totality of absolute facts about mass are grounded in the totality of relational facts about mass (2014*a*). But he does not focus on the question of which comparative facts, exactly, are fundamental, or on which comparative relations are fundamental. As we have seen, a focus on such questions leads us to difficult questions about existence assumptions and intrinsic laws—questions that Dasgupta simply doesn't address.[24] Perhaps this is appropriate given his ground-theoretic focus, or perhaps not. But in any case it illustrates how the ultimate verdict on the metaphysics of quantity is intertwined with the question of the conceptual tools with which the issue should be framed.

[24]Here is another issue that only arises when the comparativist asks which relations are fundamental (though it strikes me as less important than those on which we focused here. Is there a difference between a world of just two massive objects where one is three times as massive as the other, and a world of just two massive objects where one is seventeen times as massive as the other? The difference can be acknowledged if the fundamental comparativist predicate is one relating massive objects to numbers representing their mass ratios (recall the end of the previous section), but not if the fundamental predicates are $\succeq$ and C.

Chapter 4

The rejection of individuals

Belief in individuals may not be quite universal, but it comes pretty close. We do disagree about cases. Should we admit points of spacetime? Objects with parts? Holes? Numbers? Propositions? Gods? But nearly everyone accepts some individuals or other. And, more fundamentally, nearly everyone accepts the concepts we use to think about individuals—ontological concepts. We believe, or presuppose, that employing these concepts to think about the world doesn't amount to making a colossal metaphysical mistake.

Nevertheless, a number of philosophers have proposed (entertained, fantasized) rejecting individuals, in one way or another. Even if these proposals are mistaken (as I think they are), studying them, and the intellectual challenges from which they arise, can teach us about the role that individuals play in our theorizing. Until we think about what life without individuals would be like, we won't understand what they are doing for us now.

Moreover, as we'll see, the plausibility of the rejection of individuals is sensitive to the metaphysical tools we use to articulate that view. In particular, rejecting individuals is the sort of structuralist position that is hardest to articulate from a postmodal point of view.

4.1 Individuals

What are "individuals", and what would it mean to reject them?

The facts concerning individuals (objects, things, particulars…) constitute "ontological structure" in Jason Turner's (2011) sense:

> Ontological structure is the sort of structure we could adequately represent

> with a pegboard and rubber bands. The pegs represent things, and the rubber bands represent ways these things are and are interrelated.

Individuals include physical objects, however large or small; artifacts (such as tables and chairs); social entities (such as governments and economies); locations (times, places); purely mental entities (such as Cartesian souls); abstract objects (such as numbers and propositions); divine beings; and more. Or rather, if there are such things then they are individuals.

Linguistically, individuals are what we signify using terms, whether singular: 'The president of the United States', or plural: 'The justices of the United States Supreme Court'. These terms are ubiquitous; individuals are deeply embedded in ordinary thought and language.

It is unsurprising, then, that the concept of an individual is also deeply embedded in predicate logic. The function of the most basic kind of sentence in predicate logic, an atomic sentence, is to refer to one or more individuals using singular terms, $t_1 \ldots t_n$, and say something about them using a predicate R: $R(t_1 \ldots t_n)$.

Individuals are also embedded in scientific thinking. This is so both on the surface, and also at a foundational level, given the centrality of predicate logic to modern foundational reconstructions of mathematics and science.

4.2 Concerns about substrata

We turn now to the putative reasons for rejecting individuals. These vary in quality, and in the sort of anti-individualist position they support. Once the reasons are on the table, we will begin to articulate various forms of anti-individualism.

Suppose you believe in the existence of properties, in some sense. What connects an individual to its properties? Probably the default view is that the individual is wholly distinct from its properties, and that it *instantiates* them. Against this view people sometimes say things like this: if an individual is a distinct thing, over and above its properties, then it has no properties—no properties "in itself", as it's often added. So it's a mysterious sort of thing—a "bare substratum". This line of thought is often used to support the bundle theory of individuals, according to which an individual is nothing more than a bundle of properties. But it's very weak. Here is Elizabeth Anscombe's withering criticism:

> One of the considerations brought forward in erecting this notion (for it is not a strawman, real humans *have* gone in for it) seems so idiotic as to be almost incredible, namely that the substance is the entity that has the properties, and so it itself has not properties. (Anscombe, 1964, p. 71)

Of course the thing has properties—it instantiates them. Instantiating is the sense of "having" that the view provides.[1]

4.3 Solving puzzles of persistence

A better line of argument in favor of the bundle theory is that various metaphysical puzzles are better resolved by a bundle theory than by a substance-attribute metaphysic. For instance, Kris McDaniel (2001) and L. A. Paul (2002) have both argued that the bundle theory best resolves the puzzle of material constitution. On the one hand, a statue seems identical to the hunk of matter from which it's made, since they seem to share exactly the same parts. On the other hand they seem distinct since they apparently have different modal properties: the hunk but not the statue could survive being squashed. McDaniel and Paul's solution is that the statue and hunk do not have the same parts after all. Each is an aggregate of properties, and in the bundles are included different properties. For instance, on Paul's view the statue includes certain modal properties that aren't included in the hunk.[2]

I have no objection to the general form of argument here. But such arguments are highly defeasible. Like anyone offering such arguments, McDaniel and Paul don't claim to show that competing resolutions of the puzzles are untenable; they just argue that their own solutions are most attractive, on various grounds. Thus if new considerations are added to the mix, they can tip the scales. Below I hope to add such new considerations, by arguing that the bundle theory itself is seriously problematic. Thus one of the competing solutions to the puzzles must be right after all.

[1] Anscombe goes on to consider things one might mean by "in itself"; I talk about this issue too, in Sider (2006)—somewhat in ignorance of past discussions.

[2] According to McDaniel, the properties in the bundles are tropes, whereas according to Paul they are universals. See also Koslicki (2008), who gives a similar solution to the problem of constitution but without assuming the bundle theory.

4.4 Concerns about the practice of ontology

There is a tradition of thinking that metaphysics has come off the rails when it asks questions like these: Are there holes? Shadows? Composite material objects? Can two things be located in the same place, or one thing be located in two places?

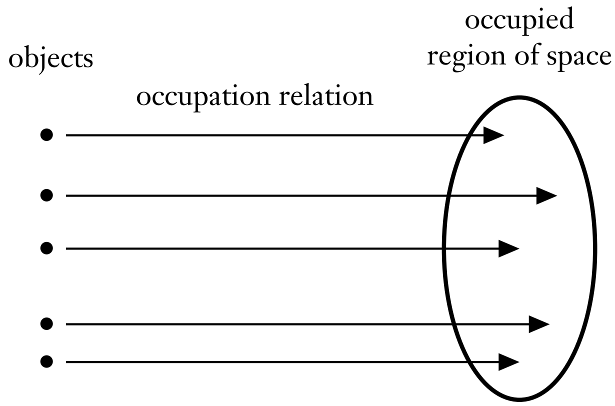
What exactly is the concern? One is epistemic: we allegedly cannot know the answers to the questions, so the questions are somehow incoherent.[3] A second is metaphysical: the competing answers are alleged to not genuinely differ, so the questions are somehow incoherent. The statement that holes exist, in addition to perforated objects, might be said to not make a genuinely different claim about the real world than the statement that only perforated objects exist.

Against either concern a direct argument may be opposed, according to which the questions *are* coherent: the questions are stated using the very same vocabulary as certain clearly coherent questions. For instance, the question of whether there exist black holes is obviously genuine; but the ontological questions mentioned above have the same form: are there any Fs? Provided we can make 'F' clear and unambiguous, it's hard to see what's wrong with ontological questions of the form "are there Fs?"

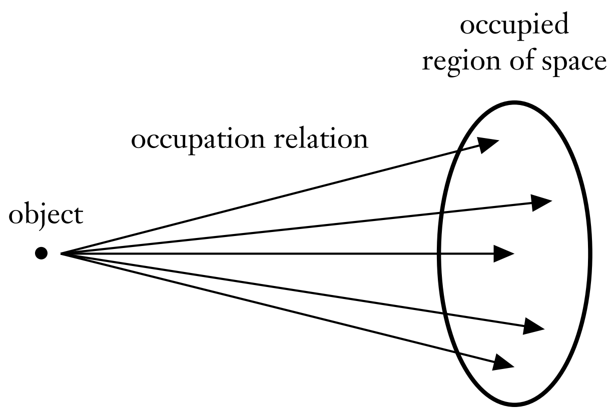
Or take the questions of multilocation and colocation.[4] Outsiders hate these topics, but it's actually very hard to avoid thinking that they're genuine. Questions about the existence and number of physical objects and where they are located are, in general, coherent questions, so how could the question of whether two things are located in the same place, or whether one thing is located in two places, be incoherent? Pushing this further: suppose you think there are such things as objects *in* space, and that they're different from space itself. Then your picture of a spatially extended object—an oval, say—is presumably this:

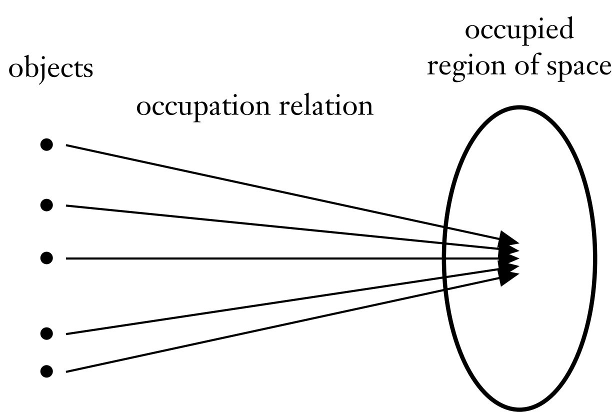
[3] This sort of objector faces well-known challenges. The objector must avoid appealing to epistemic principles that lead to skepticism. ("If your view is true, then it's metaphysically possible that I have the same evidence but am wrong about X; therefore I don't know X.") The objector must avoid giving arguments that apply to scientific inquiry just as much as metaphysical inquiry. The objector must avoid attributing more hubris to metaphysicians than they really have. (Metaphysicians might reasonably only claim to be making educated guesses.) Why do people turn into logical positivists when they encounter metaphysics? I wish I knew.

[4] **Cites to Gilmore, Hudson, Kleinschmidt, McDaniel, Parsons, Saucedo, others.



The oval is extended because it has parts that are located at the bits of an oval-shaped region of space. But then consider the following pictures:





They're constructed from the same elements as the picture you initially accept, and so seem themselves to be coherent hypotheses about the makeup of the physical world.

In each case—the case of ontological questions and the case of locational questions—we begin with an initial, ordinary description (an extended oval with many parts; the existence of a black hole); and then the metaphysician uses the very same conceptual materials to construct an extraordinary description (multilocation; the existence of shadows or the nonexistence of tables and chairs); and the philosophical question is whether the extraordinary description is true. If there's a coherent question of whether the ordinary description is true, there must surely be a coherent question of whether the extraordinary description is true.[5]

In the face of this argument, someone who *really* doesn't like the idea that metaphysicians are debating coherent questions—whether for epistemic or metaphysical reasons—might say that the metaphysician's mistake is in their construal of the initial questions. All the allegedly problematic questions involve entities. In some cases the questions are whether entities of certain types exist; in others they are about whether certain patterns of property (or relation) attributions to entities are possible; but in every case the questions are about entities. So maybe entities are the problem; and maybe the way to resist the argument above that the metaphysical questions *are* coherent is to say that, contrary to appearances, the ordinary descriptions (there is a black hole, an oval is located in a certain region of space) aren't really about entities. That way there's no way to "twist" those descriptions to arrive at coherent extraordinary descriptions.

There are problems with this sort of motivation. First, it may be that extraordinary descriptions can be obtained from the reinterpreted ordinary descriptions. After all, there is overwhelming pressure to think that the "replacement" vocabulary for thing-talk is going to be structurally very similar to thing talk—to think otherwise is to be a kind of radical skeptic. (The best-worked out views to be considered below certainly admit this. But then, they admit most of the usual metaphysical questions; their supporters are not motivated by the desire to avoid metaphysics.) Moreover, even if reality is, as it happens, entity-free, if the alternative (reality being "entity-full") is metaphysically coherent, then the various competing entity-theoretic descriptions of reality would seem themselves to be metaphysically coherent (in some good

[5] This argument assumes that the genuineness of a question is a function of its vocabulary; and one could try denying that assumption by admitting a kind of holism about genuineness.

sense), so it's hard to see how merely upholding the truth of an entity-free metaphysics would render the metaphysical questions incoherent.

4.5 Structural realism

Another set of arguments comes from the "structural realists", who say that considerations in the philosophy of science, especially the philosophy of physics, call for a new metaphysics that eliminates particular entities and replaces them with some sort of structure.[6]

4.5.1 Rescue from pessimistic metainduction

Structural realism comes most immediately from an exchange between John Worrall (1989) and James Ladyman (1998). Worrall claimed that to defend against the "pessimistic metainduction", scientific realists should become structural realists.

Science has a history of periodic drastic changes. So we should expect, according to the pessimistic metainduction, that few of the central claims of current theories are likely to be true, since those theories will likely be replaced someday by quite different theories that are inconsistent with most of the central claims of current theories—a conclusion that threatens scientific realism.

Worrall's suggestion was that this argument could be undermined by a "structuralist" position. In many cases of drastic scientific change, there is a structural similarity between the new and the superseded theory. So if all that science teaches us is structure, the superseded theory's central claims need not be regarded as inconsistent with those of the new theory.

Ladyman then noted that Worrall's proposal could be read in either an epistemic or ontic sense. "Epistemic structural realism" says merely that all we are justified in believing from science is statements about structure. "Ontic structural realism"—a metaphysical thesis—says that (in some sense) structure is all there is. Ontic structural realism (which will be our focus[7]) has since been

[6] See Ladyman (2014); Saunders and McKenzie (2015) for overviews.

[7] The literature on epistemic structural realism has focused on the "Newman problem". Let $T(P_1, \ldots P_n)$ be a scientific theory making claims about properties (or relations) $P_1 \ldots P_n$, and suppose the epistemic structural realist's claim is that the evidence in favor of T supports only its Ramsey sentence: $\exists p_1 \cdots \exists p_n T(p_1 \ldots p_n)$, where the quantifiers range over properties

argued for on other grounds, which we will take up shortly.

Our main focus will be on the position itself rather than arguments in its favor, but it must be said that those arguments seem weak. Regarding the argument from the pessimistic metainduction: first, it's hard to believe that structure (in any helpful sense) really is preserved through radical theory changes in the history of science.[8] Second, the pessimistic metainduction just isn't a serious threat to scientific realism, at least if belief is construed as coming in degrees. One can recognize the likelihood of a future scientific revolution overturning current theory, and thus have a low degree of belief in the truth of the current theory, while still holding that the current theory is much more likely than every rival theory we know about. Worrall might reply that this concedes too much to the antirealist since it would undermine the "no miracles" argument for realism, according to which current theories are probably true because otherwise the truth of their predictions about observable matters would be a miracle.[9] But while my realist can't accept exactly that argument (since she thinks that current theories are unlikely to be true), she can in effect accept the no miracles argument for giving current theory the degree of belief that she in fact does. She thinks, let's suppose, that the current theory is 20% likely to be true, that it's 5% likely that some current rival theory is true, that it's 74.9% likely that some product or other of a future scientific revolution (of whose nature she has no inkling) is true; and she reserves a 0.1% probability for a grab-bag of outcomes such as that nature has no regularities at the unobservable

construed "abundantly"—i.e., at least one for each subset of the domain of individuals. The Newman problem is that the Ramsey sentence is far too weak to be of interest, since it is guaranteed to be true provided the theory has at least one model in the domain of individuals. It is bizarre that this literature almost never considers what any metaphysician would offer as the obvious response: the quantifiers ought to be restricted to properties that are "sparse" in some sense (see Lewis (1986, 59–69) for the sparse/abundant distinction). This is presumably a consequence of the unfortunate reflexive opposition to metaphysics in certain circles. Here we have a proposal that is obviously a nonstarter if restricted by an absurdly minimalist metaphysics; and the needed enhancement—a notion of sparse properties—should not be "scary" except to the most resolutely antimetaphysical; it was, after all, originally made popular by Putnam (1970) drawing on considerations from the philosophy of science.

[8]The literature does not seem to specify in any detail exactly what this shared structure is supposed to be, which makes it difficult to evaluate structural realism at the appropriate level of detail. What one would like to do is consider the arguments in its favor, consider various ways of construing the position, and evaluate whether a given construal can be supported by the arguments; but the arguments are left underspecified because the relevant sameness of structure isn't specified.

[9]See his pp. 110–11 discussion of "conjectural realism".

level and her theory "just happens" to make true observational predictions. The low degree of belief in the grab-bag reflects her disbelief in miracles—her realist faith in the value of explanations citing unobservables, taken at face value.

4.5.2 Metaphysical undetermination

Another argument for ontological structural realism is that it allegedly avoids a certain sort of "metaphysical undetermination". In his original paper on structural realism Ladyman observed that certain physical theories leave open the metaphysical nature of the entities they allegedly concern—in particular, whether those entities count as "individuals" in a certain loaded sense. He mentions two main cases: individuality in quantum mechanics and the debate over substantivalism about spacetime. In the former case, there is a question about whether particles in quantum mechanics can be regarded as individuals; and in the latter case, there is a question of whether, as Newton thought, points of space or spacetime are individuals, or whether instead, as Leibniz thought—and contemporary advocates of the hole argument think—there are only spatiotemporal relations amongst material objects. In each case, Ladyman claims, ontic structural realism is called for because it can dissolve the question of which of the two viewpoints to adopt:

> Even if we are able to decide on a canonical formulation of our theory, there is a further problem of metaphysical underdetermination… In the case of individuality, it has been shown… that electrons may be interpreted either as individuals or as non-individuals. We need to recognise the failure of our best theories to determine even the most fundamental ontological characteristic of the purported entities they feature. It is an *ersatz* form of realism that recommends belief in the existence of entities that have such ambiguous metaphysical status. What is required is a shift to a different ontological basis altogether, one for which questions of individuality simply do not arise. (Ladyman, 1998, 419–20)

The argument here is parallel to that considered in section 4.4: individuals should be rejected because they lead to the existence of certain allegedly repellent questions—in this case, according to Ladyman, repellent because they aren't answered by the scientific theories in question.

As with the first argument for structural realism, I'll say only a little bit by way of critique; our focus is primarily on the position itself. First, as mentioned in section 4.4, any general prohibition of unanswerable questions threatens to overreach. Second, Ladyman's form of the argument sets us on a quixotic quest for a metaphysical language of science in which one simply can't formulate scientifically unanswerable questions. It seems unlikely that the goal is even achievable, given the almost perverse talent we philosophers have for raising devilishly difficult questions using any vocabulary with which we're supplied (how do I know I'm not dreaming?). Why think that an imagined structuralist reconstrual of physics—if it were ever constructed—would be immune? And even if it could be achieved, it might well come at a price: the theories stated in such a language might score worse on the theoretical virtues. (A "theory" consisting of a mere list of observational consequences is perhaps metaphysics-resistant in the desired sense, but is insufficiently explanatory, which is what leads us into the realm of the unobservable in the first place.) Perhaps the generation of unanswerable questions is the inevitable result of adopting the vocabulary needed to give an explanatory theory of the world. (We will return to this theme.)

Also, why would adopting ontological structural realism alleviate metaphysical underdetermination? Adding ontological structural realism to the mix just adds another metaphysical theory, alongside the entity-based ones, which is consistent with the science in question; and we still lack purely scientific means to resolve the question of which theory is correct.[10] Ontic structural realism, conceived as a distinctive metaphysics but supported by the undetermination argument, is an unstable position.

In a nutshell, the argument from metaphysical underdetermination is weak because it assumes that metaphysical underdetermination is avoidable and objectionable. A structural realist might concede this, regroup, and offer a first-order variant of the argument, claiming that structural realism is the best metaphysical account of the scientific theory in question. For instance, instead of touting structural realism as a way to dissolve the question of substantivalism versus relationalism, one might instead claim that it best solves the problems that these traditional views were responding to. One might, for instance, buy the hole argument against substantivalism while rejecting standard relationalism (for instance because of the modal facts it appears to require). Or one might claim that structural realism is the best account of individuality in quantum mechanics.[11]

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[10] See Pooley (2006, p. 91); Saatsi (2010, p. 262).

[11] Note, though, that the latter argument needs to establish the superiority of a structural re-

The two arguments for structural realism we have considered differ strikingly in what they support. Only the argument from metaphysical underdetermination (and its first-order variant) directly targets individuals. The argument from the pessimistic metainduction doesn't *seem* to target an individuals-based metaphysics at all. Rather, given the way it's usually presented anyway, it seems to target monadic properties, and advocates dispensing with them in favor of relations, which leaves open that the relations be orthodox—instantiated by individuals.[12] The kinds of past theory changes that undermine our confidence in current theory involve the replacement of properties proposed, or else the replacement of laws governing those properties; but all the theories in question, as usually understood, are based on individuals. Thus one would expect the proposed structural replacement of current theory to still be stated in terms of individuals, and simply replace talk of those individuals' properties with some sort of relational description. It's hard to be sure about this, however, since the relevant notion of sameness of structure is never spelled out very clearly.

Conversely, the argument from undetermination does not call in the least for replacing properties with relations. What it calls for is understanding the attribution of properties and relations both as somehow not involving individuals.

Thus it is perplexing that the literature on structural realism treats that position as a single unified position, capable of being supported in different ways. The situation seems rather to involve two completely distinct arguments leading to two completely distinct conclusions. (Not that the conclusions are incompatible; one could both replace properties with relations and dispense with individuals in various ways.)

4.6 Dasgupta against individuals

The strongest argument for a position in the vicinity of structural realism, in my view, is that of Shamik Dasgupta, according to which we should dispense with individuals for the same reason that absolute velocities should be purged from Newtonian gravitational theory: they are "physically redundant and empirically

alist account of quantum mechanics over *all* individuals-theoretic accounts, including accounts that dispense with particles but retain points of spacetime or configuration space bearing field values.

[12] We'll discuss below the suggestion that ontic structural realism just is the view that individuals have no monadic properties.

undetectable" (2009, (p. 40)).

The consensus in the philosophy of physics has been that it's best to eliminate absolute velocities from Newtonian gravitational theory; if we had accepted NGT as a correct fundamental theory, we should have adopted a conception of the structure of spacetime in which there is no intrinsic fact of the matter about which objects are absolutely at rest, and hence no intrinsic facts about absolute velocities, only intrinsic facts about bodies' velocities relative to one another. Instead of combining NGT with Newtonian spacetime, in which absolute velocities are well-defined, it's thought that it should instead be combined with Galilean ("neo-Newtonian") spacetime, in which only absolute accelerations, not absolute velocities, are intrinsically well-defined.

