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```

type A2, s=0, subset=[]

```
i+j=0 \mid \mathbb{C}
i+j=2 \mid \mathbb{C}^2
i+j=0
i+j=2 | 2
i+j=4 | 2
i+j=6 \ | \ 1
  h^{i,j} \mid j-i=0 j-i=2 j-i=4 j-i=6
```

module	multiplicity	dimension
all		16
\mathbb{C}	16	1

type A2, s=1, subset=[]

```
i+j=1 \mid \mathbb{C}L_{1,1}
i+j=1 | 9
i+j=3 | 19 9
```

module	multiplicity	dimension
all		67
\mathbb{C}	11	1
$L\left(\alpha_1+\alpha_2\right)$	7	8

type A2, s=2, subset=[]

module	multiplicity	dimension
all		211
$L\left(\alpha_1+\alpha_2\right)$	16	8
\mathbb{C}	3	1
$L\left(2\alpha_1+\alpha_2\right)$	4	10
$L\left(\alpha_1+2\alpha_2\right)$	4	10

type A2, s=3, subset=[]

module	multiplicity	dimension
all		521
\mathbb{C}	5	1
$L\left(\alpha_1+\alpha_2\right)$	15	8
$L\left(2\alpha_1+\alpha_2\right)$	9	10
$L\left(\alpha_1+2\alpha_2\right)$	9	10
$L\left(2\alpha_1+2\alpha_2\right)$	8	27

type A2, s=4, subset=[]

	j-i=0	j-i=2	j-i=4	j-i=6
i+j=0 $i+j=2$ $i+j=4$ $i+j=6$	0	0	54	35
i+j=4	0	288	315	
i+j=2	54	315		
i+j=0	35			

module	multiplicity	dimension
all		1096
$L\left(\alpha_1+\alpha_2\right)$	14	8
$L\left(2\alpha_1+2\alpha_2\right)$	20	27
\mathbb{C}	4	1
$L\left(2\alpha_1+\alpha_2\right)$	8	10
$L\left(\alpha_1+2\alpha_2\right)$	8	10
$L\left(3\alpha_1+2\alpha_2\right)$	4	35
$L\left(2\alpha_1+3\alpha_2\right)$	4	35

type A2, s=5, subset=[]

module	multiplicity	dimension
all		2072
$L\left(\alpha_1+\alpha_2\right)$	14	8
$L\left(2\alpha_1+\alpha_2\right)$	9	10
$L\left(\alpha_1+2\alpha_2\right)$	9	10
$L\left(2\alpha_1+2\alpha_2\right)$	21	27
$L\left(3\alpha_1+2\alpha_2\right)$	10	35
$L\left(2\alpha_1+3\alpha_2\right)$	10	35
$L\left(3\alpha_1+3\alpha_2\right)$	8	64
Ĉ	1	1

type A2, s=6, subset=[]

```
i+j=0 \mid L_{2,1}L_{1,2}L_{2,2}L_{3,3}
                                                        \begin{array}{l} L^5_{1,1} L^5_{2,1} L^5_{1,2} L^9_{2,2} L^5_{3,2} L^5_{2,3} L^5_{3,3} L_{4,3} L_{3,4} \\ L^2_{3,2} L^2_{2,3} L^4_{3,3} L^2_{4,3} L^2_{3,4} \end{array} 
i+j=2 \mid L_{3,3}^2
                                                                                                                                            L_{1,1}^5 L_{2,1}^5 L_{1,2}^5 L_{2,2}^9 L_{3,2}^5 L_{2,3}^5 L_{3,3}^5 L_{4,3} L_{3,4} \\ L_{3,3}^2
i+j=4
                                                                                                                                                                                                                                 L_{2,1}L_{1,2}L_{2,2}L_{3,3}
i+j=6
    h^{i,j}
                j - i = 0
                                                        j-i=2
                                                                                                                                             i-i=4
                                                                                                                                                                                                                                  i-i=6
i+j=0
               111
i+j=2
                128
                                 1215
```

module	multiplicity	dimension
all		3628
$L\left(2\alpha_1+\alpha_2\right)$	12	10
$L\left(\alpha_1+2\alpha_2\right)$	12	10
$L\left(2\alpha_1+2\alpha_2\right)$	20	27
$L\left(3\alpha_1+3\alpha_2\right)$	20	64
$L\left(\alpha_1+\alpha_2\right)$	10	8
$L\left(3\alpha_1+2\alpha_2\right)$	12	35
$L\left(2\alpha_1+3\alpha_2\right)$	12	35
$L\left(4\alpha_1+3\alpha_2\right)$	4	81
$L\left(3\alpha_1+4\alpha_2\right)$	4	81

720

i+j=4

 $i+j=6 \mid 0$

1215

111

128

j-i=0 j-i=2 j-i=4 j-i=6

type A2, s=7, subset=[]

i+j=3	702	2715	
i+j=3 $i+j=5$	0	702	936
$h^{i,j}$	j-i=1	j - i = 3	j-i=5

i+j=1 | 936

module	multiplicity	dimension
all		5991
$L\left(\alpha_1+\alpha_2\right)$	7	8
$L\left(2\alpha_1+\alpha_2\right)$	9	10
$L\left(\alpha_1+2\alpha_2\right)$	9	10
$L\left(2\alpha_1+2\alpha_2\right)$	23	27
$L\left(3\alpha_1+2\alpha_2\right)$	15	35
$L\left(2\alpha_1+3\alpha_2\right)$	15	35
$L\left(3\alpha_1+3\alpha_2\right)$	22	64
$L\left(4\alpha_1+3\alpha_2\right)$	10	81
$L\left(3\alpha_1+4\alpha_2\right)$	10	81
$L\left(4\alpha_1+4\alpha_2\right)$	8	125
$L\left(4\alpha_1+2\alpha_2\right)$	1	28
$L\left(2\alpha_1+4\alpha_2\right)$	1	28

type A2, s=8, subset=[]

 $L\left(4\alpha_1+3\alpha_2\right)$

 $L\left(3\alpha_1+4\alpha_2\right)$

 $L\left(5\alpha_1+4\alpha_2\right)$

 $L\left(4\alpha_1+5\alpha_2\right)$ 4

81

81

154

154

```
L_{2,2}L_{3,2}L_{2,3}L_{3,3}L_{4,4}
                                                   L^2_{1,1}L^3_{2,1}L^3_{1,2}L^{10}_{1,2}L^8_{3,2}L^8_{2,3}L^2_{4,2}L^{11}_{3,3}L^2_{2,4}L^5_{4,3}L^5_{3,4}L^5_{4,4}L_{5,4}L_{4,5}\\L^2_{4,3}L^2_{3,4}L^4_{4,4}L^2_{5,4}L^2_{4,5}
i+j=2
            L_{4.4}^2
                                                                                                                                                           L_{1,1}^2L_{2,1}^3L_{1,2}^3L_{1,2}^{10}L_{2,2}^8L_{3,2}^8L_{2,3}^8L_{4,2}^2L_{3,3}^{11}L_{2,4}^2L_{4,3}^5L_{3,4}^5L_{4,4}^5L_{5,4}L_{4,5}
i+j=4
            0
                                                                                                                                                                                                                                                                  L_{2,2}L_{3,2}L_{2,3}L_{3,3}L_{4,4}
i+j=6
  h^{i,j}
            j-i=0
                                                    j-i=2
                                                                                                                                                           j-i=4
                                                                                                                                                                                                                                                                 i-i=6
            286
i+j=0
            250
                          3465
i+j=2
                          1440
i+j=4
            0
                                       3465
                                       250
                                                    286
i+j=6
  h^{i,j}
            j-i=0 j-i=2 j-i=4
                                                   j-i=6
                          multiplicity
                                                   dimension
           module
                   all
                                                   9442
L\left(2\alpha_1+2\alpha_2\right)
                                                    27
L\left(3\alpha_1+2\alpha_2\right)
                                                    35
L\left(2\alpha_1+3\alpha_2\right)
                                                    35
L\left(3\alpha_1+3\alpha_2\right)
                                                    64
L\left(4\alpha_1+4\alpha_2\right)
                                                    125
   L(\alpha_1 + \alpha_2)
 L\left(2\alpha_1+\alpha_2\right)
                                                    10
 L\left(\alpha_1+2\alpha_2\right) 6
                                                    10
L\left(4\alpha_1+2\alpha_2\right) 4
                                                    28
L\left(2\alpha_1+4\alpha_2\right)
                                                    28
```

type A2, s=9, subset=[]

i+j=1 | $L_{2,1}L_{1,2}L_{2,2}^3L_{3,2}^4L_{3,2}^4L_{2,3}^4L_{4,2}L_{3,3}^6L_{2,4}L_{4,3}^3L_{3,4}^3L_{4,4}^4L_{5,4}L_{4,5}L_{5,5}$

 $L_{1,1}L_{2,1}^4L_{1,2}^4L_{1,2}^{10}L_{2,2}^{10}L_{3,2}^{10}L_{2,3}^{10}L_{4,2}^4L_{3,3}^{17}L_{2,4}^4L_{4,3}^{10}L_{3,4}^{10}L_{5,3}L_{4,4}^{10}L_{3,5}L_{5,4}^4L_{4,5}^4L_{5,5}^2$

 $L_{4.4}^2 L_{5.4}^2 L_{4.5}^2 L_{5.5}^2$

j-i=3

 $L_{2,1}L_{1,2}L_{2,2}^3L_{3,2}^4L_{2,3}^4L_{4,2}L_{3,3}^6L_{2,4}L_{4,3}^3L_{3,4}^3L_{4,4}^4L_{5,4}L_{4,5}L_{5,5}$

j-i=5

```
i+j=3 L_{4,4}^2L_{5,4}^2L_{4,5}^{2'}L_{5,5}^{2'}
i + j = 5
  h^{i,j}
          j-i=1
i+j=1 | 2331
i+j=3 | 1298
                       7064
i+j=5 | 0
                       1298
                                   2331
  h^{i,j}
           i-i=1 i-i=3 i-i=5
          module multiplicity
                                              dimension
                  all
                                              14322
  L(2\alpha_1 + \alpha_2) 6
                                              10
  L\left(\alpha_1+2\alpha_2\right) 6
                                              10
L(2\alpha_1 + 2\alpha_2) 16
L\left(3\alpha_1+2\alpha_2\right) 18
                                              35
L(2\alpha_1 + 3\alpha_2) 18
                                              35
L\left(4\alpha_1+2\alpha_2\right) 6
L(3\alpha_1 + 3\alpha_2) 29
                                              64
L\left(2\alpha_1+4\alpha_2\right)
                                              28
L\left(4\alpha_1+3\alpha_2\right)
                                              81
L\left(3\alpha_1+4\alpha_2\right)
                                              81
L\left(4\alpha_1+4\alpha_2\right)
                                              125
L\left(5\alpha_1+4\alpha_2\right)
                                              154
L\left(4\alpha_1+5\alpha_2\right)
                                              154
L\left(5\alpha_1+5\alpha_2\right) 8
                                              216
   L(\alpha_1 + \alpha_2) 1
L\left(5\alpha_1+3\alpha_2\right) 1
                                              80
L(3\alpha_1 + 5\alpha_2) 1
                                              80
```

type A2, s=10, subset=[]

216

10

10

28

28 80

80

154

154

260

260

 $L\left(5\alpha_1+5\alpha_2\right)$ 20

 $L(2\alpha_1 + \alpha_2)$ 4

 $L\left(\alpha_1+2\alpha_2\right)$ 4

 $L(2\alpha_1 + 2\alpha_2) \quad 10$ $L(4\alpha_1 + 2\alpha_2) \quad 6$

 $L\left(2\alpha_1+4\alpha_2\right)$ 6

 $L(5\alpha_1 + 3\alpha_2) \quad 4$ $L(3\alpha_1 + 5\alpha_2) \quad 4$

 $L\left(5\alpha_1+4\alpha_2\right)$ 12

 $L(4\alpha_1 + 5\alpha_2)$ 12

 $L\left(6\alpha_1+5\alpha_2\right)$ 4

 $L\left(5\alpha_1+6\alpha_2\right)$ 4

0 1	,	,	LJ				
i+j=0 $i+j=2$ $i+j=4$	$L_{3,2}L_{2,3}L_{3,3}L_{5,5}$ $L_{5,5}^2$	$L_{4,3}L_{3,4}L_{4,4}$	$L_{5,5}$ $L_{2,1}^2 L_{1,2}^2 L_{2,}^5$ $L_{5,4}^2 L_{4,5}^2 L_{5,}^4$	$_{,2}L_{3,2}^{8}L_{2,3}^{8}L_{4,2}^{3}L_{3,3}^{13}L_{3,4}^{3}L_{4,3}^{10}L_{3,4}^{10}L_{5,3}^{2}L_{4,4}^{11}L_{3,5}^{2}L_{5,4}^{5}L_{4,5}^{5}L_{5,5}^{5}I_{5,5}^{2}L_{5,6}^{2}$	$S_{5,5}L_{6,5}L_{5,6}$	$L^2_{2,1}L^2_{1,2}L^5_{2,2}L^8_{3,2}L^8_{2,3}L^3_{4,2}L^{13}_{3,3}L^3_{2,4}L^{10}_{4,3}L^{10}_{3,4}L^2_{5,3}L^{11}_{4,4}L^2_{3,5}L^5_{5,4}L^5_{5,5}L^5_{6,5}L_{6,5}L_{5,6}$	
i+j=6	0		0	, , ,		$L_{5,5}^2$	$L_{3,2}L_{2,3}L_{3,3}L_{4,3}L_{3,4}L_{4,4}L_{5,5}$
	j-i=0		j-i=2			j-i=4	j-i=6
'							
i+j=0	637						
i+j=2	432 8190						
i+j=4	0 2520	8190					
i+j=6	0 0	432 6	37				
$h^{i,j}$	j-i=0 $j-i=2$	j-i=4 j	-i=6				
·							
1	module multi	- v	imension				
	all		1038				
	$+2\alpha_2$) 18	3					
	$+3\alpha_2$) 18	3					
	$+3\alpha_2$) 28	6					
	$+3\alpha_2$) 22	8					
	$+4\alpha_2$) 22	8					
$L\left(4\alpha_{1}\right)$	$+4\alpha_2$) 24	1:	25				

type A2, s=0, subset=[1]

```
i+j=2 1 1 1 1 1 1 i+j=4 1 1 1 h^{i,j} j-i=0 j-i=2 j-i=4
```

module	multiplicity	${\rm dimension}$
all		6
\mathbb{C}	6	1

type A2, s=1, subset=[1] $\frac{i+j=1 \atop i+j=3}{\|\mathbb{C}L_{1,1} \atop h^{i,j} \mid_{j-i=1}} \mathbb{C}L_{1,1} \atop_{j-i=3}$

module	multiplicity	dimension
all		19
\mathbb{C}	3	1
$L\left(\alpha_1+\alpha_2\right)$	2	8

type A2, s=2, subset=[1]

 $i+j=0 \mid L_{1,1}$

module	multiplicity	dimension
all		52
$L\left(\alpha_1+\alpha_2\right)$	4	8
$L\left(2\alpha_1+\alpha_2\right)$	1	10
$L\left(\alpha_1+2\alpha_2\right)$	1	10

type A2, s=3, subset=[1]

module	multiplicity	dimension
all		110
$L\left(\alpha_1+\alpha_2\right)$	2	8
$L\left(2\alpha_1+\alpha_2\right)$	2	10
$L\left(\alpha_1+2\alpha_2\right)$	2	10

27

 $L\left(2\alpha_1+2\alpha_2\right)$ 2

type A2, s=4, subset=[1]

i+j = 0	$L_{2,2}$		
i+j=0 $i+j=2$ $i+j=4$	0	$L_{2,1}L_{1,2}L_{2,2}^2L_{3,2}L_{2,3}$	
i+j=4	0	0	$L_{2,2}$
$h^{i,j}$	j-i=0	j-i=2	j-i=4

i + j = 0	27		
i+j=2	0	144	
i+j=0 $i+j=2$ $i+j=4$	0	0	27
$h^{i,j}$	j-i=0	j-i=2	j-i=4

module	multiplicity	dimension
all		198
$L\left(2\alpha_1+2\alpha_2\right)$	4	27
$L\left(2\alpha_1+\alpha_2\right)$	1	10
$L\left(\alpha_1+2\alpha_2\right)$	1	10
$L\left(3\alpha_1+2\alpha_2\right)$	1	35
$L\left(2\alpha_1+3\alpha_2\right)$	1	35

type A2, s=5, subset=[1]

i+j=3	0	161	
$h^{i,j}$	j-i=1	j-i=3	
	modulo	multiplicity	dimonsio

module	multiplicity	dimension
all		322
$L\left(2\alpha_1+2\alpha_2\right)$	2	27
$L\left(3\alpha_1+2\alpha_2\right)$	2	35
$L\left(2\alpha_1+3\alpha_2\right)$	2	35
$L\left(3\alpha_1+3\alpha_2\right)$	2	64

type A2, s=6, subset=[1]

	Ι τ		
i+j=0	$L_{3,3}$		
i+j=2	0	$L_{3,2}L_{2,3}L_{3,3}^2L_{4,3}L_{3,4}$	
i+j=0 $i+j=2$ $i+j=4$	0	0	$L_{3,3}$
$h^{i,j}$	j-i=0	j-i=2	j-i=4

multiplicity	dimension
	488
4	64
1	35
1	35
1	81
1	81
	4

type A2, s=7, subset=[1]

module	multiplicity	dimension
all	T T	702
$L\left(3\alpha_1+3\alpha_2\right)$	2	64
$L\left(4\alpha_1+3\alpha_2\right)$	2	81
$L(3\alpha_1+4\alpha_2)$	2	81

125

 $L(4\alpha_1 + 4\alpha_2)$ 2

type A2, s=8, subset=[1]

	l -		
i+j=0	$L_{4,4}$		
i+j=2	0	$L_{4,3}L_{3,4}L_{4,4}^2L_{5,4}L_{4,5}$	
i+j=0 $i+j=2$ $i+j=4$	0	0	$L_{4,4}$
$h^{i,j}$	j-i=0	j-i=2	j-i=4

module	multiplicity	dimension
all		970
$L\left(4\alpha_1+4\alpha_2\right)$	4	125
$L\left(4\alpha_1+3\alpha_2\right)$	1	81
$L\left(3\alpha_1+4\alpha_2\right)$	1	81
$L\left(5\alpha_1+4\alpha_2\right)$	1	154
$L\left(4\alpha_1+5\alpha_2\right)$	1	154

type A2, s=9, subset=[1]

i+j=3		649		
$h^{i,j}$	j-i=1	j-i=3		
		1	١.	.,

module	multiplicity	dimension
all		1298
$L\left(4\alpha_1+4\alpha_2\right)$	2	125
$L\left(5\alpha_1+4\alpha_2\right)$	2	154
$L\left(4\alpha_1+5\alpha_2\right)$	2	154
$L\left(5\alpha_1+5\alpha_2\right)$	2	216

type A2, s=10, subset=[1]

i+j=0 $i+j=2$ $i+j=4$	0	$L_{5,4}L_{4}$ 0	$L_{5,5}^2L_{5,5}^2L_{6}$	$_{5,5}L_{5,6}$	$L_{5,5}$
$h^{i,j}$	j-i=0	j-i=2			j-i=4
i+j=0	216				
i+j=0 $i+j=2$	0	1260			
i+j=4	0	0	216		
$h^{i,j}$	j-i=0	j-i=2	j-i=4		

module	multiplicity	dimension
all		1692
$L\left(5\alpha_1 + 5\alpha_2\right)$	4	216
$L\left(5\alpha_1+4\alpha_2\right)$	1	154
$L\left(4\alpha_1+5\alpha_2\right)$	1	154
$L\left(6\alpha_1+5\alpha_2\right)$	1	260
$L(5\alpha_1+6\alpha_2)$	1	260

type A2, s=11, subset=[1]

i+j=1	1079	
i + j = 3	0	1079
$h^{i,j}$	j-i=1	j-i=3

module	multiplicity	dimension
all		2158
$L\left(5\alpha_1 + 5\alpha_2\right)$	2	216
$L\left(6\alpha_1+5\alpha_2\right)$	2	260
$L\left(5\alpha_1+6\alpha_2\right)$	2	260
$L\left(6\alpha_1+6\alpha_2\right)$	2	343

type A2, s=12, subset=[1]

i + j = 0	$L_{6,6}$				
i+j=0 $i+j=2$	0	$L_{6,5}L_{5}$	$L_{6,6}^2 L_{6,6}^2 L_{7}$	$L_{6,7}$	
i+j=4	0	0	,		$L_{6,6}$
$h^{i,j}$	j-i=0	j-i=2			j-i=4
i+j=0	343				
i+j=0 $i+j=2$	0	2016			
	0	0	343		
$h^{i,j}$	j-i=0	j-i=2	j-i=4		

module	multiplicity	dimension
all		2702
$L\left(6\alpha_1+6\alpha_2\right)$	4	343
$L\left(6\alpha_1+5\alpha_2\right)$	1	260
$L\left(5\alpha_1+6\alpha_2\right)$	1	260
$L\left(7\alpha_1+6\alpha_2\right)$	1	405
$L\left(6\alpha_1+7\alpha_2\right)$	1	405

type A2, s=13, subset=[1] $_{i+j=1} \mid L_{6,6}L_{7,6}L_{6,7}L_{7,7}$

i+j=1	1665	
i+j=3	0	1665
$h^{i,j}$	j-i=1	j-i=3

module	multiplicity	dimension
all		3330
$L\left(6\alpha_1+6\alpha_2\right)$	2	343
$L\left(7\alpha_1+6\alpha_2\right)$	2	405
$L\left(6\alpha_1+7\alpha_2\right)$	2	405
$L\left(7\alpha_1+7\alpha_2\right)$	2	512

