

GAME THEORY



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A brief look at what we will discuss on this report



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01

In this auction, steps are decided initially, and the bids increase in subsequent steps or, in multiples of the defined steps.

02

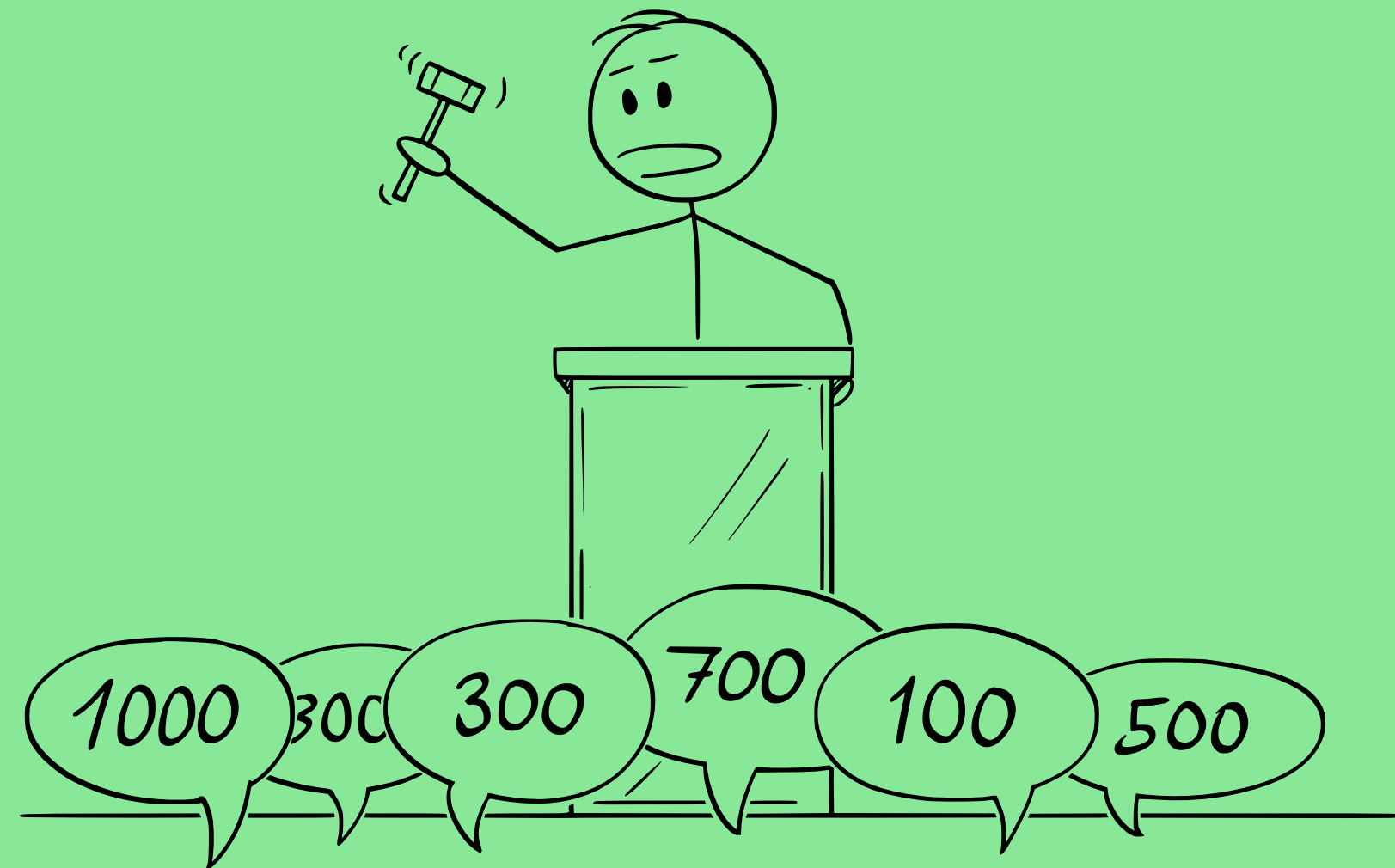
After the three initial consecutive bids, we begin to involve “weights”. Weight is some percentage of the amount which would be paid by the 2nd highest bidder on the highest bid offered by the highest bidder and remaining amount will be paid by the highest bidder.

