

# Compiling without continuations

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# Peter Landin 1930-2009



"I recall knocking on Peter's door one evening to show him some observations about the *zip* function. As I was writing down the observation on Peter's blackboard, I asked him if he was familiar with this function; he replied "I think I may have invented it".

Paul Boca

"Around Easter 1961, a course on ALGOL 60 was offered, with Peter Naur, Edsger W. Dijkstra, and Peter Landin as tutors. ... It was there that I wrote the procedure, immodestly named QUICKSORT, on which my career as a computer scientist is founded. Due credit must be paid to the genius of the designers of ALGOL 60 who included recursion in their language and enabled me to describe my invention so elegantly to the world. I have regarded it as the highest goal of programming language design to enable good ideas to be elegantly expressed."

Tony Hoare



## Peter Landin 1930-2009

“His approach is best described by John Reynolds: ‘*Peter Landin remarked long ago that the goal of his research was to tell beautiful stories about computation*’ Reynolds (1999). Furthermore, Peter aimed for precision, to be contrasted with formality, in story telling.

...

“My recollection of Peter is that he was funny, generous, haughty, egalitarian, shy, uncertain, curious and ferociously intellectual. But, *boy* what beautiful stories!”

Tony Clark

# Peter Landin 1930-2009



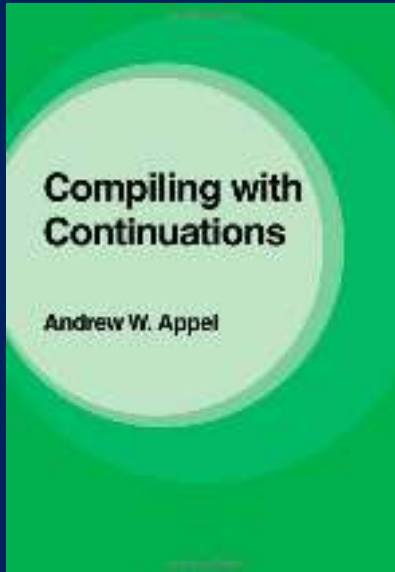
“To be able to translate Algol 60 into applicative expressions, Landin later extended these expressions and their interpreter with an assignment operation, and also a ***control operator J used to express the translation of goto's and labels*** [15, 16]. In the extended SECD machine, the result of applying J was a value containing a dump.

***Thus, in modern terminology, the J operator provided a means of embedding continuations in values*** and was an ancestor of operations such as Reynolds's escape [36], and catch [44] and call/cc [8] in Scheme.”

John Reynolds

Functional programming is all about data flow  
But Landin recognised the importance of control flow too

**Continuation passing style (CPS):**  
clever, but complicated



Zap!

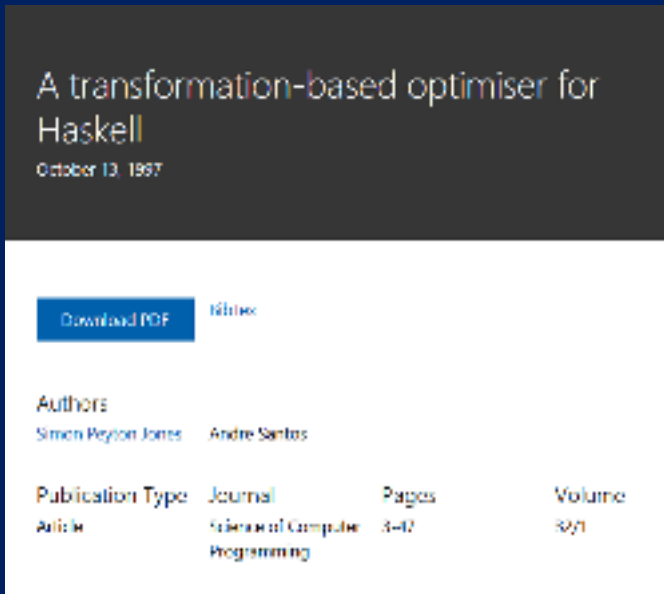
Flanagan et al, PLDI 1993

Blam!

Kennedy et al, ICFP 2007

Pow!

