

Renewable-Energy-Systems-Module-3-Problems

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⚠️ Only some of the problems are included

1. A propeller type wind turbine has following data:

- Speed of free wind at a height of 10 m = 12 m/s
- Air density = 1.226 kg/m³
- $\alpha = 0.14$
- Height of tower = 100 m
- Diameter of rotor = 80 m
- Wind velocity at the turbine reduces by 20%
- Generator efficiency = 85%
- Find the following:
 - (i) Total power available in wind
 - (ii) Power extracted by the turbine
 - (iii) Electrical power generated
 - (iv) Axial Thrust on the turbine

Answer

Formulas:

1. Total Power in Wind:

$$P_{\text{wind}} = 0.5 \cdot \rho \cdot A \cdot V^3$$

where $A = \pi r^2$ is the swept area.

2. Power Extracted by Turbine:

$$P_{\text{turbine}} = \text{Turbine Efficiency} \cdot P_{\text{wind}}$$

3. Electrical Power Generated:

$$P_{\text{electric}} = \text{Generator Efficiency} \cdot P_{\text{turbine}}$$

4. Axial Thrust on Turbine:

$$T = 0.5 \cdot \rho \cdot A \cdot (V^2 - V_1^2)$$



(i) Total Power Available in Wind

- Area swept by the rotor:
 - Given rotor diameter is 80, which makes radius 40m
 - $A = \pi \cdot r^2 = \pi \cdot 40^2 = 1600\pi \text{ m}^2$
- Total Power:
 - $P_{\text{wind}} = 0.5 \cdot \rho \cdot A \cdot V^3$
 - Air density (ρ): **1.226 kg/m³**
 - $A = 1600\pi$
 - $V = 12 \text{ m/s}$
 - $P_{\text{wind}} = 0.5 \cdot 1.226 \cdot 1600\pi \cdot 12^3$
 - $P_{\text{wind}} = 0.5 \cdot 1.226 \cdot 1600\pi \cdot 1728$
 - $P_{\text{wind}} \approx 5328950.906 \text{ W}$



(ii) Power Extracted by the Turbine

- $P_{\text{turbine}} = \text{Turbine Efficiency} \cdot P_{\text{wind}}$
- Assuming turbine efficiency as 85%:
- For finding $P_{\text{wind}} = 0.5 \cdot \rho \cdot A \cdot V^3$
 - Wind velocity at turbine reduces by **20%** ($V_1 = 0.8 \times 12 = 9.6 \text{ m/s}$)
 - $P_{\text{wind}} = 0.5 \cdot 1.226 \cdot 1600\pi \cdot 9.6^3$
 - $P_{\text{wind}} = 0.5 \cdot 1.226 \cdot 1600\pi \cdot 884.736$
 - $P_{\text{wind}} \approx 4441758.456 \text{ W}$
- $P_{\text{turbine}} = 0.85 \cdot 4441758.456$
- $P_{\text{turbine}} \approx 3775494.688 \text{ W}$



(iii) Electrical Power Generated

- $P_{\text{electric}} = \text{Generator Efficiency} \cdot P_{\text{turbine}}$
- Given generator efficiency as 85%:
- $P_{\text{electric}} = 0.85 \cdot 3775494.688$
- $P_{\text{electric}} \approx 3209170.4856 \text{ W}$



(iv) Axial Thrust on the Turbine

- Axial Thrust $T = 0.5 \cdot \rho \cdot A \cdot (V^2 - V_1^2)$
- V = Wind Velocity
- V_1 = Wind Velocity for wind turbine
 - Wind velocity reduction: $V_1 = 0.8 \cdot 12 = 9.6 \text{ m/s}$
- Axial thrust:
 - $T = 0.5 \cdot 1.226 \cdot 1600\pi \cdot (12^2 - 9.6^2)$
 - $T = 0.5 \cdot 1.226 \cdot 1600\pi \cdot (144 - 92.16)$
 - $T = 0.5 \cdot 1.226 \cdot 1600\pi \cdot 51.84$
 - $T \approx 318921.76 \text{ N}$



2. Three-bladed wind rotor with blade length of 52 m is operating in a wind stream having wind velocity of 13 m/s. Air density is 1.23 kg/m³ and power coefficient may be taken as 0.3. Calculate the extractable power from the wind.

