A GR approach to the dynamics of inflationary reheating in the early universe

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The phase of inflationary reheating during the very early universe can shed light on the production of Standard Model particles that formed the basis of the universe. This paper reviews the phases of cosmic inflation and reheating, with a focus on particle dynamics in the context of Einstein's general relativity. The concepts of inflation, preheating and inflationary reheating are also discussed in terms of the inflaton field. Connections are made between these concepts and the Standard Model of matter through the mathematical theory that is presented. This paper then reviews the universe's transition from being inflaton dominated to radiation dominated, as well as the evolution of the inflaton field. The paper then concludes with a brief overview of the connection between reheating and large-structure formation.

I. INTRODUCTION

During the beginnings of the universe, there were several key phases that immediately followed the Big Bang. One of these is the phase of Inflationary Reheating, which occurred after the end of cosmic inflation. To understand the processes that took place during reheating, it is necessary to first define the inflationary epoch and the significance of the inflaton field ϕ [1].

The inflation epoch saw the universe undergo a rapid expansion, which can be described by the theory of inflation. This theory can be explained using inflationary models that provide methods to parametrize the early universe [2]. However, the classical nature of the inflationary models argument implies that it is ill-suited to offer a comprehensive description of the universe's beginning [2]. This incompatibility comes from how classical physics breaks down at quantum scales. To remedy this, the presence of an inflaton field is considered [2].

First introduced by Alan Guth, the inflaton field is a hypothesized scalar field which is believed to have been the driving factor behind inflation in the early universe [3, 4]. In the physical sense, a field is a function of spacetime, where its quantum fluctuations are observed to be particles [4]. During inflation, all matter is redshifted to low densities, with the exception of particles (inflatons) in the inflaton field [5]. Over time, the inflaton field begins to oscillate around the minimum of its potential, where the energy densities of the inflatons decay into SM matter [5]. This process is known as preheating, and it occurs at the start of reheating.

Using perturbation theory, inflaton decay via quantum fluctuations can be understood as the progenitor of the reheating phase [1, 2]. These fluctuations give insights into the physical characteristics of the early universe that allowed for its rapid expansion, and they also describe the reheating process [5].

At a glance, reheating simply describes the production of SM matter after inflation [1]. In more specific terms, reheating occurs when the inflaton energy density is converted to the energy density of other fields, causing a decrease in its oscillation [6]. The analysis of reheating through this lens shows it is probable that reheating involves a parametric resonance instability [1]. Parametric resonance occurs during preheating, when the oscillation of the inflaton field leads to the energy transfer between inflatons and SM matter particles.

This review paper seeks to provide a description of cosmic inflation and reheating, along with the dynamics of particle interactions during this time. It then builds on this description by considering current research through relevant literature, in order to discuss the dynamics of reheating in the context of Einstein's general relativity. Various models of particle physics along with some elementary string theory are also examined. In Section II, the paper approaches the definitions of preheating, reheating, and the inflaton field in the context of general relativity, and connections to the SM of matter are made. Some mathematical theory is also included for support. Scalar field theory in curved spacetime will be introduced to help support explanations of the inflaton field (and others). In addition, the Klein-Gordon equation will also be introduced, along with its connection to the reheating process. The paper will then shift towards explaining the transition of the universe from being inflaton dominated to radiation dominated in Section III, and will also mention the evolution of the inflaton field and other particles. Connections between the dynamics of reheating and large-structure formation will also be discussed. The paper concludes with a summary of key points in Section IV.

II. A GENERAL RELATIVISTIC PRECURSOR TO REHEATING

In Section I, the concepts of inflation, preheating, inflaton field, and inflationary reheating are briefly defined. Here, the paper attempts to take a general relativistic approach to discuss the theories of inflation, preheating, and scalar fields with more detail. These lead to a discussion of dynamics that took place during reheating.

A. Epoch of Cosmic Inflation

A fundamental question cosmologists often ask concerns the origin of the universe's structures, such as galaxies and star clusters [7]. While the Big Bang theory answered several of cosmology's most important questions, the puzzles of the universe's flatness and homogeneity were left unanswered [8]. Consequently, inflation was proposed as a solution to both of these issues, and it also provides the most widely accepted explanation for the origin of the universe's structures [7, 8]. According to the theory of inflation, these structures begin as quantum fluctuations due to the presence of an inflaton field [7]. Quantum fluctuations can be thought of as the temporary energy change in a particular spatial point, which can be described using Heisenberg's energy-time and position-momentum uncertainty principles

$$\Delta E \Delta t \geq \hbar \ , \tag{1}$$

$$\Delta x \Delta p \geq \hbar \ , \tag{2}$$

where $\Delta E, \Delta t, \Delta x, \Delta p$ are the uncertainties in energy, time, position, and momentum respectively, and $\hbar$ is the reduced Planck's constant [7, 9]. During a quantum fluctuation, virtual particle-antiparticle pairs of energy ΔE separate during the interval Δt, where Δx is a measure of their separation [7]. If Δx grows greater than the event horizon during the interval Δt, the virtual particles cannot re-pair, eventually becoming real matter particles in the classical sense [7].

