Lab 1: PyLab - Radioactive Decay

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Abstract

This lab examined the intensity of radiation omitted by a Barium 137m isotope in order to determine its half-life. Using a Geiger Counter, data samples at 20 second intervals over 20 minutes were taken and analyzed using Python code. The `scipy.curve_fit` function was used to fit model functions to the data, with each model producing a unique half-life value. Compared to the theoretical half life of about 156 seconds, it was found that the nonlinear model fit resulted in a value of 163 seconds, which was closest to the theoretical. Further analysis was conducted for the radioactive count of a Fiesta plate, as well as background data that was collected. It was then found that a Poisson mass function fits the collected data moderately well, and it was determined that there were enough data points for the Poisson and Gaussian functions to look similar.

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1 Introduction

Radioactivity is the process randomly undergone by unstable atomic nuclei as they emit mass-energy. While the occurrence of a radiation event is random, radioactive isotopes maintain a constant half-life in which half the amount of the isotope decays. The intensity of radiation, that is, the number of radioactive events per second can thus be found with

$$I(t) = I_0 \frac{1}{2}^{\frac{t}{t_{1/2}}} \tag{1}$$

where t is time and $t_{1/2}$ is the half-life of the isotope in question. Radioactive events also, theoretically, follow a Poisson distribution, where the probability of a discrete number events n occurring in a given time is given by

$$P_\mu(n) = e^{-\mu} \frac{\mu^n}{n!} \tag{2}$$

where μ is the mean number of events occurring in that time interval. In this laboratory, we measure the radioactivity of a quickly decaying substance to determine its half-life. To do so, a Geiger counter is used to count the number of radioactive events occurring over time intervals of a given length, for a given measurement period. Curve fits and analysis are then performed. Using the same experimental setup, we measure the radioactivity of a slowly decaying substance to consider its statistical distribution. We analyse the accuracy of a Poisson and Gaussian distribution. This report can therefore be divided into two experiments, 'fast decay' and 'slow decay.'

By measuring the radiation of any substance, we are also taking account of background radiation. Thus, we measure background radiation in the same way as described before. Not having been done simultaneously to the measurements of the substances, we assume that the mean background radiation is constant, and subtract it from the measured counts:

$$N = N_m - \overline{N_b} \tag{3}$$

where N is the number of counts for a substance, N_m is the measured counts and N_b is the measured background counts. This applies for both the fast and slowly decaying substances.

Division of labour was split equally between the authors. Ritik was responsible for the entirety of Exercise 2 and its findings, while John was responsible for the entirety of Exercise 5 and its findings. The introduction was written by John, and the Abstract was written by Ritik. Results and data analysis were conducted together.

2 Materials and Methods

For both experiments, the lab group did not collect data personally. Instead, descriptions of the experiments and their materials were provided in the lab handouts. We describe these here.

2.1 Materials

2.1.1 Fast Decay

- Sample of metastable Barium (Ba-137m)
- Generator containing Cesium (Ca-137)
- Acid
- Geiger Counter

2.1.2 Slow Decay

- Fiesta plate glazed with uranium oxide

The two most naturally abundant isotopes of Uranium (U^{238} and U^{235}) having half-lives greater than 100 million years [1], the decay of uranium is justified as an example of slow decay. We use a Fiesta plate glazed with uranium oxide as a source of uranium. Otherwise, no additional materials were used.

2.2 Methods

2.2.1 Fast Decay

For the Fast Decay experiment, the Barium 137m was obtained from the decay of Cesium 137. Using the Geiger Counter, 60 measurements were taken at 20 second intervals over 20 minutes. At each interval, a Barium count was recorded. The background radiation present with each count was also recorded. Before conducting the data analysis, the background radiation was subtracted from each data point. Python was then used to define a linear function, with a linear regression being performed on the set of data. The same process was repeated for a non-linear regression.

Following these regressions, the half-life was calculated using each model along with uncertainties. A second plot was then created using a logarithmic y-axis to verify the model of decay. Each process compared the theoretical half-life to the value obtained in each method and conclusions were drawn with respect to the optimized method of analysis.