Why should absolute velocities be purged from NGT? Dasgupta mentions two reasons: they're physically redundant, and they're empirically undetectable. Each reason is based on the following observation. Suppose the universe is configured in a certain way at a time t_0. Given the laws of NGT, a certain series of configurations at later times results. Now, if the universe at t_0 had instead been given a "velocity boost"—if a certain constant velocity were added to each thing's actual velocity at that time—then the later configurations would have been correspondingly different in terms of absolute velocities. But this is the only difference there would be. Adding a velocity boost at t_0 would not affect the masses, or inter-particle distances, or relative velocities, or anything else we can detect, at later times. So, it would seem, we have no way to detect absolute velocities, since any experiment we could perform would yield the same result whether or not the universe initially received the velocity boost. This is the sense in which absolute velocities are empirically undetectable. Moreover, since the masses, inter-particle distances, relative velocities, and so forth, evolve in the same way regardless of the initial velocity boost, the absolute velocities of particles seem to be playing no role in the laws of nature. This is the sense in which absolute velocities are physically redundant.

According to Dasgupta, individuals are likewise empirically undetectable and physically redundant. Suppose that the universe is given a permutation rather than a velocity boost at t_0; two individuals exchange their qualitative roles. This of course doesn't have any effect on the masses, inter-particle distances, relative velocities, and so forth, at t_0; and given the laws in *any* physical theory, it won't have any effect on such matters at any later time either. The reason is that physical laws are general. The law of gravitation, for example, says that *any* two entities that are separated by a certain distance and have certain masses are subject to a certain gravitational force. It doesn't say: if particular particles

Joe and Frank are separated by a certain distance then they'll be subject to a certain gravitational force, whereas certain particular duplicate particles Callie and Iola in duplicate circumstances will not be thus subject.[13] Thus the initial permutation will result in no differences other than permutational differences at later times, just as the velocity boost affected nothing other than future absolute velocities. Now, we cannot directly observe individuals' identities (just as we can't directly observe absolute velocities). And so, since the initial permutation would have no other observational effects, Dasgupta concludes that individuals are empirically undetectable. Moreover, since the laws generate the same evolution of qualitatively defined states, regardless of the initial permutation, Dasgupta concludes that individuals are physically redundant.

So: according to Dasgupta, the same reasoning that led us to reject absolute velocities in NGT should lead us to reject individuals as well—to adopt a fundamental metaphysics in which there are no differences corresponding to a permutation of individuals.

4.7 Mathematical structuralism

A certain sort of mathematical structuralism can also be viewed as being opposed to individuals: think "mathematical objects are just positions in structures". (Structural realists often view mathematical structuralists as kindred spirits.)

Mathematical structuralism begins with the thought that *all that matters* to mathematics is structure. Consider three issues in the philosophy of mathematics.

First, Benacerraf's (1965) famous problem. Arithmetic can be "reduced to set theory" in the sense that we can provide definitions of the key arithmetic concepts under which the truths of arithmetic come out true. There are, for

[13] Given supervenience of observation/perception on what the fundamental theory says, we'll get at most that worlds that differ by a permutation of individuals will contain analogous individuals experiencing the same things. We might not get the same individuals experiencing the same things, since they may have been permuted. Maybe the idea is that the world is a whole is observationally indistinguishable in the sense that if you were outside the world you'd see (hear, etc.) the same stuff.

example, the von Neumann definitions:

$$\begin{aligned} 0 &=_{\mathrm{df}} \varnothing \\ s(n) &=_{\mathrm{df}} n \cup \{n\} \\ Nn &=_{\mathrm{df}} \forall x((0 \in x \wedge x \text{ is closed under } s) \to n \in x) \end{aligned}$$

But other definitions "work" as well; Zermelo, for example, defined $s(n)$ as $\{n\}$. These definitions generate different sequences of "numbers":

$$\underset{0}{\varnothing}, \underset{1}{\{\varnothing\}}, \underbrace{\{\varnothing,\{\varnothing\}\}}_{2}, \underbrace{\{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\}\}\}}_{3}, \ldots \qquad \text{(von Neumann numbers)}$$

$$\underset{0}{\varnothing}, \underset{1}{\{\varnothing\}}, \underset{2}{\{\{\varnothing\}\}}, \underset{3}{\{\{\{\varnothing\}\}\}}, \ldots \qquad \text{(Zermelo numbers)}$$

The question then, is this. The ability to reduce numbers to sets, one would have thought, shows that numbers are sets. But if numbers are sets, then which set is the number 2? Is it von Neumann's 2: $\{\varnothing,\{\varnothing\}\}$? Is it Zermelo's 2: $\{\{\varnothing\}\}$? It can't be both, since they're distinct from each other. It can't be one rather than the other; that would be arbitrary. So it seems that we must say that it's neither. So numbers aren't sets after all.

Second, the Caesar problem. The problem, as originally raised by Frege, was particular to his conception of numbers, but it has a more general application. Is the number 3 identical to Julius Caesar? It seems that mathematics doesn't settle the question. Mathematics settles questions like whether $3 = 1$, because such questions pertain to the structure of the natural numbers. But whether $3 =$ Caesar isn't about the structure of the natural numbers. Mathematical practice doesn't care whether 3 is identical to or distinct from Caesar.

The third problem is similar: is the natural number two, 2_N, identical to the rational number two, $2_{\mathbb{R}}$? Again, this isn't a question that normal mathematics cares about.

The problem in each case is the same. All that arithmetic "cares about" is that it's dealing with an "ω-sequence"—some entities together with a relation ordering them that look like this:

$$\bullet \quad \bullet \quad \bullet \quad \cdots$$

It doesn't matter what those entities are, and it doesn't matter which relation over them is chosen, provided it has the right form.

There are responses to these problems that are broadly structuralist in spirit but don't demand a deeply structuralist metaphysics. For example, one familiar, and conservative, response to Benacerraf is that ordinary number words are semantically indeterminate over various sets. The von Neumman reduction yields one acceptable way of making arithmetical language determinate, the Zermelo reduction yields another, but neither is determinately correct; it simply isn't semantically settled which of these reductions gives the meanings of arithmetical language. Though it's not mandatory, the view can be combined with the supervaluational account of truth in indeterminate languages, with pleasing results. "$2 = s(1)$" and other normal mathematical statements come out true (since they come out true under any acceptable reduction), but "2 has two members" and "3 = Caesar" come out indeterminate since they're true on some reductions and not on others. Such a view is fine as far as it goes, but it is hard to see how it could be extended to set theory itself. Thus structuralist concerns about set theory itself are left unaddressed; for all the response is concerned, set theory is about a distinctive range of entities, about a distinguished relation $\in$ over those entities, and although set theory provides no answer as to whether $\varnothing$ is identical to Julius Caesar, there still must be an answer to that question, and so on.

Another response that doesn't demand a deeply structuralist metaphysics is to say that number words don't really function as singular terms, numerical quantifiers aren't really quantifiers, and so on. One version of this is modal structuralism (Hellman, 1989). Here, we paraphrase an arithmetic statement $A(N, 0, s)$ along these lines:

$$\Box\forall X\forall y\forall f((X,y,f \text{ satisfy the axioms of arithmetic}) \to A(X,y,f))$$

But what's relevant for our purposes here—what can be considered a structuralist view about individuals in the current sense—is "non-eliminativist" structuralism, according to which there exist certain distinctive sorts of entities, mathematical structures, and that mathematical objects are "just positions" (in some sense) in these structures. Øystein (Linnebo, 2008, p. 60) describes the view as follows:

> [Mathematical objects] are really just positions in abstract mathematical structures. The natural number 2, for instance, is just the second (or on some approaches, the third) position in the abstract structure instantiated by all systems of objects satisfying the second-order Dedekind-Peano axioms.

Its defenders include Michael Resnik (1981) and Stewart Shapiro (1997).

Now, on the face of it, this view doesn't appear to solve any of the problems at all. The view seems to say that there really are these things, structures; there really are such things as *positions* in structures; and these structures and the positions they contain are what mathematics is about. But now consider the third position in the natural-number structure. Is it Julius Caesar? Is it a set? is it identical to any of the positions in the rational-number structure? These questions seem perfectly well-formed, and ought to have answers, if we take the talk of structures and positions in them at face value. Yet the structuralist concerns about the more face-value construal of mathematical objects were precisely that these sorts of questions are illegitimate: they're not answered by standard mathematics, and they shouldn't be regarded as being answered by anything external to standard mathematics.

Relatedly, a straightforward platonist outlook on mathematical entities can be regarded as implying that "structures" and positions in them really exist. Suppose you believe in a *sui generis* set of natural numbers, with a sui generis successor relation. It's natural to describe this as the natural numbers structure, and the individual natural numbers as positions in this structure! So what's distinctive about structuralism; what would it mean to say that numbers and other mathematical entities are "just" positions in structures? These questions need to be answered by a distinctive structuralist metaphysics.

4.8 Antihaecceitism

Let's turn now from the arguments to the views themselves, to an investigation of what it would mean to reject individuals, or to treat individuals as "just positions" in patterns or structures, or to otherwise privilege structures over the individuals in those structures.

Let's begin with a quite conservative view that barely counts at all as rejecting individuals: antihaecceitism. Antihaecceitism is a modal thesis, according to which the nonqualitative globally supervenes on the qualitative—possible worlds that are alike qualitatively are alike simpliciter.

Antihaecceitism has been perceived to be useful in cases where the problem to be addressed has been regarded as modal. Consider, for instance, the debate over substantivalism: the Leibnizian "shift" arguments and the more recent hole argument. In the case of the latter, for instance, the argument is that substantivalism implies indeterminism. Choose a bounded region—a "hole—

somewhere in spacetime; choose a diffeomorphism over the points of all of spacetime that maps each point outside the hole to itself, but (smoothly) maps the points inside the hole to slightly displaced ones inside the hole; and consider a description of reality that's just like actuality except that all features of matter and spacetime—including those of the metric field—that are possessed at a given point in the first description are in the second description possessed by the image of the point under the diffeomorphism. Given the diffeomorphism-invariance of the laws of general relativity, since the first world obeys the laws of general relativity, so does the second world; but since the worlds are identical in all times before the hole (before some chosen Cauchy surface that's before the hole), determinism fails. Antihaeccitism to the rescue: since the two worlds are qualitatively alike, they are identical given antihaecceitism, and so the threat to determinism vanishes.[14]

As we saw, it is common amongst the postmodalists to insist that modal phenomena have some underlying basis—to regard modality as a kind of epiphenomenon. Given this viewpoint, the antihaecceitist's response to the hole argument, for instance, is unsatisfying. The identification of qualitatively indistinguishable worlds ought to hold *because of* some fact about the contents of those worlds; it should not be a "bare necessity", in Dasgupta's terminology.[15] So let us turn to nonmodal ways of articulating structuralism about individuals.

4.9 Radical and moderate anti-individualism

The most radical way to reject individuals would be to reject the entire predicate-logic apparatus of reference—including quantification as well as naming—and to reject anything that's structurally similar as well. Reality simply does not have Turner's pegboard structure, on this view. Such a rejection brings a particularly heavy explanatory burden: how to describe the world without quantifying? (For example, the view itself cannot be stated as the claim that "there exists nothing at all", since that would require embracing quantification, "for no x does x exist".)

A more moderate form accepts the conceptual scheme of predicate logic, and thus accepts talk of "entities", broadly construed—an entity is anything at all; for all x, x is an entity—but rejects the existence of certain sorts of

[14]Notes…(Pooley, Brighouse, Butterfield); cf. "sophisticated substantivalism".

[15]Dasgupta also argues against the modal response, by arguing that it leaves the substantivalist without a defense against a similar arguments, namely his own.

entities, the "individuals" (and perhaps also certain concepts that are distinctive of individuals).

Consider, for example, the old bundle theory. The bundle theorist rejects particulars.[16] There is no such thing as a particular that instantiates properties P, Q, and R, according to the bundle theorist; there is just the bundle $P+Q+R$. By quantifying over and referring to universals (and bundles of universals) the bundle theorist embraces the conceptual apparatus of predicate logic, and thus counts as a moderate. She embraces entities in the broadest sense, but rejects a subclass of entities: the particulars. (She also rejects certain concepts that are distinctive of particulars, such as instantiation, that relation that holds between a putative particular and the universals that it "has".)

As examples in the recent literature of moderate opposition to individuals, there are McDaniel and Paul who defend the bundle theory, Dasgupta (2009, 2015), who embraces an ontology of universals, and the view discussed in Turner (2016), which is based on an ontology of facts. It's harder to find clear cases of radical opposition to individuals. Randall Dipert (1997), for instance, is a pleasingly wild manifesto opposing predicate logic as a foundation for metaphysics, but does not provide a clear positive alternative. And as we will see in the next section, the same is true for much of the literature on structural realism.

Other positions beyond our moderate and radical ones are possible. One might, for instance, embrace a mere part of, or a variant of, the conceptual scheme of predicate logic. One might, for instance, accept the quantified sentences of predicate logic while rejecting the sentences with proper names; this position has been recently discussed under the heading of "quantifier generalism".[17] Or one might replace the "thing quantifiers" of standard predicate logic with allegedly different "mass quantifiers".[18]

[16]There is a question of whether the bundle theorist should really be regarded as rejecting particulars, or instead accepting them and identifying them with bundles of universals. Depending on the details, concepts of ground or fundamentality might be useful here; the bundle theorist might say, for instance, that facts about particulars are grounded in facts that mention only universals. But notice that if particulars are regarded as being identical to bundles of universals, then there's a sense in which facts that mention bundles of universals *don't* "mention only universals", but also mention particulars. What I think is more at the core of the bundle theory is the rejection of fundamental concepts that are distinctive of particulars, such as instantiation.

[17]**Note to Dasgupta.

[18]Ned Markosian (2015) defends a distinction between stuff and things, but does not reject the standard quantifiers.

4.10 Eliminativist structural realism

Let us next consider what Stathis Psillos (2001) has called "eliminative structural realism". This position results from taking ontic structural realists at face value, as simply rejecting the existence of individuals. Objects do not exist; only structure exists. In some sense. Here are some representative quotations:

Ladyman (1998, p. 420):

> What is required is a shift to a different ontological basis altogether, one for which questions of individuality simply do not arise. Perhaps we should view the individuals and nonindividuals packages, like particle and field pictures, as different representations of the same structure. There is an analogy here with the debate about substantivalism in general relativity. Recently it has been suggested that this issue also calls for a different fundamental ontology within which to assess the reality of spacetime. Robert DiSalle (1994) has suggested that the structure of spacetime be accepted as existent without being supervenient on the existence of spacetime points. This is a restatement of the position developed by Stein in his famous exchange with Grünbaum, according to which spacetime is neither a substance, nor a set of relations between substances, but a structure in its own right.
>
> So we should seek to elaborate structural realism in such a way that it can diffuse the problems of traditional realism, with respect to both theory change and underdetermination. This means taking structure to be primitive and ontologically subsistent.

French (2006, p. 176):

> However, the comparison with mathematical structuralism is misleading. The quantum structure, say, does not exist independently of any exemplifying concrete system, as in the *ante rem* case, it *is* the concrete system! But this is not to say that such a structure is simply *in re*, because the ontic structural realist does not—or at least should not—accept that the system, composed of objects and relations, is ontically prior to the structure. Indeed, the central claim of OSR is that it is the structure that is both (ultimately) ontically prior and also concrete.

French and Ladyman (2003, p. 37):

> However, a realist alternative can be constructed. The locus of this metaphysical underdetermination is the notion of an object so one way of

> avoiding it would be to reconceptualise this notion entirely in structural terms. The metaphysical packages of individuality and non-individuality would then be viewed in a similar way to that of particle and field in QFT, namely as two different (metaphysical) representations of the same structure. One way of explicating the relevant structure in mathematical terms is through group theory and Castellani, for example, has begun to explore the ontological representation of the fundamental objects of physics in terms of sets of group-theoretic invariants by Weyl, Wigner, Piron, Jauch and others (Castellani 1998). This would leave no unknowable objects lurking in the shadows, as it were and it would retain a sense of objectivity understood structurally, of course…

French and Ladyman (2003, p. 41):

> The second point pertains to the above reversal of the relationship between structures and objects. How—it might be asked—can the former be regarded as primary and in some sense prior to the latter, when structures—understood as a system of relations—can only be defined in the first place in terms of objects—the relata? If the structural realist cannot answer this question, then the whole metaphysical project threatens to come undone…
>
> This question forms the kernel of an objection to the ontic form of SR which has been voiced to us by Redhead (in private discussion): If structure is understood in relational terms—as it typically is—then there needs to be relata and the latter, it seems, cannot be relational themselves. In other words, the question is, how can you have structure without (non-structural) objects? Here the structuralist finds herself hamstrung by the descriptive inadequacies of modern logic and set theory which retains the classical framework of individual objects represented by variables and which are the subject of predication or membership respectively (cf. Zahar (1994)). In lieu of a more appropriate framework for structuralist metaphysics, one has to resort to a kind of "spatchcock" approach, treating the logical variables and constants as mere placeholders which allow us to define and describe the relevant relations which bear all the ontological weight.

Ladyman and Ross (2007, p. 130):

> Ontic Structural Realism (OSR) is the view that the world has an objective modal structure that is ontologically fundamental, in the sense of not supervening on the intrinsic properties of a set of individuals. According

> to OSR, even the identity and individuality of objects depends on the relational structure of the world. Hence, a first approximation to our metaphysics is: "There are no things. Structure is all there is."

These quotations all suggest that reality is, in some sense, nothing but a qualitative structure, a network of relations with no objects standing in these relations.[19] Now, the first and most flat-footed objection to this statement of the position is that relations without relata (or properties without instantiators) are incoherent. We are told to subtract the particular objects from the grid of relations, leaving only the pattern behind, like the Cheshire Cat's smile; but as Redhead is reported by French and Ladyman as objecting, this would seem to make no sense. Many have made similar complaints.[20]

This complaint may be perceived as arising from metaphysical conservatism, an unwillingness to "think outside the box" and reimagine metaphysical categories such as structure and relation, or perhaps as blind reliance on "intuition". But either reaction would be profoundly misguided. The complaint is just the insistence that some metaphysical account be clearly specified.

What are the basic notions? What are the proposed rules governing those notions? And how can those notions then be used in a foundational account of scientific theories?[21] Standard predicate logic is the usual home for talk about relations, and gives clear answers to these questions. You can't just continue as if you accepted this framework—by speaking of relations—but subtract the entities and hope for the best. Individuals are too embedded within the standard framework; predicate logic provides no sentences about relations that don't also concern individuals. You need to properly specify a replacement framework, some replacement inventory of basic notions, rules governing those notions, and methods for using those notions in foundational contexts.

As Frege and Russell and other pioneers in the foundations of mathematics understood, great care is needed to develop the most basic framework for foundational work. Predicate logic isn't just a mindless projection of our linguistic/conceptual scheme. It was developed, with great labor, in a very unforgiving area, the foundations of mathematics, where errors were bound

[19] Recall the point made earlier, that if the position were motivated solely by the argument from underdetermination, there would be no need to regard the "structures" in question as consisting solely as involving relations; they could also involve properties. But in fact, the structural realists have tended to take the additional step of excluding properties.

[20] See for instance Greaves (2011); Pooley (2006); Psillos (2006).

[21] See Dasgupta (2011, pp. 131–4), who complains about the failure of structural realists to answer such questions, and Dorr (2010*b*).

to (and did) have huge consequences. It took a long time to reach the modern viewpoint.

Sure, it *might* be that this approach involves some illicit projection of our conceptual scheme. Rethinking everything from the ground up—great plan. But it needs to be done with the care of the Fregean tradition in logic: with clear choices made about vocabulary and theory, and a demonstration that the new proposed framework is adequate to the foundations of mathematics and science.

From a postmodal point of view, the demand for clear choices of this sort will be accompanied by a corresponding demand that some particular metaphysics of fundamental reality be given, as articulated with the tools of choice. What is reality ultimately like, according to the eliminative structural realist?[22] The friend of concept-fundamentality, for instance, will ask: if we cannot describe structures, in fundamental terms, as involving the instantiation of relations by objects, then what fundamental concepts can be used to describe structures? No answer whatsoever is given to this question in passages like these (French and Ladyman, 2003, p. 41):

> In lieu of a more appropriate framework for structuralist metaphysics, one has to resort to a kind of "spatchcock" approach, treating the logical variables and constants as mere placeholders which allow us to define and describe the relevant relations which bear all the ontological weight.

It's fine to say that a certain vocabulary is second-rate, and imperfectly represents the truths that bear all the metaphysical weight. But one must say exactly what those weight-bearing truths are, and exactly how the second-rate vocabulary answers them. Similarly, many structural realists emphasize conditions under which theories say the same thing, by structuralist lights, without giving a clear structuralist metaphysics of the shared content of the theories.

Now, one might reject the demand for a metaphysics, thus construed; a structural realist might say that there is no need to say "what reality is ultimately like". In chapter 5 we will consider a view that rejects such a demand in general; and in a way, this view strikes me as the best fit for structural realism. But for now, let us continue to construe structural realists as accepting the demand to provide a metaphysical account of ultimate reality. What, then, might that account be?

[22] Compare French's (2014) discussion of "Chakravartty's Challenge".

One might have expected ontic structural realists to defend some form of the "radical" rejection of individuals—the rejection of the ideology of predicate logic altogether. Although some informal commentary suggests such a position, this radical rejection is just whistling Dixie. No one has even begun to articulate a serious utterly quantifier-free account of fundamental reality. I'll be instead considering more conservative attempts.

4.11 Bundle theory

Talk of relations without relata naturally suggests the view that only relations exist. A number of views of this sort are possible, but it's best to start with the bundle theory—the most straightforward "properties and relations only" metaphysics.[23]

The usual bundle theory says that what we normally think of as individuals are just bundles of properties. (We'll discuss relations shortly.) Let's understand "bundling" as mereological summation (obeying the usual laws). But not just any mereological sum counts as a bundle: the sum gold+mountain shouldn't count as a bundle since there may be no golden mountain. And even if there is a gold mountain, that sum might be disqualified because it's incomplete—a gold mountain must have some particular mass and shape.