Now, it is important to address some of the mathematical theory behind cosmic inflation. The following treatment is provided using [2] and [8] as references:

Consider first that the current description of inflation assumes a homogeneous and isotropic universe [2]. The metric used to describe this space is then the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, given by

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -dt^2 + a^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin\theta d\phi^2 \right) \right] , \end{aligned} \tag{3}$$

where $g_{\mu\nu}$ is the metric tensor, $a(t)$ is the scale factor as a function of time t, and K is a discrete parameter that determines the 3D curvature of the universe [8].

With solving the Einstein equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} , \tag{4}$$

where $R_{\mu\nu}$, R, $T_{\mu\nu}$, G, and Λ are the Ricci tensor, Ricci scalar, energy-momentum tensor, gravitational constant, and cosmological constant ($\Lambda = 0$ in this case) respectively, the field equations (5) and (6) can be obtained:

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}\rho - \frac{K}{a^2} , \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{\rm pl^2}}(\rho + 3P) . \tag{6}$$

In the above, $H \equiv \dot{a}/a$ is the Hubble expansion rate, $m_{\rm pl}$ is the Planck energy, $\rho = \rho(t)$ is the energy density, and $P = P(t)$ is the pressure. Equations (5) and (6), together with the Bianchi identity $\nabla_\mu T^{\mu\nu} = 0$ and the energy-momentum tensor

$$T^\mu_\nu = \mathrm{diag}(-\rho, P, P, P) , \tag{7}$$

yield the continuity equation

$$\dot{\rho} + 3H(\rho + P) = 0 . \tag{8}$$

Rewriting Equation (5) gives

$$\Omega - 1 = \frac{K}{a^2 H^2} , \tag{9}$$

where $\Omega \equiv (8\pi\rho)/(3H^2 m_{\rm pl}^2)$ is the spatial ratio of the energy density to the critical density [2].

Now, the above mathematical derivation gives insight into the theory behind the conclusion of rapid expansion in the early universe. To demonstrate this, assume that there did indeed exist a stage of accelerated expansion during the early universe (inflation), such that $\ddot{a} > 0$ [2]. Considering Equation (6) in this context gives the following:

$$\begin{aligned} \ddot{a} &> 0 \\ \implies -\frac{4\pi a}{3{m_{\mathrm{pl}}}^2}(\rho + 3P) &> 0 \\ \implies \rho + 3P &< 0 \ , \end{aligned}$$

where the condition on the last line implies a violation of the strong energy condition [2].

From the assumption $\ddot{a} > 0$, it is then possible to conclude that $\dot{a} = aH$ increases during inflation [2]. This further implies that the co-moving Hubble radius, $(aH)^{-1}$, decreases during the inflation epoch [2]. This is an important result in the verification of the existence of an inflationary stage in the early development of the universe.

B. Preheating and the Inflaton Field

Near the end of the inflation, the universe began to shift towards the reheating phase. Prior to this, however, the concepts of preheating and the inflaton field come into play.

As briefly discussed in Section I, the inflaton field is a hypothetical scalar field that is thought to have been responsible for the accelerated expansion of the early universe [3, 4]. The inflaton field following the inflation epoch was also thought to have a large potential energy density, the decay of which converted quantum fluctuations into SM matter [4]. The process of preheating refers to the decay of this energy density as the field oscillates about the minimum of its potential [4]. Figure 1 (taken from [4]) illustrates this phenomenon.