2.2.2 Slow Decay

For the Slow Decay experiment, the Geiger counter was held just above the centre of the Fiesta plate by a clamp and was set to record counts over 6 second intervals for 10 minutes. The plate was then removed, and the same measurements were taken of background radiation. Importantly, the background radiation was measured in the same location as the plate. After subtracting background radiation and dividing by 6 seconds to yield rate in counts per second, Python was used to create a histogram of the results a to plot the theoretical Poisson and Gaussian distributions, with all weights normalized. The same analysis was performed on the background radiation individually. For both statistical models, the mean rate μ was used as the maximum likely hood estimation of the mean. The standard deviation used for the Gaussian distribution was $\sqrt{\mu}$ as instructed by the lab manual.

Complications arose with the Gaussian distribution of the later, as described in Section 3.3. Bin edges for the histogram was ensured to be integers since the Poisson distribution is only defined for discrete numbers of successes (here, a successful event is a radioactive event). This same discrete condition necessitated (evenly) rounding rates to the nearest integer. Section 3.3 provides greater detail on how the discrete condition was satisfied.

A mainly qualitative analysis was performed on how well the collected data fit the expected distributions and the similarity between the Poisson and Gaussian distributions, as predicted by the central limit theorem.

3 Results

3.1 Fast Decay

Results for the Fast Decay experiment were obtained by fitting the collected data provided alongside the Lab Handout, using the various fitting methods mentioned in Section 2.2.1.

From Figure 1, we see that the various fitting methods are defined by the legend, the linear model function `f(x, a, b)` is represented by the orange curve, while the green and red curves represent the non-linear function `g(x, a, b)` and the theoretical curve respectively.

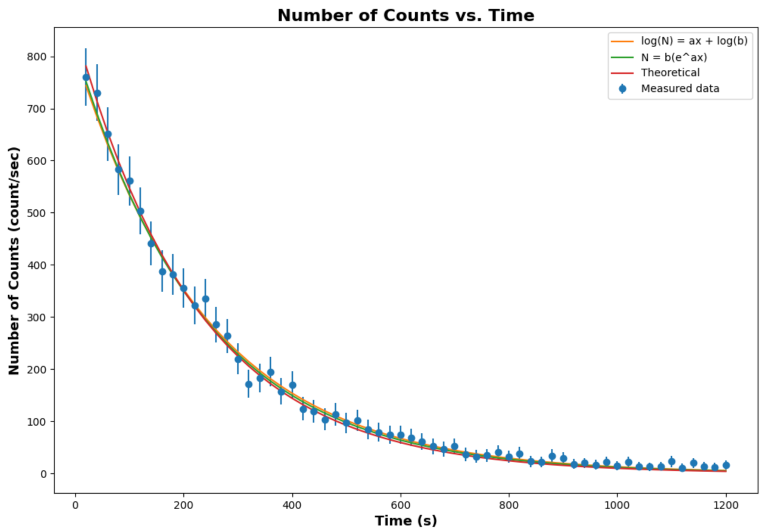

Figure 1: Non-log plot fitted with regressions and theoretical curve.

Figure 2 represents the same data as in Figure 1, only now using a logarithmic y-axis. Once again, the various fitting methods are defined by the same colours.

3.2 Fast Decay Uncertainty

The uncertainty in the count rate was determined using the formula

$$R = \frac{N}{\Delta t} \pm \frac{\sqrt{N}}{\Delta t} \tag{4}$$

R is the count rate, N is the number of data, and Δt is the time interval in seconds.

Errors in both plots were represented using errorbars, the values of which were determined using the formula

$$\sigma_s = \sqrt{N_T + \overline{N}_b} \tag{5}$$

where N_T is each data point and $\overline{N}_b$ is the mean of the background data. The uncertainty of the data after adjusting (by subtracting the mean of the background data) was found to be about 0.2 counts/second.