type A3, s=0, subset=[]

```
\mathbb{C}
  i+j=0
                \mathbb{C}^3
  i+j=2
                \mathbb{C}^5
                               \mathbb{C}^4
  i+j=4
                \mathbb{C}^6
                               \mathbb{C}^9
                                               \mathbb{C}^4
  i+j=6
                                              \mathbb{C}^9
                                                              \mathbb{C}^4
                \mathbb{C}^5
                               \mathbb{C}^{11}
 i + j = 8
                                                             \mathbb{C}^9
                                                                             \mathbb{C}^4
                \mathbb{C}^3
                               \mathbb{C}^8
                                              \mathbb{C}^{11}
i + j = 10
                                \mathbb{C}^3
                                               \mathbb{C}^5
                                                              \mathbb{C}^6
                                                                             \mathbb{C}^5
                                                                                            \mathbb{C}^3
                \mathbb{C}
                                                                                                             \mathbb{C}
i + j = 12
     h^{i,j}
                j-i=0
                               j-i=2
                                              j-i=4
                                                              j-i=6
                                                                             j-i=8
                                                                                            j - i = 10
                                                                                                             j - i = 12
                1
 i+j=0
  i+j=2
                3
                               1
                5
                                4
                                              1
 i+j=4
                6
                                9
                                              4
                                                              1
 i + j = 6
                5
                                11
                                               9
                                                                             1
                                                              4
 i + j = 8
                3
                                8
                                               11
                                                              9
                                                                             4
                                                                                            1
i + j = 10
                1
                                3
                                               5
                                                                             5
                                                                                            3
                                                                                                             1
i + j = 12
                                                              6
     h^{i,j}
                j-i=0
                               i-i=2
                                              i-i=4
                                                              j-i=6
                                                                             j-i=8
                                                                                            i - i = 10
                                                                                                             i - i = 12
```

module	multiplicity	dimension
all		125
\mathbb{C}	125	1

type A3, s=1, subset=[]

```
\mid \mathbb{C}L_{1,1,1}
 i+j=1
           \mathbb{C}^4 L^3_{1,1,1}
                            \mathbb{C}L_{1,1,1}
 i+j=3
                            \mathbb{C}^5L^4_{1,1,1}
                                                    \mathbb{C}L_{1,1,1}
 i+j=5
                            i + j = 7
                                                                                             \mathbb{C}L_{1,1,1}
 i + j = 9
                                                                              \mathbb{C}^9L_{1,\underline{1,\underline{1}}}^{5}
                                                     \mathbb{C}^{11}L_{1,1,1}^{3}
                                                                                                             \mathbb{C}L_{1,1,1}
i + j = 11
   h^{i,j}
            i-i=1
                            i-i=3
                                                     i-i=5
                                                                              i-i=7
                                                                                             i-i=9
                                                                                                             i - i = 11
 i+j=1
            16
            49
                       16
 i+j=3
                       65
                                  16
           84
 i+j=5
           56
                      193
                                  85
                                             16
 i+j=7
                       152
                                             65
 i + j = 9
                                  193
                                                        16
                       8
                                  56
                                                        49
                                                                   16
i + j = 11
                                             84
```

i-i=9

i - i = 11

module	multiplicity	dimension
all		1246
\mathbb{C}	131	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	61	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	10	20

i-i=5 i-i=7

i-i=3

 $h^{i,j}$

i-i=1

type A3, s=2, subset=[]

```
i+j=0 \mid L_{1,1,1}
                           \mathbb{C}L^3_{1,1,1}L_{2,2,1}L_{1,2,2}
                           \begin{array}{c} \mathbb{C}_{1,1,1}\mathbb{L}_{2,2,1}\mathbb{L}_{1,2,2} \\ \mathbb{C}^{5}L_{1,1,1}^{11}L_{2,2,1}^{3}L_{1,2,2}^{3} \\ \mathbb{C}^{7}L_{1,1,1}^{18}L_{1,2,1}^{3}L_{2,2,1}^{5}L_{1,2,2}^{5} \\ \mathbb{C}^{6}L_{1,1,1}^{8}L_{1,2,1}^{3}L_{2,2,1}^{3}L_{1,2,2}^{3} \\ \mathbb{C}^{3} \end{array} 
                                                                           \mathbb{C}L_{1.1.1}^3L_{2,2,1}L_{1,2,2}
 i+j=4
                                                                        i + j = 6
                                                                                                                                                                            \mathbb{C}L_{1,1,1}^3L_{2,2,1}L_{1,2,2}
 i + j = 8
                                                                                                                                                                            i+j=10 \ 0
i + j = 12
    h^{i,j}
             j-i=0 j-i=2
                                                                           j-i=4
                                                                                                                             i-i=6
                                                                                                                                                                            j-i=8
 i+j=0 | 15
                            136
 i+j=2
                           440
                                         136
 i+j=4
                            787
                                         601
                                                      136
 i+j=6
                           456
                                         1312
                                                      601
 i+j=8
                                                                    136
i+j=10 | 0
                                         456
                                                      787
                                                                    440
                                                                                 136
                                                                                                15
i+j=12
```

 $\mathbb{C}L_{1,1,1}^3L_{2,2,1}L_{1,2,2}$

 $L_{1,1,1}$

i - i = 12

 $L_{1,1,1}^3$

j - i = 10

module	multiplicity	dimension
all		6923
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	163	15
\mathbb{C}	68	1
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	43	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	43	45
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	27	20

i-i=2 i-i=4 i-i=6 i-i=8 i-i=10 i-i=12

 $h^{i,j}$

type A3, s=3, subset=[]

module	multiplicity	dimension
all		27730
\mathbb{C}	96	1
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	207	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	68	20
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	113	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	113	45
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	76	84
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	27	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	27	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	27	175

i-i=3 i-i=5

i + j = 11

736

type A3, s=0, subset=[1]

i+j=0 i+j=2 i+j=4 i+j=6 i+j=8 i+j=10	$ \begin{array}{c} \mathbb{C} \\ \mathbb{C}^2 \\ \mathbb{C}^3 \\ \mathbb{C}^3 \\ \mathbb{C}^2 \\ \mathbb{C} \end{array} $	$ \begin{array}{c} \mathbb{C} \\ \mathbb{C}^3 \\ \mathbb{C}^5 \\ \mathbb{C}^4 \\ \mathbb{C}^2 \end{array} $	\mathbb{C} \mathbb{C}^3 \mathbb{C}^5 \mathbb{C}^3	\mathbb{C} \mathbb{C}^3 \mathbb{C}^3	\mathbb{C} \mathbb{C}^2	$\mathbb C$
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10
i+j=0	1					
i+j=2	2	1				
i+j=4	3	3	1			
i + j = 6	3	5	3	1		
i + j = 8	2	4	5	3	1	
i+j=10	1	2	3	3	2	1
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10

module	$\operatorname{multiplicity}$	dimension
all		50
\mathbb{C}	50	1

type A3, s=1, subset=[1]

module	multiplicity	dimension
all		521
\mathbb{C}	51	1
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	26	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	4	20

type A3, s=2, subset=[1]

```
i+j=0 \mid L_{1,1,1}
                                                                   \begin{array}{cccc} L^{2,-,-}_{1,1,1} & \mathbb{C}L^3_{1,1,1}L_{2,2,1}L_{1,2,2} \\ L^2_{1,1,1} & \mathbb{C}^2L^7_{1,1,1}L_{1,2,1}L^2_{2,2,1}L^2_{1,2,2} & \mathbb{C}L^3_{1,1,1}L_{2,2,1}L_{1,2,2} \end{array} 
                                                                                                                                     \begin{array}{c} \mathbb{C} L_{1,1,1}L_{1,2,1}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}^2 L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C} \\ \mathbb{C} \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C}^2 L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C}^2 L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C}^2 L_{1,1,1}^5 L_{1,2,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,1}^2 L_{1,2,2}^2 L_{1,2,2}^2 \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 L_{1,2,2}^2 \\ \mathbb{C} L_{1,1,1}^5 L_{1,2,2}^2 L_{1,2,
       i+j=6
       i+j=8
i + j = 10
                       h^{i,j}
                                                                     j-i=0 j-i=2
                                                                                                                                                                                                                                                                                                                                                                          i-i=4
       i+j=0
                                                                                                                                      136
       i+j=2
                                                                                                                                      307
                                                                                                                                                                                                      136
       i+j=4
                                                                                                                                     297
                                                                                                                                                                                                      489
       i+j=6
                                                                                                                                                                                                                                                                      136
                                                                                                                                                                                                      297
                                                                                                                                                                                                                                                                      307
                                                                                                                                                                                                                                                                                                                                      136
       i+j=8
                                                                                                                                                                                                                                                                                                                                                                                                    15
i + j = 10
                                                                     i-i=0 i-i=2 i-i=4
                                                                                                                                                                                                                                                          i-i=6
```

 $L_{1,1,1}^2$

i-i=6

 $L_{1.1.1}^2$

i-i=8

 $L_{1,1,1}$

i - i = 10

module	multiplicity	dimension
all		2392
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	55	15
\mathbb{C}	17	1
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	15	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	15	45
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	10	20

type A3, s=3, subset=[1]

```
\mathbb{C}L_{1,1,1}^4L_{1,2,1}L_{2,2,1}^2L_{1,2,2}^2L_{2,2,2}^2
_{i+j=5} \mid L_{1,1,1}^{2} L_{2,2,1}^{2} L_{1,2,2}^{2} L_{2,2,2}^{2}
                                                         L_{1,1,1}^3L_{1,2,1}^2L_{2,2,1}^4L_{1,2,2}^4L_{3,2,1}^2L_{2,2,2}^2L_{1,2,3}^2L_{2,3,2}^2
i + j = 7
i+j=9
   h^{i,j}
           i-i=1
                                                         i-i=3
i+j=1 | 240
i+j=3
           429
                      816
                      1690
                                 817
i+j=5
                                 1690
i+j=7 | 0
                      1103
i+j=9 | 0
           j-i=1 j-i=3 i-i=
```

 $L\left(\alpha_1+\alpha_2+\alpha_3\right)$ $L\left(\alpha_1+2\alpha_2+\alpha_3\right)$ $L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$ $L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$ $L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$ $L(3\alpha_1 + 2\alpha_2 + \alpha_3)$ $L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$ $L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$

 $i+j=1 \mid \mathbb{C}L_{1,1,1}^3 L_{1,2,1} L_{2,2,1} L_{1,2,2} L_{2,2,2}$

1103	1690	816		
0	378	429	240	
1 j-i=3	j - i = 5	j-i=7	j-i=9	
module	multip	licity	dimension	
all			9026	
\mathbb{C}	22		1	
$\alpha_2 + \alpha_3$	59		15	
$\alpha_2 + \alpha_3$	20		20	
$\alpha_2 + \alpha_3$	37		45	
$_{2}+2\alpha_{3})$	37		45	
$_{2}+2\alpha_{3})$	26		84	
$\alpha_2 + \alpha_3$	9		35	
$_{2}+3\alpha_{3})$	9		35	
$_{2}+2\alpha_{3}$	9		175	

```
\mathbb{C}^4 L_{1,1,1}^{10,1} L_{1,2,1}^{4,7,1} L_{2,2,1}^{7,7,1} L_{1,2,2}^{7,7,2} L_{3,2,1}^2 L_{2,2,2}^{4,7,7} L_{1,2,3}^2 L_{2,3,2}^2
                                                                                                                                     \mathbb{C}^3L_{1,1,1}^6L_{1,2,1}^2L_{2,2,1}^3L_{1,2,2}^3L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}
                                                                    L_{1,1,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 L_{2,2,2}^2
                                                                                                                                      \mathbb{C}L_{1,1,1}^4L_{1,2,1}L_{2,2,1}^2L_{1,2,2}^2L_{2,2,2}^2
                                                                   i-i=5
                                                                                                                                      i-i=7
```

 $\mathbb{C}L_{1,1,1}^3L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}$

i-i=9

type A3, s=4, subset=[1]

```
i+j=0 \mid L_{1,1,1}L_{1,2,1}L_{2,2,2}
                                                                                                \begin{array}{l} \mathbb{C}^2 L_{1,1,1}^7 L_{1,2,1}^5 L_{2,2,1}^4 L_{1,2,2}^4 L_{3,2,1} L_{2,2,2}^5 L_{1,2,3} L_{2,3,2}^2 L_{3,3,2} L_{2,3,3} \\ \mathbb{C} L_{1,1,1}^4 L_{1,2,1}^2 L_{2,2,1}^5 L_{1,2,2}^5 L_{3,2,1}^2 L_{2,2,2}^8 L_{1,2,3}^2 L_{2,3,2}^2 L_{3,3,2}^2 L_{2,3,3}^2 \\ L_{2,2,1}^2 L_{1,2,2}^2 L_{3,2,1}^2 L_{2,2,2}^4 L_{1,2,3}^2 L_{2,2,2}^2 L_{2,3,2}^2 L_{2,3,3}^2 L_{2,3,3}^2 \end{array} 
    i+j=2 L_{1,2,1}L_{2,2,2}^2
                                                                                                                                                                                                                                                                                                                     \begin{array}{l} \mathbb{C}^5L_{1,1,1}^{13}L_{1,2,1}^{8}L_{2,2,1}^{7}L_{1,2,2}^{7}L_{3,2,1}^{3}L_{2,2,2}^{8}L_{1,2,3}^{3}L_{2,3,2}^{5}L_{3,3,2}^{2}L_{2,4,2}L_{2,3,3}^{2}\\ \mathbb{C}^3L_{1,1,1}^{8}L_{1,2,1}^{5}L_{2,2,1}^{8}L_{1,2,2}^{8}L_{3,2,1}^{6}L_{2,2,2}^{11}L_{1,2,3}^{6}L_{2,3,2}^{7}L_{3,3,2}^{4}L_{3,3,2}^{4}L_{2,4,2}L_{2,3,3}^{4}\\ L_{2,2,1}^{2}L_{1,2,2}^{2}L_{3,2,1}^{2}L_{2,2,2}^{4}L_{1,2,3}^{2}L_{2,3,2}^{2}L_{2,3,2}^{2}L_{2,3,3}^{2}L_{2,3,3}^{2}\end{array}
    i+j=4 \mid L_{2,2,2}^2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              \begin{array}{c} \mathbb{C}^5 L^{13}_{1,1,1} L^8_{1,2,1} L^7_{2,2,1} L^7_{1,2,2} L^3_{3,2,1} L^8_{2,2,2} L^3_{1,2,3} L^5_{2,3,2} L^2_{3,3,2} L_{2,4,2} L^2_{2,3,3} \\ \mathbb{C} L^4_{1,1,1} L^2_{1,2,1} L^5_{2,2,1} L^5_{1,2,2} L^2_{3,2,1} L^8_{2,2,2} L^2_{1,2,3} L^3_{2,3,2} L^2_{3,3,2} L^2_{2,3,3} \end{array} 
    i+j=6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          \mathbb{C}^2L^7_{1,1,1}L^5_{1,2,1}L^4_{2,2,1}L^4_{1,2,2}L_{3,2,1}L^5_{2,2,2}L_{1,2,3}L^2_{2,3,2}L_{3,3,2}L_{2,3,3}
    i+j=8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  L_{2,2,2}^2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          L_{1,2,1}L_{2,2,2}^2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  L_{1,1,1}L_{1,2,1}L_{2,2,2}
i+j=10 0
        h^{i,j} \mid j-i=0
                                                                                                 j-i=2
                                                                                                                                                                                                                                                                                                                       j-i=4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 j-i=6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          j-i=8
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                j - i = 10
```

$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=1
i+j=1 $i+j=6$ $i+j=8$ $i+j=10$	0	0	0	168	188	119
i+j=8	0	0	2030	2912	1919	
i+j=6	0	2030	5665	3876		
i+j=2 $i+j=4$	168	2912	3876			
i+j=2	188	1919				
i+j=0	119					

module	multiplicity	dimension
all		28089
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	58	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	39	20
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	71	84
\mathbb{C}	19	1
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	44	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	44	45
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	22	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	22	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	31	175
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	18	256
$L(2\alpha_1 + 3\alpha_2 + 3\alpha_3)$	18	256
$L(2\alpha_1+4\alpha_2+2\alpha_3)$	3	105

type A3, s=0, subset=[1, 2]

```
i+j=0
i+j=2
i+j=4 \mid \mathbb{C}
i+j=6 \mid \mathbb{C}
        j-i=0 j-i=2 j-i=4 j-i=6
i+j=0
i+j=2
i+j=4 \mid 1
i+j=6
```

module	multiplicity	dimension
all		10

j-i=0 j-i=2 j-i=4 j-i=6

 $h^{i,j}$

 \mathbb{C} 10

type A3, s=1, subset=[1, 2]

i+j=1	16	$ \begin{array}{c} 36 \\ 1 \\ \hline j-i=3 \end{array} $	
i + j = 3	1	36	
i+j=5	1	1	16
$h^{i,j}$	j-i=1	j - i = 3	j - i = 5

module	multiplicity	dimension
all		71
\mathbb{C}	6	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	3	15
$L\left(\alpha_1 + 2\alpha_2 + \alpha_3\right)$	1	20

type A3, s=2, subset=[1, 2]

```
i+j=0 \mid L_{1,1,1}
                  L_{1,1,1}^2 L_{1,2,1} L_{2,2,1} L_{1,2,2}
i+j=2
                                                  L_{1,1,1}^2L_{1,2,1}L_{2,2,1}L_{1,2,2}
i+j=4 | 0
i+j=6 | 0
                                                                                  L_{1,1,1}
 h^{i,j} j-i=0 j-i=2
                                                   j-i=4
                                                                                   i - i = 6
i+j=0
         15
                140
i+j=2
                   0
                             140
i+j=4
```

module	multiplicity	dimension
all		310
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	6	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	2	20
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	2	45
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3\right)$	2	45

 $h^{i,j} \mid j-i=0 \quad j-i=2 \quad j-i=4 \quad j-i=6$

0 15

i+j=6 | 0

type A3, s=3, subset=[1, 2]

all		922
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	3	15
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	4	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	4	45
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	3	84
$L\left(\alpha_1 + 2\alpha_2 + \alpha_3\right)$	1	20
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	1	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	1	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	1	175

type A3, s=4, subset=[1, 2]

 $L_{2,2,1}L_{1,2,2}L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}L_{3,3,2}L_{2,3,3}$

j-i=4

 $L_{2,2,2}$

i-i=6

j-i=0 j-i=2 j-i=4 j-i=6

module	multiplicity	dimension
all		2198
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	6	84
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	2	45
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3\right)$	2	45
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	2	35
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3\right)$	2	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	2	175
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	2	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	2	256

type A3, s=5, subset=[1, 2]

i+j=1	896		
i + j = 3	0	2760	
i+j=1 $i+j=3$ $i+j=5$	0	0	896
$h^{i,j}$	j-i=1	j-i=3	j-i=5

module	multiplicity	dimension
all		4552
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	3	84
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	4	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	4	256
$L\left(3\alpha_1+3\alpha_2+3\alpha_3\right)$	3	300
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	1	35
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3\right)$	1	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	1	175
$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	1	189
$L\left(2\alpha_1+3\alpha_2+4\alpha_3\right)$	1	189
$L\left(3\alpha_1+4\alpha_2+3\alpha_3\right)$	1	729

type A3, s=6, subset=[1, 2]

 $L_{3,3,2}L_{2,3,3}L_{4,3,2}L_{3,3,3}^2L_{2,3,4}L_{3,4,3}L_{4,4,3}L_{3,4,4}$

j-i=4

 $L_{3,3,3}$

i-i=6

```
i+j=0 \mid L_{3,3,3}
                  L_{3,3,2}L_{2,3,3}L_{4,3,2}L_{3,3,3}^2L_{2,3,4}L_{3,4,3}L_{4,4,3}L_{3,4,4}
i+j=2
i+j=4
i+j=6 | 0
  h^{i,j}
         j-i=0
                  j-i=2
         300
i+j=0
                  3969
i+j=2
                           3969
i+j=4
                                     300
i + j = 6
         j-i=0 j-i=2 j-i=4
                                   j-i=6
```

module	multiplicity	dimension
all		8538
$L\left(3\alpha_1+3\alpha_2+3\alpha_3\right)$	6	300
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	2	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	2	256
$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	2	189
$L\left(2\alpha_1+3\alpha_2+4\alpha_3\right)$	2	189
$L\left(3\alpha_1+4\alpha_2+3\alpha_3\right)$	2	729
$L\left(4\alpha_1+4\alpha_2+3\alpha_3\right)$	2	875
$L\left(3\alpha_1+4\alpha_2+4\alpha_3\right)$	2	875

type A3, s=7, subset=[1, 2]

i+j=1	2875		
i+j=3	0	9120	
i+j=1 $i+j=3$ $i+j=5$	0	0	2875
$h^{i,j}$	j-i=1	j-i=3	j-i=5

module	multiplicity	dimension
all		14870
$L\left(3\alpha_1+3\alpha_2+3\alpha_3\right)$	3	300
$L\left(4\alpha_1+4\alpha_2+3\alpha_3\right)$	4	875
$L\left(3\alpha_1+4\alpha_2+4\alpha_3\right)$	4	875
$L\left(4\alpha_1+4\alpha_2+4\alpha_3\right)$	3	825
$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	1	189
$L\left(2\alpha_1+3\alpha_2+4\alpha_3\right)$	1	189
$L\left(3\alpha_1+4\alpha_2+3\alpha_3\right)$	1	729
$L\left(5\alpha_1+4\alpha_2+3\alpha_3\right)$	1	616
$L\left(3\alpha_1+4\alpha_2+5\alpha_3\right)$	1	616
$L\left(4\alpha_1+5\alpha_2+4\alpha_3\right)$	1	2156

type A3, s=8, subset=[1, 2]

```
|L_{4,4,4}|
i+j=0
                  L_{4,4,3}L_{3,4,4}L_{5,4,3}L_{4,4,4}^2L_{3,4,5}L_{4,5,4}L_{5,5,4}L_{4,5,5}
i+j=2
i+j=4
i+j=6 | 0
  h^{i,j}
                  i-i=2
         825
i+j=0
                  11396
i+j=2
i+j=4 \mid 0
                            11396
i+j=6
                                      825
         j-i=0 j-i=2 j-i=4
```

module

 $L (4\alpha_1 + 4\alpha_2 + 4\alpha_3)$ $L (4\alpha_1 + 4\alpha_2 + 3\alpha_3)$ $L (3\alpha_1 + 4\alpha_2 + 4\alpha_3)$ $L (5\alpha_1 + 4\alpha_2 + 3\alpha_3)$ $L (3\alpha_1 + 4\alpha_2 + 5\alpha_3)$ $L (4\alpha_1 + 5\alpha_2 + 4\alpha_3)$ $L (5\alpha_1 + 5\alpha_2 + 4\alpha_3)$ $L (4\alpha_1 + 5\alpha_2 + 5\alpha_3)$ $L (4\alpha_1 + 5\alpha_2 + 5\alpha_3)$

all

$$\begin{array}{c|cc}
 j-i=4 & j-i=6 \\
\hline
 multiplicity & dimension \\
 \hline
 24442 \\
 6 & 825 \\
 2 & 875
\end{array}$$