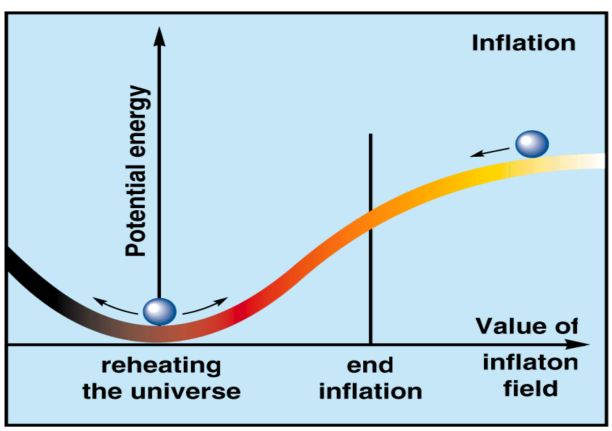

FIG. 1: Figure 3 from [4]. The inflaton field can be represented as a ball rolling down a hill. During inflation, the energy density is roughly constant, driving the rapid expansion of the universe. As the energy density of the inflaton field decreases due to inflation, the ball begins to roll down the hill, reaching a minimum. When the ball starts to oscillate around the minimum, inflation ends and the inflaton energy density decays into particles. This "preheats" the universe and prepares it for the reheating phase.

The mathematical theory behind the significance of Figure 1 is provided below. In particular, the derivation of inflaton field dynamics is followed while referencing [6] and [10].

To understand what occurs during the transfer of energy via quantum fluctuations in the inflaton field, it is first important to note that the inflation epoch requires the presence of scalar field matter. That is to say, the energy-momentum tensor $T^{\mu\nu}_{\text{matter}}$ must be dominated by the approximately constant energy density of the inflaton field ϕ [1].

Consequently, the dynamics of the inflaton field can be derived starting with the FRW equation coupled with the Klein-Gordon equation

$$H^2 = \frac{8\pi}{3m_{\text{pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) , \tag{10}$$

$$0 = \ddot{\phi} + 3H\dot{\phi} + m^2\phi , \tag{11}$$

where $V(\phi) = \frac{1}{2}m^2\phi^2$ (m is the mass of the inflaton field) is the effective potential of the scalar field in the chaotic inflation model [6, 10]. In this scenario, inflation occurs when $\phi \gtrsim m_{\text{pl}}$, and large-scale structure formation begins when $\phi \sim 3 - 4m_{\text{pl}}$ [10].

Following this, observe that Equation (10) can be parametrized using H and the angular variable θ defined as (from [6]),

$$\dot{\phi} = \sqrt{\frac{3}{4\pi}}Hm_{\text{pl}}^2 \sin\theta , \tag{12}$$

$$m\phi = \sqrt{\frac{3}{4\pi}}Hm_{\text{pl}}^2 \cos\theta . \tag{13}$$

In conjunction with Equation (11), dynamical equations for each variable can be obtained,

$$\dot{H} = -3H^2 \sin^2\theta , \tag{14}$$

$$\dot{\theta} = -m - \frac{3}{2}H \sin 2\theta . \tag{15}$$

The $-\frac{3}{2}H\sin 2\theta$ term in Equation (15) is negligible for large times $mt \gg 1$, implying that the inflaton field oscillates with an angular frequency of $\omega \simeq m$ [6]. A relation for the Hubble rate in terms of time can then be derived from Equation (14):

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right)^{-1} . \tag{16}$$

Taylor expanding $(mt)^{-1}$ gives the resulting behaviour of the inflaton field with respect to time $\phi(t)$,

$$\phi(t) \simeq \Phi(t)\cos(mt)\left(1 + \frac{\sin(2mt)}{2mt}\right) , \tag{17}$$

where the amplitude of oscillations $\Phi(t)$ with respect to time is defined as

$$\Phi(t) = \frac{m_{\text{pl}}}{\sqrt{3\pi}mt} . \tag{18}$$

A visual interpretation of the above result is seen in Figure 2 [6, 10].