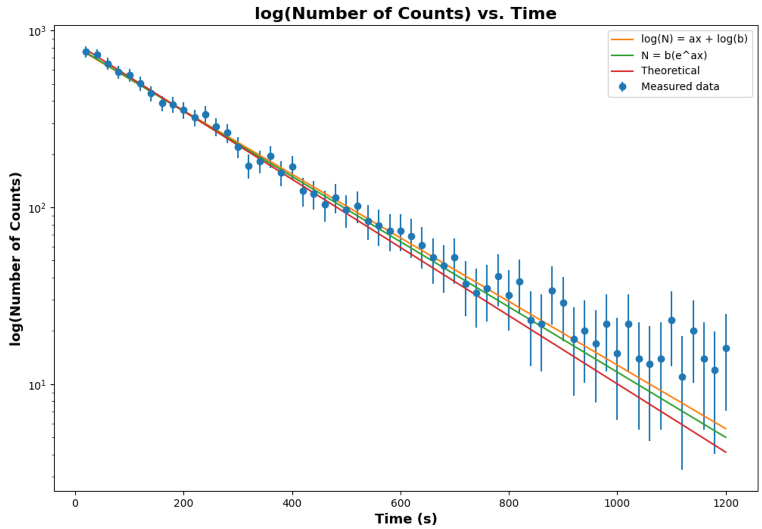

Figure 2: Log plot fitted with regressions and theoretical curve.

3.3 Slow Decay

For our purposes, the measurements of the slow decay are perhaps best illustrated with a histogram. As mentioned in Section 2.2.2, it was necessary that bin edges and rates be integers. This comes from the Poisson model's assumption of a discrete success condition (here, a radioactive event occurring). To accomplish this, the following measures were taken

- Each rate was evenly rounded to the nearest integer.
- The mean rate μ used as the maximum likely hood estimation for the Poisson distribution was evenly rounded to the nearest integer.
- The edges of the bins were set to be integers. This was accomplished by setting the range of the histogram as

  $$N_{\text{min}} \text{ to } N_{\text{min}} + \left\lceil \frac{N_{\text{max}} - N_{\text{min}}}{b} \right\rceil \cdot b$$

  where N_{min} is the lowest rate, N_{max} is the highest rate and b is the desired number of bins.

See Figure 3 which shows the histogram and statistical models for non-rounded rates and non-rounded mean. The Poisson probability is zero for all rates, illustrating the need for the above measures.

To compute the Poisson probability mass function for each bin, the probability for each integer in each bin was summed to obtain a total bin probability. For example, the bin of

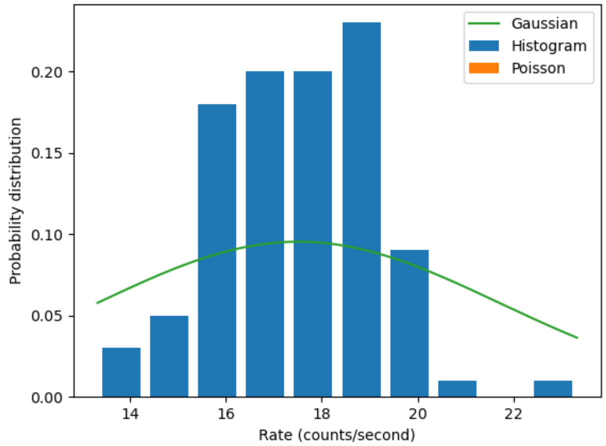

Figure 3: Histograms of non-rounded relative rates with Poisson and Gaussian distributions, Fiesta plate.

rates in $[80, 83) \cap \mathbb{N}$ has a Poisson probability of

$$P(80) + P(81) + P(82)$$

The Gaussian distribution is continuous and so issues around integer number are not relevant. Still, Gaussian analysis was performed on the same data rounded according to the aforementioned measures. As mentioned before, all weights in the histograms are normalized.

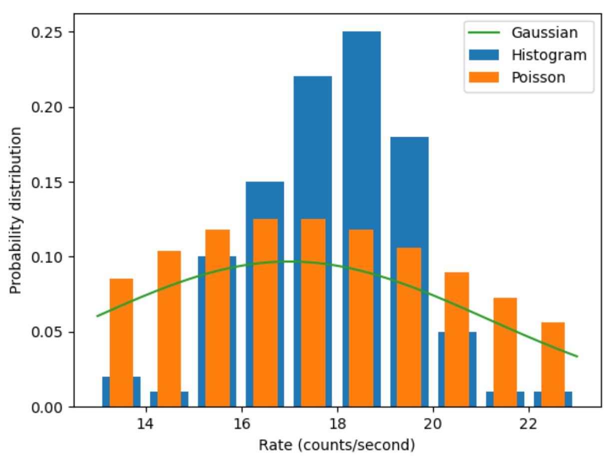

Figure 4: Histograms of relative rates with Poisson and Gaussian distributions, Fiesta plate

The same analysis was performed on just the background data, with the same provisions on rounding and bin edges. However, given the low mean background rate of 0.01 counts per second, this was rounded to zero and led to errors in drawing the Gaussian distribution. Therefore, we considered rate per six seconds, as it was collected. This led to a rounded μ of 1 count per six seconds.