Many bundle theorists deal with these issues by employing a primitive plural predicate 'compresent'. We can then define a bundle as the sum of some maximal plurality of compresent properties (i.e. the sum of some things that are compresent and are not properly among any other compresent things), and we can say that a bundle "possesses a property" iff that property is one of a maximal plurality whose sum is the bundle.[24]

Bundle theorists have traditionally divided over whether to identify individuals with bundles of "universals", or with bundles of "tropes". The concept of a universal is the familiar one: if two objects have exactly the same charge, then they share a single putative charge universal. A trope, on the other hand, is a

[23]Dorr (2010*b*) notes the fit between the bundle theory and structural realism, as well as noting how unpromising the Bundle theory is. See McDaniel (2001); Hawthorne and Sider (2002); Paul (2002) for recent discussions of the bundle theory.

[24]One might say instead that a bundle "possesses" any property it contains as a part, but that could lead to trouble depending on how part-whole relations between bundles themselves are treated. (See Paul (2002, pp. ??) for discussion.) Since the bundle theorist's treatment of relations in general is problematic, let's set this issue to the side.

"particularized" property or relation: two objects with the same charge have two numerically distinct—but perfectly resembling, in some sense—charge tropes.[25]

Bundles of tropes behave in many ways like individuals. For instance, since tropes can be numerically distinct despite being exactly alike, bundles of tropes can be numerically distinct despite being exactly alike. Certain opponents of individuals will, therefore, regard a bundle theory based on tropes as being insufficiently radical. For instance, a bundle theory of tropes wouldn't give Dasgupta what he wants, since individual tropes would be unobservable and redundant in his sense: just as permuting the identities of individuals amongst their qualitative roles is a symmetry of the laws, so, permuting the identities of distinct duplicate tropes is a symmetry.

On the other hand, its admission of distinct exactly similar objects has been regarded as a powerful reason in favor of the tropes approach, as against the universals approach. Consider an electron in my pocket and an electron on the table. If each is identified with the bundle of its universals, and if as one might naturally think they have the same universals, then they will be wrongly identified with each other. Each can be unproblematically identified with the bundle of its tropes, on the other hand, because the tropes of one are distinct from the tropes of the other.

The next bit of the usual dialectic is familiar. Some bundle theorists bring in relational properties at this point. The electrons have different bundles after all: only the electron in my pocket has the property *being in a pocket* in its bundle. The usual counter is to consider the case where the electrons are alone in the world (Black's 1952 spheres), so that the relational property in question isn't available. Properties of location at points of substantival space might then be invoked: only one of the electrons has the property *located at point* p. But on the face of it, this gives up on the bundle theory in the case of points. For it presupposes that point p is distinct from point q; but points of space are presumably all intrinsically alike; and it doesn't seem possible to use universals of location to distinguish them; so how could the points be bundles?[26]

[25] Citations…

[26] The universals theorist might deny that the properties of location have the form *located at point* p, and say instead that they are primitive properties. In effect this would make the properties of location play the role of points of space (or spacetime): there would need to be a continuum of the properties (one for each point of space, or spacetime); the properties would need to have a geometry and hence would need to stand in spatiotemporal relations to one another, just as points would, etc. Since the locational properties now play the role of

But there is a more basic objection to the bundle theory that is under the surface in this familiar dialectic. What account is to be given of relations? On the face of it, the bundle theory can't accommodate relations at all, since a relation between two things doesn't "fit" into either of the bundles with which the things are identified.

The need for an account of relations is particularly pressing if the bundle theory is in service of a structural realism based exclusively on relations! But it's of course mandatory for any bundle theory.

The silence of bundle theorists on the matter of relations is really striking.[27] At best they tend to speak of relational properties, such as the property of being in a pocket or the property of being located at point p in the above attempts to deal with duplicate particulars. But the relational property *being in a pocket* is surely ultimately a matter of the relation *being in* (and the property *being a pocket*) (though we will consider the contrary viewpoint below), and we are still left wanting an account of relations.

It's really not ok to simply speak of relational properties, as many bundle theorists do, without giving any systematic account of this talk. If one employs names for properties like "the property of bearing R to something that is F", and makes essential use of these names in giving a reconstruction of ordinary and scientific discourse, one must give an account, in terms one takes to be fundamental, about how these names work exactly, what their inferential relations are, and so forth.

How, then, might a bundle theorist attempt to handle relations? A flat-footed approach would be to include a relation in x's bundle if and only if x bears that relation to something.[28] (Alternate approach one: …iff something bears the relation to x. Alternate approach two: iff something bears the relation to x or x bears the relation to something.) These suggestions are certainly intuitively off-base, and they're clearly nonstarters. Consider a situation that involves, as we would normally say, two things, each of which bears relation R to something. The flat-footed approach (and each of the alternates) describes the situation as involving two bundles, each including R; and there is no room to include more information. But in fact the situation remains underspecified—

individuals, one wonders whether they now raise the same concerns that motived the opponent of individuals in the first place.

[27] Paul (2012*a*, 251–5) is a notable exception, but I'm not sure I follow the details.

[28] More careful statement: "some monadic and polyadic universals count as compresent exactly in the scenarios that we would normally describe as containing some individual that has all the monadic universals and bears each of the relations to some individuals or other".

does each bear R to itself, for instance, or do the things bear R to each other? These subcases cannot be distinguished by the proposal.

A second approach gives up on trying to incorporate relations between bundles. Instead, it identifies individuals with bundles of compresent monadic universals only, but then ascribes relations to the bundles as if those bundles were particulars, by saying that the bundles thus construed instantiate relations. Such a view may seem like an unattractive hybrid, with its different treatment of properties and relations, the former with compresence and the latter with instantiation. But more importantly, it leaves unanswered the problem of duplicate individuals: an electron in my pocket and an electron on the table will be identified with the same bundle of monadic universals and hence with each other.

John Hawthorne (1995) once suggested a solution to the problem of Black's spheres: there really is only one bundle, but it's multiply located in the sense that it's, say, five feet from itself. (In addition to being zero feet from itself—the two are consistent on this view.) After all, Hawthorne points out, the believer in universals already accepts that universals—in the "immanent sense"—can be multiply located; Black's spheres are just more of the same, as are more mundane pairs of duplicate bundles.

Let's view Hawthorne's suggestion as a way to defend the second approach for incorporating relations.[29] We identify particulars with bundles of compresent monadic universals, we speak of those universals instantiating relations, and we insist that the impossibility of duplicate bundles is not a problem because their role can be taken over by a single multiply located bundle.

But their role cannot be thus taken over. The problem is that distinct facts involving the instantiation of relations by bundles cannot be "linked" on this approach.[30] Consider three duplicate particulars arranged on a line, with

[29]Hawthorne himself doesn't quite accept this:

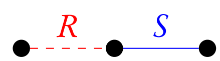
> The bundle theory thus holds that at the metaphysical groundfloor, there are universals standing in relations to each other. Some are clustered together ('compresent' in Russell's lingo), some are at other spatiotemporal relations to each other and to themselves. (p.193)

This suggests utilizing 'instantiation' alone, dispensing with compresence, and defining a bundle as a fusion of a maximal plurality of monadic universals that are spatiotemporally co-located (i.e. at zero distance from one another). (McDaniel (2001) also takes this approach to a bundle theory of tropes.) This is inadvisable, I think; it would preclude co-location by distinct particles; see Paul (2002, p. 580).

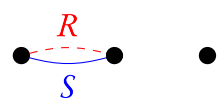
[30]See Hawthorne and Sider (2002) for a more extension discussion of the problem of linkage.

adjacent particulars separated by five feet:

Grant for the sake of argument that Hawthorne's approach can account for the geometry of the situation by saying that a certain bundle B is both five feet from itself and ten feet from itself.[31] But now consider two possibilities based on this setup, each involving the instantiation of a pair of symmetric relations, R and S. In the first case R holds between the left two objects—as we would ordinarily say, presupposing particulars—and S holds between the right two:



In the second case R and S each hold between the left two objects:



In each case, the facts Hawthorne provides are the same:

B is five feet from itself

B is ten feet from itself

B bears R to itself

B bears S to itself

Thus Hawthorne's account cannot distinguish these two—obviously distinct—possibilities. (Any structural realist will want to distinguish the cases; they correspond to clearly distinct relational structures.) What is missing is any way

[31] It's not clear that this should in fact be granted. Hawthorne's use of predicates of the form 'is x feet from' for arbitrary real numbers x must be underwritten by a representation and uniqueness theorem of some sort (chapter 3). But the usual theorems are based on axioms which will be falsified by Hawthorne's claim that a bundle can be both five feet from itself and zero feet from itself. For instance, in Mundy's (1987) theory, it is assumed that the determinate quantities are pairwise incompatible. It is an open question whether an attractive substitute theory of quantity consistent with Hawthorne's metaphysics can be developed. Similarly, it's an open question whether any attractive—relationalist, presumably—theory of geometry can be based on Hawthorne's metaphysics, since the axioms of familiar approaches will be falsified by the identification of duplicate bundles.

to "link" distinct facts involving the instantiation of relations. The distinction between the two possibilities, intuitively, has to do with the relationship between the third and fourth facts on this list. In the second possibility, there is a "case" of R holding (on the left of the diagram) and S holds *in that very same case*, whereas this is not so in the first possibility.

Particulars (or tropes) would provide the needed links. The third and fourth facts on the list would then be, in the first possibility:

Particular a bears R to particular b

Particular b bears S to particular c

whereas in the second possibility they would be:

Particular a bears R to particular b

Particular a bears S to particular b

The linkage of these facts by particulars recurring in them is what does the work: the different possibilities result from different patterns in which the linkage occurs. The absence of such linkage dooms the Hawthornian bundle theory, since it cannot account for the difference between the two possibilities, or myriad other such pairs.[32]

It should be clear that the problem isn't limited to isolated examples. The Hawthornian bundle theory has no means to represent *any* links between distinct attributions of relations between duplicate particulars. Insofar as our world consists, ultimately, of massive numbers of duplicate particulars standing in a network of relations, the collapse of possibilities will be widespread.

The problem here is not merely one of mismatch with "modal intuition", since the collapse of possibilities will make hash of the laws of nature. Assuming the possibilities are distinct, the first possibility might give rise, via the dynamical laws, to some outcome (or chance distribution) O_1, whereas the second might give rise to some outcome O_2. So if the possibilities are collapsed, the same laws could not hold. At best there could be weaker, disjunctive laws saying that the collapsed possibility gives rise either to outcome O_1 or outcome O_2. If this sort of collapsing is widespread, the "laws" would become so disjunctive and weak as to not deserve the name.

We have been discussing the problems that a universals-based bundle theory has with accommodating relations. Tropes-based bundle theories also face

[32]This problem was pointed out in a joint paper written by me and Hawthorne himself (Hawthorne and Sider, 2002).

the problem, since relational tropes, just like relational universals, don't seem to "fit" into any one bundle. The move to tropes, however, can help with the second approach to the problem. In Kris McDaniel's (2001) approach to the bundle theory, bundles are made of monadic tropes, and relations are then instantiated by these bundles, as in the second approach; but now that the bundles are made of tropes, the problem of possibility-collapse does not arise since there can be duplicate distinct bundles of tropes. Tropes, like particulars, provide the linkage between distinct relational facts.

But McDaniel's approach won't give many of the opponents of individuals what they want, because of the ways in which tropes behave like individuals. We've already noted that individual tropes would be unobservable and redundant in Dasgupta's sense. But further, notice that since McDaniel applies relations to bundles of monadic tropes, his approach won't work at all to eliminate entities in purely relational structures—i.e. networks of individuals where the individuals have no monadic properties at all.[33] (Nor will Hawthorne's.) Thus McDaniel's approach cannot be employed by structural realists who eliminate all properties in favor of relations. Nor can it be employed by structuralists about entities that seem to lack intrinsic properties, such as points of spacetime or mathematical entities.

4.12 The rejection of intrinsic properties

The slogan "individuals are just positions in a structure" is commonly used to articulate a certain sort of opposition to individuals, especially by structuralists of various sorts, including structural realists and mathematical structuralists. One way to articulate the slogan is this: begin with a structure—some objects having properties and standing in relations—and now delete the *properties* leaving only the relations.

This view is a somewhat natural fit for structural realists, insofar as their guiding thought is that all we learn from science concerns relations. (It is thus better motivated by the argument from the pessimistic metainduction than the argument from metaphysical underdetermination.) It is also a somewhat natural fit for mathematical structuralists, who emphasize the importance of relations to mathematical practice, and the unimportance of properties.

[33] McDaniel is explicit about the assumption that each thing has at least one monadic property (pp. 271–2).

Michael Esfeld and Vincent Lam (2006) have in fact defended a view along these lines, and called it "moderate structural realism.[34] Even Ladyman and Don Ross say things that come close, such as the first bit of this statement: "there are objects in our metaphysics but they have been purged of their intrinsic natures, identity, and individuality, and they are not metaphysically fundamental" (2007, p. 131).[35] In the case of mathematical structuralism, Øystein Linnebo (2008) considers the idea that "mathematical objects are incomplete in the sense that they have no 'internal nature' and no non-structural properties" (p. 61), and quotes Michael Resnik:

> In mathematics, I claim, we do not have objects with an 'internal' composition arranged in structures, we have only structures. The objects of mathematics ... are structureless points or positions in structures. As positions in structures, they have no identity or features outside a structure. (Resnik, 1981, p. 530)

The view needs to be refined.[36] It cannot be understood as saying that objects lack properties in the "abundant" sense (Lewis, 1986, 59–69), since any object x would have the property of self-identity, the property of being such that $2+2=4$, and so forth.

A somewhat better formulation would be that objects have no intrinsic properties. Talk of internal or intrinsic natures is perhaps getting at this formulation. But this is wrong as well, because a thing's intrinsic nature—the way it is "in itself"—includes its negative nature—the way it isn't, in itself. So even objects that are, intuitively, stripped of their internal nature will have *some* intrinsic properties.[37] Argument 1: the intrinsic properties are surely closed under negation: for any intrinsic property P, any object has either P or the property $\sim P$, each of which is intrinsic. Maybe the argument's assumption that that there exists an intrinsic property P will be challenged? In that case, argument 2: a property concerning a thing's part-whole structure is clearly intrinsic; and anything has a part-whole structure, even if it's a "mereological atom", lacking all proper parts (as Resnick suggests is true of mathematical objects)—the property of being a mereological atom is intrinsic. What if the

[34] See also Esfeld (2003, 2004).

[35] In his SEP survey of structural realism (2014), Ladyman categorizes a number of views in the vicinity of "no properties" as versions of structural realism.

[36] The refinements avoid Linnebo's objections.

[37] Compare Bricker (1992).

existence of a part-whole relation is also denied? Well, the property of being self-identical is arguably intrinsic.

In any case, there is a better formulation that sidesteps all these issues about intrinsicality: that neither the objects in question nor their parts possess any fundamental (monadic) properties; at most, they and their parts instantiate fundamental (polyadic) relations.[38] Call entities of this sort "bare particulars".

I have no objections to the truth of this sort of view, in principle anyway. Indeed I suspect it's likely true of mathematical entities given a robust Platonism, and perhaps of points of space, or time, or other physical spaces. The main question is whether it gives opponents of individuals what they want. But first I should address the concern that there's something metaphysically objectionable about bare particulars.

Many of the structural realists seem to think there is. Even the advocates of the position regard it as metaphysically daring[39], and opponents argue that it's no good; one comes across worries about how bare particulars would be "individuated", whether "haecceities" would be needed to individuate them, worries about what "gives them their individuality", and so forth.[40] These concerns are misguided, I think.[41]

The concerns about individuation seem to presuppose something like the following picture. Reality is fundamentally undifferentiated, not divided into individuals. So if you want to say anything about individuals, you first have to individuate them, by specifying some way in which reality is to be carved up into individuals.

But it is entirely unclear what the initial undifferentiated picture is meant to be. It cannot be that of a universe of propertied regions of space or portions of matter[42] for that universe is a universe of individuals: points and/or regions of space, or portions of matter. It's a perfectly coherent picture that in such a

[38] French (2010, p. 100) makes roughly this suggestion.. A mathematical structuralism should probably add two additional claims: that mathematical objects don't have any proper parts, and that in any particular mathematical structure, the only relations in which the entities stand are the distinctive relations of that structure—the successor relation, perhaps, for the natural numbers.

[39] Cf. Ladyman on the standard view, esfeld

[40] See, for example, Chakravartty (2012, p. 197), and also—in a different part of the metaphysics of physics literature, but illustrating the same sensibility—French and Redhead (1988, p. 235).

[41] What I think is that many of the structural realists were brought up on too much 1980s UK metaphysics.

[42] As in JubienOMFR, say.

universe, there is no *further* privileged carving into the individuals of ordinary thought; indeed, this is the standard view of most "four-dimensionalists".[43] But this picture *begins* with an initial description of the universe in individuals-theoretic terms. The problem with articulating an undifferentiated, "pre-objectual" picture of the world is in fact very similar to the problem with articulating a coherent structuralism about individuals.

And once it's conceded that the fundamental description of reality is to be given in individual-theoretic terms, the concerns about individuation then evaporate. Someone who believes that individuals exist at the fundamental level holds that certain concepts of predicate logic are metaphysically fundamental. (Perhaps the quantifiers, perhaps names too, or names instead—there are subtleties.) Accepting these concepts at the fundamental level is quite natural, given their proven value in foundations of mathematics and science. And to someone who accepts this, the complaints about bare particulars will seem completely off-base. The statement that there exist certain objects standing in certain relations to other objects is, if the relations in question are fundamental, phrased in wholly fundamental terms; why then should the objects' existence need to be grounded in monadic properties?[44]

The felt need for haecceities also seems confused. Given the view that individuals-theoretic concepts (such as quantifiers and names) are fundamental, a purely relational description of the world, containing such sentences as "Rab", is perfectly acceptable. There's no need to add "haecceities", fundamental monadic predicates or properties A and B possessed by a and b that enable their existence. Why think that:

$$Rab, Aa, Bb$$

(or: "a bears R to b, a instantiates A, b instantiates B") is any better than

$$Rab$$

(or: "a bears R to b")?

Bare particulars are perfectly good metaphysics, and would give some structuralists some of what they want. It would give mathematical structuralists the ability to deny that the number zero has any nontrivial intrinsic properties; its only fundamental features are the relations it bears to the other natural

[43] References

[44] I talk more about this in Sider (2006).

numbers. But it wouldn't give them all of what they want. If the successor relation on the natural numbers is a fundamental relation, then "flipping" zero and one in that succession would count as a genuinely different scenario, even though it's structurally identical to our own. It flatly implies that the number 3 is *not* Caesar, since Caesar has proper parts. It leaves inter-system queries, such as whether $2 = \{\{\varnothing\}\}$, whether $2_{\mathbb{Q}} = 2_{\mathbb{R}}$, and so forth, as intelligible but unanswered. It would be coherent, for example, to the advocate of bare particulars to hold that there are two nonoverlapping "sui generis" structures, the rationals and the reals, the former structured by relations $<_{\mathbb{N}}, +_{\mathbb{Q}}, \cdot_{\mathbb{Q}}$, the latter structured by distinct relations $<_{\mathbb{R}}, +_{\mathbb{R}}, \cdot_{\mathbb{R}}$. Or she could accept just the structure of real numbers and regard talk of the rationals as picking out the appropriate substructure of the reals. Or she should accept only a structure corresponding to sets, and regard talk of other mathematical entities as picking out substructures of the set-theoretic hierarchy. Thus bare particulars do not deliver what the mathematical structuralist wanted, which is that there simply are no genuinely distinct possibilities of this sort. The problem is that according to the bare particulars approach, there *are* distinguished entities and distinguished relations that mathematics is about, even though those entities don't have "innards"—qualitatively speaking, there's nothing more to them than their structure.

Nor would bare particulars give Dasgupta what he wants. Of course, he wouldn't have wanted in the first place to treat particles with mass and charge as bare particulars, but the case worth discussing is spacetime. In a paper on the hole argument (2011), Dasgupta gives arguments for the case of spacetime that are similar to those he gives against individuals in general: permutations of points of spacetime are symmetries of the laws and hence, Dasgupta argues, show that particular points of space are unobservable and redundant. But treating points of space as bare particulars would do nothing to block the conclusion that permutations of points of space correspond to distinct fundamental possibilities.

Would bare particulars give structural realists what they want? Perhaps it does with the argument from the pessimistic metainduction, but not for any of the other arguments, as far as I can tell.

4.13 Structural realism, ground, and monism

The intuitive problem with the bare particulars approach is this. In articulating the slogan that objects are "just positions in structures", structuralists don't merely want to strip individuals of their monadic properties; they also want to strip of them of their very "identities" or "individuality", so that permuting the individuals is a distinction without a difference. This is brought out very clearly in two of the quotations considered in the previous section: Resnik says that positions in structures "have no *identity* or features outside a structure", and Ladyman and Ross speak of objects being "purged of their intrinsic natures, *identity, and individuality*" (my emphasis in each case).

Now, it's very hard to make any literal sense out of this. What would it mean to speak of "having identity" "in" or "outside" structures? What would it mean to purge objects of their "identity and individuality"? These things surely make no literal sense. At best, they yield more vague slogans that themselves need to be precisely articulated.

One way of articulating them is as follows. "Positions" in structures don't exist at all, since "positions" are just individuals and individuals don't exist, not fundamentally anyway. That is: a fundamental description of reality will not mention individuals at all. To be sure, fundamental reality's nature *does* underwrite our speaking, at the nonfundamental level, of individuals and their standing in relations—a structure. (Though when we're doing so, we are not speaking in a metaphysically perspicuously, since reality is ultimately individuals-free.) But when we speak in this nonfundamental way, we must not speak of there being merely permutational differences. For fundamental reality's nature does not underwrite such differences; descriptions that differ merely permutationally do not correspond to differences in fundamental reality.

If an individuals-free fundamental metaphysics could really be articulated, this position would be just fine. But what *is* the individuals-free description of fundamental reality? That's exactly the problem we've been struggling with in this chapter, so far without success.

However, there is another way of making the slogans precise. Instead of saying that fundamental reality lacks individuals, we could say instead that the facts about individuals in a structure are *grounded* in facts about the structure. And indeed, some structural realists and mathematical structuralists have made suggestions along these lines. E.g.:

> Each mathematical object is a place in a particular structure. There is

> thus a certain priority in the status of mathematical objects. The structure is prior to the mathematical objects it contains, just as any organization is prior to the offices that constitute it. The natural-number structure is prior to 2, just as "baseball defense" is prior to "shortstop" and "U.S. Government" is prior to "vice president." (Shapiro, 1997, p. 77)

It is thus natural to ask whether structural realism (and other forms of structuralism about individuals) should be understood in terms of ground.[45]

One line of thought from section 4.11 converges with this suggestion. There I assumed that talk of relational properties presupposed a prior account of relations. But this might be denied: it might be suggested that, on the contrary, facts about individuals standing in relations are to be grounded in properties possessed by larger objects containing those properties as parts. So again, we have arrived at the suggestion that the distinctive structuralist thesis is that the arrow of grounding runs from wholes to parts, rather than from parts to wholes.