Z444Z
825
875
875
616
616
2156
2304
2304

 ${}^{L_{4,4,3}L_{3,4,4}L_{5,4,3}L_{4,4,4}^2L_{3,4,5}L_{4,5,4}L_{5,5,4}L_{4,5,5}}_{0}$

i-i=4

 $L_{4,4,4}$

i-i=6

type A3, s=9, subset=[1, 2]

i+j=1	7344		
i + j = 3	0	23660	
i+j=1 $i+j=3$ $i+j=5$	0	0	7344
$h^{i,j}$	j-i=1	j - i = 3	j-i=5

module	multiplicity	dimension
all		38348
$L\left(4\alpha_1+4\alpha_2+4\alpha_3\right)$	3	825
$L\left(5\alpha_1 + 5\alpha_2 + 4\alpha_3\right)$	4	2304
$L\left(4\alpha_1+5\alpha_2+5\alpha_3\right)$	4	2304
$L\left(5\alpha_1 + 5\alpha_2 + 5\alpha_3\right)$	3	1911
$L\left(5\alpha_1+4\alpha_2+3\alpha_3\right)$	1	616
$L\left(3\alpha_1+4\alpha_2+5\alpha_3\right)$	1	616
$L\left(4\alpha_1+5\alpha_2+4\alpha_3\right)$	1	2156
$L\left(6\alpha_1 + 5\alpha_2 + 4\alpha_3\right)$	1	1560
$L\left(4\alpha_1+5\alpha_2+6\alpha_3\right)$	1	1560
$L\left(5\alpha_1 + 6\alpha_2 + 5\alpha_3\right)$	1	5200

type A3, s=0, subset=[1, 3]

```
i+j=0
i+j=2
                     \mathbb{C}^2
          \mathbb{C}^2
i+j=4
                     \mathbb{C}^2
                                \mathbb{C}^2
         \mathbb{C}
i+j=6
         \mathbb{C}
i+j=8
  \overline{h^{i,j}}
          j-i=0 j-i=2 j-i=4 j-i=6
                                                      j-i=8
i+j=0
i+j=2
i+j=4
i + j = 6
i+j=8
  h^{i,j}
          i-i=0 i-i=2 i-i=4 i-i=6
                                                      i-i=8
```

module	multiplicity	dimension
all		20
$\mathbb C$	20	1

type A3, s=1, subset=[1, 3]

i+j=1	16			
i+j=3	17	16		
i + j = 5	2	53	16	
i+j=1 $i+j=3$ $i+j=5$ $i+j=7$	1	2	17	16
$h^{i,j}$	j-i=1	j-i=3	j-i=5	j-i=7

module	multiplicity	dimension
all		156
\mathbb{C}	16	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	8	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	1	20

type A3, s=2, subset=[1, 3]

```
i+j=0 \mid L_{1,1,1}
i+j=2 \mid L_{1,1,1} \quad L_{1,1,1}^3 L_{2,2,1} L_{1,2,2}
                \mathbb{C}L_{1,1,1}^{\overline{3},-}L_{1,2,1}L_{2,2,1}L_{1,2,2} \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{2,2,1}L_{1,2,2}
i+j=4
                                                                \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{2,2,1}L_{1,2,2} \quad L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2}
i+j=6
i + j = 8
                                                                                                         L_{1,1,1}
  h^{i,j}
           j-i=0
                      i-i=2
                                                                j-i=4
                                                                                                         i-i=6
           15
i+j=0
           15
                      135
i+j=2
                      156
i+j=4
                                   156
                       0
                                   156
                                               135
i+j=6
```

15

 $L_{1,1,1}$

i-i=8

module	multiplicity	dimension
all		798
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	19	15
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	5	45
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3\right)$	5	45
\mathbb{C}	3	1
$L\left(\alpha_1 + 2\alpha_2 + \alpha_3\right)$	3	20

0

j-i=0 j-i=2 j-i=4 j-i=6 j-i=8

 $\frac{i+j=8}{h^{i,j}}$

15

type A3, s=3, subset=[1, 3]

 $i+j=1 \mid L_{1,1,1}^2 L_{1,2,1} L_{2,2,1} L_{1,2,2} L_{2,2,2}$

 $L_{1,1,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}$

```
L_{1,1,1}^2L_{1,2,1}L_{2,2,1}^2L_{1,2,2}^2L_{3,2,1}L_{2,2,2}L_{1,2,3}L_{2,3,2}
i+j=5
i + j = 7
  h^{i,j}
        j-i=1
                                              j-i=3
i+j=1 | 224
         189
                  784
i+j=3
                   559
                            784
i+j=5
i+j=7
                            189
                                      224
  h^{i,j}
```

 $\mathbb{C}L_{1,1,1}^4L_{1,2,1}^2L_{2,2,1}^3L_{1,2,2}^3L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}$

 $\mathbb{C}L_{1.1,1}^4L_{1,2,1}^2L_{2,2,1}^3L_{1,2,2}^3L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}$

 $L_{1,1,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}$

i-i=5

 $L_{1.1.1}^2 L_{1,2,1} L_{2,2,1} L_{1,2,2} L_{2,2,2}$

i-i=7

module	multiplicity	dimension
all		2953
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	16	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	7	20
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	12	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	12	45
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	9	84
\mathbb{C}	2	1
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	3	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	3	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	3	175
*		

type A3, s=4, subset=[1, 3]

 $L_{1,1,1}^2L_{1,2,1}^2L_{2,2,1}^3L_{1,2,2}^3L_{3,2,1}L_{2,2,2}^4L_{1,2,3}L_{2,3,2}^2L_{3,3,2}L_{2,3,3}$

 $\mathbb{C}L_{1,1,1}^4L_{1,2,1}^4L_{2,2,1}^4L_{1,2,2}^4L_{3,2,1}^3L_{2,2,2}^6L_{1,2,3}^3L_{2,3,2}^4L_{2,3,2}^2L_{2,4,2}L_{2,3,3}^2$

 $L_{1,1,1}^2L_{1,2,1}^2L_{2,2,1}^3L_{1,2,2}^3L_{3,2,1}L_{2,2,2}^4L_{1,2,3}L_{2,3,2}^2L_{3,3,2}L_{2,3,3}$

 $L_{1,2,1}L_{2,2,2}$

j-i=8

 $L_{2,2,2}$

j-i=6

 $L_{2,2,1}L_{1,2,2}L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}L_{3,3,2}L_{2,3,3}$

j-i=4

 $L_{2,2,1}L_{1,2,2}L_{3,2,1}L_{2,2,2}^2L_{1,2,3}L_{2,3,2}L_{3,3,2}L_{2,3,3}$

```
i+j=6 | 0
                    0
i+j=8 | 0
 h^{i,j}
       j-i=0
                    i-i=2
i+j=0 | 104
i+j=2 | 84
              1608
i+j=4 \mid 0
              1015
                     3044
i+j=6 | 0
                      1015
                             1608
i+j=8 | 0
                             84
                                    104
 h^{i,j}
              j-i=2 j-i=4 j-i=6
                                   j-i=8
```

 $i+j=0 \mid L_{1,2,1}L_{2,2,2}$

 $i+j=2 \mid L_{2,2,2}$

 $i+j=4 \mid 0$

module	multiplicity	dimension
all		8666
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	10	20
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	22	84
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	8	15
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	12	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	12	45
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	7	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	7	35
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	10	175
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	6	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	6	256
Ĉ	1	1
$L\left(2\alpha_1+4\alpha_2+2\alpha_3\right)$	1	105
,		

type A3, s=5, subset=[1, 3]

 $\begin{array}{c} L_{1,1,1}^2 L_{1,2,1}^2 L_{2,2,1}^4 L_{1,2,2}^4 L_{3,2,1}^2 L_{2,2,2}^6 L_{1,2,3}^2 L_{2,3,2}^6 L_{3,3,2}^5 L_{2,4,2} L_{2,3,3}^5 L_{4,3,2} L_{3,4,2} L_{3,3,3}^2 L_{2,4,3} L_{2,3,4} L_{3,4,3} \\ L_{3,2,1} L_{2,2,2} L_{1,2,3} L_{2,3,2} L_{3,3,2}^2 L_{2,3,3}^2 L_{4,3,2} L_{3,3,3} L_{2,3,4} L_{3,4,3} \\ 0 \\ \hline j-i=3 \end{array}$

 $\frac{L_{1,1,1}^{2}L_{1,2,1}^{2}L_{4,2,1}^{4}L_{1,2,2}^{4}L_{3,2,1}^{4}L_{2,2,2}^{6}L_{2,3,2}^{2}L_{2,3,2}^{6}L_{3,3,2}^{5}L_{2,4,2}L_{2,3,3}^{5}L_{2,4,2}L_{3,3,2}^{5}L_{2,4,2}L_{3,3,3}^{2}L_{2,4,3}L_{2,3,4}L_{3,4,3}}{L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}^{2}L_{2,3,2}^{2}L_{2,3,2}L_{2,3,3}L_{3,3,3}}\\ \frac{L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}^{2}L_{2,3,2}L_{2,3,3}L_{3,3,3}}{j-i=7}$

i+j=1	1440			
i + j = 3	896	7056		
i + j = 5	0	2760	7056	
i+j=1 $i+j=3$ $i+j=5$ $i+j=7$	0	0	896	1440
$h^{i,j}$	j-i=1	j - i = 3	j-i=5	j-i=7

module	multiplicity	dimension
all		21544
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	6	20
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	10	45
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3\right)$	10	45
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	19	84
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	17	175
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	16	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	16	256
$L\left(3\alpha_1+3\alpha_2+3\alpha_3\right)$	9	300
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	4	15
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	5	35
$L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$	5	35
$L\left(2\alpha_1+4\alpha_2+2\alpha_3\right)$	2	105
$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	3	189
$L\left(3\alpha_1+4\alpha_2+2\alpha_3\right)$	2	280
$L\left(2\alpha_1+4\alpha_2+3\alpha_3\right)$	2	280
$L(2\alpha_1 + 3\alpha_2 + 4\alpha_3)$	3	189
$L\left(3\alpha_1+4\alpha_2+3\alpha_3\right)$	3	729

type A3, s=6, subset=[1, 3]

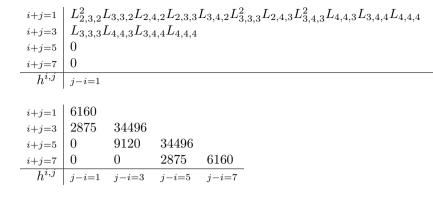
i+j=0	475				
i+j=2	300	9177			
i+j=4	0	3969	19735		
i + j = 6	0	0	3969	9177	
i+j=0 i+j=2 i+j=4 i+j=6 i+j=8	0	0	0	300	475
		i-i=2	j-i=4	i-i=6	i-i=8

module	multiplicity	dimension
all		47577
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	20	175
$L\left(3\alpha_1+3\alpha_2+3\alpha_3\right)$	22	300
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	7	45
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	7	45
$L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$	9	84
$L\left(3\alpha_1+3\alpha_2+2\alpha_3\right)$	18	256
$L\left(2\alpha_1+3\alpha_2+3\alpha_3\right)$	18	256
$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	7	189
$L\left(3\alpha_1+4\alpha_2+2\alpha_3\right)$	7	280
$L\left(2\alpha_1+4\alpha_2+3\alpha_3\right)$	7	280
$L\left(2\alpha_1+3\alpha_2+4\alpha_3\right)$	7	189
$L\left(3\alpha_1+4\alpha_2+3\alpha_3\right)$	11	729
$L\left(4\alpha_1+4\alpha_2+3\alpha_3\right)$	6	875
$L\left(3\alpha_1+4\alpha_2+4\alpha_3\right)$	6	875
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	1	15
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	2	20
$L\left(3\alpha_1+2\alpha_2+\alpha_3\right)$	1	35
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3\right)$	1	35
$L\left(2\alpha_1+4\alpha_2+2\alpha_3\right)$	2	105
$L\left(4\alpha_1+4\alpha_2+2\alpha_3\right)$	1	360
$L\left(2\alpha_1+4\alpha_2+4\alpha_3\right)$	1	360
$L\left(3\alpha_1+5\alpha_2+3\alpha_3\right)$	1	735

type A3, s=7, subset=[1, 3]

j-i=3

 $L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}^2L_{2,3,2}^6L_{3,3,3,2}L_{2,4,2}^2L_{2,3,3}^6L_{4,3,2}L_{3,4,2}^4L_{3,4,2}L_{3,3,3}^4L_{2,4,4}L_{2,3,4}^2L_{4,4,2}^2L_{3,4,3}^8L_{2,4,4}^2L_{4,4,3}^5L_{3,5,3}L_{3,4,4}^5L_{4,5,4}L_{5,4,3}L_{4,5,4}L_{4,5,4}L_{4,5,4}L_{4,3,2}L_{3,3,3}L_{2,3,4}L_{3,4,3}L_{3,4,4}L_{3,4,4}L_{3,4,5}L_{4,5,4}L_{4,5,4}L_{4,5,4}L_{4,4,2}L_{3,4,4}L_{3,4,4}L_{3,4,5}L_{4,5,4}L_{4,4,4}L_{3,4,5}L_{4,5,4}L_{4,4,4}L_{4,4,4}L_{4,4,5}L_{4,5,4}L_{4,4,4}L_{4,4,5}L_{4,5,4}L_{4,5,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_{5,4,4}L_$



all 96182 $L(2\alpha_1 + 3\alpha_2 + 2\alpha_3) 16 \qquad 175$ $L(3\alpha_1 + 3\alpha_2 + 2\alpha_3) 14 \qquad 256$ $L(2\alpha_1 + 4\alpha_2 + 2\alpha_3) 6 \qquad 105$ $L(2\alpha_1 + 3\alpha_2 + 3\alpha_3) 14 \qquad 256$ $L(3\alpha_1 + 4\alpha_2 + 2\alpha_3) 10 \qquad 280$ $L(3\alpha_1 + 4\alpha_2 + 2\alpha_3) 10 \qquad 280$ $L(3\alpha_1 + 3\alpha_2 + 3\alpha_3) 19 \qquad 300$ $L(2\alpha_1 + 4\alpha_2 + 3\alpha_3) 10 \qquad 280$ $L(3\alpha_1 + 4\alpha_2 + 3\alpha_3) 10 \qquad 280$ $L(3\alpha_1 + 4\alpha_2 + 3\alpha_3) 16 \qquad 875$ $L(3\alpha_1 + 4\alpha_2 + 3\alpha_3) 16 \qquad 875$ $L(3\alpha_1 + 4\alpha_2 + 4\alpha_3) 16 \qquad 875$ $L(4\alpha_1 + 4\alpha_2 + 4\alpha_3) 9 \qquad 825$ $L(\alpha_1 + 2\alpha_2 + \alpha_3) 2 \qquad 20$ $L(2\alpha_1 + 2\alpha_2 + \alpha_3) 2 \qquad 45$ $L(2\alpha_1 + 2\alpha_2 + 2\alpha_3) 2 \qquad 45$ $L(2\alpha_1 + 2\alpha_2 + 2\alpha_3) 4 \qquad 84$ $L(4\alpha_1 + 3\alpha_2 + 2\alpha_3) 5 \qquad 189$ $L(2\alpha_1 + 3\alpha_2 + 4\alpha_3) 5 \qquad 189$ $L(2\alpha_1 + 3\alpha_2 + 4\alpha_3) 5 \qquad 189$ $L(2\alpha_1 + 4\alpha_2 + 2\alpha_3) 4 \qquad 360$ $L(2\alpha_1 + 4\alpha_2 + 4\alpha_3) 4 \qquad 360$ $L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$ $L(2\alpha_1 + 5\alpha_2 + 5\alpha_3) 2 \qquad 1280$	module	multiplicity	dimension
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	all		96182
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	16	175
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(3\alpha_1 + 3\alpha_2 + 2\alpha_3)$	14	256
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(2\alpha_1+4\alpha_2+2\alpha_3)$	6	105
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(2\alpha_1 + 3\alpha_2 + 3\alpha_3)$	14	256
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(3\alpha_1+4\alpha_2+2\alpha_3)$	10	280
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(3\alpha_1 + 3\alpha_2 + 3\alpha_3)$	19	300
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(2\alpha_1 + 4\alpha_2 + 3\alpha_3)$	10	280
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(3\alpha_1 + 4\alpha_2 + 3\alpha_3)$	21	729
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(4\alpha_1 + 4\alpha_2 + 3\alpha_3)$	16	875
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(3\alpha_1 + 4\alpha_2 + 4\alpha_3)$	16	875
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(4\alpha_1 + 4\alpha_2 + 4\alpha_3)$	9	825
$\begin{array}{cccccccccc} L\left(\alpha_{1}+2\alpha_{2}+2\alpha_{3}\right) & 2 & 45 \\ L\left(2\alpha_{1}+2\alpha_{2}+2\alpha_{3}\right) & 4 & 84 \\ L\left(4\alpha_{1}+3\alpha_{2}+2\alpha_{3}\right) & 5 & 189 \\ L\left(2\alpha_{1}+3\alpha_{2}+4\alpha_{3}\right) & 5 & 189 \\ L\left(4\alpha_{1}+4\alpha_{2}+2\alpha_{3}\right) & 4 & 360 \\ L\left(2\alpha_{1}+4\alpha_{2}+4\alpha_{3}\right) & 4 & 360 \\ L\left(3\alpha_{1}+5\alpha_{2}+3\alpha_{3}\right) & 4 & 735 \\ L\left(5\alpha_{1}+4\alpha_{2}+3\alpha_{3}\right) & 3 & 616 \\ L\left(4\alpha_{1}+5\alpha_{2}+3\alpha_{3}\right) & 2 & 1280 \\ \end{array}$		2	20
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$L(2\alpha_1 + 2\alpha_2 + \alpha_3)$	2	45
$L(4\alpha_1 + 3\alpha_2 + 2\alpha_3) 5 \qquad 189$ $L(2\alpha_1 + 3\alpha_2 + 4\alpha_3) 5 \qquad 189$ $L(4\alpha_1 + 4\alpha_2 + 2\alpha_3) 4 \qquad 360$ $L(2\alpha_1 + 4\alpha_2 + 4\alpha_3) 4 \qquad 360$ $L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$	$L(\alpha_1 + 2\alpha_2 + 2\alpha_3)$	2	45
$L(2\alpha_1 + 3\alpha_2 + 4\alpha_3) 5 \qquad 189$ $L(4\alpha_1 + 4\alpha_2 + 2\alpha_3) 4 \qquad 360$ $L(2\alpha_1 + 4\alpha_2 + 4\alpha_3) 4 \qquad 360$ $L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$	$L(2\alpha_1+2\alpha_2+2\alpha_3)$	4	84
$L(4\alpha_1 + 4\alpha_2 + 2\alpha_3) 4 \qquad 360$ $L(2\alpha_1 + 4\alpha_2 + 4\alpha_3) 4 \qquad 360$ $L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$	$L\left(4\alpha_1+3\alpha_2+2\alpha_3\right)$	5	189
$L(2\alpha_1 + 4\alpha_2 + 4\alpha_3) 4 \qquad 360$ $L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$	$L(2\alpha_1 + 3\alpha_2 + 4\alpha_3)$	5	189
$L(3\alpha_1 + 5\alpha_2 + 3\alpha_3) 4 \qquad 735$ $L(5\alpha_1 + 4\alpha_2 + 3\alpha_3) 3 \qquad 616$ $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3) 2 \qquad 1280$	$L(4\alpha_1 + 4\alpha_2 + 2\alpha_3)$	4	360
$L(5\alpha_1 + 4\alpha_2 + 3\alpha_3)$ 3 616 $L(4\alpha_1 + 5\alpha_2 + 3\alpha_3)$ 2 1280	$L(2\alpha_1+4\alpha_2+4\alpha_3)$	4	360
$L(4\alpha_1 + 5\alpha_2 + 3\alpha_3)$ 2 1280	$L\left(3\alpha_1+5\alpha_2+3\alpha_3\right)$	4	735
	$L\left(5\alpha_1+4\alpha_2+3\alpha_3\right)$	3	616
	$L\left(4\alpha_1+5\alpha_2+3\alpha_3\right)$	2	1280
$L\left(3\alpha_1+3\alpha_2+4\alpha_3\right)$ Z 1280	$L(3\alpha_1 + 5\alpha_2 + 4\alpha_3)$	2	1280
$L(3\alpha_1 + 4\alpha_2 + 5\alpha_3)$ 3 616	- /		616
$L(4\alpha_1 + 5\alpha_2 + 4\alpha_3)$ 3 2156	` '		2156