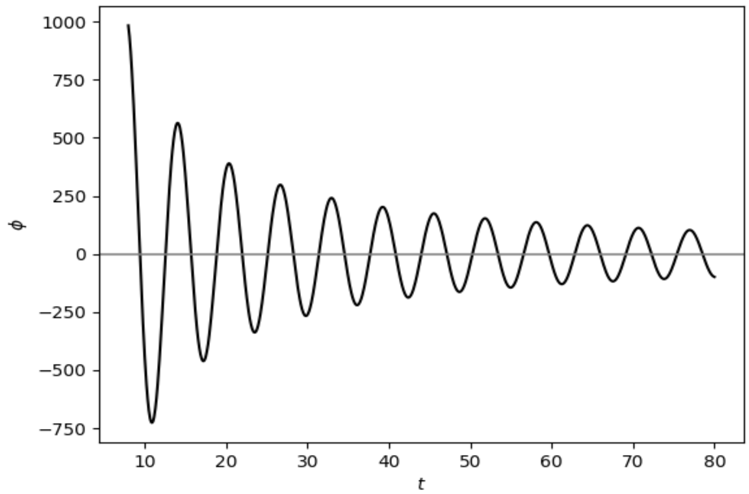

FIG. 2: (Recreated from [10]) Oscillations (what occurs at the bottom of the potential in Figure 1) of the inflaton field ϕ after inflation, under the assumption of a quadratic scalar field potential (i.e $V(\phi) = \frac{1}{2}m^2\phi^2$). The value of the scalar field is measured in units of $m_{\rm pl}$ and time is measured in units of m^{-1}. Code used to create the plot is provided in the Appendix.

Referring back to Equation (11), the "friction" term $3H\dot{\phi}$ eventually becomes negligible at the end of inflation, implying that Equation (11) becomes

$$0 = \ddot{\phi} + m^2\phi \ , \tag{19}$$

which represents a damped oscillator. Its amplitude is determined by the rate of the universe's expansion [6].

Now, the behaviour of the energy density of the inflaton field ϕ can be extrapolated as a result of the scale factor $a(t)$. This can be found by manipulating Equation (16):

$$\begin{aligned}
\frac{\dot{a}}{a} &= \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right) \\
\frac{1}{a}\frac{da}{dt} &= \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right) \\
\frac{da}{a} &= \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right) dt \\
\int \frac{da}{a} &= \int \frac{2}{3t}\left(1 - \frac{\sin(2mt)}{2mt}\right) dt \\
\ln(a) + C &= \int \frac{2}{3t}\, dt - \int \frac{\sin(2mt)}{3mt^2}\, dt \\
\ln(a) + C &= \frac{2\ln(|t|)}{3} + D - \int \frac{\sin(2mt)}{3mt^2}\, dt \\
\ln(a) &= \frac{2\ln(|t|)}{3} + E - \int \frac{\sin(2mt)}{3mt^2}\, dt \ ,
\end{aligned}$$

where C, D, E are constants of integration. Exponentiating both sides then yields

$$
\begin{aligned}
& a = \exp\left(\frac{2\ln(|t|)}{3} + E - \int \frac{\sin(2mt)}{3mt^2}\, dt\right) \\
\implies & a \propto t^{2/3} \ .
\end{aligned} \tag{20}
$$

The oscillations of the scalar field in this model are sinusoidal (Figure 2), with the decrease in amplitude being described by,

$$
\Phi(t) = \frac{m_{\text{pl}}}{3}\left(\frac{a_0}{a}\right)^{3/2} \ , \tag{21}
$$

where a_0 is a scaling proportionality constant [6, 10, 11].

Consider then the energy density of the inflaton field ρ_ϕ, which is the sum of its kinetic and potential energies,

$$
\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \ . \tag{22}
$$

With Equations (20) and (22) in hand, the energy density of the inflaton field can be determined to be decreasing in the same way as the energy density of non-relativistic particles of mass m,

$$
\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \sim a^{-3} \ . \tag{23}
$$

Oscillations of the inflaton field can be broken down into a set of discrete independent scalar particles, all oscillating at a frequency $\omega \simeq m$ [6, 10].

Following this, reheating occurs when the inflaton energy density ρ_ϕ is transferred to the energy density of other fields through decay via quantum fluctuations. This leads to a decrease in oscillation amplitude faster than what is modeled by Equation (17) [6, 10].

The theory of the inflaton field is useful in attempts to explain how large structures of the universe form, as well as its homogeneity and flatness [1]. Its decay into SM matter (a process known as preheating) is also used to explain how the universe enters the reheating phase, after inflation. The following derivation regarding the preheating phase is based on the work done in [1], [10], and [12].

To begin, consider the Lagrangian of a model describing the interaction between the inflaton field ϕ, a scalar field χ, and a spinor field ψ:

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}\phi_{,i}\phi^{,i} - V(\phi)\frac{1}{2}\chi_{,i}\chi^{,i} - \frac{1}{2}m_\chi^2(0)\chi^2 + \frac{1}{2}\xi R\chi^2 \\
& + \bar{\psi}\left(i\gamma^i\partial_i - m_\psi(0)\right)\psi - \frac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\phi \ ,
\end{aligned} \tag{24}
$$

where g, h, ξ are coupling constants, R is the spacetime curvature, and $V(\phi)$ is the effective potential of the inflaton field [10]. For generality, assume that V has a minimum at $\phi = \sigma$, and the potential remains quadratic near the minimum (in accordance with Figure 1) [6, 10]. Under this assumption, note that the potential resembles

$$
V(\phi) \sim \frac{1}{2}m^2(\phi - \sigma)^2 \ , \tag{25}
$$

where m^2 is the squared effective mass of the inflaton field [10].