The most extreme view of this sort is Jonathan Schaffer's (2010) "priority monism", according to which the arrows of grounding all originate from a single entity, the largest entity of all, the entire Cosmos. The objects of ordinary life and science, which are parts of the Cosmos, are posterior to the Cosmos itself; and facts about them hold in virtue of facts about the Cosmos.

Monism is in many ways a natural fit for many opponents of individuals. As we've seen, many structural realists and mathematical structuralists want to dispense with (in some sense) the nodes in structures and just speak of the structure itself. So perhaps what they should do is say that fundamental reality contains just one entity, a structure, the Cosmos, and claim that everything else derives from it. (They might even prefer what Schaffer calls "existence monism", according to which nothing at all exists other than the Cosmos.) So let's focus our discussion on monism. (What we say will apply to less extreme ways to understand structuralism in terms of ground.)

However, monism thus understood is unsatisfying for reasons similar to my concerns about the essence-theoretic formulation of nomic essentialism from section 2.3.[46] *How* does this grounding work? What in particular is it about the Cosmos, at the fundamental level, that enables it to ground sub-Cosmos facts? And how does this work in a systematic way?

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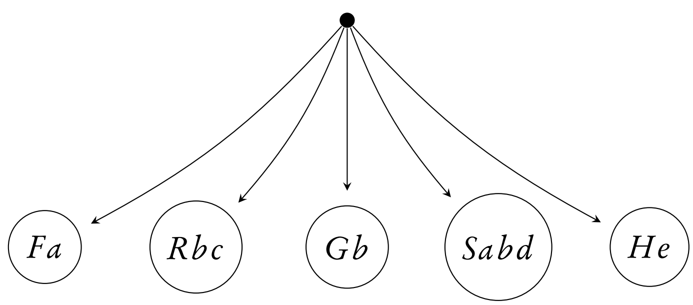
[45] Fine (1995, p. 270): "...holism, in one of its many versions, may be taken to be the doctrine that the parts of a whole can depend upon the whole itself." See, more recently, Linnebo (2008); McKenzie (2014).

[46] See Sider (2011, sections 8.5–8.6) for more on some of the following issues.

These questions are particularly pressing given the way Schaffer understands grounding: as a relation connecting entities, not facts. It's the Cosmos itself, not certain facts about the Cosmos, that grounds other entities (like me), facts about such entities (like the fact that I'm a human being), and so forth.

Without answers to these questions, monism becomes either underspecified or magical, depending on how grounding is understood. It becomes underspecified if the monist concedes that the Cosmos does have a fundamental nature which explains how it is able to ground all the sub-Cosmos facts. For then the monist would be conceding that an account of the fundamental nature of the Cosmos ought to be possible after all. Monism itself—the mere claim that the Cosmos grounds everything else—would therefore leave unanswered the crucial question of what it is that's monistic about fundamental reality.

Monism would become "magical" if the monist resists the suggestion that the Cosmos must have a fundamental nature that explains how it can ground everything else. The monist's picture would be as follows:



The dot at the top of the diagram represents the Cosmos; the arrows represent grounding, running from the Cosmos to the myriad facts of sub-Cosmos reality, represented in the diagram by circles containing sentences. What is magical is how the Cosmos, which is given no fundamental description at all, manages to ground all this immense complexity. The grounding arrows aren't enabled or underwritten by anything fundamental at all.

What is disturbing about this picture of grounding can be brought out by a further example. Suppose someone claimed that fundamental reality consisted exclusively of just a single electron, one in my left pocket, say. This claim is of course absurd; but what makes it absurd? It is, I think, the fact that the electron's states are insufficiently rich to give rise to all of the complexities of reality. The arrows of grounding are not "unmediated" or "magical", and cannot be hypothesized to be present in such a complex way unless the entities or facts from which they emerge are themselves correspondingly complex.

Now, the monist's claim that the Cosmos is doing all of the grounding might seem different. The Cosmos is a more complex entity than the electron, after all; it is the most complex entity of all. But none of this complexity is recognized at the fundamental level, according to the monist we are now considering. In order to bypass the concerns raised above about what the fundamental monistic concepts are, our monist has refused to admit the demand for an account of the fundamental nature of the Cosmos that underwrites all the grounding work it must do. The grounding arrows countenanced by this monist, therefore, are no less magical than those that would be needed to ground all facts in the electron in my pocket.[47]

In sections 1.6.2 and 1.6.3 a contrast was drawn between a conception of ground as a "super-added force" and a conception on which facts about ground are themselves grounded. A monist might reply that my complaints about magical ground are apt only if the latter conception is presupposed, and say that the grounding facts are unproblematic given the super-added force view.

But holding that these grounding facts are fundamental would amount to giving up on monism. For then, the fundamental facts would include a complete specification of the entire world, in sub-Cosmos terms! For any sub-Cosmos fact F, the monist would be accepting a fundamental fact that the Cosmos grounds F. (It might be held that only some of these facts are fundamental, but the point remains about those facts that are held to be fundamental.) As a result, the monist could no longer regard permutations of sub-Cosmos individuals over qualitative roles as inarticulable at the fundamental level. If "*the Cosmos grounds* $\phi(a,b)$" is a statement about fundamental reality, then we can construct the following statement using the same vocabulary that's also about fundamental reality "the Cosmos grounds $\phi(b,a)$".

Also the proposal would require denying something like the Purity principle: it would require fundamental facts to have non-fundamental facts—the facts being grounded—as constituents.

The hand against ground-theoretic monism should not be overplayed. It isn't objectionable on its own to claim that facts are grounded in a certain source without specifying a fundamental account of that source; one might simply be refraining from saying, on that occasion, what the source is. Similarly, as we saw in section 2.3, it isn't objectionable on its own to claim that certain

[47]If anything they are more magical, for at least the electron in my pocket is in various fundamental states, whereas the current monist denies there are any fundamental states of the Cosmos that play a role in explaining the arrows of grounding.

facts are of the essence (in Fine's sense) of a certain entity, without specifying the fundamental facts about the thing in question that mediate this essentialist claim. But sometimes such claims are objectionable, because sometimes a fundamental account is indeed called for. The use of monism to articulate structuralism about individuals is just such a case, since structuralism about individuals is (surely) meant to be an ultimate account of the nature of reality, and thus in this context a fundamental account must be given (if the possibility of such an account is acknowledged).

My main argument has been that ground-theoretic monism of Schaffer's sort is metaphysically underspecified. But in fact I think the situation is worse; I think there is reason to doubt that an attractive monistic account of the nature of the fundamental facts is available.

What would monism look like, if viewed through the lens of fundamentality? Intuitive idea: everything fundamental is at the Cosmos level. In terms of concept-fundamentality, it's natural to construe this as the rejection of fundamental concepts applying to proper parts of the Cosmos. Thus monists would reject fundamental properties or relations over particles, points of substantival spaces of any sort, parts of fields, and so forth.

What fundamental concepts would remain? Fundamental properties appropriate to possession by the entire cosmos, for one; but also one could admit higher-order properties and relations of such properties (and still higher-order properties, etc.). Thus what the monist has to work with at the fundamental level is a statespace, consisting of properties available to the cosmos, structured by higher-order properties and relations over those properties, and containing a distinguished point, the property that is in fact instantiated by the Cosmos.[48]

The problem with such a theory is that it will surely be highly complex. First, a great many fundamental properties comprising the points in the statespace will be needed. The pluralist can get by with a small number of fundamental properties and relations because the statespace for her is combinatorially generated as all the distributions of sub-Cosmos properties and relations over sub-Cosmos individuals; but for the monist, the properties of the Cosmos are fundamental. Second, it is doubtful that there will be simple laws, when stated in terms of the properties and relations recognized as fundamental by the monist. At the very least, there is no guarantee that the kinds of simple

[48] Were the higher-order properties and relations eliminated, the view would draw near to the "propositional nihilism" that Jason Turner formulates and criticizes convincingly, appropriately named the "bag of facts" metaphysics by Russell (2014).

laws with which we are familiar have simple monist analogs. For the undefined concepts in the simple laws that scientists have actually proposed apply to particles or points or other proper parts of the Cosmos. This is a banal point and should not be controversial: an undefined concept giving field values at points, for example, is the sort of concept that a monist (of the type currently under discussion) cannot recognize as fundamental, and hence cannot admit in a fundamental law.[49]

How might a monist resist these criticisms? My best guess is that the resistance will be to my presupposition that a fundamental account of the nature of the Cosmos would require specification of a simple set of Cosmos-level fundamental properties and relations, obeying simple laws. Perhaps it will be held that the Cosmos does have fundamental features that mediate the grounding arrows emanating from it (grounding is not super-added), but that no simple account needs to be given of these features. I find this response unsatisfying; surely we should demand a more forthcoming account of fundamental reality.[50] But at the very least, this discussion should again make clear the importance of general questions about the tools of metaphysics.

Might there be some ground-theoretic formulation of structuralism that's milder than monism? Linnebo (2008) discusses the idea that mathematical structuralism should be understood in terms of ontological dependence. He first considers the idea that every mathematical entity ontologically depends on every other in its structure, which he rejects on the grounds that ontological dependence cannot have cycles. But he goes on to consider the idea that all mathematical entities in a structure depend on that structure.

Linnebo's main idea seems to be the following. Suppose you already believe in set theory, and you're not a structuralist about it. Let R be a set of n-tuples. Then we can say that *in virtue of* R existing, there exist some further entities: a *structure* S and some *offices* o_i. The idea is that structures are obtained by abstraction from relations by this abstraction principle:

$$\overline{R} = \overline{R'} \text{ iff } R \cong R'$$

Here $\overline{R}$ is the structure obtained by abstraction from R under the relation $\cong$ of being isomorphic. And offices are given by abstraction from rigid relations (i.e., relations where the only isomorphism between the relation's field and itself is

[49]See Sider (2008*a*) for more on all this.

[50]It's interesting to compare this response to "quotienting out by hand", discussed in chapter 5.

the identity function) and members of their fields by this abstraction principle:

$$\tau(x,R)=\tau(x',R') \text{ iff } \exists f(f:R\cong R'\wedge f(x)=x')$$

The general idea is that abstraction principles generate objects that exist in virtue of the objects from which they're abstracted. So then, finally, we can say that in a kind of indirect way, the offices depend on the structures: there are some entities (R and x_i) such that *both* the offices $\tau(x_i,R)$ and the structure $\overline{R}$ depend on them. Linnebo calls this weak dependence (of the offices on the structures, and the structures on the offices), though it might be better called ontological common-cause.

It's hard to see why this gives mathematical structuralists what they want. It has the consequence that $2_N \neq 2_Q$ (since $\mathbb{N} \ncong \mathbb{Q}$). It also doesn't have any answer to the Caesar problem, since Caesar is not $\tau(x,R)$ for any x,R (the Caesar problem, after all, is in the first instance a problem for abstractionism). Notice also that this view just couldn't be used in a defense of universal structuralism, since we need some nonstructural mathematical entities to perform abstraction on. (Linnebo concedes this; his argument is that structuralism should be defended for only some mathematical entities.) Notice finally that this view also assumes that we understand what abstraction is. But many questions about what structuralism is supposed to be are being packed into the operation of abstraction.

But setting aside the details of Linnebo's discussion, is there some other interesting form of abstractionism that we could state using the notion of ontological dependence?

We need to decide how we're talking about dependence. Suppose first we do it Schaffer's way: we speak of one entity x as grounding another, y. Now, one view here would be analogous to Schaffer's priority monism: $2_{\mathbb{N}}$ is grounded in $\mathbb{N}$. What entity is $\mathbb{N}$, exactly? If the suggestion is analogous to monism, $\mathbb{N}$ something like the fusion of all the natural numbers.

But now, as before, we should ask: *how* does $\mathbb{N}$ ground various entities and facts? Depending on how the question is answered, we would return to several discussions we've had already. For instance, if the fundamental facts about the structures were said to be existentially quantified, we would again have a conflict with the claim that existentials are grounded in their instances. If the demand for an account of how the grounding works were resisted, the complaint would be like that in section 4.13: grounding has become objectionably magical. Other answers would simply lead away from (non-eliminativist) structuralism.

For instance, one might claim, in the spirit of Hellman, that statements about entities in structures are true in virtue of certain modal facts. And even if the claim were admitted as acceptable metaphysics, it isn't clear whether it would really give mathematical structuralists what they want. How exactly would it block cross-structure identities (for example)? What will it say about the Caesar problem? (Maybe it just says that $3 \neq$ Caesar since the latter isn't dependent on any structures; but how is that an advance from what the defender of bare particulars would say?) And how would it block the existence of duplicate structures?

4.14 Indeterminate cross-structure identities

We saw that one motivation for mathematical structuralism is to "block" questions like "is $2_N = 2_Q$?" "Is $3 =$ Caesar?" Some structuralists say things like "there's no fact of the matter" whether these identities hold. It's hard to see how such claims would on their own constitute any sort of structuralism, but perhaps they could supplement some of the views we've already considered, in order to give structuralists more of what they want.

A certain bad habit in the philosophy of language has often been remarked on: dumping everything that's ill-understood into the pragmatics bin. The following is also a bad habit, and for similar reasons: if you don't know how to answer a question, just say "no fact of the matter", and leave what that amounts to to the metaphysicians to clean up. There are hard and pressing questions about what "no fact of the matter" would amount to—especially at the fundamental level; that card should not be played lightly.

The clearest understanding of "no fact of the matter" involves semantic underspecification. There's no fact of the matter whether ϕ iff ϕ is true on some sharpenings and false on others. That would be helpful to mathematical structuralists if they adopted a limited structuralism but exempted set-theory, say. But we are looking for an alternative.

No-fact-of-the-matter is very tempting in certain sorts of fictionalist contexts. Is Buffy of the television series identical to Buffy of the first movie? To comic book Buffy? No fact of the matter, we want to say. (It's not just identities where we want to say "no fact of the matter". Does Holmes have a mole on his left shin? Again, NFoTM.)

This could be combined with a semantic underspecification approach to indeterminacy as follows. In a fundamental language, it's true to say that there are no fictional characters. But in such a language we can provide rules for speaking as if there are fictional characters. We can have rules like this:

> say 'there is a fictional character who does so-and-so' if some fiction contains sentences that entail 'someone does so-and-so'
>
> say 'fictional character C does so-and-so' if the dominant fiction containing the name C contains sentences that entail 'C does so-and-so'

Imagine putting forward enough rules along these lines so that in a large range of cases, it was clear what the rules require one to say "about fictional characters". We might say that we have then introduced a new language, and that people who speak that language speak truly when they say things like "there is a fictional character who…". (Acknowledging this wouldn't require that we ourselves—speakers of a fundamental language—admit fictional characters, since the speakers wouldn't mean the same thing by 'there is' as we do.)

But now, what of sentences about fictional characters that fall through the cracks? E.g., suppose that the Conan Doyle stories entail neither 'Holmes had a mole' nor 'Holmes did not have a mole'. What do the rules then prescribe?

They neither prescribe saying 'fictional character Sherlock Holmes has a mole' nor do they prescribe saying its negation. Indeed, it would be natural to include a further meta-rule instructing us to say "there's no fact of the matter whether fictional character Sherlock Holmes has a mole" in such a case.

We have, then, a model of how "no fact of the matter" sentences could be true in nonfundamental languages. Moreover, this story also fits in with an account of what "makes true" sentences about fictional characters.

A kind of fictionalist about mathematics could say similar things. Recall that Hellman recommends replacing $A(N,0,s)$ with

$$\Box\forall X\forall y\forall f((X,y,f \text{ satisfy the axioms of arithmetic})\rightarrow A(X,y,f)) \qquad (*)$$

We could imagine Hellman dressing up his theory in fictionalist garb: he could introduce a language in which $A(N,0,s)$ is *true* if (*) is true in a fundamental language. And then a "no fact of the matter" operator could be introduced, in cases where neither the sentence nor its negation was necessitated by the axioms of arithmetic. (e.g. $3=\text{Caesar}$).

Another model for construing indeterminacy as underspecificity comes from quantifier variance a la Hirsch (2002). According to the quantifier variantist, there are many equally good meanings for quantifiers. Which of these do we mean? We pick out one by talking in a certain way; whichever of those meanings fits our talk best is the one we mean. So when we adopt mathematical axioms, we select a meaning for the quantifiers (as well as the mathematical symbols) on which the axioms come out true. But there may be multiple quantifier meanings that agree on the axioms but differ on other sentences, such as "$3 = \text{Caesar}$". We would be free to adopt a convention as to whether this sentence is to come out true; this would be cutting down further on the candidate quantifier meanings fitting our linguistic stipulations. But before such a conventional decision, there would be semantic indeterminacy in our quantifiers, and hence our singular terms, and hence the sentence "$3 = \text{Caesar}$".

Benacerraf actually says things in this vicinity. He defends the idea that there's no fact of the matter whether $3 = \text{Caesar}$, saying:

> One might conclude that identity is systematically ambiguous, or else one might agree with Frege, that identity is unambiguous, always meaning sameness of object, but that (contra-Frege now) the notion of an *object* varies from theory to theory, category to category—and therefore that his mistake lay in failure to realize this fact. (Benacerraf, 1965, 66)

As we've seen so far, the ways for making sense of no-fact-of-the-matter all require mathematical language to be nonfundamental. But this is not what the structuralist wanted, I take it. What models of no-fact-of-the-matter are there on which mathematical entities fundamentally exist?

One would be a kind of maximalist view, a la Bricker (1992); Eklund (2006). Here the idea is that we believe in many, many abstract entities, each with their own distinctive natural properties and relations. Abundance of Platonism. This would result in a kind of semantic structuralism, since there would (or might, anyway) be no single entities we are referring to with the expressions of any bit of mathematics. (Even if the axioms were categorical, as in second-order Peano arithmetic, there might be duplicate structures.)

Another would invoke worldly indeterminacy. We might have a fundamental nonclassical logic of some sort, or a fundamental notion of determinacy. But all this strikes me as metaphysically extravagant, and not worth what it buys. (People don't generally respond to the underdetermination of theory by data in the philosophy of science by saying "no fact of the matter")

4.15 Weak discernibility

Some structural realists have emphasized the distinction between "weak" and "strong" discernibility.[51] Individuals are strongly discernible iff some monadic predicate applies to one but not the other; individuals are weakly discernible iff some binary predicate applies to the two in different patterns (e.g. Rxx but not Rxy, Rxy but not Ryx, etc.) It is claimed that structuralism demands that objects be "individuated" by their position in the qualitative grid, but that all pairs of objects being weakly discernible suffices for this.

The focus on weak discernibility seems to be misplaced, if the goal is a structural realist account of the fundamental nature of reality.[52] For even it's granted that distinct objects are always weakly discernible, this doesn't lead to any distinctive structuralist account of the nature of the fundamental facts. Objects that are merely weakly discernible can't be bundles of monadic universals, since then they'd be strongly discernible as well. Bundles of relational universals might seem good candidates for being merely weakly discernible, but this is no good without an account of relational universals. Dasgupta's account, to be discussed in a moment, requires no restriction to weak discernibility, nor do bare particulars. Weak discernibility seems like a metaphysical epiphenomenon, whose pursuit is encouraged by uncritical use of the language of individuation.

Simon Saunders points out the distinction between weak and strong discernibility, and then writes as if, in a group of entities that are pairwise weakly discernible, there is a sense in which the entities have somehow been reduced to the pattern of relations: "bodies can be identified by their relations to one another; then a particular body is no more than a particular pattern-position" (Saunders, 2003, p. 163). This claim is unjustified, for the reasons just explained: the claim that objects are weakly discernible doesn't suggest any particular reductive account of those objects. It certainly doesn't imply, for instance, that there's no difference between scenarios differing solely by a permutation (which one would expect if an individual is "no more than a particular pattern-position"). Saunders's discussion is driven by an analogy between the principle of the identity of indiscernibles and the principle that distinct objects are weakly discernible, which he regards as a sort of successor to the former principle given modern logic, i.e., logic with relations. But the former, if true, really would enable a distinctive metaphysics of individuals that would disallow

[51] See Saunders (2003) on that distinction.

[52] Dasgupta makes this point as well.

permutations: the bundle theory. (Though there would remain a question of the status of the monadic attributes bundled together, if some of them presupposed relations.) The latter just doesn't. Ultimately, Saunders seems to be thinking that if one can "individuate" objects, by which he means give a sufficient condition for their pairwise distinctness, this is a kind of accounting for what those individuals are in structuralist-friendly terms.

But perhaps weak discernibility is significant, not because it leads to a distinctive structural realist metaphysics of individuals, but rather because it leads to a distinctive view of identity—specifically, that identity can be defined and hence be regarded as nonfundamental.[53]

As Quine (1970, p. 63) points out, one can define identity, "or a serviceable facsimile" in a given language by an "exhaustion of combinations" of the atomic predicates of that language. For instance, if the language has just two predicates, a one-place predicate F and a two-place predicate R, one can define a predicate Ixy as follows: $Ixy =_{\text{df}} (Fx \leftrightarrow Fy) \wedge \forall z((Rxz \leftrightarrow Ryz) \wedge (Rzx \leftrightarrow Rzy))$. Quine points out that I thus defined will be identity-like with respect to the language in the sense that it will obey the usual laws: reflexivity and Leibniz's Law with respect to the language. Now, I might not really be the identity relation, if there are some objects that are utterly indiscernible with respect to that language's predicates. However, suppose that, as it happens, distinct objects areÊalways weakly discernible with respect to the fundamental properties and relations. In that case a definition of I, in a language with predicates for all the fundamental properties and relations, will in fact coincide with identity. For example, if Rxx but not Rxy then the definition above implies that Ixy does not hold (letting z be x, the rightmost biconditional does not hold). Thus the definition could be regarded as a reduction of identity in such cases.

[Unfinished]

- Objection 1: This doesn't deliver what, e.g., Dasgupta wants. Let a and b be two distinct objects (they are weakly discernible). Suppose the actual world/model to be $W(a,b)$, and consider another world/model $W(b,a)$ in which a and bÕs positions in the qualitative array have been swapped. Even though identity is defined, it's still true that $a \neq b$, and so it's true that $W(a,b)$ and $W(b,a)$ are distinct worlds/models. Moreover, these distinct models are given in perfectly fundamental terms—remember we're considering a view that is structuralist solely in its treatment of the

[53]Thanks for Nick Huggett for discussion here.

identity relation, and so it allows individual constants in fundamental descriptions. The situation here is a little subtle. Since the fact that $a \neq b$ is nonfundamental, there's a sense in which the distinctness of $W(a, b)$ and $W(b, a)$ is nonfundamental. Nevertheless, those two descriptions/worlds/models are both i) given in perfectly fundamental terms, and ii) distinct.