 $\begin{array}{c} L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}^2L_{2,3,2}^6L_{3,3}^6L_{2,4,2}L_{2,3,3}^6L_{2,4,2}L_{3,3,4}^6L_{2,4,3}L_{3,4,2}^2L_{3,3,3}^4L_{4,4,2}L_{3,4,3}^8L_{2,4,4}^2L_{3,4,3}^8L_{2,4,4}^2L_{3,4,3}^5L_{2,4,4}^5L_{3,4,3}L_{3,4,4}^2L_{4,4,4}L_{3,5,4}L_{4,5,3}L_{4,4,4}^2L_{3,5,4}L_{4,4,4}L_{3,5,4}L_{4,4,4}L_{4,4,4}\\ L_{2,3,2}L_{3,3,2}L_{2,4,2}L_{2,3,3}L_{2,4,2}L_{2,3,3}L_{2,4,2}L_{2,3,3}L_{2,4,3}L_{2,4,4}L_{4,4,4}\\ j-i=5 \\ \end{array}$

type A3, s=8, subset=[1, 3]

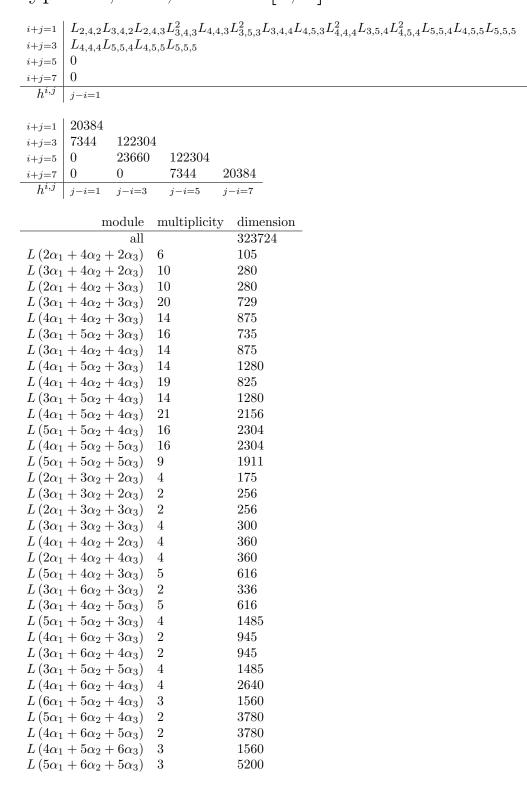
i+j=2	$L_{4,4,4}$	$L_{3,4,3}L_{4,}$	L_2^2	L^2	$_{2}L_{2}^{2}$ $_{4}$ $_{2}L_{2}^{2}$ $_{3}$	${}_{,3}L_{3,4,2}^3L_{3,3,3}^2L_{2,4,3}^3L_{4,4,2}L_{3,4,3}^6L_{2,4,4}L_{4,4,3}^4L_{3,5,3}^2L_{3,4,4}^4L_{5,4,3}L_{4,5,3}^2L_{4,4,4}^4L_{3,5,4}^3L_{3,4,5}L_{4,5,4}^2L_{5,5,4}L_{4,5,5}$	
	0					$\begin{array}{cccccccccccccccccccccccccccccccccccc$	L
	0		0	1,0 0,1,	4,4	4 0,4,0 4,0,4 0,0,4 4,0,0	L
	0		0				0
	j-i=0		j-i=	=2			<i>j</i> -
1	1050						
i+j=0		20000					
I	825	36000	91606				
- 1	0	11396 0	81696 11396	26000)		
	0	0	0	36000 825	1659		
			j-i=4	j-i=6			
16	<i>j-i</i> =0	j-i=2	j-i=4	<i>j-i=</i> 0	<i>j-i=</i> 8		
	n	nodule	multipl	icity	dimension		
		all			181456	-	
	$+4\alpha_2$ -		10		105		
	$+4\alpha_2$ -		26		729		
`		$+4\alpha_3$	22		825		
		$+2\alpha_3$	9		175		
`		$+2\alpha_3$			256		
		$+3\alpha_3$	8		256		
	$+4\alpha_2$ - $+3\alpha_2$ -	$+2\alpha_3$	9		280		
`		$+$ $3\alpha_3)$			300 280		
,	$+4\alpha_2$ - $+4\alpha_2$ -	- /	6		360		
`	$+4\alpha_2$	- /	6		360		
		$+3\alpha_3$			875		
	$+5\alpha_2$ -		9		735		
		$+4\alpha_3$	18		875		
		$+3\alpha_3$			616		
$L(4\alpha_1)$	$+5\alpha_2$ -	$+3\alpha_3$	8		1280		
		$+4\alpha_3$			1280		
`		$+5\alpha_3$)			616		
`		$+4\alpha_3$			2156		
		$+4\alpha_3$			2304		
$L(4\alpha_1)$					2304		
		$+\alpha_3$			20		
$L(2\alpha_1)$ $L(4\alpha_1)$		$+2\alpha_3$			84 189		
$L(4\alpha_1)$ $L(2\alpha_1)$					189		
$L(3\alpha_1)$					336		
$L(5\alpha_1)$					1485		
$L(3\alpha_1)$					1485		
		$+4\alpha_3$			2640		
/ 1	2						

5,5	$L_{1,2,1}L_{2,2,2}L_{2,3,2}^{5}L_{3,3,2}L_{2,4,2}^{4}L_{2,4,3}^{4}L_{2,4,3}L_{4,3,2}L_{3,4,3}^{4}L_{2,3,4}L_{4,4,2}^{4}L_{3,4,3}L_{2,4,4}L_{4,4,2}^{8}L_{3,4,3}L_{4,4,2}^{5}L_{3,4,3}L_{4,5,3}L_{3,4,4}L_{5,4,3}L_{3,4,5}L_{3,5,4}L_{3,4,4}L_{3,5,4}L_{3,4,5}L_{5,5,3}L_{4,5,4}^{5}L_{2,5,5,4}L_{4,6,4}L_{4,5,5}^{2}$		
	$L_{4,4,3}L_{3,4,4}L_{5,4,3}L_{4,4,4}^2L_{3,4,5}L_{4,5,5}L_{4,5,5}L_{4,5,5}$	$L_{2,3,2}^2 L_{3,3,2}^2 L_{2,4,2}^2 L_{2,3,3}^2 L_{3,4,2}^3 L_{3,3,3}^3 L_{2,4,3}^3 L_{4,4,2} L_{3,4,3}^6 L_{2,4,4} L_{4,4,3}^4 L_{3,5,3}^2 L_{3,4,4}^4 L_{5,4,3} L_{4,5,3}^2 L_{4,4,4}^4 L_{3,5,4}^2 L_{3,5,4} L_{3,4,5} L_{4,5,4}^2 L_{5,5,4} L_{4,5,5} L_{4,5,5}^4 L_{4,5,5} L_{4,5,5}^4 L_{4,5,5} L_{4,5,5}^4 L_{5,5,5,4} L_{4,5,5,5} L_{4,5,5,5}^4 L_{5,5,5,5} L_{5,5,5,5}^4 L_{5,5,5,5} L_{5,5,5} L_{5,5,5} L_{5,5,5} L_{5,5,5} L_{5,5,5$	
	0	$L_{4,4,4}$	L_2
	$j{-}i{=}4$	$j{-}i{=}6$	j-

 $L_{2,4,2}L_{3,4,3}L_{4,4,4}$

type A3, s=9, subset=[1, 3]

j-i=3



 $L^2_{2,3,2}L_{3,3,2}L^2_{2,4,2}L_{2,3,3}L^4_{3,4,2}L^2_{3,3,3}L^4_{4,4,2}L^2_{3,4,3}L^2_{2,4,4}L^6_{4,4,3}L^6_{3,5,3}L^6_{3,4,4}L^2_{5,4,3}L^6_{4,5,3}L_{3,6,3}L^6_{4,4,4}L^6_{3,5,4}L^2_{3,4,5}L^2_{5,5,3}L_{4,6,3}L^8_{4,5,4}L^2_{3,5,5}L^5_{5,5,4}L^2_{4,6,4}L^5_{5,5,5}L_{4,5,6}L^2_{5,5,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,6,5}L_{4,5,6}L_{5,5,5}L_{5,5,5}L_$ j-i=5

 $L_{2,3,2}^2L_{3,3,2}L_{2,4,2}^2L_{2,3,3}L_{3,4,2}^4L_{2,3,3}L_{3,4,2}^4L_{2,3,3}L_{4,4,2}^4L_{3,4,3}^8L_{2,4,4}^2L_{4,4,3}^8L_{3,5,3}^6L_{3,4,4}^6L_{5,4,3}^2L_{4,5,3}^6L_{4,5,4}L_{3,5,4}^6L_{3,5,4}^2L_{2,5,5,3}^2L_{4,6,3}L_{4,5,4}^8L_{3,5,5}L_{5,5,4}^2L_{4,5,5}^2L_{4,5,5}L_{4,5,5$ $L_{4,4,4}L_{5,5,4}L_{4,5,5}L_{5,5,5}$

 $L_{2,4,2}L_{3,4,2}L_{2,4,3}L_{3,4,3}^2L_{4,4,3}L_{3,5,3}^2L_{3,4,4}L_{4,5,3}L_{4,4,4}^2L_{3,5,4}L_{4,5,4}^2L_{5,5,4}L_{4,5,5}L_{5,5,5}$

type A3, s=0, subset=[2]

i+j=0 i+j=2 i+j=4 i+j=6 i+j=8 i+j=10	$ \begin{array}{c} \mathbb{C} \\ \mathbb{C}^2 \\ \mathbb{C}^3 \\ \mathbb{C}^3 \\ \mathbb{C}^2 \\ \mathbb{C} \end{array} $	$ \begin{array}{c} \mathbb{C} \\ \mathbb{C}^3 \\ \mathbb{C}^5 \\ \mathbb{C}^4 \\ \mathbb{C}^2 \end{array} $	\mathbb{C} \mathbb{C}^3 \mathbb{C}^5 \mathbb{C}^3	\mathbb{C} \mathbb{C}^3	\mathbb{C} \mathbb{C}^2	$\mathbb C$
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	$j\!-\!i\!=\!10$
i+j=0	1					
i+j=2	2	1				
i+j=4	3	3	1			
i + j = 6	3	5	3	1		
i + j = 8	2	4	5	3	1	
i+j=10	1	2	3	3	2	1
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j - i = 6	j-i=8	j - i = 10

module	multiplicity	dimension
all		50
\mathbb{C}	50	1

type A3, s=1, subset=[2]

module	multiplicity	dimension
all		456
\mathbb{C}	46	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	22	15
$L\left(\alpha_1 + 2\alpha_2 + \alpha_3\right)$	4	20

type A3, s=2, subset=[2]

```
i+j=0 \mid L_{1,1,1}
                                                                     \begin{array}{cccc} L_{1,1,1}^{2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ L_{1,1,1}^{2} & \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2}^{2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \end{array} 
                                                                                                                   \begin{array}{c} \mathbb{C} \stackrel{L_{1},1,L_{1},2,1}{L_{1},2,1} \stackrel{L_{2},2,1}{L_{2},2,1} \stackrel{L_{1},2,2}{L_{1},2,2} & \mathbb{C}^{1},1,1-2,2,1-1,2,2} \\ \mathbb{C}^{2}L_{1,1,1}^{5}L_{1,2,1}^{2}L_{2,2,1}^{2}L_{1,2,2}^{2} & \mathbb{C}^{4}L_{1,1,1}^{9}L_{1,2,1}^{4}L_{2,2,1}^{3}L_{1,2,2}^{3} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C} & \mathbb{C}^{2}L_{1,1,1}^{5}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2}^{2} & \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2}^{2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2}^{2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}^{2}L_{1,1,1}^{7}L_{1,2,1}L_{2,2,1}^{2}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{1,2,1}^{3}L_{2,2,1}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{2,2,1}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{1,2,1}^{3}L_{2,2,1}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{1,2,1}^{3}L_{2,2,1}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,1}L_{1,2,2} & \mathbb{C}L_{1,1,1}^{3}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,2} & \mathbb{C}L_{1,1}^{3}L_{1,2,2} \\ \mathbb{C}L_{1,1,1}^{3}L_{1,2,2} & \mathbb{C}L_{1,1}^{3}L_{1,2} \\ \mathbb{C}L_{1,1}^{3}L_{1,2} & \mathbb{C}L_{1,1}^{3}L_{1,2} \\ \mathbb{C}L_{1,1}^{3}L_{1,2
       i+j=6
       i+j=8
i + j = 10
                        h^{i,j}
                                                                         j-i=0 j-i=2
                                                                                                                                                                                                                                                                                                                                                                                              i-i=4
       i+j=0
                                                                                                                                              136
       i+j=2
                                                                                                                                              307
                                                                                                                                                                                                                 136
       i+j=4
                                                                                                                                            297
                                                                                                                                                                                                                 489
       i+j=6
                                                                                                                                                                                                                                                                                    136
                                                                                                                                                                                                                 297
                                                                                                                                                                                                                                                                                    307
                                                                                                                                                                                                                                                                                                                                                        136
       i+j=8
                                                                                                                                                                                                                                                                                                                                                                                                                          15
i + j = 10
                                                                         i-i=0 i-i=2 i-i=4
                                                                                                                                                                                                                                                                        i-i=6
```

 $L_{1,1,1}^2$

i-i=6

 $L_{1.1.1}^2$

i-i=8

 $L_{1,1,1}$

i - i = 10

module	multiplicity	dimension
all		2392
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	55	15
\mathbb{C}	17	1
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	15	45
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3\right)$	15	45
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	10	20

type A3, s=3, subset=[2]

 $\mathbb{C}L_{1,1,1}^4L_{1,2,1}L_{2,2,1}^2L_{1,2,2}^2L_{2,2,2}^2$

```
i+j=5 L_{1,1,1}^2L_{2,2,1}^2L_{1,2,2}^2L_{2,2,2}^2
                                  L_{1,1,1}^3L_{1,2,1}^2L_{2,2,1}^4L_{1,2,2}^4L_{3,2,1}^2L_{2,2,2}^2L_{1,2,3}^2L_{2,3,2}^2
i + j = 7
i+j=9
 h^{i,j}
      i-i=1
                                  i-i=3
i+j=1 | 240
i+j=3
      429
             816
             1690
                   817
i+j=5
                   1690
i+j=7 | 0
             1103
                          816
                          429
i+j=9 | 0
      j-i=1 j-i=3 j-i=5 j-i=7 j-i=9
           module multiplicity
               all
```

 $L\left(\alpha_1+\alpha_2+\alpha_3\right)$ $L\left(\alpha_1+2\alpha_2+\alpha_3\right)$ $L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$ $L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$

 $L\left(2\alpha_1+2\alpha_2+2\alpha_3\right)$ $L(3\alpha_1 + 2\alpha_2 + \alpha_3)$

 $L\left(\alpha_1+2\alpha_2+3\alpha_3\right)$

 $L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$

 $i+j=1 \mid \mathbb{C}L_{1,1,1}^3 L_{1,2,1} L_{2,2,1} L_{1,2,2} L_{2,2,2}$

```
240
dimension
9026
45
35
35
175
```

 $\mathbb{C}^4 L_{1,1,1}^{10,1} L_{1,2,1}^{4,7,1} L_{1,2,2}^{7,2,1} L_{1,2,2}^{7,2,2} L_{3,2,1}^{2,1} L_{2,2,2}^{4,7,2} L_{1,2,3}^{2,2} L_{2,3,2}^{2,2} \quad \mathbb{C}^3 L_{1,1,1}^6 L_{1,2,1}^2 L_{2,2,1}^3 L_{1,2,2}^3 L_{3,2,1} L_{2,2,2}^2 L_{1,2,3} L_{2,3,2} \\ \mathbb{C}^3 L_{1,1,1}^6 L_{1,2,1}^6 L_{2,2,1}^6 L_{1,2,2}^6 L_{2,2,2}^6 L_{1,2,3}^6 L_{2,3,2} \\ \mathbb{C}^3 L_{1,1,1}^6 L_{1,2,1}^6 L_{2,2,1}^6 L_{1,2,2}^6 L_{2,2,2}^6 L_{1,2,3}^6 L_{2,2,2}^6 L_{2,2}^6 L_{2,2}$

i-i=7

 $\mathbb{C}L_{1,1,1}^4L_{1,2,1}L_{2,2,1}^2L_{1,2,2}^2L_{2,2,2}^2$

 $L_{1,1,1}^2 L_{2,2,1}^2 L_{1,2,2}^2 L_{2,2,2}^2$

i-i=5

 $\mathbb{C}L_{1,1,1}^3L_{1,2,1}L_{2,2,1}L_{1,2,2}L_{2,2,2}$

i-i=9

type A4, s=0, subset=[]

0	\mathbb{C}										
i+j=0	\mathbb{C}^4	\mathbb{C}									
i+j=2	\mathbb{C}_{9}	\mathbb{C}^5	\mathbb{C}								
i+j=4	\mathbb{C}^{15}	\mathbb{C}^{14}									
i+j=6	_		\mathbb{C}^5	\mathbb{C}_{5}							
i + j = 8	\mathbb{C}^{20}	\mathbb{C}^{29}	\mathbb{C}^{15}	\mathbb{C}^5	$\mathbb{C}_{\mathbb{C}^{5}}$	~					
i + j = 10	\mathbb{C}^{22}	\mathbb{C}^{44}	\mathbb{C}^{33}	\mathbb{C}^{15}	\mathbb{C}^5	\mathbb{C}					
i + j = 12	\mathbb{C}^{20}	\mathbb{C}^{51}	\mathbb{C}^{54}	\mathbb{C}^{34}	\mathbb{C}^{15}	\mathbb{C}^5	\mathbb{C}_{\downarrow}				
i+j=14	\mathbb{C}^{15}	\mathbb{C}^{46}	\mathbb{C}^{66}	\mathbb{C}^{58}	\mathbb{C}^{34}	\mathbb{C}^{15}	\mathbb{C}^5	\mathbb{C}			
i+j=16	\mathbb{C}^9	\mathbb{C}^{31}	\mathbb{C}^{56}	\mathbb{C}^{66}	\mathbb{C}^{54}	\mathbb{C}^{33}	\mathbb{C}^{15}	\mathbb{C}^5	\mathbb{C}		
i + j = 18	\mathbb{C}^4	\mathbb{C}^{15}	\mathbb{C}^{31}	\mathbb{C}^{46}	\mathbb{C}^{51}	\mathbb{C}^{44}	\mathbb{C}^{29}	\mathbb{C}^{14}	\mathbb{C}^5	\mathbb{C}	
i + j = 20	\mathbb{C}	\mathbb{C}^4	\mathbb{C}_{9}	\mathbb{C}^{15}	\mathbb{C}^{20}	\mathbb{C}^{22}	\mathbb{C}^{20}	\mathbb{C}^{15}	\mathbb{C}_{9}	\mathbb{C}^4	\mathbb{C}
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10	j-i=12	j-i=14	j-i=16	j-i=18	j-i=20
	!										
i + j = 0	1										
i+j=0 $i+j=2$	$\begin{vmatrix} 1 \\ 4 \end{vmatrix}$	1									
i+j=0 $i+j=2$ $i+j=4$		1 5	1								
i+j=2	4		1 5	1							
i+j=2 $i+j=4$ $i+j=6$	4 9 15	5 14	5		1						
i+j=2 $i+j=4$ $i+j=6$ $i+j=8$	4 9 15 20	5 14 29	5 15	5	1 5	1					
i+j=2 i+j=4 i+j=6 i+j=8 i+j=10	4 9 15 20 22	5 14 29 44	5 15 33	5 15	5	1 5	1				
i+j=2 i+j=4 i+j=6 i+j=8 i+j=10 i+j=12	4 9 15 20 22 20	5 14 29 44 51	5 15 33 54	5 15 34	5 15	5	1 5	1			
$i+j=2 \\ i+j=4 \\ i+j=6 \\ i+j=8 \\ i+j=10 \\ i+j=12 \\ i+j=14$	4 9 15 20 22 20 15	5 14 29 44 51 46	5 15 33 54 66	5 15 34 58	5 15 34	5 15	5	1 5	1		
$i+j=2 \\ i+j=4 \\ i+j=6 \\ i+j=8 \\ i+j=10 \\ i+j=12 \\ i+j=14 \\ i+j=16$	4 9 15 20 22 20 15 9	5 14 29 44 51 46 31	5 15 33 54 66 56	5 15 34 58 66	5 15 34 54	5 15 33	5 15	5	1 5	1	
$i+j=2 \\ i+j=4 \\ i+j=6 \\ i+j=8 \\ i+j=10 \\ i+j=12 \\ i+j=14 \\ i+j=16 \\ i+j=18$	4 9 15 20 22 20 15 9 4	5 14 29 44 51 46 31 15	5 15 33 54 66 56 31	5 15 34 58 66 46	5 15 34 54 51	5 15 33 44	5 15 29	5 14	5	1 4	1
$i+j=2 \\ i+j=4 \\ i+j=6 \\ i+j=8 \\ i+j=10 \\ i+j=12 \\ i+j=14 \\ i+j=16 \\ i+j=18 \\ i+j=20$	4 9 15 20 22 20 15 9 4	5 14 29 44 51 46 31 15 4	5 15 33 54 66 56 31 9	5 15 34 58 66 46 15	5 15 34 54 51 20	5 15 33 44 22	5 15 29 20	5 14 15	5 9	4	1
$i+j=2 \\ i+j=4 \\ i+j=6 \\ i+j=8 \\ i+j=10 \\ i+j=12 \\ i+j=14 \\ i+j=16 \\ i+j=18$	4 9 15 20 22 20 15 9 4	5 14 29 44 51 46 31 15	5 15 33 54 66 56 31	5 15 34 58 66 46	5 15 34 54 51	5 15 33 44	5 15 29	5 14	5		$\frac{1}{j-i=20}$