Taking the typical shift $\phi - \sigma \to \phi$ in the context of symmetry breaking, it follows that

$$V(\phi) \sim \frac{1}{2}m^2(\phi - \sigma)^2 \to V(\phi) \sim \frac{1}{2}m^2\phi^2 \ ,$$

which adds a linear interaction term with respect to ϕ to Equation (24). This results in

$$\Delta\mathcal{L} = -g^2\sigma\phi\chi^2 - h\bar{\psi}\psi\phi \ , \tag{26}$$

leading to

$$\Gamma(\phi \to \chi\chi) = \frac{g^4\sigma^2}{8\pi m_\phi} \ , \tag{27}$$

$$\Gamma(\phi \to \bar{\psi}\psi) = \frac{h^2 m_\phi}{8\pi} \ , \tag{28}$$

where m_ϕ is the mass of the inflaton [1, 6, 10].

Equations (27) and (28) represent the decay rates of inflatons into scalar field particles and spinor field particles respectively, and their sum $\Gamma = \Gamma(\phi \to \chi\chi) + \Gamma(\phi \to \bar{\psi}\psi)$ represents the total decay rate of inflaton field particles [1, 10].

Turning now to the effects of inflaton decay in terms of the universe's expansion and particle production, consider the Klein-Gordon equation describing a scalar field in a homogeneous universe

$$\ddot{\phi} + 3H(t)\dot{\phi} + (m^2 + \Pi(\omega))\phi = 0 \ , \tag{29}$$

where $\Pi(\omega)$ is the inflaton polarization (complex) operator with 4-momentum $k^i = (\omega, 0, 0, 0)$ for $\omega = m$ [6, 10]. For simplicity, assume that $m^2 \gg H^2, m^2 \gg \operatorname{Im}\Pi$. These conditions are satisfied near the end of inflation, so it is reasonable to assume that they are true [10]. Then, neglecting the time-dependence of H and $\operatorname{Im}\Pi$ as a result of the universe's expansion, the solution for Equation (29) can be obtained:

$$\phi(t) = \phi_0 \exp\left[-\frac{1}{2}\left(3H + \frac{\operatorname{Im}\Pi(m)}{m}\right)t + imt\right] \ , \tag{30}$$

The amplitude of the inflaton field's oscillations decrease as

$$\phi(t) \propto \exp\left[-\frac{1}{2}(3H + \Gamma)t\right] \ , \tag{31}$$

due to the decay of inflatons and the resulting particle production [10]. The appearance of χ-particles and ψ-particles, through the decay of inflatons derived above, is what drives the phase of preheating [12].

III. DYNAMICS OF REHEATING

With explanations of the inflationary epoch, inflaton field, and preheating phase outlined in the previous section, the paper now shifts its focus onto the dynamics of inflationary reheating.

To begin, recall that the phase of reheating takes place following the decay of inflatons into SM matter particles, via preheating. The preheating phase occurs as a precursor for the universe to "ready itself" for reheating, on its way towards entering the Big-Bang nucleosynthesis era [13].

Initially, reheating was studied as a perturbative process, in which individual particles of the inflaton field decay independently [13]. More recent studies instead placed greater importance on collective, nonperturbative resonances in the transfer of energy from inflatons to SM matter (i.e parametric resonance) [13].

An important parameter in the study of inflationary dynamics is the reheating temperature T_{rh}. Preliminary studies of this topic estimated the value of T_{rh} by equating $H \sim \Gamma$, where H is the Hubble parameter and Γ is the decay rate of inflatons as before [13]. The reheating process is thought to be completed when the equality is satisfied [14].

Assuming then that the decay products of χ-particles and ψ-particles are light relative to H, it follows that they should behave like radiation [13]. From this assumption, the following is obtainable:

$$\rho = \frac{\pi^2}{30} g_* T^4 = 3m_{\rm pl}^2 H^2 \ , \tag{32}$$

where $\rho = \rho(t_{rh})$ is the energy density at the reheating time t_{rh}, and $g_* \sim 100$ is the number of relativistic degrees of freedom at $t = t_{rh}$ [14]. From this, it is found that

$$T_{rh} \sim 0.1\sqrt{H m_{\rm pl}} \tag{33}$$

gives the temperature T_{rh} during reheating [14].