Given:

- Blade length, $r = 52 \text{ m}$
- Wind velocity, $V = 13 \text{ m/s}$
- Air density, $\rho = 1.23 \text{ kg/m}^3$
- Power coefficient, $C_p = 0.3$

Formula:

The extractable power is given by:

$$P_{\text{extractable}} = 0.5 \cdot \rho \cdot A \cdot V^3 \cdot C_p$$

Step 1: Calculate the swept area (A):

$$A = \pi r^2 = \pi \cdot (52)^2 = \pi \cdot 2704 = 8494.04 \text{ m}^2$$

Step 2: Substitute values into the formula:

$$P_{\text{extractable}} = 0.5 \cdot 1.23 \cdot 8494.04 \cdot (13)^3 \cdot 0.3$$

Step 3: Simplify:

1. Wind velocity cubed:

$$V^3 = 13^3 = 2197$$

2. Multiply:

$$P_{\text{extractable}} = 0.5 \cdot 1.23 \cdot 8494.04 \cdot 2197 \cdot 0.3$$

$$P_{\text{extractable}} = 0.5 \cdot 1.23 \cdot 8494.04 \cdot 659.1$$

$$P_{\text{extractable}} \approx 3443029.385 \text{ W (or 3.443 MW)}$$

Final Answer:

The extractable power from the wind is approximately:

$$P_{\text{extractable}} = 3.443 \text{ MW}$$



3. A wind turbine rotor has a blade diameter of 20m. At a wind speed of 16m/s the turbine produces a power output of 270kW. Given that the density of air is 1.2 kg/m³ determine the maximum power coefficient of the turbine.

Given:

- Blade diameter, $D = 20 \text{ m}$
- Wind speed, $V = 16 \text{ m/s}$
- Power output, $P = 270 \text{ kW} = 270,000 \text{ W}$
- Air density, $\rho = 1.2 \text{ kg/m}^3$

Formula:

The formula for the power extracted from the wind is:

$$P_{\text{extractable}} = 0.5 \cdot \rho \cdot A \cdot V^3 \cdot C_p$$

Where:

- $A = \pi r^2$ is the swept area of the rotor, with $r = \frac{D}{2}$ being the radius of the rotor.
- C_p is the power coefficient (which we need to find).

Step 1: Calculate the swept area (A):

The radius of the rotor is:

$$r = \frac{D}{2} = \frac{20}{2} = 10 \text{ m}$$

Now, calculate the swept area:

$$A = \pi r^2 = \pi \cdot (10)^2 = 314.16 \text{ m}^2$$

Step 2: Use the formula to find C_p :

Rearranging the power formula to solve for C_p :

$$C_p = \frac{2P}{\rho A V^3}$$

Substitute the known values:

$$C_p = \frac{2 \cdot 270,000}{1.2 \cdot 314.16 \cdot 16^3}$$

Step 3: Simplify the expression:

1. Wind velocity cubed

$$1. V^3 = 16^3 = 4096$$

2. Now, calculate the value of C_p :

$$1. C_p = \frac{540,000}{1.2 \cdot 314.16 \cdot 4096}$$

$$2. C_p = \frac{540,000}{1,544,159.232}$$

$$3. C_p \approx 0.35$$

Final Answer:

The maximum power coefficient of the turbine is approximately:

$$C_p \approx 0.35$$

4. Define the capacity factor of a wind energy system. What is the capacity factor of a WES delivering average energy of 10000GWh for the last 10 years if its rated power capacity is 300MW?

Definition of Capacity Factor:

The **capacity factor** of a wind energy system (WES) is a measure of how efficiently the system is operating relative to its maximum potential output. It is defined as the ratio of the actual energy produced to the maximum possible energy the system could produce if it were operating at full capacity all the time.

The formula for the capacity factor is:

$$\text{Capacity Factor} = \frac{\text{Actual Energy Output}}{\text{Maximum Possible Energy Output}}$$

Where:

- **Actual Energy Output** is the total energy delivered by the system over a period of time.
- **Maximum Possible Energy Output** is the total energy the system would produce if it operated at its rated power capacity for the entire period.

Given:

- Average energy delivered, $E_{\text{actual}} = 10,000 \text{ GWh}$
- Rated power capacity, $P_{\text{rated}} = 300 \text{ MW}$
- Time period, $T = 10 \text{ years}$

Step 1: Calculate the maximum possible energy output:

The maximum possible energy output over 10 years if the system operated at full capacity is:

- $E_{\text{max}} = P_{\text{rated}} \cdot \text{hours per year} \cdot \text{number of years}$
 - Given $P_{\text{rated}} = \text{Power capacity} = 300 \text{ MW}$
 - There are $365 \times 24 = 8760$ hours in a year
 - $E_{\text{max}} = 300 \text{ MW} \cdot 8760 \text{ hours/year} \cdot 10 \text{ years}$
 - $E_{\text{max}} = 300 \cdot 8760 \cdot 10 = 26,280,000 \text{ MWh}$

- Since $1 \text{ GWh} = 1000 \text{ MWh}$, we convert to GWh:
- $E_{\max} = 26,280 \text{ GWh}$

Step 2: Calculate the capacity factor:

Now, we can calculate the capacity factor:

- Capacity Factor = $\frac{E_{\text{actual}}}{E_{\max}}$
- Substitute the values:
 - Given Average energy delivered = 10,000
 - We found the maximum energy = 26280
 - Capacity Factor = $\frac{10,000}{26,280}$
 - Capacity Factor ≈ 0.38

Final Answer:

The capacity factor of the wind energy system is approximately:

Capacity Factor ≈ 0.38 or 38%