**Direct style:**  
simple,  
but misses  
some tricks



**Direct style with join points:**

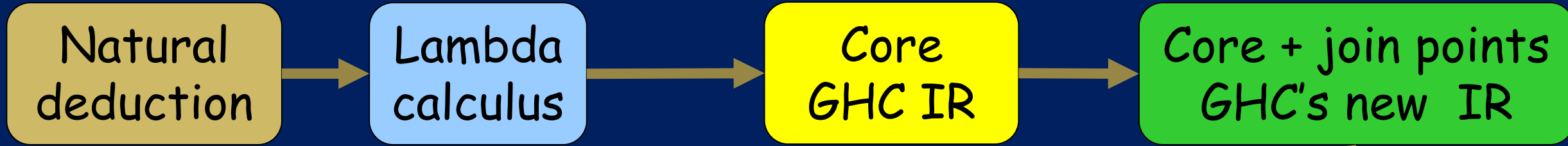
Simple, simple, simple

All the goodness of CPS  
with none of the pain

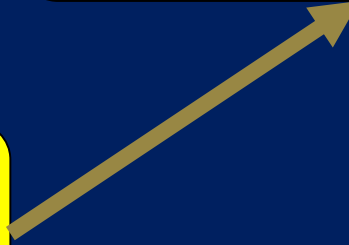
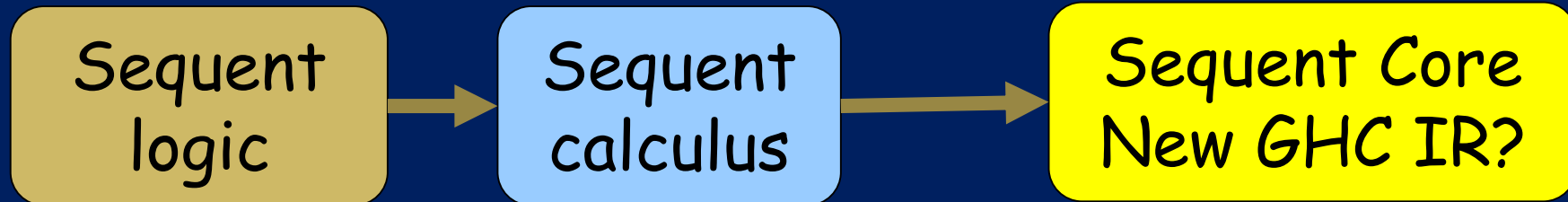
Unexpected new wins  
(fusion)

Works at scale (GHC)

[PLDI 2017]



Curry-Howard isomorphism



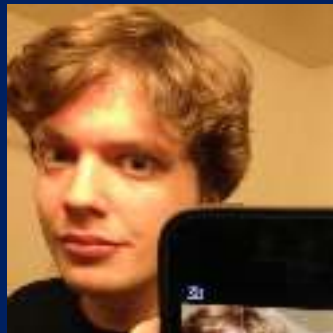
[ICFP 2016]



Zena  
Ariola



Paul  
Downen



Luke  
Maurer

# How GHC works



## Haskell

Massive language.  
Hundreds of  
pages of user  
manual.

Syntax has dozens  
of data types  
100+ constructors



Typecheck



Desugar



Core language:  
optimisation  
happens here



Rest of GHC

# Example

```
module Prelude where
```

```
not :: Bool -> Bool
```

```
not True = False
```

```
not False = True
```

```
null :: [a] -> Bool
```

```
null [] = True
```

```
null (x:xs) = False
```

```
data Bool = False | True
```

```
data [a] = [] | a:[a]
```

Haskell

```
module Prelude where
```

```
not :: Bool -> Bool
```

```
= \ (b::Bool) . case b of
```

```
    True  -> False
```

```
    False -> True
```

```
null :: [a] -> Bool
```

```
= \ (xs::[a]) . case xs of
```

```
    []      -> True
```

```
    (x:xs) -> False
```

Core



# Example

```
data [a] = [] | a : [a]

map :: (a->b) -> [a] -> [b]
map f []      = []
map f (x:xs) = f x : map f xs
```

```
map = \ (f::a->b) . \ (xs:[a]) .
      case xs of
        []      -> []
        y:ys    -> f y : map f ys
```