The details of the above analysis, combined with the derivation done in Section II B for the inflaton field, form a majority of the perturbative analysis for the dynamics of inflationary reheating. However, it fails to take into account the coherent nature of the oscillating inflaton field about its minimum [14]. For the remainder of this section, the paper discusses changes in the theory of reheating when the inflaton's coherent oscillation is taken into consideration, while referencing [14].

When this aspect of the inflaton is accounted for, it becomes possible for the time-dependent classical inflaton background ϕ to induce the quantum mechanical production of matter particles χ [14]. Using quantum field theory in a classical, time-dependent background, the quantum field $\hat{\chi}$ is given by

$$\hat{\chi}(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} (\hat{a_{\mathbf{k}}} \chi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger \chi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}}) \ , \tag{34}$$

where $a_{\mathbf{k}}^\dagger$ and $\hat{a}_{\mathbf{k}}$ are the creation and annihilation operators respectively, and $\chi_k(t)$ is the mode function [14]. By ignoring expansion of the universe, and setting $z \equiv mt$, this quantum field satisfies the Mathieu equation

$$\chi_k'' + (A_k - 2q\cos 2z)\chi_k = 0 \ , \tag{35}$$

where $A_k \equiv \frac{k^2}{m^2} + 2q$ and $q \equiv \frac{g^2\Phi^2}{4m^2}$ [14].

The Mathieu equation gives two solutions; one for $q << 1$ that gives narrow resonance, and one for $q > 1$ that gives broad resonance. For the sake of brevity, the mathematical analysis of the resonance bands will be omitted, and only the main results will be discussed.

The narrow resonance solution closely represents the elementary theory of reheating, where two $\phi-$particles of mass m_ϕ condense into two $\chi-$particles with momenta $k = m$ [14]. The difference lies in how the effect relies on Bose condensation of the produced particles, in that it dominates over elementary decays when $qm > 3H + \Gamma$" [14]. Similar results can be found for both the Minkowski space and for Bose Condensation.

For broad resonance, on the other hand, the oscillation frequency varies with the oscillation ϕ without losing resonance. There are still many other aspects and cases that require more research.

IV. CONCLUSIONS

In this paper, a review of inflationary reheating and its applications to Standard Model matter is presented. The phases of cosmic inflation, preheating, and reheating, and the role of the inflaton field were described. It was discussed how the oscillation of the inflaton field around the minimum of its potential leads to the energy densities of the inflatons to decay into SM particles within the reheating phase, due to parametric resonance instability. It was then explained how the theory of inflation was used to address the questions about the universe's flatness and homogeneity. According to this theory, the universe's structures began as quantum fluctuations due to the presence of an inflaton field ϕ. Supporting mathematical theory was presented to explain cosmic inflation and to verify the existence of an inflationary stage under the assumption of a homogeneous and isotropic universe. Next, the mechanisms of preheating were explained by using the FRW and Klein-Gordon equations to demonstrate how the decay of inflatons allows for the production of $\chi-$ and $\psi-$particles via the rapid transfer of energy density. Then, the dynamics of reheating were described through the mathematical analysis of the reheating temperature, T_{rh}, and how the theory of reheating is changed when accounting for the inflaton's coherent oscillation. Inflationary reheating is an area of research that can be very insightful for the evolution of the universe, but it still needs to be studied further to consider the intricacies of the inflaton field and the potential cases.

Appendix: Written Code

The below shows code written for Figure 2.

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.constants import G

m_pl = (8 * np. pi * G) ** (-1/2)

m = 1  # assumed for simplicity


def damped(t):
    return (m_pl / (np.sqrt(3 * np. pi) * m * t)) * np.sin(m * t)


time = np.linspace(8, 80, 1000)

plt.plot(time, damped(time), color='black')
plt.axhline(y=0, color='grey', linestyle='-', alpha=0.8)
plt.xlabel(r'$t$')
plt.ylabel(r'$\phi$')
plt.savefig('recreated damped.png')
plt.show()
```

This code represents the damped oscillation of the inflaton field ϕ as a function of time t. Using the equation

$$\phi(t) = \frac{m_{\rm pl}}{\sqrt{3\pi} mt} \sin mt \ , \tag{A.1}$$

the function `damped` returns the value of ϕ on a scale of $t \in [0, 80]$. The term with t^{-1} dependence is what causes the decrease in amplitude as t grows.

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