module	multiplicity	dimension
all		1296
\mathbb{C}	1296	1

type A4, s=0, subset=[1]

```
i+j=0
                   \mathbb{C}^3
  i+j=2
                   \mathbb{C}^6
                                    \mathbb{C}^4
  i+j=4
                   \mathbb{C}^9
                                    \mathbb{C}^{10}
                                                    \mathbb{C}^4
  i+j=6
                   \mathbb{C}^{11}
                                    \mathbb{C}^{18}
                                                    \mathbb{C}^{11}
                                                                     \mathbb{C}^4
  i+j=8
                   \mathbb{C}^{11}
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{21}
                                                                     \mathbb{C}^{11}
                                                                                      \mathbb{C}^4
i + j = 10
                   \mathbb{C}^9
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{29}
                                                                     \mathbb{C}^{22}
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^4
                                                                                                                          \mathbb{C}
i + j = 12
                                                                                                       \mathbb{C}^{11}
                   \mathbb{C}^6
                                    \mathbb{C}^{17}
                                                    \mathbb{C}^{28}
                                                                     \mathbb{C}^{29}
                                                                                      \mathbb{C}^{21}
                                                                                                                          \mathbb{C}^4
                                                                                                                                             \mathbb{C}
i + j = 14
                                    \mathbb{C}^9
                   \mathbb{C}^3
                                                    \mathbb{C}^{17}
                                                                     \mathbb{C}^{23}
                                                                                      \mathbb{C}^{23}
                                                                                                       \mathbb{C}^{18}
                                                                                                                          \mathbb{C}^{10}
                                                                                                                                             \mathbb{C}^4
                                                                                                                                                                \mathbb{C}
i + j = 16
                                    \mathbb{C}^3
                                                    \mathbb{C}^6
                                                                     \mathbb{C}^9
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^{11}
                                                                                                                           \mathbb{C}^9
                                                                                                                                             \mathbb{C}^6
                                                                                                                                                                \mathbb{C}^3
                                                                                                                                                                                   \mathbb{C}
i + j = 18
      h^{i,j}
                  j-i=0
                                                                                                       i - i = 10
                                                                                                                                                                j - i = 16
                                    i-i=2
                                                    i-i=4
                                                                     i-i=6
                                                                                      i-i=8
                                                                                                                          i - i = 12
                                                                                                                                             i - i = 14
                                                                                                                                                                                   i - i = 18
  i+j=0
                   3
                                    1
  i+j=2
                                                    1
                  6
                                    4
  i+j=4
                                    10
                                                    4
                                                                     1
  i+j=6
                   11
                                    18
                                                    11
                                                                     4
                                                                                       1
  i+j=8
                   11
                                    23
                                                    21
                                                                     11
                                                                                      4
                                                                                                       1
i + j = 10
                   9
                                    23
                                                    29
                                                                     22
                                                                                      11
                                                                                                       4
                                                                                                                           1
i + j = 12
                   6
                                    17
                                                     28
                                                                     29
                                                                                      21
                                                                                                       11
                                                                                                                          4
                                                                                                                                             1
i + j = 14
                   3
                                    9
                                                    17
                                                                     23
                                                                                      23
                                                                                                       18
                                                                                                                          10
                                                                                                                                             4
i + j = 16
                                                                                                                                                                1
                                    3
                                                                                                                                                                3
                                                    6
                                                                     9
                                                                                      11
                                                                                                       11
                                                                                                                          9
                                                                                                                                             6
                                                                                                                                                                                   1
i + j = 18
      h^{i,j}
                   j-i=0
                                    j-i=2
                                                    j-i=4
                                                                     j-i=6
                                                                                      j-i=8
                                                                                                       j - i = 10
                                                                                                                          j-i=12
                                                                                                                                             j-i=14
                                                                                                                                                                j-i=16
                                                                                                                                                                                   j - i = 18
```

module	multiplicity	dimension
all		540
\mathbb{C}	540	1

type A4, s=0, subset=[1, 2]

```
\mathbb{C}
 i+j=0
                 \mathbb{C}^2
 i+j=2
                 \mathbb{C}^3
                                 \mathbb{C}^3
 i+j=4
                                 \mathbb{C}^5
                                                 \mathbb{C}^3
                 \mathbb{C}^4
 i + j = 6
                 \mathbb{C}^4
                                 \mathbb{C}^7
                                                 \mathbb{C}^6
                                                                 \mathbb{C}^3
                                                                                 \mathbb{C}
 i + j = 8
                                 \mathbb{C}^6
                                                 \mathbb{C}^8
                 \mathbb{C}^3
                                                                 \mathbb{C}^6
                                                                                 \mathbb{C}^3
i + j = 10
                                                 \mathbb{C}^6
                                                                 \mathbb{C}^7
                                                                                 \mathbb{C}^5
                                                                                                 \mathbb{C}^3
                 \mathbb{C}^2
                                 \mathbb{C}^4
i + j = 12
                                 \mathbb{C}^2
                                                 \mathbb{C}^3
                                                                 \mathbb{C}^4
                                                                                 \mathbb{C}^4
                                                                                                 \mathbb{C}^3
                                                                                                                                     \mathbb{C}
i + j = 14
     h^{i,j}
                 j-i=0
                                                                                                                                     j - i = 14
                                 j-i=2
                                                 j-i=4
                                                                 j-i=6
                                                                                 j-i=8
                                                                                                 j - i = 10
                                                                                                                   j - i = 12
 i+j=0
                 1
                 2
                                 1
 i+j=2
                 3
                                 3
                                                 1
 i+j=4
                                 5
                                                 3
                                                                 1
                 4
 i + j = 6
                                                 6
                                                                 3
                                                                                 1
                 4
 i+j=8
                 3
                                 6
                                                 8
                                                                 6
                                                                                 3
                                                                                                 1
i + j = 10
                                                                                 5
                                 4
                                                 6
                                                                                                 3
i + j = 12
                  1
                                                 3
                                                                 4
                                                                                 4
                                                                                                 3
                                                                                                                   2
                                                                                                                                     1
i + j = 14
     h^{i,j}
                 j-i=0
                                 j-i=2
                                                 j-i=4
                                                                 j-i=6
                                                                                 j-i=8
                                                                                                 j - i = 10
                                                                                                                   j - i = 12
                                                                                                                                     j - i = 14
```

module	multiplicity	dimension
all		120
\mathbb{C}	120	1

type A4, s=0, subset=[1, 2, 3]

```
i+j=0
i+j=2
         \mathbb{C}
i+j=4
i+j=6
         \mathbb{C}
                   \mathbb{C}
i+j=8
  \overline{h^{i,j}}
         j-i=0 j-i=2
                             j-i=4 j-i=6
                                                 j-i=8
i + j = 0
          1
i+j=2
          1
i+j=4
          1
i+j=6
i+j=8
  h^{i,j}
          j-i=0
                  i-i=2
                             i-i=4 i-i=6
                                                 i-i=8
```

module	multiplicity	dimension
all		15
\mathbb{C}	15	1

type A4, s=1, subset=[1, 2, 3]

25

i+j=1

module	multiplicity	dimension
all		256
$\mathbb C$	10	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4\right)$	4	24
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4\right)$	2	75

type A4, s=2, subset=[1, 2, 3]

```
i+j=0 \mid L_{1,1,1,1}
                      L_{1,1,1,1}^2 L_{1,2,2,1} L_{2,2,2,1} L_{1,2,2,2}
i+j=2
                                                               L^2_{1,1,1,1}L^2_{1,2,2,1}L_{2,2,2,1}L_{1,2,2,2}L_{2,3,2,1}L_{1,2,3,2}
i+j=4 \mid 0
i+j=6
i+j=8 | 0
                      i-i=2
                                                               j-i=4
i+j=0
                    375
i+j=2
                              800
i+j=4
i + j = 6
                                        375
```

24

 $L_{1,1,1,1}^2L_{1,2,2,1}L_{2,2,2,1}L_{1,2,2,2}$

j-i=6

 $L_{1,1,1,1}$

j-i=8

module	multiplicity	dimension
all		1598
$L\left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4\right)$	8	24
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4\right)$	4	75
$L\left(2\alpha_1+2\alpha_2+2\alpha_3+\alpha_4\right)$	3	126
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4\right)$	3	126
$L\left(2\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	1	175
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 2\alpha_4\right)$	1	175

j-i=2 j-i=4 j-i=6

i+j=8

type A4, s=3, subset=[1, 2, 3]

 $i+j=1 \mid L_{1,1,1,1}L_{2,2,2,1}L_{1,2,2,2}L_{2,2,2,2}$

 $L(2\alpha_1 + 3\alpha_2 + 2\alpha_3 + \alpha_4)$ 2

 $L(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 2\alpha_4)$ 2

 $L(3\alpha_1 + 3\alpha_2 + 2\alpha_3 + \alpha_4)$ 2

 $L(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4)$ 2

 $L(2\alpha_1 + 3\alpha_2 + 3\alpha_3 + 2\alpha_4)$ 2

i+j=3

 $i+j=5 \mid 0$

i+j=7	0				0	
$h^{i,j}$	j-i=1				j-i=	=3
i+j=1	476					
i + j = 3	0	2625				
i+j=5	0	0	2625			
i+j=7			-	476		
$h^{i,j}$	j-i=1	j - i = 3	j-i=5	j - i = 7		
		m	odule	multipli	icity	dimension
			all			6202
L	$(\alpha_1 + \alpha$	$\alpha_2 + \alpha_3$	$+\alpha_4)$	4		24
L(2a	$\alpha_1 + 2\alpha_2$	$+2\alpha_3$	$+\alpha_4)$	6		126
$L(\alpha_1)$	$+2\alpha_2$	$+2\alpha_3 +$	$-2\alpha_4)$	6		126
$L\left(2\alpha_1\right)$	$+2\alpha_2$	$+2\alpha_3 +$	$2\alpha_4$	4		200
L(a)	$\alpha_1 + 2\alpha_2$	$+2\alpha_3$	$+\alpha_4$	2		75

175

175

224

224

1024

 $L_{1,1,1,1}L_{1,2,2,1}L_{2,2,2,1}^2L_{1,2,2,2}^2L_{2,3,2,1}L_{2,2,2,2}L_{1,2,3,2}L_{3,3,2,1}L_{1,2,3,3}L_{2,3,3,2}$

 $L_{1,1,1,1}L_{1,2,2,1}L_{2,2,2,1}^2L_{1,2,2,2}^2L_{2,3,2,1}L_{2,2,2,2}L_{1,2,3,2}L_{3,3,2,1}L_{1,2,3,3}L_{2,3,3,2}$

j-i=5

 $L_{1,1,1,1}L_{2,2,2,1}L_{1,2,2,2}L_{2,2,2,2}$

j-i=7

type A4, s=4, subset=[1, 2, 3]

$ \begin{array}{c c} i+j=0 & L_{2,2,2,2} \\ i+j=2 & 0 \end{array} $	$L_{2,2,2,1}L_{1,2,2,2}L_{2,2,2,2}^2L_{3,3,2,1}L_{1,2,3,3}L_{2,3,3,2}L_{3,3,3,2}L_{2,3,3,3}$			
i+j=4 0	0	$L_{2,2,2,1}L_{1,2,2,2}L_{2,3,2,1}L_{2,2,2,2}^2L_{1,2,3,2}L_{3,3,2,1}^2L_{1,2,3,3}^2L_{4,3,2,1}L_{2,3,3,2}^2L_{1,2,3,4}L_{3,3,3,2}L_{2,3,3,3}L_{3,4,3,2}L_{2,3,4,3}$		
i+j=6 0	0	0	$L_{2,2,2,1}L_{1,2,2,2}L_{2,2,2,2}^2L_{3,3,2,1}L_{1,2,3,3}L_{2,3,3,2}L_{3,3,3,2}L_{2,3,3,3}$	
$i+j=8 \mid 0$	0	0	0	$L_{2,2,2,2}$
$h^{i,j} \mid_{j-i=0}$	j-i=2	j-i=4	j-i=6	j-i=8

$\frac{i+j-6}{h^{i,j}}$	j-i=0	O	j-i=4	j-i=6	
<i>i</i> ⊥ <i>i</i> −8	0	0	0	0	200
i + j = 6	0	0	0	4224	
i+j=4	0	0	9700		
i+j=2	0	4224			
i+j=0 i+j=2 i+j=4 i+j=6 i+j=8	200				

module	multiplicity	dimension
all		18548
$L\left(2\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4\right)$	8	200
$L\left(2\alpha_1+2\alpha_2+2\alpha_3+\alpha_4\right)$	3	126
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4\right)$	3	126
$L\left(3\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	4	224
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4\right)$	4	224
$L\left(2\alpha_1+3\alpha_2+3\alpha_3+2\alpha_4\right)$	4	1024
$L\left(3\alpha_1+3\alpha_2+3\alpha_3+2\alpha_4\right)$	3	1050
$L\left(2\alpha_1+3\alpha_2+3\alpha_3+3\alpha_4\right)$	3	1050
$L\left(2\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	1	175
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 2\alpha_4\right)$	1	175
$L\left(4\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	1	126
$L\left(\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4\right)$	1	126
$L\left(3\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4\right)$	1	1701
$L\left(2\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4\right)$	1	1701

type A4, s=5, subset=[1, 2, 3]

module	multiplicity	dimension
all		46902
$L\left(2\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4\right)$	4	200
$L\left(3\alpha_1+3\alpha_2+3\alpha_3+2\alpha_4\right)$	6	1050
$L\left(2\alpha_1+3\alpha_2+3\alpha_3+3\alpha_4\right)$	6	1050
$L\left(3\alpha_1+3\alpha_2+3\alpha_3+3\alpha_4\right)$	4	1000
$L\left(3\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	2	224
$L\left(\alpha_1+2\alpha_2+3\alpha_3+3\alpha_4\right)$	2	224
$L\left(4\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	2	126
$L\left(2\alpha_1+3\alpha_2+3\alpha_3+2\alpha_4\right)$	2	1024
$L\left(\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4\right)$	2	126
$L\left(3\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4\right)$	2	1701
$L\left(2\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4\right)$	2	1701
$L\left(4\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4\right)$	2	1750
$L(2\alpha_1 + 3\alpha_2 + 4\alpha_3 + 4\alpha_4)$	2	1750
$L(3\alpha_1 + 4\alpha_2 + 4\alpha_3 + 3\alpha_4)$	2	6125

j-i=1 j-i=3 j-i=5 j-i=7

type A4, s=6, subset=[1, 2, 3]

$i \perp i = 0$	1000				
t J=0	1000	0000			
i+j=2	0	23625			
i+j=4	0	0	56000		
i + j = 6	0	0	0	23625	
i+j=0 i+j=2 i+j=4 i+j=6 i+j=8	0	0	0	0	1000
		j-i=2	j-i=4	j-i=6	j-i=8

		_
module	multiplicity	dimension
all		105250
$L\left(3\alpha_1 + 3\alpha_2 + 3\alpha_3 + 3\alpha_4\right)$	8	1000
$L\left(3\alpha_1+3\alpha_2+3\alpha_3+2\alpha_4\right)$	3	1050
$L\left(2\alpha_1+3\alpha_2+3\alpha_3+3\alpha_4\right)$	3	1050
$L\left(4\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4\right)$	4	1750
$L\left(2\alpha_1+3\alpha_2+4\alpha_3+4\alpha_4\right)$	4	1750
$L\left(3\alpha_1+4\alpha_2+4\alpha_3+3\alpha_4\right)$	4	6125
$L\left(4\alpha_1+4\alpha_2+4\alpha_3+3\alpha_4\right)$	3	4950
$L\left(3\alpha_1+4\alpha_2+4\alpha_3+4\alpha_4\right)$	3	4950
$L\left(4\alpha_1+3\alpha_2+2\alpha_3+\alpha_4\right)$	1	126
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4\right)$	1	126
$L\left(3\alpha_1+4\alpha_2+3\alpha_3+2\alpha_4\right)$	1	1701
$L\left(2\alpha_1+3\alpha_2+4\alpha_3+3\alpha_4\right)$	1	1701
$L\left(5\alpha_1 + 4\alpha_2 + 3\alpha_3 + 2\alpha_4\right)$	1	924
$L\left(2\alpha_1+3\alpha_2+4\alpha_3+5\alpha_4\right)$	1	924
$L\left(4\alpha_1+5\alpha_2+4\alpha_3+3\alpha_4\right)$	1	8624
$L(3\alpha_1 + 4\alpha_2 + 5\alpha_3 + 4\alpha_4)$	1	8624

type A4, s=0, subset=[1, 2, 3, 4]
$$\frac{1+j=0}{1+j} | \mathbb{C}$$

$$h^{i,j} \mid j-i=0$$

module multiplicity dimension

all 1

type A4, s=0, subset=[1, 2, 4]

```
i+j=0
                \mathbb{C}
 i+j=2
                              \mathbb{C}^2
                \mathbb{C}^2
 i+j=4
                                            \mathbb{C}^2
                \mathbb{C}^2
                              \mathbb{C}^3
 i+j=6
                                                           \mathbb{C}^2
                \mathbb{C}^2
                              \mathbb{C}^3
                                            \mathbb{C}^4
 i + j = 8
                                                           \mathbb{C}^3
                                                                          \mathbb{C}^2
                              \mathbb{C}^2
                                            \mathbb{C}^3
                \mathbb{C}
i + j = 10
                                            \mathbb{C}^2
                                                           \mathbb{C}^2
                \mathbb{C}
i + j = 12
    h^{i,j}
                j-i=0
                              j-i=2
                                            j-i=4
                                                           j-i=6
                                                                          j-i=8
                                                                                        j - i = 10
                                                                                                        j - i = 12
 i+j=0
 i+j=2
                1
                              1
                                             1
 i+j=4
                              3
 i + j = 6
                              3
                                             4
                                                                          1
 i+j=8
                1
                              2
                                             3
                                                           3
i + j = 10
                1
                              1
                                                                          2
                                                                                         1
                                                                                                         1
i + j = 12
     h^{i,j}
                j-i=0
                              i-i=2
                                            i-i=4
                                                           j-i=6
                                                                          j-i=8
                                                                                        i - i = 10
                                                                                                        i - i = 12
```

module	multiplicity	dimension
all		50
\mathbb{C}	50	1

type A4, s=0, subset=[1, 3]

```
i + j = 0
                  \mathbb{C}^2
 i+j=2
                  \mathbb{C}^4
                                  \mathbb{C}^3
 i+j=4
                                  \mathbb{C}^7
                                                   \mathbb{C}^3
                  \mathbb{C}^5
                                                                   \mathbb{C}
 i + j = 6
                  \mathbb{C}^6
                                  \mathbb{C}^{10}
                                                   \mathbb{C}^8
                                                                   \mathbb{C}^3
 i+j=8
                                                   \mathbb{C}^{12}
                                                                   \mathbb{C}^8
                  \mathbb{C}^5
                                  \mathbb{C}^{11}
                                                                                    \mathbb{C}^3
i + j = 10
                                                   \mathbb{C}^{14}
                  \mathbb{C}^4
                                   \mathbb{C}^9
                                                                   \mathbb{C}^{12}
                                                                                    \mathbb{C}^8
                                                                                                    \mathbb{C}^3
                                                                                                                       \mathbb{C}
i + j = 12
                                                                   \mathbb{C}^{11}
                                                                                                    \mathbb{C}^7
                  \mathbb{C}^2
                                   \mathbb{C}^5
                                                   \mathbb{C}^9
                                                                                    \mathbb{C}^{10}
                                                                                                                       \mathbb{C}^3
i + j = 14
                                  \mathbb{C}^2
                                                   \mathbb{C}^4
                                                                   \mathbb{C}^5
                                                                                    \mathbb{C}^6
                                                                                                    \mathbb{C}^5
                                                                                                                                         \mathbb{C}^2
i + j = 16
     h^{i,j}
                  j - i = 0
                                  j-i=2
                                                   j-i=4
                                                                   j-i=6
                                                                                    j-i=8
                                                                                                    j-i=10
                                                                                                                       j - i = 12
                                                                                                                                         j-i=14
                                                                                                                                                           i - i = 16
 i + j = 0
                                  1
 i+j=2
                                  3
                                                   1
 i+j=4
                                                   3
                  5
                                                                   1
 i + j = 6
                                                                   3
                  6
                                  10
                                                   8
                                                                                    1
 i + j = 8
                  5
                                  11
                                                   12
                                                                   8
                                                                                    3
                                                                                                    1
i + j = 10
                  4
                                  9
                                                   14
                                                                   12
                                                                                    8
                                                                                                    3
                                                                                                                       1
i + j = 12
                                  5
                                                   9
                                                                   11
                                                                                    10
                                                                                                                       3
                                                                                                                                         1
i + j = 14
                                  2
                                                   4
                                                                   5
                                                                                    6
                                                                                                                       4
                                                                                                                                                           1
i + j = 16
     h^{i,j}
                  j-i=0
                                  i-i=2
                                                   j-i=4
                                                                   i-i=6
                                                                                   i-i=8
                                                                                                    i - i = 10
                                                                                                                       i - i = 12
                                                                                                                                         i - i = 14
                                                                                                                                                           j - i = 16
```

module	multiplicity	dimension	
all		225	
\mathbb{C}	225	1	

1 1

type A4, s=0, subset=[1, 3, 4]

```
i+j=0
                \mathbb{C}
 i+j=2
                             \mathbb{C}^2
                \mathbb{C}^2
 i+j=4
                              \mathbb{C}^3
                                            \mathbb{C}^2
                \mathbb{C}^2
 i+j=6
                                                           \mathbb{C}^2
               \mathbb{C}^2
                              \mathbb{C}^3
                                           \mathbb{C}^4
 i + j = 8
                                            \mathbb{C}^3
                                                          \mathbb{C}^3
                                                                         \mathbb{C}^2
                              \mathbb{C}^2
                \mathbb{C}
i + j = 10
                                            \mathbb{C}^2
                                                           \mathbb{C}^2
                \mathbb{C}
i + j = 12
    h^{i,j}
                j-i=0
                              j-i=2
                                            j-i=4
                                                           j-i=6
                                                                         j-i=8
                                                                                       j - i = 10
                                                                                                        j - i = 12
 i+j=0
 i+j=2
                1
                              1
                              2
                                            1
 i+j=4
                              3
 i + j = 6
                              3
                                            4
                                                                         1
 i+j=8
                1
                              2
                                            3
                                                           3
                                                                                        1
i + j = 10
                1
                              1
                                                                         2
                                                                                        1
                                                                                                        1
i + j = 12
     h^{i,j}
               j-i=0
                              i-i=2
                                            i-i=4
                                                           j-i=6
                                                                         i-i=8
                                                                                       i - i = 10
                                                                                                        i - i = 12
```