Haskell

Core

# Example

```
data [a] = [] | a : [a]

filter :: (a->Bool) -> [a] -> [a]
filter p [] = []
filter p (x:xs) = if p x
                  then x : filter p xs
                  else   filter p xs
```

```
filter
= \ (p::a->Bool) . \ (xs:[a]) .
  case xs of
    [] -> []
    y:ys -> case p y of
              True  -> y : filter p ys
              False ->   filter p ys
```

Haskell

Core

# Commuting conversions

# Case of case

```
notNull xs = not (null xs)
```

```
= case (null xs) of  
    True  -> False  
    False -> True
```

```
= case (case xs of  
        []      -> True  
        (p:ps) -> False) of  
    True  -> False  
    False -> True
```



```
module Prelude where
```

```
null xs = case xs of  
    []      -> True  
    (x:xs) -> False
```

```
not b = case b of  
    True  -> False  
    False -> True
```

```
case e of  
    True  -> r1  
    False -> r2
```

means

```
if e  
then r1  
else r2
```

CASE:  
case of  
case

```
notNull xs = not (null xs)
```

```
= case (case xs of  
      []      -> True  
      (p:ps) -> False) of  
  True  -> False  
  False -> True
```

```
= case xs of  
  []      -> case True of  
              True -> False  
              False -> True  
  
  (p:ps) -> case False of  
              True  -> False  
              False -> True
```

```
= case xs of  
  []      -> False  
  (p:ps) -> True
```



Commuting conversions. All good compilers do them.

A worry

```
= case (case xs of
        []      -> True
        (p:ps) -> False) of
  True  -> BIG1
  False -> BIG2
```

```
= case xs of
  []      -> case True of
    True  -> BIG1
    False -> BIG2

  (p:ps) -> case False of
    True  -> BIG1
    False -> BIG2
```



Duplicates  
arbitrary  
amounts of code  
:-)

# Join points

```
= case (case xs of
      []      -> True
      (p:ps) -> False) of
  True  -> BIG1
  False -> BIG2
```

```
= let j1 () = BIG1
    j2 () = BIG2
  in case xs of
    []      -> case True of
      True  -> j1 ()
      False -> j2 ()

    (p:ps) -> case False of
      True  -> j1 ()
      False -> j2 ()
```

No duplication :-)

So far, a "join point" is just an ordinary function

# Join points

```
= case (case xs of
      []      -> True
      (p:ps) -> False) of
  True  -> BIG1
  False -> BIG2
```

```
= let j1 () = BIG1
    j2 () = BIG2
  in case xs of
    []      -> case True of
      True  -> j1 ()
      False -> j2 ()

    (p:ps) -> case False of
      True  -> j1 ()
      False -> j2 ()
```

```
= case xs of
  []      -> BIG1
  (p:ps) -> BIG2
```



Ordinary  
inlining  
applies

```
= let j1 () = BIG1
    j2 () = BIG2
  in case xs of
    []      -> j1 ()
    (p:ps) -> j2 ()
```

Join points. All good compilers do them.



# Arbitrary patterns

Pattern binds variable  $x$

```
= case (case xs of
      []      -> Nothing
      (p:ps) -> Just p) of
  Just x  -> BIG1[x]
  Nothing -> BIG2
```