03

The weights would increase in defined steps as well, which implies that the percentage of amount on the highest bid for the asset being paid by the 2nd highest bidder increases in these steps as well.

COMPLETE STUDY OF THE GAME

What is an AUCTION ?



Auction is an arrangement in which potential buyers place bids on assets or services.

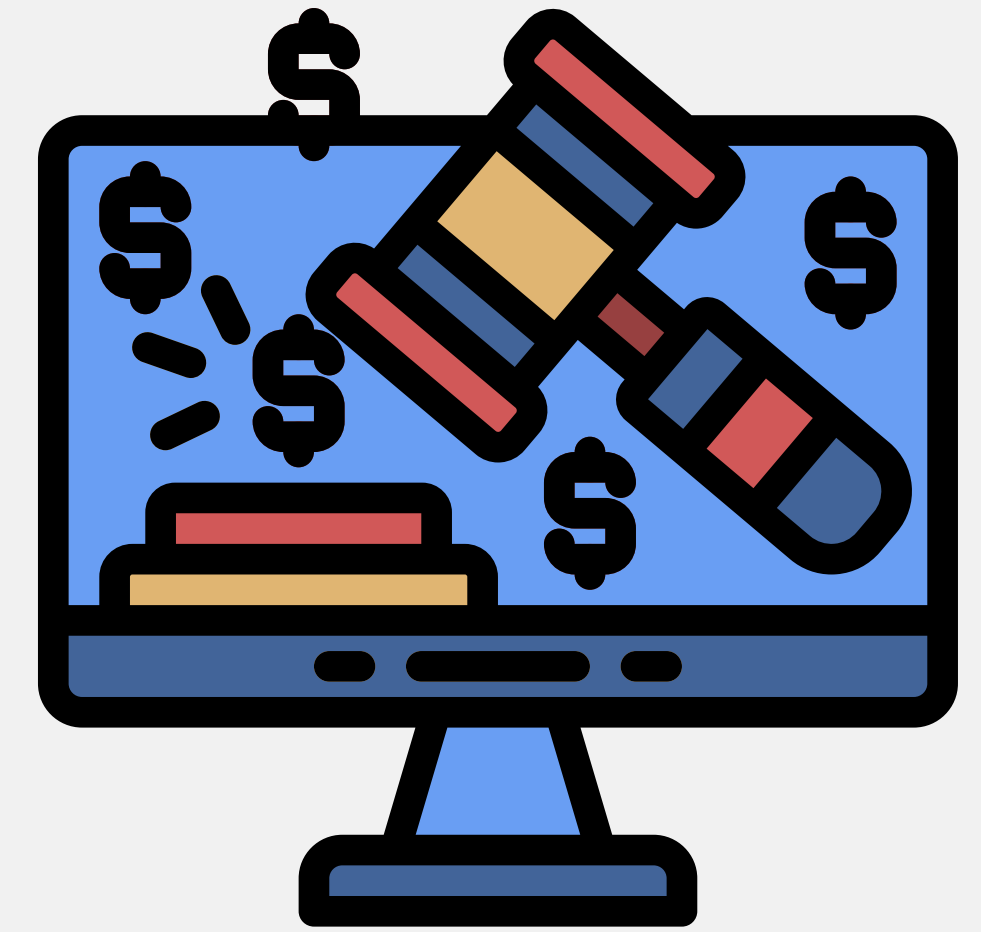
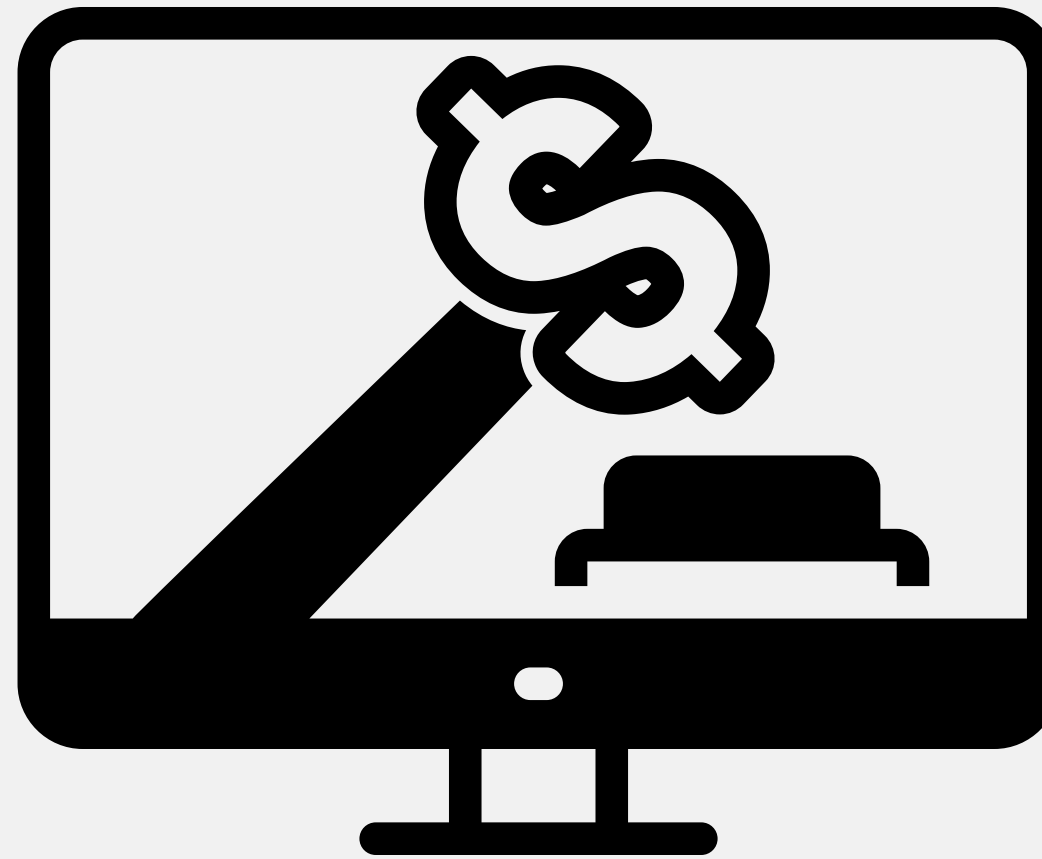
The asset in the auction is sold at the highest bid offered by the buyers in a competitive mechanism. An auction can be of two types – open auction, Dutch auction, sealed bid auction and Vickery sealed bid auction.