module	multiplicity	dimension
all		50
\mathbb{C}	50	1

type A4, s=0, subset=[1, 4]

```
i + j = 0
                  \mathbb{C}^2
 i+j=2
                  \mathbb{C}^4
                                   \mathbb{C}^3
 i+j=4
                                   \mathbb{C}^7
                                                   \mathbb{C}^3
                  \mathbb{C}^5
                                                                   \mathbb{C}
 i + j = 6
                  \mathbb{C}^6
                                   \mathbb{C}^{10}
                                                   \mathbb{C}^8
                                                                   \mathbb{C}^3
 i+j=8
                                                   \mathbb{C}^{12}
                                                                   \mathbb{C}^8
                  \mathbb{C}^5
                                   \mathbb{C}^{11}
                                                                                    \mathbb{C}^3
i + j = 10
                                                   \mathbb{C}^{14}
                  \mathbb{C}^4
                                   \mathbb{C}^9
                                                                   \mathbb{C}^{12}
                                                                                    \mathbb{C}^8
                                                                                                     \mathbb{C}^3
                                                                                                                       \mathbb{C}
i + j = 12
                                                                   \mathbb{C}^{11}
                                                                                                     \mathbb{C}^7
                  \mathbb{C}^2
                                   \mathbb{C}^5
                                                   \mathbb{C}^9
                                                                                    \mathbb{C}^{10}
                                                                                                                       \mathbb{C}^3
i + j = 14
                                   \mathbb{C}^2
                                                   \mathbb{C}^4
                                                                   \mathbb{C}^5
                                                                                    \mathbb{C}^6
                                                                                                     \mathbb{C}^5
                                                                                                                                          \mathbb{C}^2
i + j = 16
     h^{i,j}
                  j - i = 0
                                  j-i=2
                                                   j-i=4
                                                                   j-i=6
                                                                                    j-i=8
                                                                                                    j - i = 10
                                                                                                                       j - i = 12
                                                                                                                                         j-i=14
                                                                                                                                                            j - i = 16
 i + j = 0
                                   1
 i+j=2
                                   3
                                                   1
 i+j=4
                                                   3
                  5
                                                                   1
 i + j = 6
                                                                   3
                  6
                                   10
                                                   8
                                                                                    1
 i + j = 8
                  5
                                   11
                                                   12
                                                                   8
                                                                                    3
                                                                                                    1
i + j = 10
                  4
                                   9
                                                   14
                                                                    12
                                                                                    8
                                                                                                    3
                                                                                                                       1
i + j = 12
                                   5
                                                   9
                                                                    11
                                                                                    10
                                                                                                                       3
                                                                                                                                          1
i + j = 14
                                   2
                                                   4
                                                                   5
                                                                                    6
                                                                                                                       4
                                                                                                                                                            1
i + j = 16
     h^{i,j}
                  j-i=0
                                  i-i=2
                                                   j-i=4
                                                                   i-i=6
                                                                                    i-i=8
                                                                                                    i - i = 10
                                                                                                                       i - i = 12
                                                                                                                                         i - i = 14
                                                                                                                                                            j - i = 16
```

module	multiplicity	dimension
all		225
\mathbb{C}	225	1

type A4, s=0, subset=[2]

```
i+j=0
                   \mathbb{C}^3
  i+j=2
                   \mathbb{C}^6
                                    \mathbb{C}^4
  i+j=4
                   \mathbb{C}^9
                                    \mathbb{C}^{10}
                                                    \mathbb{C}^4
  i+j=6
                   \mathbb{C}^{11}
                                    \mathbb{C}^{18}
                                                    \mathbb{C}^{11}
                                                                     \mathbb{C}^4
  i+j=8
                   \mathbb{C}^{11}
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{21}
                                                                     \mathbb{C}^{11}
                                                                                      \mathbb{C}^4
i + j = 10
                   \mathbb{C}^9
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{29}
                                                                     \mathbb{C}^{22}
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^4
                                                                                                                          \mathbb{C}
i + j = 12
                                                                                                       \mathbb{C}^{11}
                   \mathbb{C}^6
                                    \mathbb{C}^{17}
                                                    \mathbb{C}^{28}
                                                                     \mathbb{C}^{29}
                                                                                      \mathbb{C}^{21}
                                                                                                                          \mathbb{C}^4
                                                                                                                                             \mathbb{C}
i + j = 14
                                    \mathbb{C}^9
                   \mathbb{C}^3
                                                    \mathbb{C}^{17}
                                                                     \mathbb{C}^{23}
                                                                                      \mathbb{C}^{23}
                                                                                                       \mathbb{C}^{18}
                                                                                                                          \mathbb{C}^{10}
                                                                                                                                             \mathbb{C}^4
                                                                                                                                                                \mathbb{C}
i + j = 16
                                    \mathbb{C}^3
                                                    \mathbb{C}^6
                                                                     \mathbb{C}^9
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^{11}
                                                                                                                          \mathbb{C}^9
                                                                                                                                             \mathbb{C}^6
                                                                                                                                                                \mathbb{C}^3
                                                                                                                                                                                   \mathbb{C}
i + j = 18
      h^{i,j}
                  j-i=0
                                                                                                       i - i = 10
                                                                                                                                                                j - i = 16
                                    i-i=2
                                                    i-i=4
                                                                     i-i=6
                                                                                      i-i=8
                                                                                                                          i - i = 12
                                                                                                                                             i - i = 14
                                                                                                                                                                                   i - i = 18
  i+j=0
                   3
                                    1
  i+j=2
                                                    1
                  6
                                    4
  i+j=4
                  9
                                    10
                                                    4
                                                                     1
  i+j=6
                   11
                                    18
                                                    11
                                                                     4
                                                                                       1
  i+j=8
                   11
                                    23
                                                    21
                                                                     11
                                                                                      4
                                                                                                       1
i + j = 10
                   9
                                    23
                                                    29
                                                                     22
                                                                                      11
                                                                                                       4
                                                                                                                          1
i + j = 12
                   6
                                    17
                                                     28
                                                                     29
                                                                                      21
                                                                                                       11
                                                                                                                          4
                                                                                                                                             1
i + j = 14
                   3
                                    9
                                                    17
                                                                     23
                                                                                      23
                                                                                                       18
                                                                                                                          10
                                                                                                                                             4
i + j = 16
                                                                                                                                                                1
                                    3
                                                                                                                                                                3
                                                    6
                                                                     9
                                                                                      11
                                                                                                       11
                                                                                                                          9
                                                                                                                                             6
                                                                                                                                                                                   1
i + j = 18
      h^{i,j}
                   j-i=0
                                    j-i=2
                                                    j-i=4
                                                                     j-i=6
                                                                                      j-i=8
                                                                                                       j - i = 10
                                                                                                                          j-i=12
                                                                                                                                             j-i=14
                                                                                                                                                               j-i=16
                                                                                                                                                                                   j - i = 18
```

module	multiplicity	dimension
all		540
\mathbb{C}	540	1

type A4, s=0, subset=[2, 3]

```
\mathbb{C}
 i + j = 0
                 \mathbb{C}^2
 i+j=2
                 \mathbb{C}^3
                                  \mathbb{C}^3
 i+j=4
                                  \mathbb{C}^5
                                                 \mathbb{C}^3
                 \mathbb{C}^4
 i + j = 6
                 \mathbb{C}^4
                                  \mathbb{C}^7
                                                 \mathbb{C}^6
                                                                 \mathbb{C}^3
                                                                                 \mathbb{C}
 i + j = 8
                                  \mathbb{C}^6
                                                 \mathbb{C}^8
                 \mathbb{C}^3
                                                                 \mathbb{C}^6
                                                                                  \mathbb{C}^3
i + j = 10
                                                 \mathbb{C}^6
                                                                 \mathbb{C}^7
                                                                                 \mathbb{C}^5
                                                                                                 \mathbb{C}^3
                 \mathbb{C}^2
                                  \mathbb{C}^4
i + j = 12
                                  \mathbb{C}^2
                                                  \mathbb{C}^3
                                                                 \mathbb{C}^4
                                                                                  \mathbb{C}^4
                                                                                                 \mathbb{C}^3
                                                                                                                                     \mathbb{C}
i + j = 14
     h^{i,j}
                 j-i=0
                                                                                                                                     j - i = 14
                                  j-i=2
                                                 j-i=4
                                                                 j-i=6
                                                                                 j-i=8
                                                                                                 j - i = 10
                                                                                                                   j - i = 12
 i+j=0
                 1
                 2
                                  1
 i+j=2
                 3
                                  3
                                                 1
 i+j=4
                                  5
                                                 3
                                                                 1
                 4
 i + j = 6
                                                 6
                                                                 3
                                                                                  1
                 4
 i+j=8
                 3
                                  6
                                                 8
                                                                 6
                                                                                 3
                                                                                                 1
i + j = 10
                                                                                 5
                                  4
                                                 6
                                                                                                 3
i + j = 12
                  1
                                                 3
                                                                 4
                                                                                 4
                                                                                                 3
                                                                                                                   2
                                                                                                                                      1
i + j = 14
     h^{i,j}
                 j-i=0
                                  j-i=2
                                                 j-i=4
                                                                 j-i=6
                                                                                 j-i=8
                                                                                                 j - i = 10
                                                                                                                   j - i = 12
                                                                                                                                     j - i = 14
```

module	multiplicity	dimension
all		120
\mathbb{C}	120	1

type A4, s=0, subset=[2, 3, 4]

```
i+j=0
i+j=2
         \mathbb{C}
i+j=4
i+j=6
         \mathbb{C}
                   \mathbb{C}
i+j=8
  \overline{h^{i,j}}
         j-i=0 j-i=2
                             j-i=4 j-i=6
                                                 j-i=8
i + j = 0
          1
i+j=2
          1
i+j=4
          1
i+j=6
i+j=8
  h^{i,j}
          j-i=0
                   i-i=2
                             i-i=4 i-i=6
                                                 i-i=8
```

module	multiplicity	dimension
all		15
\mathbb{C}	15	1

type A4, s=0, subset=[3]

```
i+j=0
                   \mathbb{C}^3
  i+j=2
                   \mathbb{C}^6
                                    \mathbb{C}^4
  i+j=4
                   \mathbb{C}^9
                                    \mathbb{C}^{10}
                                                    \mathbb{C}^4
  i+j=6
                   \mathbb{C}^{11}
                                    \mathbb{C}^{18}
                                                    \mathbb{C}^{11}
                                                                     \mathbb{C}^4
  i+j=8
                   \mathbb{C}^{11}
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{21}
                                                                     \mathbb{C}^{11}
                                                                                      \mathbb{C}^4
i + j = 10
                   \mathbb{C}^9
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{29}
                                                                     \mathbb{C}^{22}
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^4
                                                                                                                          \mathbb{C}
i + j = 12
                                                                                                       \mathbb{C}^{11}
                   \mathbb{C}^6
                                    \mathbb{C}^{17}
                                                    \mathbb{C}^{28}
                                                                     \mathbb{C}^{29}
                                                                                      \mathbb{C}^{21}
                                                                                                                          \mathbb{C}^4
                                                                                                                                             \mathbb{C}
i + j = 14
                                    \mathbb{C}^9
                   \mathbb{C}^3
                                                    \mathbb{C}^{17}
                                                                     \mathbb{C}^{23}
                                                                                      \mathbb{C}^{23}
                                                                                                       \mathbb{C}^{18}
                                                                                                                          \mathbb{C}^{10}
                                                                                                                                             \mathbb{C}^4
                                                                                                                                                                \mathbb{C}
i + j = 16
                                    \mathbb{C}^3
                                                    \mathbb{C}^6
                                                                     \mathbb{C}^9
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^{11}
                                                                                                                          \mathbb{C}^9
                                                                                                                                             \mathbb{C}^6
                                                                                                                                                                \mathbb{C}^3
                                                                                                                                                                                   \mathbb{C}
i + j = 18
      h^{i,j}
                  j-i=0
                                                                                                       i - i = 10
                                                                                                                                                                j - i = 16
                                    i-i=2
                                                    i-i=4
                                                                     i-i=6
                                                                                      i-i=8
                                                                                                                          i - i = 12
                                                                                                                                             i - i = 14
                                                                                                                                                                                   i - i = 18
  i+j=0
                   3
                                    1
  i+j=2
                                                    1
                  6
                                    4
  i+j=4
                  9
                                    10
                                                    4
                                                                     1
  i+j=6
                   11
                                    18
                                                    11
                                                                     4
                                                                                       1
  i+j=8
                   11
                                    23
                                                    21
                                                                     11
                                                                                      4
                                                                                                       1
i + j = 10
                   9
                                    23
                                                    29
                                                                     22
                                                                                      11
                                                                                                       4
                                                                                                                          1
i + j = 12
                   6
                                    17
                                                     28
                                                                     29
                                                                                      21
                                                                                                       11
                                                                                                                          4
                                                                                                                                             1
i + j = 14
                   3
                                    9
                                                    17
                                                                     23
                                                                                      23
                                                                                                       18
                                                                                                                          10
                                                                                                                                             4
i + j = 16
                                                                                                                                                                1
                                    3
                                                                                                                                                                3
                                                    6
                                                                     9
                                                                                      11
                                                                                                       11
                                                                                                                          9
                                                                                                                                             6
                                                                                                                                                                                   1
i + j = 18
      h^{i,j}
                   j-i=0
                                    j-i=2
                                                    j-i=4
                                                                     j-i=6
                                                                                      j-i=8
                                                                                                       j - i = 10
                                                                                                                          j-i=12
                                                                                                                                             j-i=14
                                                                                                                                                               j-i=16
                                                                                                                                                                                   j - i = 18
```

module	multiplicity	dimension
all		540
\mathbb{C}	540	1

type A4, s=0, subset=[4]

```
i+j=0
                   \mathbb{C}^3
  i+j=2
                   \mathbb{C}^6
                                    \mathbb{C}^4
  i+j=4
                   \mathbb{C}^9
                                    \mathbb{C}^{10}
                                                    \mathbb{C}^4
  i+j=6
                   \mathbb{C}^{11}
                                    \mathbb{C}^{18}
                                                    \mathbb{C}^{11}
                                                                     \mathbb{C}^4
  i+j=8
                   \mathbb{C}^{11}
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{21}
                                                                     \mathbb{C}^{11}
                                                                                      \mathbb{C}^4
i + j = 10
                   \mathbb{C}^9
                                    \mathbb{C}^{23}
                                                    \mathbb{C}^{29}
                                                                     \mathbb{C}^{22}
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^4
                                                                                                                          \mathbb{C}
i + j = 12
                                                                                                       \mathbb{C}^{11}
                   \mathbb{C}^6
                                    \mathbb{C}^{17}
                                                    \mathbb{C}^{28}
                                                                     \mathbb{C}^{29}
                                                                                      \mathbb{C}^{21}
                                                                                                                          \mathbb{C}^4
                                                                                                                                             \mathbb{C}
i + j = 14
                                    \mathbb{C}^9
                   \mathbb{C}^3
                                                    \mathbb{C}^{17}
                                                                     \mathbb{C}^{23}
                                                                                      \mathbb{C}^{23}
                                                                                                       \mathbb{C}^{18}
                                                                                                                          \mathbb{C}^{10}
                                                                                                                                             \mathbb{C}^4
                                                                                                                                                                \mathbb{C}
i + j = 16
                                    \mathbb{C}^3
                                                    \mathbb{C}^6
                                                                     \mathbb{C}^9
                                                                                      \mathbb{C}^{11}
                                                                                                       \mathbb{C}^{11}
                                                                                                                          \mathbb{C}^9
                                                                                                                                             \mathbb{C}^6
                                                                                                                                                                \mathbb{C}^3
                                                                                                                                                                                  \mathbb{C}
i + j = 18
      h^{i,j}
                  j-i=0
                                                                                                       i - i = 10
                                                                                                                                                                j - i = 16
                                    i-i=2
                                                    i-i=4
                                                                     i-i=6
                                                                                      i-i=8
                                                                                                                          i - i = 12
                                                                                                                                             i - i = 14
                                                                                                                                                                                  i - i = 18
  i+j=0
                   3
                                    1
  i+j=2
                                                    1
                  6
                                    4
  i+j=4
                  9
                                    10
                                                    4
                                                                     1
  i+j=6
                   11
                                    18
                                                    11
                                                                     4
                                                                                      1
  i+j=8
                   11
                                    23
                                                    21
                                                                     11
                                                                                      4
                                                                                                       1
i + j = 10
                   9
                                    23
                                                    29
                                                                     22
                                                                                      11
                                                                                                       4
                                                                                                                          1
i + j = 12
                   6
                                    17
                                                     28
                                                                     29
                                                                                      21
                                                                                                       11
                                                                                                                          4
                                                                                                                                             1
i + j = 14
                   3
                                    9
                                                    17
                                                                     23
                                                                                      23
                                                                                                       18
                                                                                                                          10
                                                                                                                                             4
i + j = 16
                                                                                                                                                                1
                                    3
                                                                                                                                                                3
                                                    6
                                                                     9
                                                                                      11
                                                                                                       11
                                                                                                                          9
                                                                                                                                             6
                                                                                                                                                                                  1
i + j = 18
      h^{i,j}
                   j-i=0
                                    j-i=2
                                                    j-i=4
                                                                     j-i=6
                                                                                      j-i=8
                                                                                                       j - i = 10
                                                                                                                         j-i=12
                                                                                                                                            j-i=14
                                                                                                                                                               j-i=16
                                                                                                                                                                                  j - i = 18
```

module	multiplicity	dimension
all		540
\mathbb{C}	540	1

type A5, s=0, subset=[1, 2, 3, 4]

```
i+j=0
 i+j=2
 i+j=4
            \mathbb{C}
                      \mathbb{C}
 i+j=6
            \mathbb{C}
                      \mathbb{C}
 i + j = 8
i + j = 10
   h^{i,j}
            j-i=0
                      j-i=2
                                 i-i=4
                                            i-i=6
                                                      i-i=8
                                                                  i - i = 10
           1
 i+j=0
           1
 i+j=2
            1
 i + j = 4
            1
 i + j = 6
 i + j = 8
            1
i + j = 10
   h^{i,j}
           j-i=0 j-i=2
                                 j-i=4 j-i=6 j-i=8
                                                                 i - i = 10
```

module	multiplicity	dimension
all		21
\mathbb{C}	21	1

type A5, s=1, subset=[1, 2, 3, 4]

```
_{i+j=1} \mid \mathbb{C}L_{1,1,1,1,1}
                              \mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,1}
i+j=3
i+j=5 \mid \mathbb{C}
                                                             \mathbb{C}L_{1,1,1,1}L_{1,2,2,2,1}L_{1,2,3,2,1}
                                                                                                         \mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,1}
i+j=7
i+j=9 \mid \mathbb{C}
  h^{i,j}
          j-i=1
                              j-i=3
                                                             j-i=5
                                                                                                         j - i = 7
i+j=1 | 36
                      225
i+j=3
                                  400
i+j=5
                                             225
i+j=7
                                                         36
i+j=9
```

 $\mathbb{C}L_{1,1,1,1,\underline{1}}$

i-i=9

module	multiplicity	dimension
all		932
$\mathbb C$	15	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5\right)$	5	35
$L\left(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 + \alpha_5\right)$	3	189
$L\left(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 2\alpha_4 + \alpha_5\right)$	1	175

j-i=1 j-i=3 j-i=5 j-i=7 j-i=9

 $h^{i,j}$

type A6, s=0, subset=[1, 2, 3, 4, 5]

```
i + j = 0
 i+j=2
                     \mathbb{C}
 i+j=4
 i + j = 6
 i + j = 8
i + j = 10
i + j = 12
   h^{i,j}
           j-i=0
                     j-i=2
                                j-i=4
                                          j-i=6
                                                                           j - i = 12
                                                    j-i=8
                                                               j - i = 10
 i + j = 0
 i+j=2
 i+j=4
 i + j = 6
 i+j=8
i + j = 10
                                                                           1
i + j = 12
                                                               1
   h^{i,j}
           j-i=0
                     j-i=2
                                i-i=4
                                          j-i=6
                                                    j-i=8
                                                               j-i=10
                                                                           i - i = 12
```

module	multiplicity	dimension
all		28
\mathbb{C}	28	1

type A6, s=1, subset=[1, 2, 3, 4, 5]

 $\mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,2,1}$

 $i+j=1 \mid \mathbb{C}L_{1,1,1,1,1,1}$

i+j=3

 $i+j=5 \mid \mathbb{C}$

```
i+j=7
 i + j = 9
i+j=11
  h^{i,j}
        j-i=1
                      i-i=3
                                              j-i=5
i+j=1 | 49
                441
 i+j=3
i+j=5 | 1
                       1225
i+j=7 | 1
                               1225
                                      441
i+j=9
                                              49
i + j = 11
  h^{i,j}
        j-i=1 j-i=3 j-i=5 j-i=7 j-i=9
                              module multip
                                   all
```