Simply abstract over the pattern bound variables

```
= let j1 x = BIG1[x]
   j2 () = BIG2
   in case xs of
     []      -> case Nothing of
       Just x  -> j1 x
       Nothing -> j2 ()

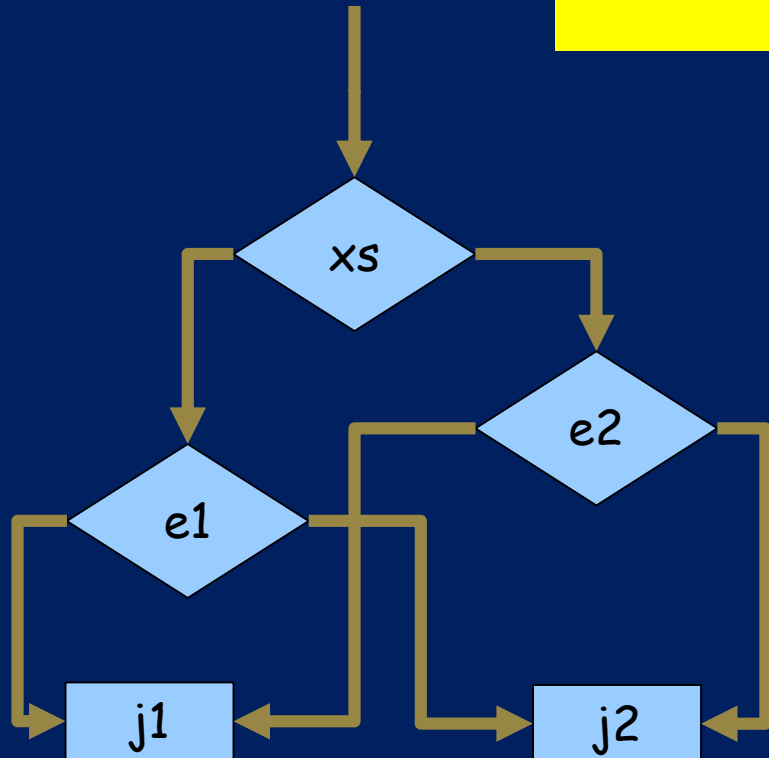
     (p:ps) -> case Just p of
       Just x  -> j1 x
       Nothing -> j2 ()
```

- Works fine for existentials, GADTs

Join points  
are like  
control-  
flow labels

```
= let j1 x = let v = x+1 in Just v
   j2 () = BIG2
in case xs of
  []      -> case e1 of
               Just x -> j1 x
               False  -> j2 ()

  (p:ps) -> case e2 of
               True   -> j1 p
               False  -> j2 ()
```



- Let bindings allocate a thunk or data constructor
- Join points allocate nothing!
- “Calling” a join point = adjust stack pointer and goto
- Typically the back end
  - Spots functions that happen to be join points
  - Implements them as jumps

```
= let j1 x  = let v = x+1 in Just v
   j2 () = BIG2
in case xs of
  []      -> case e1 of
                Just x -> j1 x
                False  -> j2 ()

  (p:ps) -> case e2 of
                True   -> j1 p
                False  -> j2 ()
```

## What characterises a join point?

- All calls are tail calls, relative to binding site
- No call is captured in a thunk or closure
- All calls saturated

**The Main Idea of this talk**

Problem:  
losing join  
points

```
case (let j x = E1
      in case xs of
          Just x  -> j x
          Nothing -> E2) of
  True  -> R1
  False -> R2
```



```
= let j x = E1
   in case xs of
       Just x  -> case (j x) of
                     True  -> R1
                     False -> R2
       Nothing -> case E2 of
                     True  -> R1
                     False -> R2
```

Bad bad  
bad!

Two bad things

1. 'j' is no longer a join point
2. The outer black case does not scrutinise E1

Keeping  
join points

```
case (let j x = E1
      in case xs of
        Just x  -> j x
        Nothing -> E2) of
  True  -> R1
  False -> R2
```

Move outer  
case into  
join point



```
= let j x =
  in case xs of
    Just x  ->
    Nothing ->
  of
    True  -> R1
    False -> R2
```

No no  
no

```
= let j' x = case E1 of
              True  -> R1
              False -> R2
  in case xs of
    Just x  -> j' x
    Nothing -> case E2 of
                  True  -> R1
                  False -> R2
```

As well as the  
case RHS

1. 'j' remains a join point
2. The outer black case now scrutinises E1

Keeping  
join points

```
case (join j x = E1
      in case xs of
        Just x  -> jump j x
        Nothing -> E2) of
  True  -> R1
  False -> R2
```



```
= join j' x = case E1 of
  True  -> R1
  False -> R2
  in case xs of
    Just x  -> jump j' x
    Nothing -> case E2 of
      True  -> R1
      False -> R2
```

Make join  
points part of  
the syntax

Very like let!