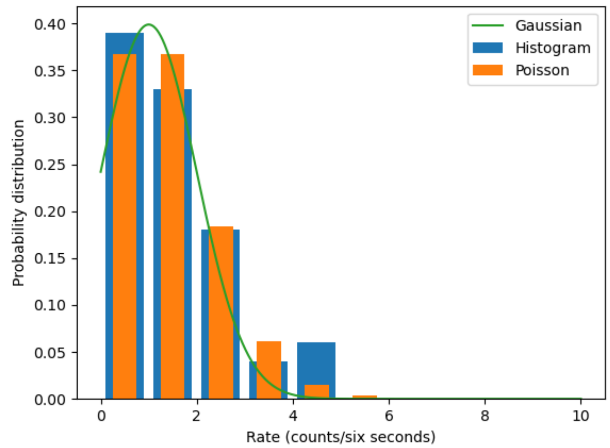

Figure 5: Histogram of relative rates with Poisson and Gaussian distributions, background

3.4 Slow Decay Uncertainty

Just as with the fast decay, if we let N_s be counts with regard to background, N_m be counts measured, N_b be background counts and Δt be the measurement time interval, then rate can is found with

$$R = \frac{N_s}{\Delta t}$$

and as the lab manual states,

$$\sigma_s = \sqrt{\sigma_m^2 + \sigma_b^2} = \sqrt{N_m + N_b}$$

We also assume again assume that $N_b = \overline{N_b}$. While we do not know the uncertainty in Δt, it is negligible compared to other sources of uncertainty, since it was performed by an electronic device. Therefore,

$$u(R) = \frac{u(N_s)}{\Delta t} = \frac{\sqrt{N_m + N_b}}{\Delta t}$$

Carrying out this calculation in Python for the Fiesta plate yields an uncertainty in rate of from 1. to 2. counts per second for all rates. We can also find the statistical uncertainty in the background radiation with $\sqrt{N_b}/\Delta t$ which yields an uncertainty of 0. to 0.3 counts per second for all data points. Running the submitted Python code prints all uncertainties.

4 Analysis and Discussion

4.1 Fast Decay

After obtaining the results, they were graphed using Python and expressed by defining linear and nonlinear regression functions. The theoretical half-life of barium 137 is

$\tau = 2.6$ minutes, or 156 seconds. Results of applying each model function using `scipy.curve_fit` were then compared with τ. Before the analysis of the data, the background radiation count was considered. The mean background radiation was averaged among the 60 data points and was then subtracted from each data count of the barium sample. The uncertainty in the Geiger Counter was calculated using the standard error propagation in Equation 5.

Assuming the data fit an exponential regression, we then had

$$y = y_0 e^{ax} \implies \log(y) = ax + \log(y_0) \tag{6}$$

This was done in order to obtain a linear model from the exponential one. The sample data was then plotted using Equation 6 as a model. Since the plot was graphed in terms of a logarithm, the error of each data point was propagated in order to fit the conditions using the equation

$$\sigma_{z_i} = \left| \frac{\sigma_{y_i}}{y_i} \right| \tag{7}$$

Considering the linear model fit first, the parameters received from `scipy.curve_fit` indicated that the half-life of Barium 137 was 167.31 s $\pm$ 2.95 s. The χ^2_{red} value for this data was 1.28, implying that the curve of best fit for the data was not a perfect fit and also that the data was inconsistent. The `curve_fit` parameters were determined through the code, from which we obtained a fit equation

$$\log(y) = -0.0041t + \log(6.69) \tag{8}$$

where 6.69 represents the initial count of Barium.