The auction types that we are going to discuss are -

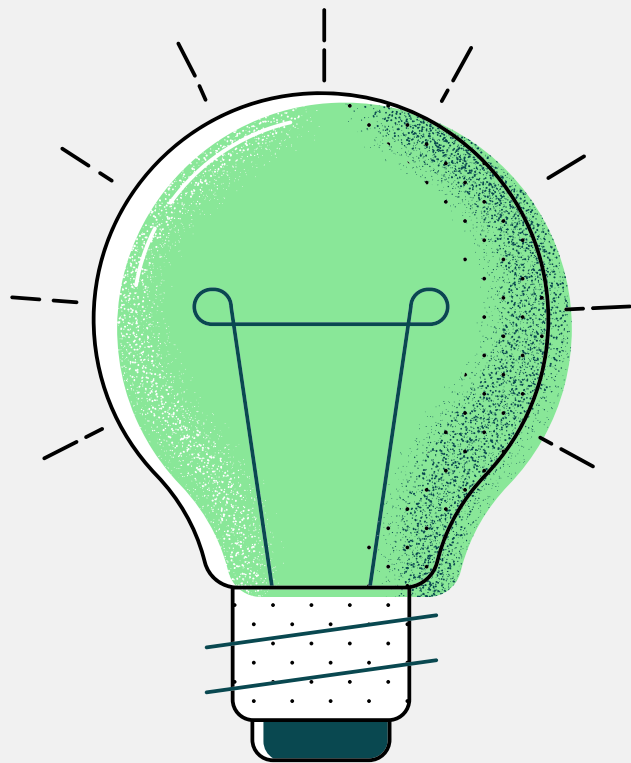
- (1) Ascending-bid (open, oral, or English) auction
- (2) Descending-bid (Dutch) auction