Move outer  
case into  
join point

Outer case  
evaporates when  
it hits a jump

Outer case  
wraps this  
RHS

PS: if R1, R2 are big, then you can bind them  
as join points before doing this.

The solution:  
formalise join points as  
language construct



## Terms

$x$	$\in$	Term variables	
$j$	$\in$	Label variables	
$e, u, v$	$::=$	$x \mid l \mid \lambda x:\sigma.e \mid e u$	
		$\mid \Lambda a.e \mid e \varphi$	Type polymorphism
		$\mid K \vec{\varphi} \vec{e}$	Data construction
		$\mid \text{case } e \text{ of } \vec{alt}$	Case analysis
		$\mid \text{let } vb \text{ in } v$	Let binding
		$\mid \text{join } jb \text{ in } u$	Join-point binding
		$\mid \text{jump } j \vec{\varphi} \vec{e} \tau$	Jump
$alt$	$::=$	$K \vec{x}:\vec{\sigma} \rightarrow u$	Case alternative

## Value bindings and join-point bindings

$vb$	$::=$	$x:\tau = e$	Non-recursive value
		$\mid \text{rec } \vec{x}:\vec{\tau} = \vec{e}$	Recursive values
$jb$	$::=$	$j \vec{a} \vec{x}:\vec{\sigma} = e$	Non-recursive join point
		$\mid \text{rec } j \vec{a} \vec{x}:\vec{\sigma} = e$	Recursive join points

$$\boxed{\langle e; s; \Sigma \rangle \mapsto \langle e'; s'; \Sigma' \rangle}$$

$$\langle F[e]; s; \Sigma \rangle \mapsto \langle e; F : s; \Sigma \rangle \quad (\text{push})$$

$$\langle \lambda x.e; \square v : s; \Sigma \rangle \mapsto \langle e; s; \Sigma, x = v \rangle \quad (\beta)$$

$$\langle \Lambda a.e; \square \varphi : s; \Sigma \rangle \mapsto \langle e\{\varphi/a\}; s; \Sigma \rangle \quad (\beta_\tau)$$

$$\langle \text{let } vb \text{ in } e; s; \Sigma \rangle \mapsto \langle e; s; \Sigma, vb \rangle \quad (\text{bind})$$

$$\langle x; s; \Sigma[x = v] \rangle \mapsto \langle v; s; \Sigma[x = v] \rangle \quad (\text{look})$$

$$\left\langle \begin{array}{c} K \vec{\varphi} \vec{v}; \\ \text{case } \square \text{ of } \overrightarrow{alt} : s; \\ \Sigma \end{array} \right\rangle \mapsto \langle u; s; \Sigma, \overrightarrow{x} = \vec{v} \rangle \quad (\text{case})$$

if  $(K \vec{x} \rightarrow u) \in \overrightarrow{alt}$

$$\left\langle \begin{array}{c} \text{jump } j \vec{\varphi} \vec{v} \tau; \\ s' ++ (\text{join } jb \text{ in } \square : s); \\ \Sigma \end{array} \right\rangle \mapsto \left\langle \begin{array}{c} u\{\varphi/a\}; \\ \text{join } jb \text{ in } \square : s; \\ \Sigma, \overrightarrow{x} = \vec{v} \end{array} \right\rangle \quad (\text{jump})$$

if  $(j \vec{a} \vec{x} = u) \in jb$

$$\left\langle \begin{array}{c} A; \\ \text{join } jb \text{ in } \square : s; \\ \Sigma \end{array} \right\rangle \mapsto \langle A; s; \Sigma \rangle \quad (\text{ans})$$

$$F ::= \square v$$

$$| \square \tau$$

$$| \text{case } \square \text{ of } \overrightarrow{p} \rightarrow \overrightarrow{u}$$

$$| \text{join } jb \text{ in } \square$$

- Claim: join points are control flow labels
- Operational semantics validates these claims

Figure 3: Call-by-name operational semantics for System  $F_J$ .