- Objection 2: even though a reduction of identity would be an advance in parsimony—a reduction of the fundamental relations/concepts—, it would undermine the simplicity of the laws. Every law concerning identity would become more complex.

4.16 Dasgupta's term functorese, and quantifier generalism

The two views we've discussed that best fit structuralism at an intuitive level, namely the bundle theory and monism, are pretty much untenable; neither provided a sufficiently rich account of the fundamental facts. What is needed is an account rich enough to fully describe physical structures, but not so rich as to allow the distinctions that structuralists eschew: "the structure, and nothing but the structure". One view that provides this is Dasgupta's (2009; 2015) "algebraic generalism".

Like the bundle theorist, Dasgupta's ontology consists exclusively of universals. But unlike the bundle theorist, Dasgupta makes no use of compresence. Instead, he provides a systematic way for combining universals to form complex universals. As we'll see, his approach incorporates relations and also provides the "linkage" that the bundle theory as augmented by Hawthorne could not.

The main trick is to employ an analog of the algebraic or predicate-functor approach to first-order logic that Quine brought to our attention in "Variables Explained Away". Suppose you were just dealing with simple existential quantifications of one-place predications, such as $\exists xFx$. In that case you wouldn't really need the "x"; you could just write $\exists F$, or rather, to use Quine's notation, Der$F$. Grammatically, Der is a "predicate functor": it attaches to a predicate and forms a complex predicate. In this particular case, it attaches to the one-place predicate F and forms a zero place predicate—i.e., a sentence—Der$F$.

We could take the negation sign to be a predicate functor as well, rather than a sentential connective. We could then write Der$\sim F$ instead of $\exists x{\sim}Fx$.

The predicate functor $\sim$ attaches to the one-place predicate F to form the one-place predicate $\sim F$; then, we stick Der in front to form the zero-place predicate Der$\sim F$. Similarly we can take the conjunction sign $\wedge$ to relate predicates rather than sentences.

Der doesn't only combine with one-place predicates. In general, it combines with an $n+1$-place predicate to form an n-place predicate, which is—intuitively—the existential quantification of the final place of the original predicate. Thus if R is a two-place predicate, Der R is a one-place predicate meaning "Rs something". Thus Der$\sim$Der R is the substitute for $\exists x \sim \exists y Rxy$.

If you add a few more operators (allowing you permute and add argument places), it turns out that the resulting language is equivalent to predicate logic in the following sense. It's easy to give a definition of what it is for a sentence of predicate functorese to be true in a model (i.e., a standard sort of model, with a domain and an interpretation function). Then you can show that for any sentence of first-order predicate logic that contains no names, there is a sentence of predicate functorese that is true in exactly the same models.

Algebraic generalism is a little different. In place of predicate functors—expressions that turn predicates into predicates—Dasgupta has term functors, expressions that turn terms into terms. The terms in question name universals, which can be either properties or relations of any finite -adicy. Given a set of primitive terms for universals, the term functors can then be used to generate terms for complex universals, whose existence Dasgupta also accepts. For instance, a name for a relation can be combined with the term functor c (the analog of Der) resulting in a complex term referring to a relation that is the existential quantification of the first place of the first relation. If L is the relation of loving, then cL is the relation of being loved by someone, and ccL is the "relation" (the zero-place relation) of someone loving someone. Dasgupta also adds a fundamental predicate 'obtains', which he applies to terms for zero-place universals: instead of saying that someone loves someone, he says that ccL obtains.

Term functors provide the "linkage" that was lacking in the Hawthornian bundle theory. Taking a new example: when two nonsymmetric relations hold between a pair of particulars, Hawthorne cannot distinguish the relations holding in the same direction from the relations holding in opposite directions:

In each case, Hawthorne has only the conjunction of the sentences "Bundle B bears R to itself" and "Bundle B bears S to itself". Dasgupta, though, can distinguish the cases. For the first he would offer the translation of $\exists x \exists y(Bx \wedge By \wedge Rxy \wedge Sxy)$ into term functorese: "$cc(\sigma pB \,\&\, pB \,\&\, R \,\&\, S)$ obtains"; and for the second he would offer the translation of $\exists x \exists y(Bx \wedge By \wedge Rxy \wedge Syx)$: "$cc(\sigma pB \,\&\, pB \,\&\, R \,\&\, \sigma S)$ obtains.[54]

Think of it this way. For Hawthorne, an attribution of a relation always takes the form "Bundle B_1 bears relation R to bundle B_2", which is true in cases we'd normally describe thus: $\exists x \exists y(B_1 x \wedge B_2 y \wedge Rxy)$. So in a pair of such attributions:

$$\exists x \exists y(B_1 x \wedge B_2 y \wedge Rxy)$$
$$\exists x \exists y(C_1 x \wedge C_2 y \wedge Sxy)$$

no "link" can be made between the statements; the variables x and y in the second sentence are not bound to the quantifier in the first sentence. Individuals would provide the needed linkage: names of individuals can recur in distinct attributions of relations, such as Rab and Sba. Dasgupta, on the other hand, can achieve the linkage without appealing to names of individuals, since arbitrary sentences of predicate logic without names, including sentences like $\exists x \exists y(Bx \wedge By \wedge Rxy \wedge Sxy)$ and $\exists x \exists y(Bx \wedge By \wedge Rxy \wedge Syx)$, can be translated into term functorese. Only a limited range of sentences of predicate logic are available to Hawthorne, namely, those sentences corresponding to the attribution of a single relation to n bundles. These sentences all take the form:

$$\exists x_1 \exists x_2 \ldots \exists x_n (B_1 x_1 \wedge B_2 x_2 \wedge \cdots B x_n \wedge R x_1 x_2 \ldots x_n)$$

(where B_i is a conjunction $F_1^i \wedge F_2^i \cdots \wedge F_m^i$ corresponding to the m compresent properties in the i^{th} bundle).

Note too that unlike all the forms of the bundle theory considered above (whether based on tropes or universals), Dasgupta's approach does not require monadic properties, and is therefore friendlier to various structuralisms. Any sentence of predicate logic without names can be translated into term

[54] σ is a term functor that rotates the argument places of a relation; thus σS is the converse of S. p is a term functor that "pads" a relation by adding a vacuous argument place on the left. Thus pB is a two-place relation that, as we'd normally say, holds between x and y iff y has the property B. Argument places were added to B to "line it up" with the two place relations with which it is conjoined. See the appendix of Dasgupta (2009) for more details.

functorese, and this includes sentences without monadic predicates such as $\exists x \exists y Rxy$, which goes into Dasgupta's "ccR obtains".

Dasgupta's account is detailed in a way that is important to our discussion in a couple ways. First, in section 4.11 I complained about the bundle theorist's appeal to relational properties without a prior account of relations. Dasgupta also accepts relational properties, but his term functors allow him to regard relations as fundamental, and relational properties as in a sense deriving from them. He can accept a range of fundamental universals, including the loving relation L, let us pretend. He then accepts complex universals like cL, loving-someone. But he can regard that property as being nonfundamental, and has a systematic way of talking about it and other nonfundamental properties in which the undefined terms—the names of fundamental universals, the term-functors, and 'obtains'—can all be recognized as fundamental concepts.

Also, I complained above about structural realist's failure to give a systematic theory. We need, I said, a theory that makes clear choices about its basic concepts and the rules governing those concepts, and which demonstrably is adequate to the foundations of scientific theories. Dasgupta's account clearly satisfies the former: its basic concepts, as just mentioned, are the names for fundamental universals, the term functors, and 'obtains', and one can write down rules governing those concepts by analogy to the standard inference rules of predicate logic. As for the latter, given the parallelism to Quine's term-functorese, Dasgupta's language in a sense has the expressive power of the name-free fragment of standard predicate logic: to every sentence of that language there is a corresponding sentence of term-functorese that is "equivalent" in the sense of being true in the same models. This makes possible a systematic reconstruction of a great many foundational theories, anyway.

Unlike the other views we've considered so far, then, Dasgupta's algebraic generalism has a fighting chance of being the correct metaphysics of the world. That's not to say that it *is* correct; soon we'll be turning to objections. But first it's worth mentioning a closely related view, what Dasgupta (2015) calls "quantifier generalism". As we saw, purely general sentences of predicate logic can be "translated" into term functorese. The reverse is true as well—term functorese can be translated into predicate logic. According to quantifier generalism, the fundamental facts are simply the facts expressed by the translations into predicate logic of Dasgupta's fundamental facts. These are quantified facts, facts such as that every electron has negative charge, that something is at least as massive as something, and so forth.

Quantifier generalism has all the advantages of algebraic generalism due to term functorese's having the expressive power of name-free predicate logic. It can account for the linkage missing in the bundle theory, can account for purely relational structures, it makes clear choices about the structure of fundamental facts, and so forth. It might seem like a strange fit for certain forms of structuralism, especially structural realism, since it accepts individuals: it accepts fundamental facts that are existentially quantified in form, that there exist an x and y such that x least as massive as y, say. Nevertheless the view does count as structuralist in a clear sense. By denying the existence of fundamental singular facts—facts about individuals, facts expressible using proper names—the quantifier generalist denies that there are distinct fundamental descriptions of reality differing only by a permutation of individuals. For in scenarios differing by a such a permutation, exactly the same general sentences are true. Thus the view would seem to be supported by Dasgupta's anti-individualistic argument, anyway, as well as the by hole argument, and perhaps the argument from quantum statistics.

Quantifier generalism seems to induce a sort of metaphysical vertigo, perhaps because it violates the principles that played a prominent role in chapter 2: that existentials are grounded in their instances, that existential sentences are never fundamental (or at least, are fundamental only if their instances are as well), and so forth. (It also violates analogous principles about the grounding and fundamentality of universally quantified facts, such as that $\forall x F x$ is grounded in all of its instances taken together plus a "totality fact".) Insofar as those principles are well-founded—the support for them in chapter 2 was pretty inconclusive—they provide a reason to prefer algebraic generalism over quantifier generalism. In any case, the arguments to be given in the following sections don't depend on the existentials principles, and thus threaten both views.

4.16.1 Holism and expressive resources

Dasgupta's view is holistic in a certain sense.[55] Suppose we want to make a pair of attributions of relations. As we saw in section 4.11, such a pair could not be linked in Hawthorne's system. Such a pair can be linked given individuals, by the recurrence of names. As for Dasgupta, although the members of such a pair cannot be linked, combined systems can be described by constructing

[55] For discussion of issues in this vicinity see Hawthorne and Sider (2002, pp. 62–3), Dasgupta (2009, pp. 55–6), and Turner (2011, section 4.2.3).

a single description for the combined system. In term-functorese, a complex system must be described using a single sentence, rather than a series of sentences. (Analogously, if one eschews names in predicate logic, then a complex system must be described using a single sentence; a list of ramsey sentences $\exists x_1 \dots \exists x_n A_1$, $\exists x_1 \dots \exists x_m A_2$, …for proper parts of the system will omit linkage between the values of the variables in different sentences of the list since each variable is bound within its sentence.)

To illustrate, consider what the believer in individuals would describe with the pair of attributions Rab and Sab. In place of the first attribution Dasgupta can offer:

ccR obtains (predicate logic equivalent: $\exists x \exists y Rxy$)

In place of the second he can offer:

ccS obtains (predicate logic equivalent: $\exists x \exists y Sxy$)

But in place of the pair of the attributions Rab and Sab, Dasgupta cannot offer the pair of his replacements. For that pair would leave out the fact that, as the individualist would put it, the two situations involve the same individuals, and the same direction of holding of the relations ($\exists x \exists y Rxy$ and $\exists x \exists y Sxy$ don't imply $\exists x \exists y(Rxy \wedge Sxy)$). Instead Dasgupta must supply a single sentence that describes the entire situation: the term-functorese translation of $\exists x \exists y(Rxy \wedge Sxy)$. Similarly, if he wants to go on to describe some larger system of which this situation is a part, he cannot simply add some further sentences describing the remaining parts of the larger system (and their relations to the smaller situation). Rather, he must begin anew, and provide a single sentence for the larger system.

All this is true of quantifier generalism as well. A complex system must be described, on this view, using a single quantified sentence; a collection of quantified statements describing the proper parts of the system will omit any linkages between those parts.

Why is this "holistic"? Because the truth about the situation cannot be regarded as emerging from multiple "smaller" facts. The individualist *can* so regard the situation, since the entities in her facts recur, providing the way in which the facts are "linked".

This holism leads to my main objection. Both forms of generalism require the truth about a system consisting of the entire universe to be describable by a *single sentence* in a fundamental language. But this requires the fundamental language to have strong expressive resources—infinitary quantification and conjunction, say—if the universe is infinite.

This commitment to strong expressive resources marks a difference with individualism. For the individualist, the set of facts stateable in a first-order language might well completely characterize the world. For each individual a, the friend of fundamental entities can admit simple fundamental facts concerning a (Fa, Rab, …). Given infinitely many entities there will be infinitely many such facts; but these facts will "link up" via recurrence of the individuals in them. Since the fundamental facts of the quantifier generalist don't link up, a single fact must characterize the whole world; and this won't be possible without powerful logical resources.

Dasgupta (2009, p. 52) does mention further expressive resources at one point:

> I assumed above that all possible general facts can be expressed in PL. Given the expressive limitations of PL one may think that a more realistic assumption is that all possible general facts can be expressed in second-order logic, or perhaps an infinitary first-order logic of some degree.

Jeffrey Sanford Russell (2014) also talks about the potential need for resources beyond those of standard predicate logic in a related setting. But Dasgupta and Russell aren't concerned with the same issue as I am (not clearly, anyway). A quite different issue is the following. There are various well-known arguments, in various theoretical contexts, for the need to bring in logical resources beyond those available in first-order logic. For example, George Boolos (1984) argues that we need plural quantification to give an attractive set theory and to do natural language semantics. So one concern one might have is this: if Boolos or someone else gives arguments that we must recognize, e.g., irreducibly plural facts, how can we generalize a given no-entity approach—algebraic generalism, for example—so as to recognize something analogous? As far as I can tell, that's what Dasgupta and Russell were interested in. My concern, on the other hand, is that both forms of generalism require the extra resources simply to state the facts about an infinite world, even if arguments like Boolos's do not convince us of the need for such resources at the fundamental level.[56] And further, the needed resources are likely to be far stronger.

Let's make the argument in more detail, in various cases. First, suppose reality consists solely of the positive integers, related by successor, addition,

[56] I argue that they should not in Sider (2011, section 9.15).

and multiplication.[57] The truths available to the generalist are the name-free sentences in the language of arithmetic, or their translations into term-functorese. But the totality of the former sentences is not "categorical": the set of these sentences has nonstandard models that are not isomorphic to the standard model. So intuitively, the totality of first-order sentences about arithmetic don't fully capture all the facts about arithmetic.

To sharpen this a bit, consider two "possible worlds", one in which the numbers are "normal", the other in which they are structured as in one of the nonstandard models. Exactly the same name-free quantified first-order sentences are true in both worlds, and thus the worlds share the same fundamental facts given either form of generalism, provided neither introduces extra logical resources into the language used to state the fundamental facts. But—premise:[58]

Supervenience The totality of facts supervenes on the totality of fundamental facts

Thus, the quantifier generalist's sentences cannot express all and only the fundamental facts. It may be objected that mathematical facts are necessary and so supervene on anything. But this seems like a red herring here; we could re-run the argument with an infinite collection of physical objects structured by a physical relation playing the role of the successor relation.

A quantifier generalist might respond by introducing more powerful logical ideology—monadic second-order quantifiers, say—into the language used to state the fundamental facts. Monadic second-order arithmetic is categorical: its models are all isomorphic to the standard model, and thus completely equivalent by the quantifier generalist's lights (since a permutation of entities is a non-difference according to her). Perhaps Dasgupta could say something similar (but see below).

Even before assessing whether the resulting package of views is attractive, notice how generalism has led to an additional fundamental commitment. In order to comply with Supervenience, the quantifier generalist had to introduce new ideology—the second-order monadic quantifiers. The individualist didn't.

[57] In this case we won't need to appeal to the assumption that the quantifier generalist needs a single sentence to capture all of reality.

[58] This principle is inadequate as a full statement of the sense in which the fundamental facts are "complete" (section 2.5.2), but its use in the present context doesn't require it to play that role; it must merely be true.

And the quantifier generalist won't be able to stop with just the monadic second-order quantifiers. The facts about arithmetic are comparatively easy to express completely in a name-free language, because the integers are so simple. But consider the whole range of possibilities for what happens in space. For simplicity, imagine that the metaphysician's favorite physics, a Democritean physics in which each point may be "on" or "off', is true. Since there are continuum-many points, each of which may be on or off, there are more than continuum many possible worlds of this sort.[59] By the supervenience principle, no two of these worlds can share exactly the same fundamental facts; thus, there must be more than continuum-many sets of potential fundamental facts. But no language with a countable vocabulary whose sentences are only finitely long can express that many potential fundamental facts. (The set of sentences in such a language is only countably infinite, so this set has only continuum-many subsets.) At best, it would seem, the quantifier generalist will need to describe such worlds (and also, presumably, our own world) by a Ramsey sentence with continuum-many existential quantifiers and (at least) continuum-many conjuncts.[60] And Dasgupta would need some sort of term-functorese substitute for such a sentence.

Why doesn't this argument apply to the friend of entities? Because the language available to her to express all the fundamental facts does not have a countable vocabulary: it can contain a name of each entity (a Lagadonian name, say). Or, to put it nonlinguistically, in terms of facts, there are more than continuum many components of the fundamental facts at such a world, if individuals can be components of fundamental facts. But for Dasgupta, only the term functors, 'obtains', and the fundamental universals, correspond to components of fundamental facts. So assuming the fundamental facts are structured like the sentences in the linguistic form of the argument, Dasgupta

[59] Hitch: translations and rotations of the pattern don't change the world, by the generalist's lights. Solution: just consider the points in some finite region, and add "reference objects" elsewhere in the world.

[60] Might set theory demand even greater expressive resources? Suppose the only way for the quantifier generalist to describe all the truths about the universe of sets was by "brute force"— by its ramsey sentence $\exists x \exists y \ldots (\sim\exists z\, z \in x \wedge \ldots)$. Then *really* strong expressive resources would be needed: the sentence couldn't be formulated in any of the languages $L_{\varkappa,\lambda}$, but would need to be formulated in the "language" $L_{\infty,\infty}$. Now, Vann McGee (1997) has proved that the axioms of second-order ZFCU plus an axiom saying that the urelements form a set is in a sense categorical, in that roughly any two models of this theory that share the same domain are isomorphic. So perhaps the quantifier generalist could get by with a finite (albeit second-order) language here.

could not admit enough fundamental facts to form a supervenience basis for all the possibilities.

Thus there is pressure on quantifier generalists to enhance their logical ideology, to include, for instance, infinitary quantification and infinitary conjunction. And there is a further question for algebraic generalism of whether there are attractive variable-free formulations of infinitary predicate logic. In particular, what would the analogs of padding and the two sorts of inversion for infinitary quantification be?[61]

It might be objected that positing infinitely many individuals—or infinitely many names—adds complexity as well. The argument against Dasgupta assumes that the additions to logical ideology that he must posit offend against Ockham more than positing infinitely many individuals; but is this assumption justified?

It's often assumed that the mere *number* of individuals is less important to parsimony than the number of *kinds* of individuals one posits; properties add more complexity than individuals.[62] If this is right, and if, further, the logical ideology Dasgupta needs should be classified with properties for these purposes, then the argument stands. But is it right; and if it is, why is it right? I think it is right, and that the reason—or part of the reason, anyway—has to do with laws. Individuals are all alike, as far as the laws are concerned. Laws don't concern particular individuals by name, but only via their properties. Laws—as they're normally understood, anyway— *do* mention particular properties by name. And further, logical laws mention particular logical constants by name. Dasgupta's additions will enter into distinctive logical laws, just as the usual constants have their own distinctive laws, and just as properties of mass and charge and the rest have their distinctive laws. But neither I, nor the Earth, nor any other individual plays any distinctive role in the laws, not in the fundamental laws anyway. Individuals only "occur as variables" in the laws; logical expressions and properties "occur as constants"; that is a reason for their differential impact on complexity.

[61] See Turner's comments on my APA talk on individuals for a helpful discussion of the challenges here. Jeff Russell (2014) suggests that the algebraic approach to logic doesn't generalize well to certain extensions of standard first-order logic; perhaps he has infinitary languages in mind. To be sure, there clearly are artificial ways to do it. One could say that each of Dasgupta's relations is associated with an ordinal, representing the sequence of its argument places, and one could then have principles of padding and permutation that operate on these ordinals. But the question is whether a more "intrinsic" approach is possible.

[62] Citations? Lewis, Nolan?

It might be objected that infinitary expressive resources are needed at the fundamental level anyway, even if we accept individuals. For example, suppose an individualist thinks that the fact that $\forall x Fx$ is grounded in the plurality of its instances plus a "totality fact" (section 2.5.2). Suppose further that the totality fact is $\forall x(x = a \vee x = b \vee \dots)$, where all individuals are listed. Then if this totality fact is fundamental (as it would seem likely to be), infinitary disjunction would be required at the fundamental level. (Though not infinitary quantification, notice.) But an individualist needn't be a friend of ground. And even a friend of ground might adopt a different view about the grounding of universally quantified facts, or take totality facts in some other way,[63] or adopt the "grounding-qua" approach to the problem of totality (section 2.5.2).

In a criticism of my book *Writing the Book of the World*, Kit Fine distinguished the "D project", that of completely describing the world, from the "E project", that of *expressing* facts in the most fundamental terms. He says :

> We can easily bring out the difference between the two projects with the case of disjunction. I can say 'p or q' and it is not clear that this can be said except by using disjunction or the like. But suppose now that I correctly describe the world by means of the sentence 'p or q'. Then the use of 'or' is dispensable, since I can alternatively describe the world by means of p or q, depending upon which is true. Thus even though 'or', or the like, may be indispensable for saying what we can say, it would not appear to be indispensable for describing what we can describe. Fine (2013, p. 730)

It might seem at first that this distinction is relevant to my criticism of generalism. For in that criticism I am assuming that it's important to have a simple fundamental ideology; Dasgupta might claim that this pertains only to the E project, where simplicity considerations are out of place; simplicity is relevant only to the D project, he might say.