 $L(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)$ $L(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 + 2\alpha_5 + \alpha_6)$ $L(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4 + 2\alpha_5 + \alpha_6)$

olicity	dimension
	3445
	1
	48
	392
	784

21

 $\mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,2,1}L_{1,2,3,3,2,1}$

 $\mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,2,1}L_{1,2,3,3,2,1}$

 $\mathbb{C}L_{1,1,1,1,1}L_{1,2,2,2,2,1}$

j-i=9

 $\mathbb{C}L_{1,1,1,1,\underline{1,1}}$

j - i = 11

 \mathbb{C}

j-i=7

type A7, s=0, subset=[1, 2, 3, 4, 5, 6]

```
i+j=0
 i+j=2
                          \mathbb{C}
 i+j=4
                           \mathbb{C}
 i+j=6
                           \mathbb{C}
 i+j=8
                          \mathbb{C}
                                       \mathbb{C}
i + j = 10
                           \mathbb{C}
                                       \mathbb{C}
i + j = 12
i + j = 14
    h^{i,j}
             j\!-\!i\!=\!0
                          j-i=2
                                                                                                          j - i = 14
                                       j-i=4
                                                    j-i=6
                                                                j-i=8
                                                                             j - i = 10
                                                                                            j - i = 12
 i+j=0
             1
 i+j=2
             1
 i+j=4
 i + j = 6
              1
 i + j = 8
i + j = 10
i + j = 12
                                                                                                          1
i + j = 14
    \overline{h^{i,j}}
             j-i=0
                           j-i=2
                                       j-i=4
                                                    j-i=6
                                                                j-i=8
                                                                             j - i = 10
                                                                                            j - i = 12
                                                                                                          j - i = 14
```

module	multiplicity	dimension
all		36
\mathbb{C}	36	1

type A7, s=1, subset=[1, 2, 3, 4, 5, 6]

 $\mathbb{C}L_{1,1,1,1,1,1}L_{1,2,2,2,2,2,1}$

 $\mathbb{C}L_{1,1,1,1,1,1}L_{1,2,2,2,2,2,1}L_{1,2,3,3,3,2,1}$

63

720

2352

 $\mathbb{C}L_{1,1,1,1,1,1}$

i+j=1

i+j=3

i+j=5

```
i + j = 7
 i+j=9
i + j = 11
i+j=13
   h^{i,j}
        j-i=1
                        i-i=3
                                                    j-i=5
 i+j=1 | 64
 i+j=3 | 1
                784
                        3136
 i+j=5
 i+j=7 | 1
                                        3136
 i + j = 9
                                                784
i + j = 11
                                                        64
i+j=13
   h^{i,j}
               i-i=3 i-i=5
                               j-i=7 j-i=9 j-i=11 j-i=13
                                     module
                                               multiplicity
                                                              dimension
                                                              7989
                                          all
                                               27
```

 $L(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7)$ 6

 $L(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6 + \alpha_7)$ 4

 $L(\alpha_1 + 2\alpha_2 + 3\alpha_3 + 3\alpha_4 + 3\alpha_5 + 2\alpha_6 + \alpha_7)$ 2

•	•	•
$j\!-\!i\!=\!7$	j-i=9	$j\!-\!i\!=\!11$

 $\mathbb{C}L_{1,1,1,1,1,1}L_{1,2,2,2,2,2,1}L_{1,2,3,3,3,2,1}$

 $\mathbb{C}L_{1,1,1,1,1,1}L_{1,2,2,2,2,2,1}$

 $\mathbb{C}L_{1,1,1,1,\underline{1,1,1}}$

j - i = 13

type B2, s=0, subset=[]

module	multiplicity	dimension
all		30
\mathbb{C}	25	1
$L\left(\alpha_1+\alpha_2\right)$	1	5

type B2, s=1, subset=[]

i+j=1 i+j=3 i+j=5 i+j=7	11			
i + j = 3	27	11		
i + j = 5	18	52	11	
i+j=7	2	18	27	11
$h^{i,j}$	j-i=1	j-i=3	j - i = 5	j-i=7

module	multiplicity	dimension
all		188
\mathbb{C}	19	1
$L\left(\alpha_1+2\alpha_2\right)$	12	10
$L\left(\alpha_1+\alpha_2\right)$		5
$L\left(2\alpha_1+2\alpha_2\right)$	1	14

type B2, s=2, subset=[]

10

module	multiplicity	dimension
all		768
$L\left(\alpha_1+2\alpha_2\right)$	33	10
$L\left(\alpha_1+\alpha_2\right)$	11	5
$L\left(2\alpha_1+3\alpha_2\right)$	10	35
\mathbb{C}	5	1
$L\left(2\alpha_1+2\alpha_2\right)$	2	14

211

j-i=0 j-i=2 j-i=4 j-i=6 j-i=8

10

65

25

61

i+j=6 | 0

 $h^{i,j}$

 $i+j=8 \mid 0 = 0$

type B2, s=3, subset=[]

h	i,j	j-i=1	j - i = 3	j - i = 5	j - i = 7
i+j	=7	109 294 80 0	80	294	109
i+j	=5	80	805	274	
i+j	=3	294	274		
i+j	=1	109			

module	multiplicity	dimension
all		2319
$L\left(\alpha_1+\alpha_2\right)$	13	5
$L\left(\alpha_1+2\alpha_2\right)$	31	10
$L\left(2\alpha_1+2\alpha_2\right)$	14	14
$L\left(2\alpha_1+3\alpha_2\right)$	27	35
$L\left(2\alpha_1+4\alpha_2\right)$	17	35
\mathbb{C}	7	1
$L\left(3\alpha_1+3\alpha_2\right)$	4	30
$L\left(3\alpha_1+4\alpha_2\right)$	1	81
,		

type B2, s=4, subset=[]

```
i+j=0 \mid L_{1,1}L_{2,2}L_{2,4}
               \begin{array}{ccc} L_{2,2}^2 L_{2,3} L_{2,4}^2 & \mathbb{C}^2 L_{1,1}^4 L_{1,2}^2 L_{2,2}^4 L_{2,3}^4 L_{3,3} L_{2,4}^4 L_{3,4} L_{3,5} \\ L_{2,4} & \mathbb{C} L_{1,1}^2 L_{1,2}^4 L_{2,2}^5 L_{2,3}^{10} L_{3,3}^2 L_{2,4}^{10} L_{3,4}^4 L_{3,5}^3 \end{array}
                                                                                                                                       \mathbb{C}^3L_{1.1}^6L_{1,2}^3L_{2,2}^6L_{2,3}^6L_{3,3}^3L_{2,4}^6L_{3,4}^2L_{3,5}
i+j=4
                                                                                                                                                                                                                            \mathbb{C}^2L^4_{1.1}L^2_{1.2}L^4_{2.2}L^4_{2,3}L_{3,3}L^4_{2,4}L_{3,4}L_{3,5}
                                        L_{2,3}L_{2,4}^2L_{3,5}
                                                                                                                                       \mathbb{C}L_{1.1}^2L_{1.2}^4L_{2.2}^5L_{2.3}^{10}L_{2.3}^2L_{3.3}^{10}L_{2.4}^4L_{3.4}^3L_{3.5}^3
i + j = 6
                                                                                                                                                                                                                            L_{2,2}^2 L_{2,3} L_{2,4}^2
                                                                                                                                                                                                                                                                                                                   L_{1,1}L_{2,2}L_{2,4}
i+j=8 | 0
                                                                                                                                       L_{2,4}
   h^{i,j}
                                                 i-i=2
                                                                                                                                                                                                                             i-i=6
                                                                                                                                       i-i=4
                                                                                                                                                                                                                                                                                                                   i-i=8
i+j=0 | 54
                133
                                  594
i+j=2
                                  1520
                                                   924
i+j=4
```

i+j=6	U	210	1920	394	
i+j=6 $i+j=8$	0	0	35	133	54
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j - i = 6	j-i=
	module	multi	plicity	dimens	sion
	all			5806	
$L(\alpha$	$(1+\alpha_2)$	20		5	
$L(2\alpha_1)$	$+2\alpha_2$	30		14	
$L(2\alpha_1)$	$+4\alpha_2$	44		35	
$L\left(2\alpha_1\right)$	$+3\alpha_2$	37		35	
	\mathbb{C}	9		1	

 $L\left(\alpha_1+2\alpha_2\right)$ 15

 $L\left(3\alpha_1+3\alpha_2\right)$

 $L\left(3\alpha_1+4\alpha_2\right)$

 $L\left(3\alpha_1+5\alpha_2\right)$

210

1520

504

10

30

81

105

type B2, s=5, subset=[]

```
i+j=1 | \mathbb{C}L_{1,1}^2L_{1,2}^2L_{2,2}^2L_{2,3}^3L_{2,4}^2L_{3,4}^2L_{3,5}L_{3,6}
                                                                                           \mathbb{C}^3L_{1,1}^6L_{1,2}^7L_{2,2}^6L_{2,3}^{11}L_{3,3}^3L_{2,4}^6L_{3,4}^7L_{3,5}^4L_{4,5}L_{3,6}^2
i+j=3 L_{2,2}^2L_{2,3}^4L_{3,3}L_{2,4}^5L_{3,4}^5L_{3,5}^4L_{3,5}^4L_{3,6}^2
                                                                                           L_{1.1}^2 L_{1.2}^1 L_{2.2}^4 L_{2.3}^{11} L_{3.3}^5 L_{2.4}^{12} L_{3.4}^{10} L_{4.4}^4 L_{3.5}^{12} L_{4.5}^2 L_{3.6}^5 L_{4.6}
                                                                                                                                                                                                     \mathbb{C}^3L_{1,1}^6L_{1,2}^7L_{2,2}^6L_{2,3}^{11}L_{3,3}^3L_{2,4}^6L_{3,4}^7L_{3,5}^4L_{4,5}L_{3,6}^2
i+j=5
               L_{2,4}L_{3,5}L_{3,6}
                                                                                                                                                                                                     L_{2,2}^2 L_{2,3}^{4} L_{3,3}^{72} L_{2,4}^{52} L_{3,4}^{52} L_{3,5}^{42} L_{3,5}^{22}
                                                                                                                                                                                                                                                                                                     \mathbb{C}L_{1,1}^2L_{1,2}^2L_{2,2}^2L_{2,3}^3L_{2,4}^2L_{3,4}^2L_{3,5}L_{3,6}
                                                                                           L_{2,4}L_{3,5}L_{3,6}
i+j=7
   h^{i,j}
               j-i=1
                                                                                           j-i=3
                                                                                                                                                                                                     j-i=5
                                                                                                                                                                                                                                                                                                    j-i=7
i+j=1 | 585
```

$h^{i,j} \mid j-i=1$	j-i=3	j-i=5	j-i=7
module	multi	plicity	dimer
all			12816
\mathbb{C}	8		1
$L\left(\alpha_1+\alpha_2\right)$	18		5
$L\left(\alpha_1+2\alpha_2\right)$	19		10
$L(2\alpha_1+2\alpha_2)$	24		14
$L(2\alpha_1 + 3\alpha_2)$	47		35
$L(2\alpha_1+4\alpha_2)$	40		35
$L(3\alpha_1 + 4\alpha_2)$	38		81
$L(3\alpha_1 + 5\alpha_2)$	32		105

2181

4104

224

2181

1366

1366

224

 $\begin{array}{ll} L \left(3\alpha_{1} + 6\alpha_{2} \right) & 17 \\ L \left(3\alpha_{1} + 3\alpha_{2} \right) & 13 \\ L \left(4\alpha_{1} + 5\alpha_{2} \right) & 4 \\ L \left(4\alpha_{1} + 4\alpha_{2} \right) & 1 \\ L \left(4\alpha_{1} + 6\alpha_{2} \right) & 1 \end{array}$

i+j=3

i+j=5

 $i+j=7 \mid 0$

-i=5	j-i=7
icity	dimension
	12816
	1
	5
	10
	14
	35
	35
	81
	105
	84
	30
	154
	55
	220

585

type B2, s=6, subset=[]

```
i+j=0 \mid L_{1,2}L_{2,3}L_{3,4}L_{3,6}
                                                                     \begin{array}{l} \mathbb{C}L^3_{1,1}L^8_{1,2}L^3_{2,2}L^{21}_{2,3}L^2_{3,3}L^4_{2,4}L^9_{3,4}L^5_{3,5}L^2_{4,5}L^4_{3,6}L_{4,6}L_{4,7} \\ L_{2,2}L^5_{2,3}L^4_{3,3}L^4_{2,4}L^{11}_{3,4}L_{4,4}L^{13}_{3,5}L^5_{4,5}L^{10}_{3,6}L^4_{4,6}L^3_{4,7} \end{array}
i+j=2 L_{3,3}L_{3,4}^2L_{3,5}L_{3,6}^2
                                                                                                                                                                                                                      \begin{array}{l} \mathbb{C}L_{1,1}^6L_{1,2}^{14}L_{2,2}^7L_{2,3}^{20}L_{3,3}^5L_{2,4}^8L_{3,4}^{16}L_{4,4}L_{3,5}^{10}L_{4,5}^5L_{3,6}^6L_{4,6}^2L_{4,7}\\ L_{2,2}L_{2,3}^5L_{3,3}^4L_{2,4}^4L_{3,4}^{11}L_{4,4}L_{3,5}^{13}L_{4,5}^4L_{3,6}^0L_{4,6}^4L_{4,7}^4 \end{array} 
i+j=4 \mid L_{3,6}
                                                                                                                                                                                                                                                                                                                                                                                 \mathbb{C}L^3_{1,1}L^8_{1,2}L^3_{2,2}L^{11}_{2,3}L^2_{3,3}L^4_{2,4}L^9_{3,4}L^5_{3,5}L^2_{4,5}L^4_{3,6}L_{4,6}L_{4,7}
                                                                       L_{3,5}L_{3,6}^{\bar{2},5}L_{4,7}
i + j = 6
                                                                                                                                                                                                                                                                                                                                                                                 L_{3,3}L_{3,4}^2L_{3,5}L_{3,6}^2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                L_{1,2}L_{2,3}L_{3,4}L_{3,6}
                                                                                                                                                                                                                      L_{3.6}
i + j = 8
    h^{i,j}
                j-\overline{i=0}
                                                                       j-i=2
                                                                                                                                                                                                                                                                                                                                                                                j-i=6
                                                                                                                                                                                                                      j-i=4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                j-i=8
_{i+i=0} \mid 210
```

-					j-i=6	
	i+j=8	0		84		210
	i+j=6	0	504	5943	3072	
	i+j=0 i+j=2 i+j=4 i+j=6 i+j=8	84	5943	5745		
	i+j=2	465	3072			
		-				

module	multiplicity	dimension
all		25797
$L\left(\alpha_1+2\alpha_2\right)$	32	10
$L\left(2\alpha_1+3\alpha_2\right)$	54	35
$L\left(3\alpha_1+4\alpha_2\right)$	62	81
$L\left(3\alpha_1+6\alpha_2\right)$	44	84
$L\left(3\alpha_1+3\alpha_2\right)$	19	30
$L\left(3\alpha_1+5\alpha_2\right)$	49	105
\mathbb{C}	3	1
$L\left(\alpha_1+\alpha_2\right)$	12	5
$L\left(2\alpha_1+2\alpha_2\right)$	15	14
$L\left(2\alpha_1+4\alpha_2\right)$	24	35
$L\left(4\alpha_1+5\alpha_2\right)$	19	154
$L\left(4\alpha_1+6\alpha_2\right)$	12	220
$L(4\alpha_1 + 7\alpha_2)$	10	231
$L\left(4\alpha_1+4\alpha_2\right)$	3	55
· · · · · · · · · · · · · · · · · · ·		

type B2, s=7, subset=[]

i+j=1	2255			
i+j=3	0	0		
i + j = 5	0	0	0	
i+j=1 $i+j=3$ $i+j=5$ $i+j=7$	0	0	0	2255
$h^{i,j}$	j-i=1	j-i=3	j-i=5	j-i=7

module	multiplicity	dimension
all		4510
$L\left(\alpha_1+\alpha_2\right)$	2	5
$L\left(\alpha_1+2\alpha_2\right)$	6	10
$L\left(2\alpha_1+2\alpha_2\right)$	4	14
$L\left(2\alpha_1+3\alpha_2\right)$	10	35
$L\left(3\alpha_1+3\alpha_2\right)$	4	30
$L\left(2\alpha_1+4\alpha_2\right)$	6	35
$L\left(3\alpha_1+4\alpha_2\right)$	8	81
$L\left(4\alpha_1+4\alpha_2\right)$	2	55
$L\left(3\alpha_1+5\alpha_2\right)$	6	105
$L\left(4\alpha_1+5\alpha_2\right)$	2	154
$L\left(3\alpha_1+6\alpha_2\right)$	4	84
$L(4\alpha_1+6\alpha_2)$	4	220
$L(4\alpha_1 + 7\alpha_2)$	2	231
$L(4\alpha_1 + 8\alpha_2)$	2	165
_/		

type B2, s=0, subset=[1]

```
i+j=0
         \mathbb{C}
i+j=2
i+j=4 \mid \mathbb{C}
i+j=6 \mid \mathbb{C}
h^{i,\overline{j}}
         j-i=0 j-i=2 j-i=4 j-i=6
i+j=0
i+j=2
i+j=4
i+j=6
  h^{i,j}
          j-i=0 j-i=2 j-i=4 j-i=6
```

module	multiplicity	dimension
all		10
\mathbb{C}	10	1

type B2, s=1, subset=[1]

i+j=1	$\mathbb{C}L_{1,1}L_{1,2}$		
i+j=3	$ \begin{vmatrix} \mathbb{C}L_{1,1}L_{1,2} \\ \mathbb{C} \\ \mathbb{C} \end{vmatrix} $	$\mathbb{C}^2 L_{1,1}^2 L_{1,2} L_{2,2}$	
i+j=5	\mathbb{C}	\mathbb{C}	$\mathbb{C}L_{1,1}L_{1,2}$
$h^{i,j}$	j-i=1	j-i=3	j - i = 5

module	multiplicity	dimension
all		71
\mathbb{C}	7	1
$L\left(\alpha_1+\alpha_2\right)$	4	5
$L\left(\alpha_1+2\alpha_2\right)$	3	10
$L\left(2\alpha_1+2\alpha_2\right)$	1	14

type B2, s=2, subset=[1]

module	muniphency	unnension
all		310
$L\left(\alpha_1+\alpha_2\right)$	8	5
$L\left(\alpha_1+2\alpha_2\right)$	10	10
\mathbb{C}	2	1
$L\left(2\alpha_1+2\alpha_2\right)$	2	14
$L\left(2\alpha_1+3\alpha_2\right)$	4	35

modulo multiplicity dimension

type B2, s=3, subset=[1]

module	multiplicity	dimension
all		922
$L\left(\alpha_1+\alpha_2\right)$	5	5
$L\left(\alpha_1+2\alpha_2\right)$	12	10
$L\left(2\alpha_1+2\alpha_2\right)$	5	14
$L\left(2\alpha_1+3\alpha_2\right)$	12	35
$L\left(2\alpha_1+4\alpha_2\right)$	5	35
\mathbb{C}	1	1
$L\left(3\alpha_1+3\alpha_2\right)$	1	30
$L(3\alpha_1+4\alpha_2)$	1	81

...

type B2, s=4, subset=[1]

i+j=4	0	0	1015	
i+j=2 $i+j=4$ $i+j=6$	0	0	0	84
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6

module	multiplicity	dimension
all		2198
$L\left(2\alpha_1+2\alpha_2\right)$	8	14
$L\left(2\alpha_1+3\alpha_2\right)$	16	35
$L\left(2\alpha_1+4\alpha_2\right)$	14	35
$L\left(\alpha_1+\alpha_2\right)$	2	5
$L\left(\alpha_1+2\alpha_2\right)$	6	10
$L\left(3\alpha_1+3\alpha_2\right)$	2	30
$L\left(3\alpha_1+4\alpha_2\right)$	6	81
$L\left(3\alpha_1+5\alpha_2\right)$	4	105

type B2, s=5, subset=[1]

	module	multiplicity	dimension
	all		4552
L	$(2\alpha_1 + 2\alpha_2)$	5	14
L	$(2\alpha_1 + 3\alpha_2)$	13	35
L	$(3\alpha_1 + 3\alpha_2)$	5	30
L	$(2\alpha_1 + 4\alpha_2)$	14	35
L	$(3\alpha_1 + 4\alpha_2)$	13	81
L	$(3\alpha_1 + 5\alpha_2)$	14	105
L	$(3\alpha_1 + 6\alpha_2)$	5	84
	$L\left(\alpha_1+\alpha_2\right)$	1	5
i	$L\left(\alpha_1+2\alpha_2\right)$	1	10
L	$(4\alpha_1 + 4\alpha_2)$	1	55
L	$(4\alpha_1 + 5\alpha_2)$	1	154
L	$(4\alpha_1 + 6\alpha_2)$	1	220

type B2, s=6, subset=[1]

i+j=2	0	3969		
i+j=2 $i+j=4$ $i+j=6$	0	0	3969	
		0	0	300
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6
	module	multi	plicity	dimensio
	all			8538

module	multiplicity	dimension
all		8538
$L\left(3\alpha_1+3\alpha_2\right)$	8	30
$L\left(3\alpha_1+4\alpha_2\right)$	16	81
$L\left(3\alpha_1+5\alpha_2\right)$	20	105
$L\left(3\alpha_1+6\alpha_2\right)$	14	84
$L\left(2\alpha_1+2\alpha_2\right)$	2	14
$L\left(2\alpha_1+3\alpha_2\right)$	6	35
$L\left(2\alpha_1+4\alpha_2\right)$	6	35
$L\left(4\alpha_1+4\alpha_2\right)$	2	55
$L\left(4\alpha_1+5\alpha_2\right)$	6	154
$L\left(4\alpha_1+6\alpha_2\right)$	6	220
$L(4\alpha_1 + 7\alpha_2)$	4	231
,		

type B2, s=7, subset=[1]