A second model was created using Equation 6 as the model function. According to the `scipy.curve_fit` parameters, the half-life of the barium was 163.17 s $\pm$ 2.99 s. Using the reduced chi squared equation

$$\chi^2_{red} = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i} \right)^2 \tag{9}$$

it was determined that the χ^2_{red} value for this curve of best fit was 1.06. This indicates that the data was almost an ideal fit, since 1.06 is extremely close to 1. From Python, the nonlinear function was determined to be

$$y = 818.5e^{-0.0042t} \tag{10}$$

where 818.5 is the initial count of Barium. These results do not match the theoretical ones, implying that there was error in the lab.

For comparison, the theoretical curve was also plotted against the data and produced a χ^2_{red} value of 1.24. This value was between the values for the linear fit and the nonlinear fit.

According to the data, the initial count of barium was 760 and each model presented an initial value higher than such. This indicates that the data points do not follow the theoretical half-life trend.

The results indicate that there was evident error in the lab. It is possible that the substance in question was not pure. Since the cesium decayed into barium, it is possible that the substance was not completely converted, and remnants could have been present while data was being collected. Since cesium has a higher half-life than barium, this mixture of isotopes could have affected the experimental data and final determinations. The results of this lab could be improved if there was a process to verify purity before the measurement were recorded.

Subsequent to the initial data analysis, a second plot was created taking the logarithm of the y-axis representing barium count. The results shown in Figure 2 show a linear plot. This verifies that the barium 137 undergoes an exponential decay. According to the above results, the non-linear fit represents the decay most accurately. Amongst the data obtained from the lab, the χ^2_{red} is closest to 1, (1.05), and the half-life value calculated using this method produced a measurement closest to that of the theoretical data.

4.2 Slow Decay

The figures of Section 3.3 show the results of the Python code which produced histograms and drew theoretical Poisson and Gaussian distributions. From here we can use qualitative judgment on how well the data fits these models.

Figure 4 shows some agreement in shape between the Poisson distribution and histogram. The plot shows a much higher (up to double) distribution in the region of 16 to 20 counts per second than theoretically predicted. The Poisson function has no standard deviation parameter like the Gaussian, so this could not be explained by high precision in measurement. Instead this might be explained by the rounding explained in Section 2.2.2, though this conflicts with Figure 3 (for non-rounded rates) exhibiting the same high peak around 16 to 21 counts per second. In summary the discrepancy has no clear explanation other than chance, or some issue with the theory being applied to the Fiesta plate in particular. The collection of more rates while repeating this experiment might reduce any random errors.

The background radiation plot (Figure 5) does not exhibit this anomaly but this may be a result of the grouping of rate in counts per six seconds.

For both the background and Fiesta plate radiation, we see agreement between the Poisson and Gaussian models, in concordance with the central limit theorem. For the Fiesta plate, there is a noticeably higher distribution for all rates of the Poisson versus the Gaussian. Higher quantities of data might bring these models closer.

Given the importance placed on background radiation in this lab, investigation into residual radiation might prove useful. The same Geiger counter was used for both experiments so there is a possibility that radioactive emissions found their way onto the counter in such a way that this produced higher counts. This might be tested by comparing background measurements in the same location but with one used and one new Geiger counter.

5 Conclusion

The purpose of this lab was to determine the half-life of a substance using different methods.

For the Fast Decay experiment, data obtained from the lab was plotted using Python with respect to various fit regressions. A linear method determined a Barium 137 half life of about 167.3 s, while a nonlinear method determined a half life of about 163.1 s. Compared to the theoretical half-life of 156 s, it is clear that the nonlinear regression presented a more accurate result. Despite a small range of values when considering their magnitude, the error range in each sample did not correspond with the theoretical half-life, indicating that there were inconsistencies in the data. For the Slow Decay experiment, data obtained from the lab was represented using Python by plotting histograms and various distributions. A Poisson mass probability function was used to determine the frequency of counts per sample, with a Gaussian distribution subsequently being fitted to the data.

References

[1] "Frequently Asked Questions about Depleted Uranium Deconversion Facilities." *U.S.NRC*, United States Nuclear Regulatory Commission, 23 Mar. 2015, www.nrc.gov/materials/fuel-cycle-fac/ur-deconversion/faq-depleted-ur-decon.html_ftnref1.