In fact I do think that Fine's distinction is often relevant to first-order concerns—and I oppose it—but here I don't think it would help. Given the holism of his view, Dasgupta must say that the fundamental facts in the sense of the D project—the facts that ground all the others—are logically complex in the sense of being the sort of facts that can only be expressed in a language with infinitary logical resources.

[63] See Fine (2012, p. 62), though Fine is neutral about the underlying metaphysics of the totality fact.

4.16.2 Holism and scientific explanation

A second objection to Dasgupta concerns the impact of holism on explanation.

Any view about what's fundamental must account for all the other facts. How will the generalist account for the fact that Obama is sitting on a chair?

One might claim that Obama's sitting on a chair has a "qualitative" ground of some sort. But it apparently could not be anything like the term-functorese translation of *some person with feature F is sitting in a chair with features C*. For grounds necessitate, and it's not necessary that anyone with features F sitting on a chair with feature C is Obama. How to respond to this problem?

One possibility is to make the grounds very "big": the translation of "there is a person with feature F, in a room with feature G, in a planet with features H…". Dasgupta (2014*a*), though, argues that any such "big" proposition cannot be a ground for Obama's sitting on a chair, since it will include irrelevant information. So we're caught: the grounds must be big in order for them to plausibly necessitate Obama's being on the chair, but they can't be big since that makes them irrelevant. Dasgupta's solution is to say that there is no ground at all of the individual fact that Obama is sitting on a chair; instead there's a ground for the totality of singular facts taken together.

On any of these views, there is no "highly local" ground of Obama's sitting on the chair. Either the ground is very big (bringing in a lot of the Earth as well, and maybe the whole solar system…) or else the ground is simultaneously a ground of all the other singular facts as well.

Next point: how should we decide what is fundamental? As I've said a number of times in this book, we ought to choose a view of fundamental concepts that enables simple and strong laws of nature. But I would also argue for a parallel constraint about scientific explanation. Intuitively: just as laws ought to look good, by ordinary scientific standards, when viewed through the lens of fundamentality, so, explanations ought to look good, from ordinary scientific standards, when viewed through the lens of fundamentality.

But now: explanations involve, in the simplest case, subsumptions of individual cases under laws; and statements about individual cases, given generalism, are going to be highly holistic. Thus an explanation of why a certain electron behaved in a certain way will turn, when viewed from a fundamental perspective, into the subsumption of a massively complex fact about the entire universe—or in Dasgupta's case, the subsumption of a massive plurality of such complex facts—under the laws of nature.

4.17 Dasgupta's argument again

My objection to Dasgupta is not meant to be decisive. Parsimony must always be weighed against all the other relevant considerations, and Dasgupta might say that infinitary ideology is a price worth paying, given the argument he gives for his theory. But let's look at that argument more carefully.

Dasgupta's complaint about individuals was that they are physically redundant and empirically undetectable. These of course aren't the same thing, and Dasgupta remains neutral on which if either is more significant.

From a certain "realist" point of view, the complaint about empirical undetectability carries little weight. For the realist, there is nothing whatsoever wrong with a theory that implies the existence of facts that we cannot—in various senses—know. We should embrace the external world, the existence of unobservables, and facts about the distant past, even if in various senses we cannot know about these things. Now, Dasgupta does not mean to be denying this broadly realist thought. His complaint is only about a specific and nuanced sort of empirical undetectability. Still, its force is undermined by the comparison to the more general complaint. If nothing is wrong with unknowability per se, why should there be something wrong with one particular sort? From the realist point of view, the complaint about unknowability—both in the case of absolute velocities and in the case of individuals—is entirely unmotivated.

To be sure, the fact that a theory implies unknowable facts of various sorts might well be a *sign* that something has gone wrong. The claim that there exist little green men who disappear whenever we look at them is problematic for reasons that have nothing to do with the unknowable facts it implies. For instance, it posits entities that play no role in explaining the evidence. Similarly, the empirical undetectability of absolute velocities in Newtonian gravitational theory is not, on its own, a strike against them, but it might be a sign that embracing them would result in some other theoretical defect—physical redundancy, perhaps. But unknowability is not *always* a sign of some such theoretical defect; theoretical improvement does not always go hand-in-hand with easier epistemology. Indeed, consider the paradigm case of an appeal to the theoretical virtues: Bertrand Russell's argument that we have reason to believe in the external world because its explanation of our sensory experience is simpler than the "idealist" hypothesis that nothing exists other than our sense data. The external world hypothesis is usually thought to imply the existence of *more* unknowable facts than the idealistic hypothesis.

Thus it is the complaint about physical redundancy, not the complaint about empirical undetectability, that is Dasgupta's strongest argument against individuals. That complaint, intuitively, is that physically redundant posits in a theory are wheels that turn without being properly incorporated into the rest of the theory's explanatory mechanisms. Absolute velocities would not be *completely* inert in NGT, of course, since absolute velocities at one time affect absolute velocities at later times. But, so the thought goes, since that's *all* they affect, they are insufficiently integrated with mass, relative velocity, etc. Simply dropping them from NGT sacrifices nothing of explanatory value, and thus results in a superior theory, even from the "realist" point of view.

So our question is whether this complaint about physical redundancy can be made about individuals. In fact this is far from clear, at least if the issues are viewed through the lens of concept-fundamentality. For through that lens, it is far from clear that individuals are physically redundant in the relevant sense—an insufficiently integrated explanatory posit. Given how integrated the concept of an individual is within individualistic theories, one cannot simply "scoop out" the individuals and leave the rest of the theory intact. The individuals-theoretic fundamental concepts—names, quantifiers, predicates—are part of every statement the theory makes. The conceptual change Dasgupta wants us to make, namely exchanging these concepts for names of universals and his term functors is not a *deletion* of structure from the theory, but rather an *exchange* of one sort of structure for another. The complaint about "redundant" structure is ultimately an Ockhamist one: if we can simply *delete* a component of a fundamental theory without sacrificing anything of explanatory importance, we should do so, since the deleted component wasn't doing any distinctive work that needed to be done. But if one component can be *exchanged* for another without sacrificing anything of explanatory importance, this doesn't show that the exchanged component was redundant; it just shows that there's a different way to get the same job done. One might of course argue that the alternate way to get the job done is better, but the mere possibility of the exchange doesn't establish this.

The key point here is that the judgment that a theory has excess or redundant structure is, when viewed through the lens of concept-fundamentality, a judgment about a privileged statement of that theory, complete with distinguished putative fundamental concepts, distinguished formulations of laws in terms of those concepts, and so forth. If there is excess or redundant structure in such a theory, that must be a fact about those privileged concepts and how they are utilized in the laws. The mere fact that a certain "aspect" of the theory (such as that of which particular individuals play which roles at a certain time) is isolated from other aspects (in the sense that, for example, a permutation of individuals over roles at a time won't affect anything at later times other than which individuals play which roles then) does not show that the theory contains redundant structure, if the aspect in question does not correspond to a single distinguished element in the theory—a distinguished fundamental concept.

(In chapter 5 we will discuss a much more metaphysically minimalist approach that rejects concept-fundamentality, and does allow "scooping out" arbitrary aspects of a theory's structure. Thus we have another illustration of the importance of the choice of metaphysical tools to these issues.)

It must be conceded that this way of viewing physical redundancy creates a problem for all judgments of physical redundancy, including the judgment that absolute velocities are physically redundant in NGT. Suppose in NGT we are choosing whether to adopt Galilean or Newtonian spacetime, and in particular choosing which of the following sets of fundamental concepts to adopt:[64]

Galilean ideology spatial-distance-at-a-time, temporal distance, affine connection

Newtonian ideology spatial-distance-at-a-time, temporal distance, affine connection, same-place-as

This particular choice is straightforward: the Galilean ideology is a proper subset of the Newtonian ideology, and for that reason is preferable on grounds of parsimony. Moreover, let us suppose, the laws of NGT can be formulated simply (and intrinsically) using only the Galilean ideology—same-place-as doesn't even appear in those laws. Given this, the same-place-as relation is redundant in a very clear sense, and may simply be dropped.

However, the geometry of Newtonian spacetime can be characterized using different fundamental concepts than those just mentioned. Consider this ideology for that geometry, mentioned in a a co-written dialog by Dasgupta and Jason Turner (2015):

Alternate Newtonian ideology cross-time-spatial-distance, temporal distance

The choice between this and the Galilean ideology is far less clear. Neither ideology is a proper subset of the other. The laws of NGT, when given an intrinsic formulation using the Alternate Newtonian ideology, would now

[64]I'm simplifying by ignoring details of how to incorporate the quantitative aspects of these notions.

employ both elements of that ideology, and so that ideology wouldn't contain an element that simply doesn't occur in the laws.

Thus a single theory can be given multiple formulations, corresponding to multiple choices of fundamental concepts, which lead to different judgments about redundancy. One obstacle this poses to judgments of redundancy viewed through the lens of concept-fundamentality is that of which formulation to choose. Now, this isn't really an obstacle, since from the point of view of concept fundamentality, the "formulations" are just different theories. (This of course creates an epistemic problem, which we'll discuss in chapter ??.) But a second obstacle arises: for some of these formulations/theories, it's unclear what the judgment should be. Is the Alternate Newtonian ideology superior or inferior to the Galilean ideology?

Dasgupta and Jason Turner bring up this alternate Newtonian ideology in an attack on the idea that absolute velocities are to be rejected because they give rise to ideological complexity. As they point out, the judgment of complexity isn't clear since Newtonian spacetime can be given a formulation—the Alternate ideology—which has fewer fundamental concepts (two) than a formulation of Galilean spacetime, namely the ideology given above, which has three fundamental concepts. Now, as Nelson Goodman and others have pointed out, one can always artificially reduce the number of primitive expressions in a theory by cooking up appropriate definitions.[65] So evaluating ideological simplicity simply by counting fundamental concepts was never in the cards. But the point remains that it is far from clear how to make judgments of ideological simplicity. And for similar reasons, it's far from clear how to make judgments of redundancy, viewed through the lens of concept-fundamentality.

The fan of concept-fundamentality has no choice but to concede that these matters are complex and multifaceted and (at present anyway) nonalgorithmic. It would be foolish to jettison the entire approach simply because of the messiness. What we are talking about here, after all, is the messy epistemology of theory choice, and there's no reason to suppose that the correct superempirical principles governing such choices are tidy (or even that there's always a fact of the matter about them). The realist is just stuck in this muck, and had better learn to live with it.

As for life in this muck, one might invoke simplicity comparisons between individual notions, rather between entire ideologies. One might argue, for instance, that the notion of cross-time-spatial-distance is intrinsically more

[65] See Goodman (1951, chapter 3) for a discussion of some of these issues.

complex than spatial-distance-at-a-time. It's certainly complex when viewed from the perspective of what I originally called the Newtonian ideology, since one can reductively define the distance between nonsimultaneous p_1 and p_2 in Alternate Newtonian terms, as the distance between p_1 and the point simultaneous to p_1 that is at the same place as p_2. But what I am saying one could argue is that cross-time spatial distance is an *absolutely* more complex notion, that facts stated using that notion are in a sense richer, contributing to a more richly structured world, than facts stated using the notion of spatial distance at a time.

It might also be useful in the muck to simultaneously evaluate for simplicity a proposed ideology and set of laws (and by the latter I mean a proposed distinguished formulation of given laws in terms of the proposed ideology). For instance, there might be a sense in which an attractive formulation of the laws of NGT only "look directly at" spatial distances at a time, and never at cross-time spatial distances, which could be taken as a strike against the latter. (This seems clear in the case of gravitational and electromagnetic force laws, but I'm not sure how to think about what the laws of motion "look at"; this is less of a fully formed thought, more of a direction in which to look.)

4.18 How far to go?

How far would the demand to remove undetectable or redundant structure lead, if given completely free rein?

The nomic essentialist's objection to unknowable facts about which properties play which nomic or causal roles (chapter 2), is akin to Dasgupta's objection to undetectable individuals. Permuting references to charge and mass within a combined NGT+Classical electromagnetism theory, say, might be argued to have no observational effect, as Turner (2015) points out. Likewise, one might argue that individual properties of charge and mass are physically redundant in that theory, because an initial permutation of charge and mass would result in correspondingly permuted outcomes.

But *can* the argument be followed this far? What would a theory that eliminated *both* individuals and properties like charge and mass—a combined structuralism about both properties and individuals—look like? Dasgupta's method for eliminating individuals relies on retaining universals, and thus cannot be used to eliminate all universals as well. His strategy for translating a theory into term-functorese was to write that theory using quantifiers over individuals and names for properties like mass and charge, and then translate it into term-functorese. For instance, if the theory were simply "$\exists x\, x$ has M", with M a name for mass, the translation would be "cM obtains". Now, suppose we wanted to replace even the name M with a quantified variable. If the original theory read "$\exists x \exists p\; x$ has p", then Dasgupta would have no way to translate it into term-functorese, since no universals are named in the theory.

Dasgupta could perhaps reify the relational tie between particulars and universals, expressed by 'has' in the original theory. Where H is a dyadic universal of "having", he could rewrite the original theory as saying that "something bears the *having* universal to something"—"$\exists x \exists p$ bears(H, x, p)" (note the continued need for a predicate, now 'bears', connecting the singular terms H, x, and p to form a sentence). This would then go into predicate functorese as "ccH obtains". More generally, instead of making statements about complex universals built up from simple universals of mass, charge, and so forth (via the term functors), Dasgupta could make statements about universals built up solely from the simple universal H of having.

This would be a very sparse basis for the fundamental facts. If a rich metaphysics of laws were accepted, it could be enriched somewhat: simple universals drawn from that metaphysics might be added to H in the basis for constructing the complex universals—a two-place universal of necessitation given Armstrong's (1983) metaphysics of laws, or a one-place universal L instantiated by zero-place universals (states of affairs), corresponding to a sentence operator 'it is a law that'. For example, suppose we begin with an initial theory concerning two particular properties, F and G, which says that F necessitates G (in Armstrong's sense) and that something has F. We could first rewrite the theory thus, where N names the necessitation relation: $\exists x(\text{bears}(N, F, G) \wedge \text{has}(x, F))$. We could then replace the names of the particular properties F and G with existentially quantified variables: $\exists p \exists q \exists x(\text{bears}(N, p, q) \wedge \text{bears}(H, x, p))$ (H is again the relation of having). We could then translate the whole thing into term-functorese: "$ccc(\sigma pN \,\&\, \iota p \sigma H)$ obtains".

Suppose Dasgupta says that this is indeed an improvement. (Dasgupta expresses openness to a combined structuralism about individuals and properties in Dasgupta and Turner (2015).) I then wonder where things will stop. Any theory will contain "constants", expressions that are not ramsified away. Couldn't one always construct descriptions in which any remaining constants are permuted, but claimed to obey each others' laws? For example, if the theory concerns conjunction and disjunction, suppose one swapped conjunction for disjunction, including in the laws governing these logical constants; would even this theory contain objectionably undetectable and redundant structure?

It may be responded—perhaps in the first place, regarding charge and mass, or later, in the case of conjunction and disjunction, say—that the imagined permutations are not possible, or, alternatively, that they are not really observationally distinguishable (e.g.: "a world in which mass rather than charge behaves in such-and-such ways just *looks* different"). But it's hard to see why these responses would be any more successful here than they would have been at the outset, in the case of individuals.

It may be responded that the case of conjunction and disjunction, anyway, is different because we have no entities to permute. 'And' and 'or' are sentence operators, not names, whose semantic function is not to stand for truth functions or any other entities. But it's hard to see why this fact makes a difference. Begin with a true theory; construct a permuted theory in which 'And' occurs everywhere 'Or' occurred and vice versa, including in the statement of logical rules, and now run the Dasguptian argument. It's hard to see why the fact that the permuted expressions were operators rather than names makes a difference.

Jeff Russell made a nice distinction concerning the example of permuting conjunction and disjunction. There's a difference between saying that permutations of situations that are allowed by a theory are allowed *by that very theory, as originally stated*, and saying that permutations of allowed situations are allowed *by a permuted version of the theory*. Only the latter is true in the conjunction/disjunction case. If a theory allows $A \vee B$ as a possibility, then a "permuted" theory that exchanges $\vee$ and $\wedge$ in the logical laws will allow $A \wedge B$, but the original theory may not allow $A \wedge B$. With that distinction in mind, we might say: what's objectionable is for a theory itself to allow indistinguishable possibilities; it isn't objectionable (or anyway, not in the same way) if a theory is such that its permutations allow indistinguishable possibilities. But now return to the case of permuting individuals in NGT: the original theory, without re-interpretation, allows the permuted possibilities: if NGT allows $\phi(a, b)$ then it allows $\phi(a, b)$. The crucial difference is that in NGT, construed so as to include the underlying logic, the laws contain no names of particular individuals, but do contain '$\wedge$' and '$\vee$' (as "constants"). Thus from this point of view, one could continue to make Dasgupta's objection to individuals without pursuing the argument so far as to apply to conjunction and disjunction. In fact, one should stop even before the rejection of charge and mass, since that argument is like the argument against conjunction and disjunction, and not like the argument against individuals. Like '$\wedge$' and '$\vee$', 'Charge' and 'mass' occur as constants in the laws,[66] so there is no guarantee that if one's original theory (NGT+Maxwell, say) allows ϕ(charge, mass)', it also allows ϕ(mass, charge)'; at best, a modified theory (which permutes 'mass' and 'charge') allows the latter. So Russell's way of thinking counsels Dasgupta to get off the boat right at the outset.

Is Russell right? It would seem to depend on whether Dasgupta's main complaint about individuals is to their undetectability or to their redundancy. If his argument is based ultimately on a concern about unknowable facts, then Russell's point seems wrong. For in that case, the complaint should be just as compelling, regardless of whether we need to permute the theory in addition to the descriptions it allows. The argument would be this: if you accept the existence of individuals, or charge and mass, or an ideology containing conjunction and disjunction, then you must admit that there are these possibilities that are observationally equivalent: theory+outcome, permuted-theory+permuted-outcome. But suppose instead that the complaint is less epistemic (as I argued it should be), but is instead based on the thought that there's something wrong with a *theory* that allows certain sorts of observationally equivalent models. Then Russell's point seems right. For then, the alleged badness pertains to the theory—the unpermuted theory—rather than to the hypothesis of individuals (or charge/mass, or conjunction/disjunction) per se.

4.19 Other forms of generalism

Some other views similar to algebraic and quantifier generalism have been discussed recently. These all share the conviction that the name-free fragment of predicate logic is in some sense the only portion that is apt for describing reality.

4.19.1 Predicate functorese nihilism

A very similar view would be like Dasgupta's, but based on predicate functorese rather than term functorese. Dasgupta's ontology is one of properties and

[66]One might wonder whether this is contentious; might a nomic essentialist think that the laws are somehow variable with respect to properties as well as individuals? But this idea evaporates on closer inspection. The idea would be that the laws look like this: "any property that ϕs is such that …" But how to fill in ϕ? Not like this: "Any property that plays role $\mathscr{L}$ in the laws is such that …", since this very statement is the statement of the laws!

relations. As for his primitive concepts, he accepts the term-functors, as well as the predicate ‘obtains’. Though this hasn’t been relevant so far, and I don’t know whether Dasgupta addresses it—he presumably also accepts all the usual concepts of predicate logic, including quantifiers and variables, except ranging only over properties and relations. Otherwise the description of ‘obtains’ as a predicate would seem odd, as would his claim to accept properties and relations in his ontology.

The predicate functorese nihilist, on the other hand, accepts no ontology at all, and she rejects the quantifiers of first-order logic. In their place she proposes to use predicates (not names of properties) and the ideology of predicate-functorese. Since predicate functorese has the same expressive resources of the name-free fragment of predicate logic (in the sense described earlier), this view is similar in many ways to Dasgupta’s. It can deal with the problem of linkage in the same way Dasgupta can; and it is subject to the same objections I gave to Dasgupta’s view.

Jason Turner (2011) has given an interesting argument that predicate functorese is a notational variant of ordinary predicate logic, and thus is not a form of opposition to individuals after all. I’d like to make a remark about this, a remark that doesn’t depend on the details of Turner’s argument.

A predicate functorist might simply concede Turner’s point, but say that this just shows that $\exists x F x$ is misleading, since it means the same as $\mathrm{Der} F$. Her picture might be the following. Suppose we can make sense of a metaphysical difference between the intended meaning of ‘Der’ and the intended meaning of ‘$\exists$’. Then if just one of these meanings “exists”, it seems likely that both ‘Der’ and ‘$\exists$’ will pick it out, as Turner’s argument shows. (If both meanings exist then presumably each expression is ambiguous between the two meanings.) But the fact that both ‘Der’ and ‘$\exists$’ pick out this one meaning leaves open which of the two meanings it is.

It seems to me that this way of looking at things pushes all the action into the metaphysical rather than metasemantic arena. The big question now is: can we really make sense of a metaphysical difference between the two intended meanings? If we can then it seems to me that the predicate functorist’s reply from the previous paragraph stands. If not, then it doesn’t, and the predicate functorese position isn’t really distinct from the ordinary quantificational picture.

About this remaining metaphysical question, I have one small further point. One of Dasgupta’s replies to Turner was to insist that the usage of ‘Der’ includes its connection to ground; and one might bring in this idea to metaphysically distinguish Der from quantification. The difference, one might say, is that $\exists$ facts are grounded by their instances whereas Der facts aren't. But my concern here is that this relies on an implausible view about ground.

Recall the "extra force" view of ground introduced in chapter 1. On that view, when we accept ground we are recognizing a new metaphysically basic phenomenon. In addition to containing (perhaps) fundamental mereological, modal, determinate-determinable structure, etc. reality also contains fundamental ground structure. In my view the extra force view is very implausible; but without that view, it's unsatisfying to say "the metaphysical difference between Der and quantification is one of grounding", since that just pushes the question back: ok, what is it exactly, at the fundamental level, which grounds this ground-theoretic difference between Der and quantification? (Compare the arguments of sections 2.3 and 4.13.)

4.19.2 Factualistic quantifier generalism

Russell (2014) discusses a view that's similar to quantifier generalism, except that the privilege enjoyed by the name-free fragment of predicate logic is characterized using his concept of "factuality" rather than ground or fundamentality. Instead of denying that singular claims like Fa are fundamental, he denies that they are factual; more exactly:

Factualistic quantifier generalism If it is a factual matter whether A then it is a qualitative matter whether A.

Factuality for Russell plays some of the same theoretical role that fundamentality does: for instance, it is used to distinguish conventional from objective content. But it is in a sense a weaker notion since the factual claims, for Russell, are not stratified into a ground level and dependent superstructure.

