THE UNIVERSITY OF WARWICK

Fourth Year Examinations: 2017/2018

Agent Based Systems

Time allowed: 2 hours.

Answer **FOUR** questions.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on **each** answer book.

Approved calculators are allowed.

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1. Each one of two bars charges its own price for a beer, choosing between £2, £4, or £5. The cost of obtaining and serving the beer can be neglected. It is expected that 6000 beers per month are drunk in a bar by tourists, who choose one of the two bars randomly, and 4000 beers per month are drunk by natives, who go to the bar with the lowest price, and split evenly in case both bars offer the same price.

   (a) Model this scenario as a normal form game. [8]

   (b) Calculate all the equilibria in strictly dominant strategies. [5]

   (c) Calculate all the pure strategy Nash Equilibria. [5]

   (d) Show the procedure of iterated elimination of strictly dominated strategies, explaining each step, until convergence. [7]

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2. Alice and Bob are deciding how to split a cake. Players take turns: Alice starts, and she can propose a fraction $p \in \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}$. Bob can either accept (and if he does the game terminates with Alice taking p and Bob $1-p$), or make a counterproposal to Alice. Alice, in turn, can accept (and the game terminates with the accepted division) or make a counterproposal (also a fraction $p \in \{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\}$) to Bob, and so on.

   When a game terminates with an accepted division $(p^*, 1-p^*)$, meaning Alice gets p^* and Bob $1-p^*$, the utility of Alice equals $p^* - t$, and that of Bob equals $(1-p^*) - t$, where t is the number of times a proposal has been rejected before. For example, if Alice proposes $(\frac{1}{2}, \frac{1}{2})$ and Bob accepts immediately, then they both get $\frac{1}{2}$.

   After a given number of rounds the game terminates. If nobody has accepted any proposal they both get $-t$.

   (a) Model this scenario as an extensive game. [10]

   (b) Assume the game terminates after at most three turns (e.g., Alice proposes, Bob rejects and counter-proposes, Alice accepts or rejects). Calculate the backwards induction outcome. [8]

   (c) Give a (strictly positive!) number of turns that the game needs to have so that Bob gets nothing in some backwards induction outcome. [7]

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3. Ann, Bob and Charles are contemplating whether to go out together.

   Ann would love hanging out with both Bob and Charles. If she stays at home, though, she'd rather have Bob and Charles go out anyway. But she would definitely not want to go out with either Bob or Charles only, especially Bob.

   Bob's ideal scenario would be to go out with Ann only. He'd otherwise prefer staying on his own, unless he finds out Charles and Ann are going out together, in which case he'd prefer to join. He has absolutely no intention of going out with Charles only.

   Charles's ideal scenario would be to go out with Bob only. Otherwise, he is ok with hanging out with Ann only. If not, he'd prefer staying on his own, unless he finds out Bob and Ann are going out together, in which case he'd prefer to join.

   (a) Describe the preference ranking of each individual, in terms of how they sort the possible outcomes. [7]

   (b) Describe the potential coalitional deviations. What can you say about the stable outcomes? [9]

   (c) Describe the value of each coalition and the core of the game. [9]

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4. Ann and Bob are participating in the auction of a car. The auction is a sealed-bid second-price auction and each can bid a number $b \in [1, 100] \cap \mathbb{N}$, i.e., a natural number between 1 and 100. The final utility of players is calculated as the difference between the amount paid and the value they assign to the object, for the winner, 0 for the other player. If players tie then they both get utility 0.

   (a) Assume Ann values the car 10, while Bob values the car 11. Suppose Ann bids 9 and Bob bids 10. Is this a Nash equilibrium strategy? Explain. [6]

   (b) Assume Ann and Bob both value the car 10. Give two Nash equilibrium strategies, one where players tie, one where they do not. [9]

   (c) Assume it is common knowledge that Bob values the car either 1 or 3, and that these values are drawn uniformly at random. Also assume that Ann's value is determined by Bob's value, and that she values the car 3 if Bob values it 1, and 1 if he values it 3. Notice that while Bob knows his own value, and thus Ann's value, Ann doesn't. Give two Nash equilibrium strategies, one where players tie, one where they do not. [10]

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5. Let $\{w_1, w_2, w_3\}$ be a set of worlds and $N = \{Ann, Bob\}$ be a set of agents.

   (a) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is distributed knowledge at w_1. [6]

   (b) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is common knowledge at w_1. [6]

   (c) Let $E = \{w_1\}$ be a fact. Find an indistinguishability relation for each agent so that, at w_1, fact E is distributed knowledge at w_1 but not common knowledge at w_1. [6]

   (d) Is it possible to find a fact and an indistinguishability relation such that (1) w_3 is disconnected from all the other worlds by the indistinguishability relation of either agent and (2) at some world, that fact is general knowledge but not common knowledge? Explain. [7]