Theory

GAME UNO



THE ENGLISH (ORAL ASCENDING) AUCTION



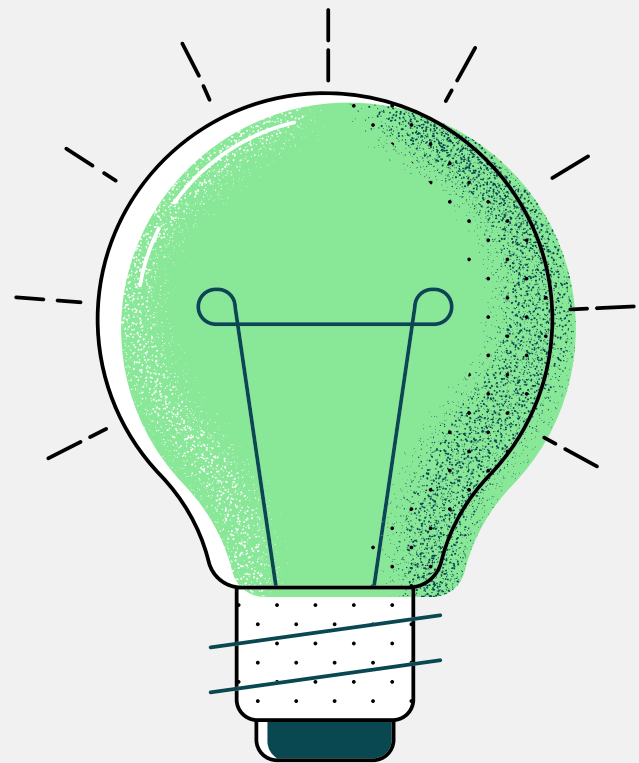
All bidders start in the auction with a price of zero. The price rises continuously, and bidders may drop out at any point in time. Once they drop out, they cannot reenter. The auction ends when only one bidder is left, and this bidder pays the price at which the second-to-last bidder dropped out.

Theory

GAME DUO



DESCENDING-BID (DUTCH) AUCTION



This describes auctions where the price is progressively lowered until a bidder accepts the current price. Dutch auctions are the most well-known kind of descending auction.

A Dutch auction has also been called a clock auction or open-outcry descending-price auction. This type of auction shows the advantage of speed since a sale never requires more than one bid.

AUCTION AS A GAME



Like all games we have studied so far, we have players who selfishly maximize their payoffs: using all the information available to them, they choose their best responses to other player's strategies.

Rules of the game include: players, strategies available for each player, payoffs for all possible combinations of strategies of all players.

Asymmetric information adds one more element: each player can be of more than one type and each player's type is his private information.