Russell gives an interesting argument against Factualistic quantifier generalism. The argument makes use of this connective:

$A := B$ "What it takes for A is B"

In summary, here is his argument:

1. The generalist's language must have a compositional semantics
2. That compositional semantics will make use of clauses like this:
   "For all a, (a satisfies 'x has mass' $:=$ a has mass)"

3. If $A := B$ then it is factual whether B (Factuality Principle)

4. Therefore, for all a, it is factual whether a has mass, contradicting generalism

Where are these premises coming from? The need for *factual* truth conditions is stated thus:

> ...if the world can be adequately characterized using certain sentences, then it should be possible to say what it takes for each sentence to be true. Furthermore it should be possible to give these truth conditions in factual terms. This would be a way of explicitly spelling out how these claims "correspond to reality", as opposed to doing some other less metaphysically loaded job. (Russell, 2014)

The need for a *compositional* truth theory (as opposed to a theory that just took each complete sentence S and said "S is true $:= S$" is that i) fundamental languages are supposed to be simple, and ii) the simplicity of a language is the simplicity of its truth theory, since this is its simplicity "as a representation of the world".

First, a parenthetical worry about the argument's assumption that truth conditions must be given using the $:=$ connective, rather than the material biconditional. To be sure, there are old puzzles (from the literature on Davidsonian theories of meaning) about what it takes for a theory that gives truth-conditions to be a good semantics for a language. E.g. a semanticist who thought that 'snow is white' meant that grass is green would accept the true material biconditional "'Snow is white' is true iff grass is green"; and there is a concern that she might base an entire theory consisting of true sentences about truth-conditions on her semantic misapprehension. But there are ways to address this concern other than bringing in the new connective $:=$. (The following is only a start: if the theory includes other T-sentences that follow the pattern of the one above, it will include false T-sentences. (E.g. if one is putting 'grass is green' on the right hand side for every sentence then one would get "'Snow is black' is true iff grass is green".) And if there is no pattern to the theory's T-sentences then it's a bad theory on general theoretical grounds.)

(Also, I don't see how to get the $:=$ connective out of the "factuality" framework—isn't it strange to add it? Seems like an intrusion from the "ground" side of the aisle.)

My main worry is that the Factuality Principle seems too strong. Why not a weaker principle, which says that if A is factual and $A := B$ then B is factual?

Compare what Fine says about ground: A grounds $A \vee B$ even when A (and B) are nonfactual; wouldn't it make sense to allow the same for $:=$? If A itself isn't factual, why should what it is for A to be the case be factual?

Given the weakened Factuality Principle, the generalist could just use the standard truth theory to provide factual truth conditions for each of the sentences in her language. Take the sentence 'for some x, x has mass', for instance. The semantic theory includes this axiom:

For all a, (a satisfies 'x has mass' $:=$ a has mass)

This axiom is itself factual, notice. It does not violate the weakened factuality principle: for any a, it isn't factual whether a satisfies 'x has mass', so the weakened principle doesn't imply that for any a, it's factual whether a has mass. We can use this axiom, plus other clauses in the truth theory, to derive this:

'For some x, x has mass' is true $:=$ for some a, a has mass

Here the right hand side *is* factual.

Chapter 5

Theoretical Equivalence

5.1 Intro

To say that theories (or statements, or models, or representations) are "equivalent" is to say that they represent the very same state of the world, that any differences between them are merely conventional or notational. The topic of this chapter is what equivalence amounts to—what it is to be equivalent.

We will focus on two approaches to equivalence. They are in a sense the most extreme positions, and perhaps in the end some intermediate view is needed. Still, it's often useful to begin with the extremes. At the very least they can serve as contrasts for intermediate views. And if intermediate views are unsatisfying, as they often are, we'll have a stark choice before us.

Philosophers of physics discussing equivalence usually talk about symmetry. Define a symmetry (of the laws) as a one-one mapping from the set of possible histories onto itself that maps solutions of the laws of nature to solutions and nonsolutions to nonsolutions. Jenann Ismael and Bas van Fraassen (2003), for example, have claimed that if some symmetry maps one history to another, and if the histories are also perceptually indistinguishable (in a certain sense), then we have reason to think that the histories are equivalent. Others have discussed other claims of the same form, the form being: "we have reason to think that histories are equivalent if they are i) related by a symmetry, and ii) X".

Claims of this form address the *epistemology* of equivalence, of when we have reason to believe in equivalence. This epistemic question appears to be the main question addressed in the literature; and some such symmetry/equivalence principle might well play a central role in answering it. Symmetry may be a guide, even our best guide, to equivalence.

But our topic is what equivalence *is*—what it is for histories (or descriptions, or theories) to be saying the same thing about the world. And I doubt very much that the answer to this question will take the form "symmetry plus X". A symmetry might map a history to an utterly dissimilar history, so long as it never maps a solution to a nonsolution or a nonsolution to a solution.[1] The actual history, for example, might get mapped to a history in which there is no sentient life at all, or a history involving just a single particle. This is why an X always gets added to symmetry/equivalence principles; but no such X, it seems to me, could turn symmetry into an account of what equivalence is. Since histories related by a symmetry can be as dissimilar as you like, the concept of a symmetry doesn't even *approach* the concept of equivalence—it isn't as if symmetry is "almost" equivalence, or a "part" of equivalence. And given this, it's hard to see how adding some further condition X would get us closer to the concept of equivalence, unless X amounts to equivalence on its own.

Also, the X that's added often prejudges things that ought not to be prejudged. For example, requiring the symmetry to be continuous with respect to the topology of the space of histories just bakes in that topological structure isn't conventional.[2] This is a plausible claim, but it shouldn't be settled by the very definition of equivalence. (For that matter, "symmetry plus X" bakes in the objectivity of the laws.) To be sure, such objections are delicate, since any contentful account of equivalence will bake in something. The objection rests on the assumption that these particular claims about equivalence shouldn't be prejudged.

Symmetry is presumably a necessary condition for equivalence (assuming that the laws are indeed objective), since the laws are presumably sensitive only to genuine features of reality, not conventional artifacts of representations of reality. Moreover, other things being equal we ought to avoid recognizing genuine features of fundamental reality that physics doesn't care about. These observations speak in favor of symmetry as a guide to equivalence. But if our interest is in what equivalence is, I think we need to set symmetry aside, and delve into some issues in general metaphysics, so as to directly investigate the concept of "saying the same thing about the world".

[1] This is a commonly made point; see Ismael and van Fraassen (2003); Belot (2013).

[2] Via the way in which the topology of the space of histories is defined in terms of the topology of physical space and time or spacetime.

5.2 Examples: quantities, metric, ontology

We all can agree, surely, that two theories that differ only in what unit of measure they employ are equivalent theories. What is it that we are all agreeing on?

Well, imagine someone who denies the claim. Imagine a person, "Kilo", who thinks that there is a "distinguished" unit of mass—kilograms, say. Just as some people think there is a distinguished direction of time, Kilo thinks that the kilogram unit is physically distinguished.

Kilo has perfectly ordinary views about what masses things have in any given scale. Kilo understands that statements about mass in grams are *true*, and takes them to be correlated with statements about mass in other scales in the same way that we all do. But a description in terms of grams isn't equivalent, Kilo says, to a description in terms of kilograms, because there are some *extra mass facts* that are left out by the grams description. If you say only that something is 1kg, and that it is 1000g, you've left out that it's *really 1* in mass; it's 1 in the *distinguished* unit. But how should we think about this alleged "extra fact"? What would it mean for a unit to be "distinguished"?

Before trying to answer, consider two other cases in which parallel issues arise. First consider someone like Grünbaum (1973), who thought that space lacks a distinguished or intrinsic metric. The points of space, he thought, do have a distinguished topology, but of the many functions from pairs of points of space to real numbers with the right formal properties to count as distance functions, no one of them is physically distinguished. And not just because the unit of distance is conventional; he also thought that even pairs of such functions where one is not a scalar transformation of the other (the result of multiplying each value by a constant positive real number) can be on a par, so far as the intrinsic geometry of space is concerned. Most of us reject this point of view, but to reject it is to think that one distance function (or rather, one equivalence class of distance functions) is "distinguished"; and what does that amount to?

Second, consider Eli Hirsch's (2011) views on (meta)ontology. There is a dispute amongst metaphysicians over whether there exist composite objects, objects with smaller parts. David Lewis (1986, pp. 212–3) said yes: there really do exist objects with parts, such as molecules, chairs, planets, and so on. (Indeed, according to Lewis, there exist arbitrarily scattered objects, since for any objects whatsoever, there exists a mereological sum of those objects.) Peter van Inwagen (1990) said no: composite objects do not exist. All that really exist are things with no smaller parts at all: subatomic particles, perhaps.[3] Then along came Hirsch, who listened to all this and said: this is a nonissue! According to him (and really, lots of people, especially nonmetaphysicians), the following two claims are equivalent:

Van Inwagen: there exist subatomic particles in a certain "chairlike" arrangement C; there does not exist any further object (a chair) containing those particles as parts

Lewis: there exist subatomic particles in arrangement C, and there also exists a further object containing those subatomic particles as parts

Hirsch's reason was, in essence, that van Inwagen and Lewis can each use his concept of existence (his existential quantifier) to define up a concept that fits the other's theory, and that there is no "distinguished" concept of existence. For example, van Inwagen can define up a new meaning of 'there are'—call it "existence$_{\text{Lewis}}$"—under which he can agree that Lewis's claims come out true. He would simply need to define 'there are$_{\text{Lewis}}$ chairs' as meaning what he (van Inwagen) ordinarily would mean by "there are subatomic particles in arrangement C".

What would it take to oppose Hirsch and regard the ontological debate as being genuine after all? A distinguished concept of existence. Then even though everyone can agree on what "exists$_{\text{Lewis}}$" and what "exists$_{\text{van Inwagen}}$" (where these are the defined-up notions of existence), there remains a question of what exists in the distinguished sense, i.e., *what really exists*.[4]

Hirsch, Grünbaum, and Kilo's claims of equivalence have been seen to turn on whether certain concepts (a unit, a metric, an existence-concept) are "distinguished". But if our goal is to clarify what we mean by equivalence, we haven't made much progress. For what does it mean to call a unit of measure or a metric or an existence-concept "distinguished"?

The idea itself is a familiar one. We don't normally think of space as having a distinguished direction or origin, whereas we (most of us anyway) *do* think of space as having a distinguished, or "intrinsic", metric. But what exactly does that amount to?

To move forward, we'll need to enter some disputed territory in metaphysics. This shouldn't be a surprise. Investigating the idea of theories "saying the same

[3]Actually van Inwagen also accepted composite living things.

[4]See Sider (2001, introduction; 2009; 2011, chapter 9).

thing about the world" leads immediately to general questions of what sameness and difference in the world consists in, very abstract questions about what the world's ultimate constituents are, about how we should think about such matters…—the deep seas of metaphysics. I doubt that a contentful metaphysics of equivalence can avoid broaching such issues.

5.3 Fundamentality

I think the best way forward makes use of the metaphysical tool of concept fundamentality: equivalent theories are those that say the same thing about the world at the fundamental level, those that give the same description of the world in terms of fundamental concepts.[5] To say that a unit, metric, or existence-concept is distinguished is to say that it is fundamental, or at any rate that it is uniquely singled out by a fundamental description of the world. A believer in a distinguished unit of measure thinks, perhaps, that there is an absolutely fundamental relation between massive objects and real numbers, the mass-in-kilograms relation, and that relations like mass-in-grams and mass-in-pounds are not absolutely fundamental. Suppose an object is 1 kilogram. Then the fact that it bears the mass-in-kilograms relation to the real number 1 is a "fundamental fact". That object will also bear the mass-in-grams relation to the real number 1000, but this fact is not fundamental; it holds in virtue of the fact that the object bears the mass-in-kilograms relation to 1.

This account needs to be sharpened. To sharpen the claim that theories are equivalent when they "say the same thing" about the world at the fundamental level, we need to choose an approach to speaking of the dependence of non-fundamental on fundamental. In terms of ground, for instance, as a first pass we might say that facts F_1 and F_2 are equivalent when, necessarily, for any facts that involve only fundamental concepts, those facts ground F_1 if and only if

[5] Miller (2005) defends a similar view.

they ground F_2.[6] [7] It's harder to sharpen the claim that a distinguished concept is one that is "uniquely singled out" by the fundamental concepts. For example, we might want to speak of distinguished concepts of chemistry, because those concepts have simple definitions in terms of fundamental concepts. But "simple definition" is itself unclear, and we may also wish to speak of distinguished concepts in domains in which simplicity of definition in fundamental concepts isn't the whole story.[8] But for present purposes, it will be enough to work with a sufficient condition: a concept is distinguished if it is fundamental.

5.4 Difficult choices

This characterization of equivalence in terms of fundamentality is the first of the two views I want to discuss. I call it "extreme" because it has certain apparently uncomfortable consequences.

Suppose we think—as we surely should—that there is not a distinguished unit of measure for mass. We should then presumably say that *none* of the relations *mass-in-unit-U* between massive objects and real numbers is fundamental. What, then, *are* the fundamental properties or relations of mass? An answer to this question ought to give a satisfying account of what mass ultimately is, and

[6] This is imperfect: it counts the fact that snow is white as being equivalent to the fact that either snow is white or $0 \neq 0$. The problem is caused by the fact that ground is a "conditional" rather than "biconditional" notion (Sider, 2013*b*). We might instead say that facts are equivalent if each is "biconditionally grounded" some one fact (or collection of facts) involving only fundamental concepts. Biconditional ground could, in turn, be taken as an additional posit alongside ground, or it could be understood in terms of my notion of metaphysical truth conditions (Sider, 2011, section 7.4), or Rayo's (2013) notion of "just is", or Dorr's (2016) notion of "to be…". Dorr and Rayo's notions yield notions of equivalence all on their own (since they may be applied to arbitrary pairs of facts or sentences), but one might doubt the legitimacy of such notion in application to pairs of nonfundamental facts while accepting their value in connecting facts to their fundamental equivalents.

[7] Defining equivalence in terms of ground (or metaphysical semantics) renders equivalence "nontransparent" in a certain sense. Suppose a psychological theory of personal identity is true. Then facts about personal identity will be equivalent to certain psychological facts—because they share the same grounds—even though an opponent of the psychological theory would deny this without thereby being rationally deficient or conceptually confused. This is due to ground (as it's usually understood) being a nontransparent, nonepistemic relation. (Thanks to Chris Hauser here.) For an account of equivalence in the philosophy of science, this feature seems acceptable, though for other purposes one might prefer a different, more transparent conception of equivalence.

[8] Parallel issues are discussed in Sider (2011, section 7.11.1).

also ought to explain why the relations to real numbers are so useful in physics.

Fortunately, these questions can be answered, with help from the theory of measurement, as we saw in chapter ??. Given certain assumptions about how these nonnumeric mass predicates behave:

- $x \succeq y$: x is at least as massive as y
- $Cxyz$: x and y's combined masses equal z's

one can prove representation and uniqueness theorems:

Representation theorem There exists a function f which assigns real numbers to concrete objects, subject to the constraints i) $f(x) \geq f(y)$ iff $x \succeq y$ and ii) $f(x)+f(y)=f(z)$ iff $Cxyz$

Uniqueness theorem Any function g obeying constraints i) and ii) is a scalar multiple of any other such function h—i.e., for some positive real number k, $g(x)=kh(x)$, for all x

As we saw, these theorems show how numerical statements about mass "code up" facts about $\succeq$ and C.

Given these theorems, one can say the following. The fundamental concepts of mass are $\succeq$ and C. The relations to real numbers in particular units are all of them *non*fundamental. The equivalence of unit-differing theories then amounts to their saying the same thing about the world fundamentally: they agree on the fundamental mass facts, namely, the facts that can be expressed using $\succeq$ and C.

So far so good: this is a pretty plausible, if controversial, view of mass. (One could say parallel things given certain other views about the nature of quantity, such as "mixed absolutism" (section 3.6).)

But notice a feature of the approach. In order to say that two theories (corresponding to different units of measure) are equivalent, we needed to find a *third* theory, more fundamental than the first two, from whose point of view the first two equivalent theories can be regarded as different equally good ways of getting at the same facts.[9] What if there is no such third theory? What if we

[9] Claims of equivalence don't always require a third theory: one of the two theories compared for equivalence might itself be a fundamental theory, which can supply grounds for the claims of the other. (See also note 11.)

have hit "rock bottom" already, when we considered the first two theories, and there is no more fundamental level of description to which we could descend?

The case of ontology is arguably just such a case. Hirsch wants to say that the theories of Lewis and van Inwagen are equivalent. But the dispute between Lewis and van Inwagen is a dispute over *what there is*, which is apparently as fundamental a concept as can be. Van Inwagen and Lewis say different things about what objects the world contains. What third way of describing the world, neutral between van Inwagen and Lewis—and thus somehow not talking about "what there is" at all—could one shift to, in order to regard van Inwagen and Lewis's uses of 'there is' as getting at the same fundamental facts?

There is a long discussion to be had here, but my own view is that there is no good third theory Hirsch could appeal to, and that Hirsch is wrong roughly because of this.[10] Differing ontological theories are not equivalent; there is a genuine issue about whether composite objects really exist. But someone deeply committed to saying that "competing" ontological theories

[10] In more detail: let Lewis's sole claim be (in his language) that "every two objects are part of a third", and let van Inwagen's sole claim be (in *his* language) that "it's not the case that every two objects are part of a third". Now suppose, first, that the language of the third theory is a standard quantificational one: it speaks of what there is. I suppose further that it contains the predicate 'is part of'. (Similar points would hold even if the language lacks 'is part of', since in certain cases van Inwagen and Lewis disagree over how many entities exist.) Then the third-view language contains the very sentence "Every two objects are part of a third", which will be either true or false. If it is true then Lewis's claim is true and van Inwagen's is false, and hence they are not equivalent. (Objection: van Inwagen's claim is still true since it is grounded in a third-theory claim in which the quantifier is restricted. Reply: van Inwagen would be willing to stipulate an understanding of his quantifiers under which this cannot be.) If the sentence is false then Lewis's claim is false and van Inwagen's is true, and so again they are not equivalent. (Objection: Lewis's claim is true because it is grounded in a third-theory claim that is not existential in form. Reply: as with van Inwagen, Lewis would be willing to stipulate an understanding of his quantifiers under which this cannot be.) For more on all this, see Sider (2009).

Suppose next that the third-view language lacks the standard quantifiers. But as we saw in chapter 3.10, it's very hard to see how an adequate fundamental theory can be based on such a language. The only views with any real plausibility that we considered there—bare particulars and generalism—are of no use here since we can raise the questions of composition using vocabulary that those views deem fundamental. For instance, if parthood is a fundamental relation, then we can translate "Every two objects are part of a third" into Dasgupta's term functorese. The resulting sentence would be a claim about fundamental reality, and would be either true or false; and either way, the argument can proceed as before. Term functorese thus does not yield the sort of third theory Hirsch needs: a theory that is neutral on the question of when composition occurs.

are equivalent might take this as an objection to the fundamentality-based approach to equivalence: that approach requires there to be a "third theory" if theories are to be equivalent; Lewis and van Inwagen's views are equivalent despite the nonexistence of a third theory; so the approach is wrong.

I'm not sure this puts much dialectical pressure on the approach since I don't think Hirsch's ontological conventionalism is particularly well-motivated, but in other cases there is more pressure. I have said that the concept of existence is fundamental. What concept is that, exactly? A natural answer is: the existential quantifier of first-order logic. But there is also the universal quantifier of first-order logic. The two are related in the familiar ways:

$$\forall x A(x) \text{ iff } {\sim}\exists x {\sim} A(x)$$
$$\exists x A(x) \text{ iff } {\sim}\forall x {\sim} A(x)$$

So I face the question: is $\exists$ the fundamental concept and $\forall$ derived using the first equivalence? Or is $\forall$ the fundamental concept and $\exists$ the derived one, via the second equivalence? My approach to equivalence leads to accepting this as a genuine question, whereas one is inclined to feel that it is not.[11]

The problem isn't restricted to logical terms like quantifiers. Take the allegedly fundamental concepts $\succeq$ and C. Why $\succeq$ rather than its converse? Perhaps this can be blocked by the—attractive—view that a relation is identical to its converse (Williamson, 1985; Fine, 2000; Dorr, 2004). But not all cases are like that one. Suppose mereology is fundamental; there is then the question of which concept of mereology to regard as the fundamental one: parthood, overlap, fusion, or some other.

In some cases there may be theoretical considerations favoring one rather than another choice, which can be regarded by the friend of fundamentality as super-empirical virtues of claiming that the chosen concept is more fundamental.[12] But it is hard to believe that there will always be some such

[11] See recent work by Michaela M. McSweeney, Stephen Steward, and Rohan Sud on this and related issues. Note: the problem isn't that my approach of equivalence falsely predicts that a theory stated using $\exists$ is inequivalent to the corresponding theory stated using $\forall$. For even if $\exists$ is fundamental and $\forall$ is not, the theories would still "say the same thing at the fundamental level"; they would each say that which one would say more "perspicuously" using $\exists$. In light of this point, we might say that theories are "weakly equivalent" if they say the same thing at the fundamental level, and "strongly equivalent" if in addition they employ concepts that are "equally fundamental". (Defining equal fundamentality in terms of fundamentality simpliciter is nontrivial; see Sider (2011, section 7.11.1).) Thus we can say that the theory stated in terms of $\exists$ is weakly but not strongly equivalent to the theory stated in terms of $\forall$.

[12] See Donaldson (2015); Torza (2017) for discussion of some related issues.

considerations.

5.5 "Quotienting out" conventional content "by hand"

What though is the alternative to the fundamentality-based approach? Let me mention one—the second extreme approach to equivalence I want to discuss—according to which, roughly, we can say *that* theories are equivalent without saying *why* they are equivalent in terms of fundamentality and underlying third theories. What *is* equivalence, according to this view? No illuminating account can be given, according to this approach (which is not to say that equivalence is somehow a metaphysically fundamental notion).

Let's consider an example. In the case of $\forall$ and $\exists$, the defender of this second approach might make the following speech:

> A good theory can be formulated using the concept of $\forall$. But one can formulate an equivalent theory using the concept of $\exists$ instead. Indeed, we can define a relation between theories that guarantees equivalence: differing solely by exchanges of formulas QvA and $\sim Q'v\sim A$ (for Q one of $\exists$ and $\forall$, and Q' the other). True, we cannot provide a third, "more fundamental" description of quantificational reality underlying this relation. But no such theory is needed; it's enough simply to say that theories standing in the relation are equivalent.