## Core with join points

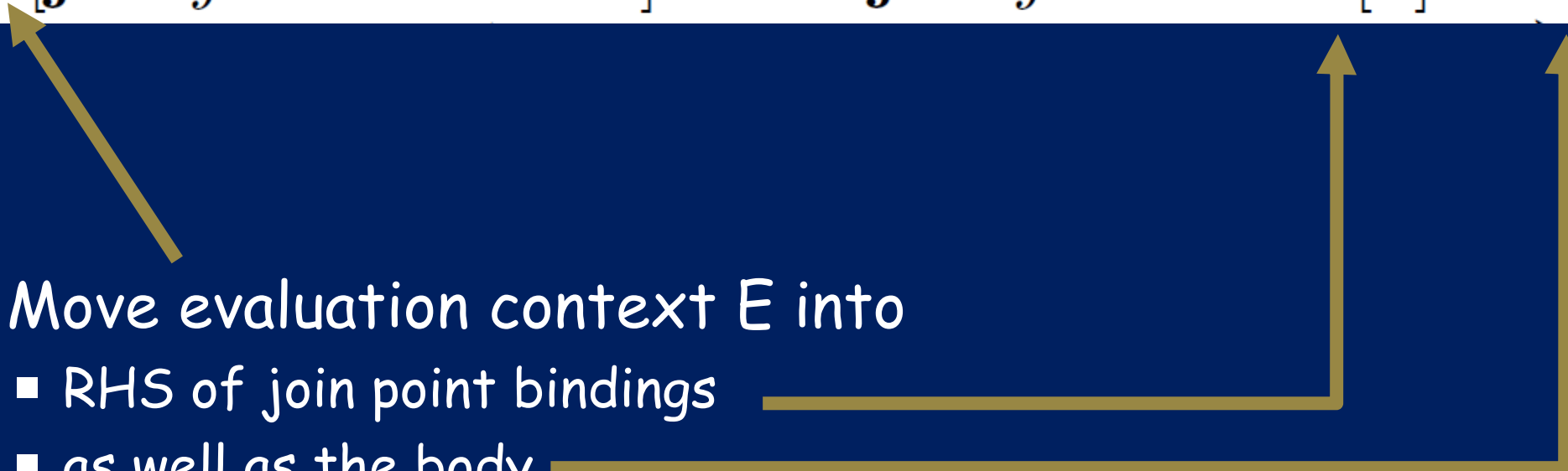
- **Identify** join points as a proper syntactic construct, with typing rules, operational semantics etc
- **Exploit** join points in commuting conversions
- **Infer** which let-bindings are in fact join points (contification)

# Optimising transformations for join points

$$E[\mathbf{let} \textit{vb in } e] = \mathbf{let} \textit{vb in } E[e]$$

$$E[\mathbf{join} \textit{j} \vec{a} \vec{x} = u \mathbf{in} e] = \mathbf{join} \textit{j} \vec{a} \vec{x} = E[u] \mathbf{in} E[e]$$

- Move evaluation context  $E$  into
  - RHS of join point bindings
  - as well as the body

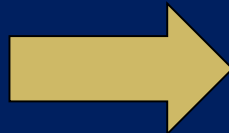


# Optimising transformations for join points

$$E[\text{let } vb \text{ in } e] = \text{let } vb \text{ in } E[e]$$

$$E[\text{join } j \vec{a} \vec{x} = u \text{ in } e] = \text{join } j \vec{a} \vec{x} = E[u] \text{ in } E[e]$$

```
case (join j x = B1
      in case xs of
        Just x  -> j x
        Nothing -> B2) of
  True  -> R1
  False -> R2
```



```
join j x = case B1 of
  True  -> R1
  False -> R2

in case xs of
  Just x  -> j x
  Nothing -> case B2 of
    True  -> R1
    False -> R2
```

Here!

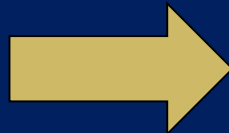
And here

# Optimising transformations for join points

$$E[\text{jump } j \vec{\varphi} \vec{e} \tau] : \tau' = \text{jump } j \vec{\varphi} \vec{e} \tau'$$

- Discard E altogether at jumps

```
case (join j x = B1
      in case xs of
        Just x  -> j x
        Nothing -> B2) of
  True  -> R1
  False -> R2
```



```
join j x = case B1 of
  True  -> R1
  False -> R2
in case xs of
  Just x  -> j x
  Nothing -> case B2 of
    True  -> R1
    False -> R2
```

Here!