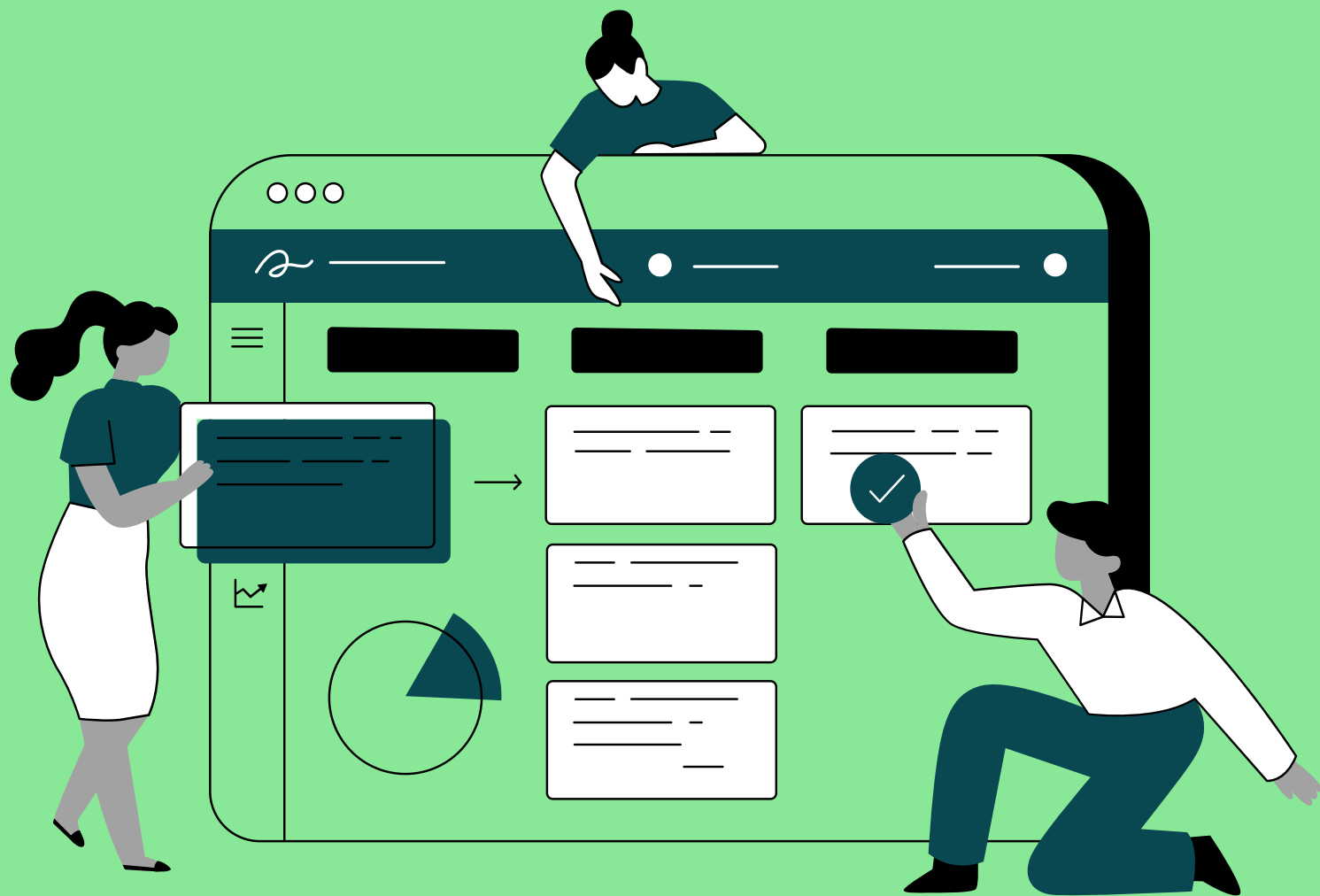
ASSUMPTIONS

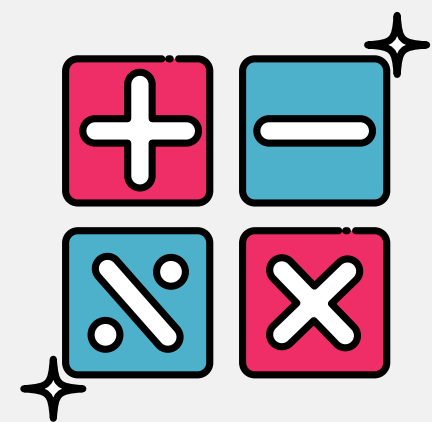
Each buyer knows for sure his own valuation and the number of buyers who participate in the auction, but not the exact valuations of the other buyers.

Each buyer knows the range of all possible valuations for the object and the probability with which each valuation happens.

Auctioneer has no clue whether the actual valuations of all buyers who came to his auction are high, low, a bunch of high and bunch of low, all valuations are exactly average, etc. He expects that any combination of valuations is possible with some probability.

Depending on all bids submitted (strategies played by all players), the auctioneer will determine the winner and announce some price, P . One buyer will win the auction and get the object. What is his utility? His utility is his private valuation that he will get to enjoy with the object minus the price he has to pay. If buyer indexed i wins the auction, then his utility, denoted U_i is $U_i = V_i - P$





MATHEMATICS

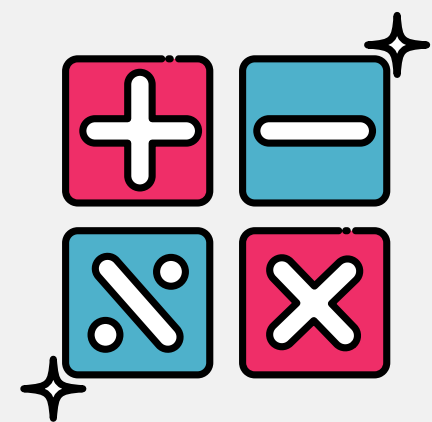
- Then bidder i 's strategy can be described by functions $b_{ik}(X_i | p_1, \dots, p_k)$ which specify the price at which bidder i will quit if, at that point, k other bidders have quit at the prices
- Now consider the strategy $b^* = (b_0^*, \dots, b_{n-2}^*)$ defined iteratively as follows.

$$(5) \quad b_0^*(x) = E[V_1 | X_1 = x, Y_1 = x, \dots, Y_{n-1} = x] \quad .$$

$$(6) \quad b_k^*(x | p_1, \dots, p_k) = E[V_1 | X_1 = x, Y_1 = x, \dots, Y_{n-k-1} = x,$$

$$b_{k-1}^*(Y_{n-k} | p_1, \dots, p_{k-1}) = p_k, \dots, b_0^*(Y_{n-1}) = p_1]$$

- The n —tuple $b^* = (b_0^*, \dots, b_{n-2}^*)$ is an equilibrium point of the English auction game.