Similarly, one could say that any theories differing solely by a unit of measure are equivalent, or that Lewis and van Inwagen's theories are equivalent, without saying *why* these theories are equivalent via a third, more fundamental theory. "But what are you saying reality is like?", you might protest. The answer will be that reality is such as to be well-described in any one of the equivalent ways, and that there is no need to say anything further.

I think of this view of equivalence as flowing from a certain view about the nature of modeling, realism, etc., concerning the way one separates representational content from "artifacts of the model". Everyone agrees that a good model can have features that aren't part of its representational content. For example, a map of the USA drawn on paper isn't representing the USA as being made of paper. For another example, suppose one chooses to represent mass using kilograms. One is then using real numbers as a kind of model, a model in which, for example, the number 1 is used to model the mass of a certain thing (a 1kg thing). But the fact that the number 1 is used isn't part of the representational content of the model; it's an artifact of the choice to use one scale rather than another for measuring mass. (The object isn't objectively 1 in mass, assuming there is no distinguished unit.)

I think that many, maybe most, metaphysicians tend to assume (perhaps implicitly) something like the following:

> It's ok to construct models of some phenomenon, with artifacts. But there must always be some way of describing the phenomenon that in some sense[13] does not have artifacts, some way of saying what is really going on. For example, modeling mass with real numbers is fine, but we need an underlying artifact-free description, such as the $\succeq$ and C description, from which one can recover a specification of which numerical models are acceptable, and a specification of which features of these models are artifacts.

Whereas the second approach to equivalence I am describing here rejects this assumption, and says instead:

> There may be no way to say what is "really" going on; maybe every good model has artifacts. It's ok to just say: this model does a good job of representing the phenomenon, but certain features of the model are artifacts. Moreover, for any model, we can say which features of the model are genuinely representational and which are artifacts. There is no need to provide some privileged, artifact-free description from which we can recover this information.

(Some of this is reminiscent of the semantic view of theories, but it seems to me that whether theories are sentence-like or model-like, there will arise the issue of whether a demand for a certain sort of best theory is legitimate.)

Think of it this way. If we have a set of theories with conventional differences, my opponent says that one can "quotient out" the conventional content and regard the best description as an equivalence class of theories. Moreover,

[13]A statement written in red ink about $\succeq$ and C ought to count as an artifact-free statement about mass, even though it isn't representing the world as being red. A first pass at the needed clarification: the only artifacts in this representation are nonsemantic in nature. (Sider (2011, pp. 221–2) wrestles with a similar issue.)

one can do this "by hand": the equivalence relation doesn't have to be induced by some more fundamental theory—as I think—but rather can simply be stipulated.

The quotienter I'll primarily have in mind rejects all talk of fundamentality. However, a milder quotienter might try to hang on to some such talk. She might, for instance, allow a distinction between fundamental and nonfundamental facts; the idea would be that some fundamental facts are equivalent to others.[14]

5.6 The significance of quotienting

In my view the question of whether quotienting is legitimate has profound implications for a wide range of questions in the metaphysics of science, and within metaphysics generally. As I see it, the question is a crucial choice point for foundational theorizing about science, which has great implications downstream. The question has generally lurked below the surface, but in retrospect can be seen to underlie various disputes, especially when some disputants are more hostile to metaphysical inquiry than others. Speaking just for myself, once I got the issue of quotienting clearly in view, a number of otherwise extremely perplexing disputes were dramatically clarified; I could finally see exactly what is at stake.

For a very simple example of this, consider language like "only such-and-such an equivalence class is real" or "what is real is that which is common to all members of the equivalence class". Language like this is quite common (especially in the philosophy of physics), but is perplexing. The first sentence is practically absurd if taken literally, and the latter uses a noun phrase ("that which is common…") without a clear referent. But they make perfect sense if taken as an expression of the quotienting view.

In the following sections we'll examine some more substantive illustrations of the importance of the question of the legitimacy of quotienting.

5.6.1 Quotienting and structuralism

As we've seen in previous chapters, structuralist theses can be difficult to articulate in postmodal terms. Sometimes there simply is no attractive view of the nature of fundamental reality that is structuralist in spirit. But a structuralist

[14] The views of Sud and Steward are not unrelated.

who likes quotienting could dismiss all those concerns as arising from an illegitimate demand for metaphysics. The structuralist slogan that nodes are not "independent of" patterns could naturally be taken as a claim of equivalence: variation of nodes while leaving the pattern intact results in an equivalent theory. And given quotienting, instead of attempting to specify a structuralist-inspired conception of the fundamental facts from which these equivalences could be derived, one could simply define, by hand, an equivalence relation on descriptions of patterns. All the problems I raised in previous chapters would immediately be avoided, just like that.

For instance, the nomic essentialist could define a relation between theories that holds when those theories differ only by a permutation of scientific properties in the laws, and claim that theories standing in this relation are equivalent. She could stop with this kind of claim, rather than trying to develop a distinctively nomic essentialist conception of the fundamental facts.

Similarly, in the case of quantity one could simply say that theories that differ by a mere choice of unit are equivalent, without saying why that's so. And if one thought that further differences are also merely conventional—for example doubling the mass of each thing, which is regarded by comparativists as a distinction without a difference—one could make further claims of this sort, again without the need for a distinctively comparativist conception of the fundamental facts.

Likewise for the case of structuralism about individuals, in which this approach may be especially appealing. Instead of struggling to find an entity-free fundamental account of reality, structural realists (for example) could simply claim that certain descriptions are equivalent—descriptions that differ solely by a permutation of individuals, for instance.

(Relatedly, recall the discussion of "scooping out" structure in section 4.17. A conception of the physical structure of reality (of space, say) might have an "aspect" that the laws don't care about—the laws don't care about the identities of points of space or particles. Nevertheless, given realism about fundamental concepts, this aspect can't simply be "scooped out" leaving the rest of the theory intact. We need to see whether the aspect corresponds to some fundamental concept or concept that can be eliminated while keeping the theory attractive. But given quotienting, arbitrary aspects *can* be scooped out in this way. For one can simply define up an equivalence relation corresponding to the aspect in question.)

As it happens, structural realists seem not to have availed themselves of the quotienting option. They seem instead to have engaged in a more traditional metaphysical project, of seeking an account of the nature of the fundamental facts that eliminates the sorts of differences that they regard as nongenuine.[15] But I wonder whether the more deeply antimetaphysical approach of quotienting might be a better fit for their overall outlook.

5.6.2 Quotienting and quantum mechanics

Another illustration of the importance of quotienting comes from the dispute over the metaphysics of the wavefunction in quantum mechanics. The most straightforward metaphysics of the wavefunction treats it as a fundamental field, living in a fundamental, substantival configuration space (Albert, 1996, 2015). But since this massively-dimensional space is wholly distinct from the three-dimensional space of ordinary experience, many regard this straightforward metaphysics as unbelievable.

One might instead regard the wavefunction as being a field in abstract configuration space, a constructed space whose "points" are set theoretic constructions that represent the locations of all particles in three-dimensional space. But this approach is unattractive on its face, and no one seems to have pursued it. Perhaps many have bypassed it because it attaches a fundamental physical magnitude to abstract entities. I myself think its main vice stems rather from the arbitrariness of the construction of abstract configuration space: on which of the many abstract entities that can with equal justice be regarded as configuration space is the wavefunction defined?[16]

But for a quotienter, the metaphysical status of the wavefunction is not at all puzzling. Indeed, the quotienter will likely regard angst over the status of the wavefunction and configuration space as deriving from an illegitimate demand for metaphysics. For one can simply say the following:[17]

> You got into trouble because you were trying to specify, once and for all, what the fundamental properties and laws were, independently of conventional choices. That's misguided. The proper procedure is instead the following. First we must make some arbitrary choices for how to construct configuration space. *Given* those choices, there

[15] McKenzie (2017) begins a survey article by saying "While a number of distinct positions go under the banner of 'ontic structural realism' (OSR), common to them is the insistence that the structural features of reality should be accorded ontologically fundamental status".

[16] For a discussion of parallel issues see Sider (1996).

[17] The quotienting outlook seems to be best conveyed through speeches.

> is a certain physically significant wavefunction on configuration space. But we cannot, and need not, say what the physical facts and laws are, independent of our conventional choices.
>
> More fully: there are various theories, $T_1, T_2 \ldots$ based on different equally acceptable constructions of abstract configuration space. In each case, the theory T_i will utilize a physical predicate Ψ_i for wavefunction values at points in abstract configuration space as constructed in T_i. And although a relation of equivalence can be defined between the theories, which will constrain the relationships between the physical predicates Ψ_i, there is no definition of any of the predicates Ψ_i in terms that don't refer to the others. This all may seem unsatisfying. "Which aspect of physical reality are these predicates Ψ_i representing", you ask? Surely physics is about how the world is, in and of itself? But I am not denying this. The world is such as to be representable by a wavefunction in an abstract space, by any of the theories T_i; and it is this way independent of any conventional choices. But there's there's nothing further to say, and no need to say anything further, than that. There is no "God's-eye" description of reality.

Thus the quotienter would be denying the need to ground the use of mathematical representations of the wavefunction with a representation theorem; we can stop with the mathematical representations.

Indeed, one might take this a step further, and regard even the question of whether the "fundamental space" is ordinary three-dimensional space or massively-dimensional configuration space as itself being conventional. On this view, there's nothing more to say about the "real" dimensionality of physical reality, and about the ontology of space, other than that reality can be described as containing a concrete configuration space with a complex-valued field on it, but can also be described as containing a concrete three-dimensional space, in which the wavefunction is modeled in some more complex way.[18]

David Wallace in fact takes something like this line. He writes of "a gap in the market for some intermediate philosophical position, one which respects scepticism about overly 'metaphysical' claims while incorporating the impossibility of any coherent theory/observation divide" (Wallace, 2012, p. 314) What

[18]Note the conventionalism about ontology. This does not require antirealism in the sense that reality is up to us; see Hirsch (2002).

he has in mind is the following. On the one hand, he thinks that the question of whether space is *really* three-dimensional is not a genuine one (it's "overly metaphysical"). Similarly, many would say, the question of whether Lewis is right that there exist tables and chairs, or whether van Inwagen is right that there do not, is not a genuine question. But the most familiar way of rejecting there being a genuine question here is that of the discredited positivists, whose main sin (according to Wallace) is upholding a sharp divide between theory and observation.

Wallace goes on to say that he suspects the gap in the market will be filled in by structural realism, but I don't think that's quite right. As I read much of the structural realist literature, it accepts a more standard "metametaphysics", thus accepting a demand for a (somewhat) artifact-free metaphysics—otherwise, why take so seriously worries about the coherence of a conception of metaphysics without objects, relations without relata, and so forth? What Wallace is really after is a much more conventionalist attitude toward ontology and metaphysics in general; quotienting would provide that.

5.6.3 Quotienting and modality: Stalnaker

Another instance of the quotienting strategy, I believe, can be found in Robert Stalnaker's book *Mere Possibilities*. Stalnaker wants to defend Kripke's possible worlds semantics for modal concepts like possibility and necessity. He is an "actualist": he rejects Lewis's (1986) modal realism and holds that possible worlds are just parts of actuality: they are properties, he says—ways reality could be. Now, there is an old problem with this approach: how to construct possible worlds containing entities that do not in fact exist? Consider the possibility of rolling a pair of dice that are exactly alike and distinct from every actual entity, so that the roll adds up to 3. There should be two possible worlds here: one where one die comes up 1 and the other comes up 2, and a second world where the first die comes up 2 and the second die comes up 1. But since these dice do not in fact exist, it is hard to see how there really could be two worlds, if worlds are just properties and actualism is true.

Stalnaker's solution to this problem is intriguing. Stalnaker accepts the usual sort of Kripke semantics, except that the set W that Kripke would call the set of "worlds" Stalnaker instead calls the set of "points". The set of points corresponds, to a first approximation, to what David Lewis would regard as all the possible worlds. And so there are two points corresponding to the possibility of our dice summing to 3: one for 2/1, the other for 1/2. What *are* these points? The particular entities that are the points, Stalnaker says, are unimportant—their identity has no particular representational significance in the model. (They could be bananas, or fish, or whatever; "it's only a model".) Moreover, neither point individually represents anything different from what the other point represents. If you ask what one of these points represents, taken apart from the rest of the model, you get the same answer as for the other: the possibility of two duplicate nonactual dice summing to 3. Neither point goes about its job of representing this possibility (in a representationally significant way) differently from how the other does; and there simply *are* no possibilities of *particular* nonactual dice A and B coming up 2/1, say, for them to differentially represent, since there do not in fact exist dice A and B. But the fact that the model contains *two* points here rather than one *is* representationally significant: it represents the modal fact that had two such dice existed, there would have been two possibilities.

Since the points corresponding to nonactual possibilities (so to speak) are not representationally significant, there are different Stalnakerian models that are representationally equivalent. Stalnaker therefore provides an account of which of his models are representationally equivalent to others—thus, I would say, quotienting out the artifactual content. And he does this "by hand", in that he does *not* give any further account of what possible worlds are, or what properties are, or what the modal facts are, that renders this class of models apt, or justifies his claims about which models are equivalent. His attitude is: modal reality is such as to be well-modeled in this way, and there's no need to give any further, artifact-free account of modality reality that shows *why* this is the case.

Stalnaker is up front that a distinctive view about the nature of metaphysics, and of the division between substantive and conventional/semantic questions is central to his approach.[19] Indeed, he compares his equivalence classes of Kripke models (differing by which entities play the role of which "mere points") to equivalence classes of models of space as a set of points that would be given by a relationalist: the relationalist will regard models differing only by e.g. translations as being representationally equivalent. What I think is going on is that Stalnaker is a quotienter.

[19]E.g. Preface, p. x.

5.7 Quotienting is unsatisfying

What can be said in defense of the fundamentality approach to equivalence, and against quotienting? I won't have anything very decisive to say; my main goal is simply to get the issues here—the possibility of quotienting, and the metaphysician's usual presupposition of the requirement of artifact-free representation—out in the open. Still, I do oppose quotienting and will say what I can against it.

In actual fact, mere quotienting out isn't normally pursued—except when there seems to be no other choice. Philosophers of science, for example, have invested a lot of effort in finding representation theorems for various sorts of measurement. Granted, this work is motivated by many different desires, but surely part of it is dissatisfaction with merely quotienting out the conventional content of a unit of measure—a desire for a more satisfying account of *why* any scalar transform of a mass function is just as good as any other. Similarly, philosophers of physics often prefer coordinate-free formulations of geometric theories, rather than coordinate-theories with quotiented-out conventional content.

This is especially evident when we consider ontology. The following attitude is *not* typical: the facts in a certain domain can be well-represented by a theory that takes there to be a certain set of objects; but we could equivalently take there to be a quite different set of objects provided we make certain adjustments. A typical—I take it!—reaction to this attitude would be that until an ontology is specified, it hasn't been yet made clear what the theory is saying about the world. (That's not to say that there aren't exceptions—David Wallace is an example.)

Second, at an intuitive level, quotienting is intuitively unsatisfying precisely because it gives no answer to the question: why *this* equivalence relation? When multiple equally good ways to represent are available, it is natural to ask why that is, to ask what it is about reality that enables it to be multiply represented.

Consider Leibniz himself, who held a view about which theories of space are equivalent: they are those that preserve the relations between material bodies. It would have been far less compelling if Leibniz had said merely that descriptions of objects in space are equivalent when and only when they differ only by some combination of translations and rotations.

Most would, I think, agree with that last claim; they would demand at least a partial answer to the question of why such descriptions are equivalent. But fewer would be happy with the full demand I would make: I would ask of Leibniz not only that he specify the ontology of his theory that generates the equivalence (material bodies), and the *kinds* of properties and relations the theory concerns (spatial relations), but also that he specify *which* spatial relations are the fundamental ones—are they relations like $\succeq$ and C or others? It's interesting that most philosophers' intuitions are thus halfway between the quotienter and my own approach.

I say that "why this equivalence relation?" demands an answer; the quotienter rejects this demand; can we make any dialectical progress? Perhaps. Suppose the quotienter acknowledges in *some* cases that it's better to "explain" relations of equivalence in terms of a deeper theory. Suppose, for instance, she concedes that moving from a family of unit-based theories of mass (together with an appropriate equivalence relation) to a single theory based on $\succeq$ and C is some sort of theoretical improvement, or that coordinate-free geometric theories are preferable to coordinate-based ones, and that this is for metaphysical reasons. Then a tension in her position has arisen, since she no longer can maintain that there is nothing at all wrong with quotienting. She may reply that quotienting is a necessary evil: to be avoided whenever possible, but sometimes unavoidable. But in so saying she concedes that what *I* say should be the goal—a theory in need of no quotienting—does indeed result in a superior theory.

My complaint about quotienting has been that it has frustratingly unexplained explainers: the equivalence relation put in "by hand". It's worth noting that my argument with the quotienter is an instance of a larger pattern of dispute between friends and foes of metaphysics. Foes accuse metaphysicians of asking questions whose weight our language cannot bear: language has clear sense when used within certain contexts (such as within science) but not otherwise. Friends reply that it is difficult to allow the familiar questions while disqualifying metaphysical questions as nonsensical. I myself regard the demand for an explanation of the quotienter's equivalence relations—an explanation given via a proposed inventory of the fundamental concepts—as akin to the demand for understanding that motivates scientific investigation: we demand an answer to the question of what are reality's ultimate constituents? But foes of metaphysics will regard the demand for metaphysical understanding as being a perversion of the legitimate demand for scientific understanding. This overarching opposition is ongoing.

5.8 Progress can be unexpected

I turn now to defensive maneuvers on behalf of the fundamentality-based approach.

As we saw, the approach leads to problematic questions, for example whether the fundamental quantificational notion is $\forall$ or $\exists$.[20] One concern about such questions is that they sure seem unanswerable. But sometimes we do, in the end, make progress on such questions.

There is an excellent recent example of this. Kit Fine's "Towards a Theory of Part" gives compelling—which is not to say irresistible—reasons to favor basing mereology on fusion rather than parthood or overlap. Such questions are, we would have antecedently thought, paradigmatically unanswerable, and indeed, questions on which one could *never* have the *slightest* reason in favor of one answer or another. But then along comes Fine giving reasons—of a completely different sort than I, anyway, had expected to be relevant—for a particular answer to the question. It can happen; we can't know in advance that it won't.

Notice that the attitude "we could never possibly have reason to favor one answer or another" is precisely the attitude that nonphilosophers tend to have to all of philosophy. It's practically the job description of a philosopher to somehow find considerations relevant to what look initially like questions that are good only for baseless speculation.

5.9 Hard choices are hard to avoid

The worrisome thing is being saddled with the choice of whether to say that, for example, it is $\exists$ or $\forall$ that is fundamental. But in fact it's hard to avoid acknowledging the existence of such choices. Recall Goodman's grue/bleen example:

> An object is *grue* iff it is green and first observed before 3000 A.D. or blue and not first observed before 3000 A.D.
>
> An object is *bleen* iff it is blue and first observed before 3000 A.D. or green and not first observed before 3000 A.D.

Goodman points out that although these equivalences define grue and bleen in terms of green and blue, one could reverse the definitions:

[20] Jessica Wilson (forthcoming) might call them "spandrel questions".

> An object is *green* iff it is grue and first observed before 3000 A.D. or bleen and not first observed before 3000 A.D.
>
> An object is *blue* iff it is bleen and first observed before 3000 A.D. or grue and not first observed before 3000 A.D.

We can play this trick with fundamental physical quantities, and we could rewrite physical theories using the cooked-up predicates. Now, if you're completely happy with quotienting, you will say that there's nothing wrong with this, so long as we specify the relevant relations of equivalence. But suppose you're at least somewhat sympathetic to the fundamentality approach, and admit that the "grueified" theories fail to properly express the physical facts. The objects attracted each other because they had opposite charges, you insist, and not because they had opposite schmarges and were first observed before 3000 A.D.! In that case you admit *some* questions of the $\exists$ versus $\forall$ variety; and if you don't like the $\exists$ vs $\forall$ question itself, the problem becomes where to draw the line.

5.10 There can be more than one

Another defensive move: the question "is it $\exists$ or $\forall$ that is fundamental?" leaves out a third possibility, namely that they are *both* fundamental.

I grant a general presumption in favor of fewer fundamental concepts—that's parsimony. But parsimony isn't the only relevant consideration: there is also avoidance of arbitrariness. Perhaps parsimony should sometimes be sacrificed to avoid arbitrariness.[21]

5.11 Why think we can know everything?

Pointing out that "both" is an available answer to "is it $\exists$ or $\forall$ that is fundamental?" mitigates one concern, which is that there should be no metaphysical asymmetry between $\exists$ and $\forall$. But another concern is simply that we have no way of knowing what the answer to the question is. That concern remains, since we seem to have no way of knowing whether it's $\exists$ or $\forall$ or both that is fundamental.

[21] See Sider (2011, section 10.2).

Here the central reply is simple: why think we can know everything? The defender of the fundamentality approach should not be shy about saying that she does not always know which concepts are fundamental. L. A. Paul (2012*b*, p. 21) put it well: "It is the fate of philosophy to have many too many options".

Though that is the central reply, more can be said to make that reply feel less abrupt, and more comfortable to embrace.

First, I really do think philosophers (nonmetaphysicians, mostly) throw around the "your proposed metaphysics leads to unknowable facts, so it should be rejected" argument far too easily. To seriously defend such an argument, one get into a very old dialectic: we need a principle that somehow bans the target metaphysical questions without also eliminating, for example, legitimate scientific questions.

In my view, unknowability does not *on its own* constitute something's having gone wrong, but it might be a sign that the theory is employing concepts that aren't in good standing. We need to look at the concepts involved in the unknowable question, to see whether there is good reason, on general systematic grounds, to think that those concepts are in good standing. In particular, do those concepts play a central role in *other* questions that are part of legitimate inquiry? If so, that is a good reason to think that they are in good standing; and then, the fact that they can also be used to raise a question we can't see how to answer (or even begin answering) is no reason at all to think that the question is somehow ill-posed. Compare, in a special relativistic setting, questions about what is going on outside our light-cone. Such questions are clearly legitimate because they're framed using concepts we have reason to think are in good standing—concepts that latch onto real features of the world—and the fact that those concepts can then be combined to raise a question that we can't answer—and in this case have reason to think is in-principle unanswerable—has no tendency at all to show that the question is illegitimate.

Second, someone who rejects the concept of fundamentality because of a desire to avoid unanswerable questions may be living in a glass house. For rejecting the notion of fundamentality is itself is a metaphysical stance, albeit a negative one, and it's not at all straightforward what justifies it.

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