# Implementing join points

# Implementing join points

- *GHC*: a big, 25-yr-old optimising compiler for Haskell
- But it was easy to add join points to *GHC*'s tiny intermediate language, *Core*

A join-point binding is almost exactly like an ordinary `letrec`

```
join j x = <rhs>  
in ... (jump j <arg>) ...
```

```
let j x = <rhs>  
in ... (j <arg>) ...
```



# Implementing join points

A join-point binding is almost exactly like an ordinary letrec

- Join points are just a variant of letrec, and share much in common (hurrah):
  - strictness analysis
  - inlining decisions
- But not all things! Every Core-to-Core pass needed review, often beneficial.
- Inferring join points is extremely easy (paper)
- Checked guarantee: join points are never lost

# Performance

Program	Allocs
fibheaps	-1.1%
ida	-1.4%
nucleic2	+0.2%
para	-4.3%
primetest	-3.6%
simple	-0.9%
solid	-8.4%
sphere	-3.3%
transform	+1.1%
(45 others)	
Min	-8.4%
Max	+1.1%
Geo. Mean	-0.4%

Rather modest performance gains, but

- *GHC* already does a lot; no low hanging fruit
- Replaces some ad-hoc hacks with simpler, more reliable transformations
- Makes optimisation much more robust; less fragile to inlining decisions
- Absolutely nails some inner loops to zero
- And, intriguingly: may affect programming style

Recursive join points

# Recursive join points

```
letrec last xs = case xs of
    [x] -> x
    (x:xs) -> last xs
in last blah
```



```
joinrec last xs = case xs of
    [x] -> x
    (x:xs) -> jump last xs
in jump last blah
```

- Join points can be recursive => loop in control flow graph
- Easy, easy. Everything just works.

# Loopification

```
module Utils( last ) where
  last xs = case xs of
             [x]   -> x
             (x:xs) -> last xs
```

- Uh oh! Now last is not a join point ☹️
- Idea: introduce a local letrec

# Loopification

```
module Utils( last ) where
  last xs = case xs of
    [x] -> x
    (x:xs) -> last xs
```



```
module Utils( last ) where
  last xs = joinrec lj xs = case xs of
    [x] -> x
    (x:xs) -> jump lj xs

    in jump lj xs
```

- The local loop is a join point 😊
- Introducing a join point directly expresses “turning tail recursion into a loop”. Better code.
- Replaces a rather ad-hoc back-end optimisation

**Fusion: an unexpected bonus**

# Streams

```
data Stream a where
  Mk :: forall a s. s -> (s->Step s a) -> Stream a

data Step s a = Done | Yield s a

filterS :: (a->Bool) -> Stream a -> Stream a
filterS p (Mk state step)
  = Mk state (\s. letrec step' s = case step s of
                    Done -> Done
                    Yield s' x | p x          -> Yield s' x
                               | otherwise -> step' s'
                  in step' s)
```

- **step' is recursive so filter2 will not fuse**
- So (Leshchinskiy et al) add a Skip constructor to Step.
- But that leads to other Bad Things.
- **With join points:** step' is now inferred as a (recursive) join point, so fusion just works without Skip.

```
filter2 p q xs
  = filterS p (filterS q xs)
```



# Stream fusion

```
case (joinrec step' s = case step s of
  Done          -> Done
  Yield s' x | p x -> Yield s' x
  | otherwise -> jump step' s') of
  in jump step' s
BLAH
```



```
joinrec step' s = case step s of
  Done          -> case Done          of BLAH
  Yield s' x | p x -> case Yield s' x of BLAH
  | otherwise -> jump step' s'
in jump step' s
```

Commuting conversions automatically work over loops!

# Inlining

GHC's single most important  
optimisation decision

# Inlining of join points: just like ordinary functions

```
join j x = <small>  
in case y of  
  True  -> jump j e1  
  False -> jump j e2
```

Inline if  
<small> is  
small

```
join j x = <BIG>  
in case y of  
  True  -> e1  
  False -> jump j e2
```

Inline if  
j is used  
only once

# Inlining into join-point RHSs

```
let v = f 22  
in join j x = ...v...  
in ...
```

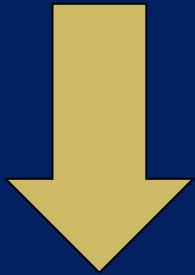
Build a  
thunk

Can we  
inline v?