MATHEMATICS

- If no information is provided by the seller, the expected price is

$$R_N^E = E[\bar{v}(Y_1, Y_1, Y_2, \dots, Y_{n-1}) | \{X_1 > Y_1\}]$$

- If the seller announces X_0 , the expected price is

$$R_I^E = E[\bar{w}(Y_1, Y_1, Y_2, \dots, Y_{n-1}; X_0) | \{X_1 > Y_1\}]$$

- Revealing information publicly raises revenues, that is $R_I^E \geq R_N^E$

GAME UNO :

THE ENGLISH (ORAL ASCENDING) AUCTION ->

RULES AND BEHAVIORAL ANALYSIS ->

Rules Of The GAME



- In this auction, steps are decided initially, and the bids increase in subsequent steps or, in multiples of the defined steps.
- After the first bid, we begin to involve “weights”. Weight is some percentage of the amount which would be paid by the 2nd highest bidder on the highest bid offered by the highest bidder and the remaining amount will be paid by the highest bidder.
- The weights would increase in defined steps as well, which implies that the percentage of amount on the highest bid for the asset being paid by the 2nd highest bidder increases in these steps as well.

Explanation of the GAME

Number of rounds in the game=4

Number of participants in the game= n (we have taken 4 players for our game)

As we can see from the spreadsheet, all 4 players have won one round each. Hence, the player with the maximum purse remaining after the game was declared the winner, i.e. A wins the game.

DATASHEET (SPREADSHEET) -

<https://docs.google.com/spreadsheets/d/ISLs95fCeFjMyYVWOHg6ra7YjJaIMFubFYSoE9jYCTEE/edit?usp=sharing>



BEHAVIORAL ANALYSIS



- In a general ascending auction each bidder tries to bid a value higher than the previous bidder's bid amount.
- The winner is the bidder who makes the highest bid.
- As the decision to stay or to exit is common knowledge among the bidders, the ascending auction makes the strategic decision of the agent public.
- Since the bidders had a certain amount only therefore the bids for a particular object were placed keeping the amount left in the purse of all the bidders in mind.
- Also, the bidder tries not to finish with the second-highest bid as he/she has to pay a certain fraction of the final price without winning the object. Hence, the bidder either drops out early in the auction or tries to bid higher, which is sometimes more than expected.

GAME DUO

DESCENDING-BID (DUTCH) AUCTION ->

RULES AND BEHAVIORAL ANALYSIS ->

Rules Of The GAME



- In this auction, we have decided on a few components per auction for which the bidders will bid against. There exists a minimum value available for bidders, which is the lowest amount that they can bid. Also, bidders are allowed to bid in pre-decided steps or multiple of these steps.
- Auction begins with one component at a time, the bidders can descend in the bidding value with the above restrictions. For the project (for which auction is conducted) there is a pre-decided price range considering practical factors like environmental factors, real-life costs, etc.
- This is done to simulate real-life situations in which bidders bid with real-life values for which they can undertake and make the project profitable.
- The winner of the auction is decided by the summation of bids, the person who has bid the closest to the cutoff value (which is determined before the auction) is declared as the winner.

Explanation of the GAME

Number of components=6

Number of participants in the game= n
(We have taken 4 players in our game)

As we can see from the spreadsheet,
Player C's purse is the closest to the pre-decided amount. Hence, Player C is the winner of the game.

DATASHEET (SPREADSHEET) -

<https://docs.google.com/spreadsheets/d/INT48M4pt8LpdFqAIWaKMvcc4CwMqfbkWahVRXQKYf9Q/edit?usp=sharing>

BEHAVIORAL ANALYSIS



- In a general descending auction each bidder tries to bid a value lower than the previous bidder's bid amount.
- The winner is the bidder who makes the lowest bid.
- Since the range of each material is fixed and the total price spent is the sum of all the materials hence the bidders try to reach a bid amount such that the total sum is above and as close as possible to the fixed amount set before the game starts.
- Therefore, the bidders try to get their total bid sum of all materials somewhere in the middle of the range of the sum amount.