# Inlining into join point RHSs

```
let v = f 22  
in join j x = ...v...  
in ...
```

Yes! Inline v! Non-recursive join points can only be called once



```
join j x = ...(f 22)...  
in ...
```

Exit floating

# Exit floating

```
f x = let v = g x
      in joinrec go 10 = h (v + 8)
                        go i  = jump go (i+1)
                        in jump go 0
```

- The local binding for *v* allocates a thunk for (*g x*)
- Call to *h* allocates a thunk for (*v+8*)
- *h* may or may not evaluate its argument

But we can do better!

# Exit floating

```
f x = let v = g x
      in joinrec go 10 = h (v + 8)
                        go i = jump go (i+1)
      in jump go 0
```

Thunk

Thunk

- The local binding for v allocates a thunk for (g x)
- Call to h allocates a thunk for (v+8)
- h may or may not evaluate its argument

Good! No  
thunk for v

We want this:

```
f x = join go 10 = h (g x + 8)
      go i = jump go (i+1)
      in jump go 0
```

OK, so why not just inline v into the join-point RHS?



# Exit floating

```
f x = let v = g x
      in joinrec go 10 = h 8
              go i   = jump go (I + v)
              in jump go 0
```

- But inlining v could be very very bad!

Inline  
v

Uses v  
every  
iteration

```
f x = join go 10 = h 8
      go i   = jump go (i + g x)
      in jump go 0
```

BAD! Evaluates (g x) on  
every iteration

# Exit floating

```
f x = let v = g x
      in joinrec go 10 = h 8
              go i   = jump go (I + v)
              in jump go 0
```

Inline  
v

Uses v  
every  
iteration

No no no

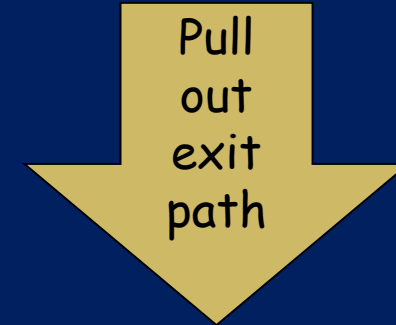
- But inlining v could be very very bad!
- So we can't unconditionally inline a thunk into a joinrec
- But sometimes we want to!
- What we want: treat exit paths specially

# Exit floating

```
f x = let v = g x
      in joinrec go 10 = h (v + 8)
                        go i = jump go (i+1)
      in jump go 0
```

- Key idea: float out the exit path as a join point

- Now we can inline  $v$ , because  $j$  is a non-recursive join point



Exit path as a join point

```
f x = let v = g x
      in join jx = h (v+8)
      in joinrec go 10 = jump jx
                        go i = jump go (i+1)
      in jump go 0
```

# Exit floating

```
f x = let v = g x
      in join jx = h (v+8)
      in joinrec go 10 = jump jx
                        go i = jump go (i+1)
      in jump go 0
```

- Now we can inline *v*, because *j* is a non-recursive join point

Inline  
*v*

Exit path  
as a join  
point

```
f x = join jx = h (g x + 8)
      in joinrec go 10 = jump jx
                        go i = jump go (i+1)
      in jump go 0
```

Wrap up

# Bottom line

- An extremely (almost embarrassingly) simple idea
- Excellent power-to-weight ratio

It's a no-brainer  
Every direct-style compiler  
should use join points

- Paper: on my home page  
<http://research.Microsoft.com/~simonpj>

# To CPS or not to CPS

CPS is extremely cool, but

- CPS fixes order of evaluation
- Some transformations much harder e.g.
  - Common subexpression elimination
  - Rewrite rules  $f(g\ x) \rightarrow h\ x$

CPS: lambda-focused

Join points: let-focused

Join points appear to  
give you all the  
advantages of CPS  
with none of the pain