35

91

260

390

455

 $L\left(2\alpha_1+4\alpha_2\right)$ 1

 $L(5\alpha_1 + 5\alpha_2)$ 1

 $L(5\alpha_1+6\alpha_2)$ 1

 $L(5\alpha_1 + 7\alpha_2)$ 1

 $L(5\alpha_1 + 8\alpha_2)$ 1

```
i+j=1 | L_{3,3}L_{3,4}^3L_{4,4}L_{3,5}^3L_{4,5}^3L_{3,6}^3L_{4,6}^3L_{4,7}^3L_{4,8}
                                                                       L_{2,2}L_{2,3}L_{3,3}^3L_{2,4}L_{3,4}^7L_{4,4}^3L_{3,5}^9L_{4,5}^7L_{3,6}^8L_{5,5}L_{4,6}^9L_{5,6}L_{4,7}^8L_{5,7}L_{4,8}^3L_{5,8}
i+j=3
                                                                                                                                                                             L_{3,3}L_{3,4}^3L_{4,4}L_{3,5}^3L_{4,5}^3L_{3,6}^3L_{4,6}^3L_{4,7}^3L_{4,8}
i+j=5
  h^{i,j}
           j-i=1
                                                                      j-i=3
                                                                                                                                                                             j-i=5
           2875
i+j=1
                       9120
i+j=3
                                   2875
i+j=5
           i-i=1 i-i=3 i-i=5
          module
                       multiplicity
                                             dimension
                 all
                                              14870
L\left(3\alpha_1+3\alpha_2\right)
                                              30
L\left(3\alpha_1+4\alpha_2\right)
                                              81
L\left(4\alpha_1+4\alpha_2\right) 5
                                              55
L(3\alpha_1 + 5\alpha_2) 15
                                              105
L(4\alpha_1 + 5\alpha_2) 13
                                              154
L(3\alpha_1 + 6\alpha_2) 14
                                              84
L(4\alpha_1 + 6\alpha_2) 15
                                              220
L(4\alpha_1 + 7\alpha_2) 14
                                              231
L(4\alpha_1 + 8\alpha_2) 5
                                              165
L\left(2\alpha_1+2\alpha_2\right) 1
                                              14
L\left(2\alpha_1+3\alpha_2\right) 1
                                              35
```

type B2, s=8, subset=[1]

 $L\left(5\alpha_1+8\alpha_2\right)$ 6

 $L\left(5\alpha_1+9\alpha_2\right)$ 4

455

429

```
L_{4,4}L_{4,5}L_{4,6}L_{4,7}L_{4,8}
                                                L_{3.3}L_{3.4}^3L_{4.4}^3L_{3.5}^3L_{4.5}^7L_{3.6}^3L_{5.5}L_{4.6}^9L_{5.6}^3L_{4.7}^9L_{5.7}^3L_{4.8}^6L_{5.8}^3L_{5.9}^2
i+j=2
                                                                                                                                                L_{3,3}L_{3,4}^3L_{4,4}^3L_{3,5}^3L_{4,5}^7L_{3,6}^3L_{5,5}L_{4,6}^9L_{5,6}^3L_{4,7}^9L_{5,7}^3L_{4,8}^6L_{5,8}^3L_{5,9}^2
i+j=4
                                                                                                                                                                                                                                               L_{4\,4}L_{4\,5}L_{4\,6}L_{4\,7}L_{4\,8}
i+j=6
  h^{i,j}
           j-i=0
                                                i-i=2
                                                                                                                                                i-i=4
                                                                                                                                                                                                                                               i-i=6
           825
i+j=0
                        11396
i+j=2
            0
                                     11396
i+j=4
                        0
                                                  825
i+j=6
  h^{i,j}
            j-i=0 j-i=2
                                     j-i=4
                                                  j-i=6
                        multiplicity
          module
                                                dimension
                  all
                                                24442
L\left(4\alpha_1+4\alpha_2\right)
                                                55
L\left(4\alpha_1+5\alpha_2\right)
                                                154
L(4\alpha_1+6\alpha_2)
                                                220
L\left(4\alpha_1+7\alpha_2\right)
                                                231
L(4\alpha_1 + 8\alpha_2) 14
                                                165
L\left(3\alpha_1+3\alpha_2\right)
                                                30
L\left(3\alpha_1+4\alpha_2\right)
                                                81
L\left(3\alpha_1+5\alpha_2\right)
                                                105
L\left(3\alpha_1+6\alpha_2\right) 6
                                                84
L\left(5\alpha_1+5\alpha_2\right)
                                                91
L\left(5\alpha_1+6\alpha_2\right)
                                                260
L\left(5\alpha_1+7\alpha_2\right) 6
                                                390
```

type B2, s=0, subset=[2]

```
i+j=0
         \mathbb{C}
i+j=2
        \mathbb{C}
                   \mathbb{C}L_{1,1} \mathbb{C}
i+j=4
        \mathbb{C}
i+j=6
  h^{i,j}
         j-i=0 j-i=2 j-i=4 j-i=6
i + j = 0
i+j=2
                    6
i+j=4
i+j=6
  h^{i,j}
         j-i=0 j-i=2 j-i=4 j-i=6
```

module	$\operatorname{multiplicity}$	dimension
all		15
\mathbb{C}	10	1
$L\left(\alpha_1+\alpha_2\right)$	1	5

type B2, s=1, subset=[2]

module	multiplicity	dimension
all		71
\mathbb{C}	6	1
$L\left(\alpha_1+2\alpha_2\right)$	5	10
$L\left(\alpha_1+\alpha_2\right)$	3	5

type B2, s=2, subset=[2]

$n^{\circ,j}$	j-i=0	j-i=2	j-i=4	j-i=6
1 1 1				
i+j=2 $i+j=4$ $i+j=6$	0	0	-0	10
i+j=4	0	60	70	
v 1 J — =	+0	• 0		

70

module	multiplicity	dimension
all		240
$L\left(\alpha_1+2\alpha_2\right)$	12	10
$L\left(\alpha_1+\alpha_2\right)$	3	5
$L\left(2\alpha_1+3\alpha_2\right)$	3	35

type B2, s=3, subset=[2]

multiplicity	dimension
	627
9	10
3	14
7	35
6	35
2	5
1	30
	3 7 6

type B2, s=4, subset=[2]

```
i+j=0 \mid L_{2,2}L_{2,4}
_{i+j=2} \mid L_{2,4} \qquad L_{1,1}L_{1,2}L_{2,2}^2L_{2,3}^3L_{3,3}L_{2,4}^4L_{3,4}L_{3,5}
i+j=4 | 0
                  L_{2,3}L_{2,4}^2L_{3,5}
                                                                              L_{1,1}L_{1,2}L_{2,2}^2L_{2,3}^3L_{3,3}L_{2,4}^4L_{3,4}L_{3,5}
i+j=6 | 0
                          0
                                                                                                                                 L_{2,2}L_{2,4}
                                                                             L_{2.4}
         j-i=0
                          i-i=2
                                                                              i-i=4
                                                                                                                                 i-i=6
i+j=0 | 49
i + i = 2 \mid 35
                      504
```

$h^{i,j}$	j-i=0	j-i=2	j-i=4	j - i = 6
i+j=2 $i+j=4$ $i+j=6$	0	0		49
i+j=4	0	210	504	
$\iota + j - 2$	00	001		

module	multiplicity	dimension
all		1386
$L\left(2\alpha_1+2\alpha_2\right)$	6	14
$L\left(2\alpha_1+4\alpha_2\right)$	14	35
$L\left(\alpha_1+\alpha_2\right)$	2	5
$L\left(\alpha_1+2\alpha_2\right)$	2	10
$L\left(2\alpha_1+3\alpha_2\right)$	7	35
$L\left(3\alpha_1+3\alpha_2\right)$	2	30
$L\left(3\alpha_1+4\alpha_2\right)$	2	81
$L\left(3\alpha_1+5\alpha_2\right)$	3	105

type B2, s=5, subset=[2]

module	multiplicity	dimension
all		2732
$L\left(2\alpha_1+2\alpha_2\right)$	3	14
$L\left(2\alpha_1+3\alpha_2\right)$	6	35
$L\left(2\alpha_1+4\alpha_2\right)$	10	35
$L\left(3\alpha_1+4\alpha_2\right)$	7	81
$L\left(3\alpha_1+5\alpha_2\right)$	8	105
$L\left(3\alpha_1+6\alpha_2\right)$	6	84
$L\left(\alpha_1+\alpha_2\right)$	1	5
$L\left(3\alpha_1+3\alpha_2\right)$	2	30
$L\left(4\alpha_1+5\alpha_2\right)$	1	154

type B2, s=6, subset=[2]

	i + j = 0	165			
	i+j=2	84	1974		
	i+j=4	0	504	1974	
	i+j=0 i+j=2 i+j=4 i+j=6	0	0	84	165
•	$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6

module	multiplicity	dimension
all		4950
$L\left(3\alpha_1+4\alpha_2\right)$	10	81
$L\left(3\alpha_1+6\alpha_2\right)$	14	84
$L\left(2\alpha_1+3\alpha_2\right)$	4	35
$L\left(3\alpha_1+3\alpha_2\right)$	2	30
$L(2\alpha_1+4\alpha_2)$	2	35
$L\left(3\alpha_1+5\alpha_2\right)$	9	105
$L\left(4\alpha_1+5\alpha_2\right)$	4	154
$L(4\alpha_1 + 6\alpha_2)$	2	220
$L(4\alpha_1 + 7\alpha_2)$	3	231

type B2, s=7, subset=[2]

i+j=1	1480		
i+j=3	480	4485	
i+j=1 $i+j=3$ $i+j=5$	0	480	1480
		j-i=3	j-i=5

module	multiplicity	dimension
all		8405
$L\left(3\alpha_1+4\alpha_2\right)$	7	81
$L\left(4\alpha_1+4\alpha_2\right)$	3	55
$L\left(3\alpha_1+5\alpha_2\right)$	7	105
$L\left(4\alpha_1+5\alpha_2\right)$	6	154
$L\left(3\alpha_1+6\alpha_2\right)$	10	84
$L(4\alpha_1+6\alpha_2)$	8	220
$L(4\alpha_1 + 7\alpha_2)$	8	231
$L(4\alpha_1 + 8\alpha_2)$	6	165
$L(2\alpha_1 + 3\alpha_2)$	1	35
$L(3\alpha_1 + 3\alpha_2)$	2	30
$L(5\alpha_1 + 5\alpha_2)$	1	91
$L(5\alpha_1 + 7\alpha_2)$	1	390

type B3, s=0, subset=[]

```
i+j=0
                 \mathbb{C}^3
 i+j=2
                 \mathbb{C}^5
                                \mathbb{C}^3
 i+j=4
                                \mathbb{C}^6
 i+j=6
                 \mathbb{C}^8
                                \mathbb{C}^{10}
 i+j=8
                                \mathbb{C}^{14}L_{1,1,1}
                                                        \mathbb{C}^{10}
                                                                                           \mathbb{C}^6
i + j = 10
                                                        \mathbb{C}^{14}L^4_{1,1,1}
                                \mathbb{C}^{15}L_{1,1,1}^3
                                                                                                                   \mathbb{C}^6
                 \mathbb{C}^7
                                                                                                                                          \mathbb{C}^3
i + j = 12
                                                                                           \mathbb{C}^{14}L_{1,1,1}^{\bar{4}}
                                \mathbb{C}^{12}L_{1,1,1}^{2}
                                                        \mathbb{C}^{15}L_{1,1,1}^{6}L_{1,2,2}
                                                                                                                   \mathbb{C}^{10}
                                                                                                                                          \mathbb{C}^6
                                                                                                                                                           \mathbb{C}^3
i + j = 14
                                                                                                                  \mathbb{C}^{14}L_{1,1,1}
                                \mathbb{C}^8
                                                        \mathbb{C}^{12}L_{1,1,1}^{2}
                                                                                           \mathbb{C}^{15}L_{1,1,1}^{3}
                                                                                                                                          \mathbb{C}^{10}
                                                                                                                                                                             \mathbb{C}^3
i + j = 16
                                 \mathbb{C}^3
                                                        \mathbb{C}^5
                                                                                           \mathbb{C}^7
                                                                                                                                          \mathbb{C}^8
                                                                                                                                                                             \mathbb{C}^5
                                                                                                                   \mathbb{C}^8
                                                                                                                                                                                              \mathbb{C}^3
i + j = 18
     h^{i,j}
                j-i=0
                                j-i=2
                                                        j-i=4
                                                                                                                  j-i=8
                                                                                                                                                           j - i = 12
                                                                                                                                                                             j - i = 14
                                                                                                                                                                                              j - i = 16
                                                                                           j-i=6
                                                                                                                                          i - i = 10
 i+j=0
 i+j=2
 i+j=4
                                                3
 i+j=6
                                                               3
 i+j=8
                                 10
                                                6
                                                                               3
                                 21
                                                10
                                                                                               1
i + j = 10
                                                                               6
                                 36
                                                42
                                                                17
i + j = 12
                                                                                                                1
                                                                42
                                 26
                                                78
                                                                               10
i + j = 14
                                                                                                                                  1
i + j = 16
                                                26
                                                                36
                                                                               21
                                                                                               10
                                 3
                                                5
                                                                               8
                                                                                               8
                                                                                                                                                    3
                                                                                                                                                                     1
i + j = 18
     h^{i,j}
                j-i=0
                                j-i=2
                                                j-i=4
                                                               j-i=6
                                                                               j-i=8
                                                                                              j - i = 10
                                                                                                                                 j - i = 14
                                                                                                                                                                    j-i=18
                                                                                                                j-i=12
                                                                                                                                                   j-i=16
```

j - i = 18

module	multiplicity	dimension
all		553
\mathbb{C}	343	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	27	7
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	1	21

type B3, s=0, subset=[1]

```
i+j=0
                  \mathbb{C}^2
  i + j = 2
                  \mathbb{C}^3
                                  \mathbb{C}^2
  i + j = 4
                                 \mathbb{C}^4
                  \mathbb{C}^4
  i+j=6
                                  \mathbb{C}^6
                  \mathbb{C}^4
                                                                                            \mathbb{C}^2
  i + j = 8
                                 \mathbb{C}^7L_{1,1,1}
                                                         \mathbb{C}^6L_{1,1,1}
                  \mathbb{C}^4
                                                                                                                  \mathbb{C}^2
i + j = 10
                                                                                           \mathbb{C}^6L_{1,1,1}
                                  \mathbb{C}^6 L_{1,1,1}
                                                         \mathbb{C}^7 L_{1,1,1}^3 L_{1,2,2}
                                                                                                                                  \mathbb{C}^2
                  \mathbb{C}^3
i + j = 12
                                                         \mathbb{C}^6L_{1,1,1}
                                                                                           \mathbb{C}^7 L_{1,1,1}
                                                                                                                  \mathbb{C}^6
                                                                                                                                   \mathbb{C}^4
                                                                                                                                                     \mathbb{C}^2
                  \mathbb{C}^2
                                   \mathbb{C}^4
                                                                                                                                                                       \mathbb{C}
i + j = 14
                                   \mathbb{C}^2
                                                         \mathbb{C}^3
                                                                                            \mathbb{C}^4
                                                                                                                  \mathbb{C}^4
                                                                                                                                   \mathbb{C}^4
                                                                                                                                                     \mathbb{C}^3
                                                                                                                                                                       \mathbb{C}^2
                                                                                                                                                                                          \mathbb{C}
i + j = 16
     h^{i,j}
                  j - i = 0
                                  j-i=2
                                                         j-i=4
                                                                                            j-i=6
                                                                                                                  j-i=8
                                                                                                                                  j - i = 10
                                                                                                                                                     j - i = 12
                                                                                                                                                                       j - i = 14
                                                                                                                                                                                          j - i = 16
                 1
  i+j=0
  i+j=2
                                                   1
  i+j=4
                                                   2
                                  4
                                                                   1
 i+j=6
                                                   4
                                  6
                                                                                   1
  i + j = 8
                                   14
                                                   13
                                                                                                    1
i + j = 10
                                  13
                                                   49
                                                                   13
                                                                                   4
                                                                                                                      1
i+j=12
                                                                                                    4
                                  4
                                                   13
                                                                   14
                                                                                   6
                                                                                                                      2
                                                                                                                                        1
i+j=14
                                                                                                                      3
                                                   3
                                                                   4
                                                                                   4
                                                                                                                                        2
i + j = 16
                                                                                                    4
                                                                                                                                                           1
     h^{i,j}
```

module	multiplicity	dimension
all		231
\mathbb{C}	147	1
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	9	7
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	1	21

j-i=4

j-i=6

j-i=8

j - i = 10

j - i = 12

j - i = 14

j-i=2

type B3, s=0, subset=[1, 2]

```
i + j = 0
 i+j=2
                           \mathbb{C}^2
\mathbb{C}^2
\mathbb{C}
\mathbb{C}
 i+j=4
             \mathbb{C}^2
 i+j=6
                                        \mathbb{C}^2 L_{1,1,1} L_{1,2,2}
\mathbb{C}^2
                                                                    \mathbb{C}^2
 i+j=8
i + j = 10
i + j = 12
    h^{i,j}
              j-i=0
                           j-i=2
                                        j-i=4
                                                                                 j - i = 8
                                                                                              j - i = 10
                                                                                                            j - i = 12
                                                                    j-i=6
 i+j=0
 i+j=2
 i+j=4
                                        1
                                                     1
 i+j=6
              1
                                        30
 i + j = 8
              1
                                                                               1
i + j = 10
                                                     2
              1
                                                                                              1
i + j = 12
    h^{i,j}
              j-i=0
                           j-i=2
                                        j-i=4
                                                     j-i=6
                                                                                              j - i = 12
                                                                  j-i=8
                                                                               j - i = 10
```

module	multiplicity	dimension
all		63
\mathbb{C}	35	1
$L\left(\alpha_1+\alpha_2+\alpha_3\right)$	1	7
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	1	21

$h^{i,j} \mid j$	-i = 0	
module	multiplicity	dimension
all		1

 $i+j=0 \mid 1$

type B3, s=0, subset=[1, 3]

```
i + j = 0
               \mathbb{C}
 i+j=2
               \mathbb{C}^2
 i + j = 4
               \mathbb{C}^2
 i + j = 6
                            \mathbb{C}^3L_{1,1,1}
               \mathbb{C}^2
                                                \mathbb{C}^3
 i + j = 8
               \mathbb{C}^2
                             \mathbb{C}^3 L_{1,1,1}
i + j = 10
                                                \mathbb{C}^3L_{1,1,1}
                                                                   \mathbb{C}^3L_{1,1,1}
                                                                                      \mathbb{C}^3
                             \mathbb{C}^2
i + j = 12
                                                \mathbb{C}^2
                                                                   \mathbb{C}^2
                                                                                      \mathbb{C}^2
i + j = 14
    h^{i,j}
               j-i=0
                             j-i=2
                                                j-i=4
                                                                   j - i = 6
                                                                                      j-i=8
                                                                                                    j - i = 10
                                                                                                                   j - i = 12
                                                                                                                                   j - i = 14
 i+j=0
 i+j=2
                             1
 i+j=4
                             3
 i+j=6
                             10
 i + j = 8
                             10
                                           17
                                                        3
                                                                                    1
i + j = 10
                                           10
                                                         10
                                                                      3
i + j = 12
                                          2
                                                        2
                                                                      2
                                                                                    2
                                                                                                                   1
i + j = 14
    h^{i,j}
               j-i=0
                             j-i=2
                                          j-i=4
                                                        j-i=6
                                                                      j-i=8
                                                                                                                   i - i = 14
                                                                                    j - i = 10
                                                                                                   j-i=12
```

module	multiplicity	dimension
all		105
\mathbb{C}	63	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	6	7

type B3, s=0, subset=[2]

```
i+j=0
                  \mathbb{C}^2
  i + j = 2
                  \mathbb{C}^3
                                  \mathbb{C}^2
  i + j = 4
                                 \mathbb{C}^4
                  \mathbb{C}^4
  i+j=6
                                  \mathbb{C}^6
                  \mathbb{C}^4
                                                                                            \mathbb{C}^2
  i + j = 8
                                 \mathbb{C}^7L_{1,1,1}
                                                         \mathbb{C}^6L_{1,1,1}
                  \mathbb{C}^4
                                                                                                                  \mathbb{C}^2
i + j = 10
                                                                                           \mathbb{C}^6L_{1,1,1}
                                  \mathbb{C}^6 L_{1,1,1}
                                                         \mathbb{C}^7 L_{1,1,1}^3 L_{1,2,2}
                                                                                                                                  \mathbb{C}^2
                  \mathbb{C}^3
i + j = 12
                                                         \mathbb{C}^6L_{1,1,1}
                                                                                           \mathbb{C}^7 L_{1,1,1}
                                                                                                                  \mathbb{C}^6
                                                                                                                                   \mathbb{C}^4
                                                                                                                                                     \mathbb{C}^2
                  \mathbb{C}^2
                                   \mathbb{C}^4
                                                                                                                                                                       \mathbb{C}
i + j = 14
                                   \mathbb{C}^2
                                                         \mathbb{C}^3
                                                                                            \mathbb{C}^4
                                                                                                                  \mathbb{C}^4
                                                                                                                                   \mathbb{C}^4
                                                                                                                                                     \mathbb{C}^3
                                                                                                                                                                       \mathbb{C}^2
                                                                                                                                                                                          \mathbb{C}
i + j = 16
     h^{i,j}
                  j - i = 0
                                  j-i=2
                                                         j-i=4
                                                                                            j-i=6
                                                                                                                  j-i=8
                                                                                                                                  j - i = 10
                                                                                                                                                     j - i = 12
                                                                                                                                                                       j - i = 14
                                                                                                                                                                                          j - i = 16
                 1
  i+j=0
  i+j=2
                                                   1
  i+j=4
                                                   2
                                  4
                                                                   1
 i+j=6
                                                   4
                                  6
                                                                                   1
  i + j = 8
                                   14
                                                   13
                                                                                                    1
i + j = 10
                                  13
                                                   49
                                                                   13
                                                                                   4
                                                                                                                      1
i+j=12
                                                                                                    4
                                  4
                                                   13
                                                                   14
                                                                                   6
                                                                                                                      2
                                                                                                                                        1
i+j=14
                                                                                                                      3
                                                   3
                                                                   4
                                                                                   4
                                                                                                                                        2
i + j = 16
                                                                                                    4
                                                                                                                                                           1
     h^{i,j}
```

module	multiplicity	dimension
all		231
\mathbb{C}	147	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	9	7
$L\left(\alpha_1+2\alpha_2+2\alpha_3\right)$	1	21

j-i=4

j-i=6

j-i=8

j - i = 10

j - i = 12

j - i = 14

j - i = 16

j-i=2

type B3, s=0, subset=[2, 3]

i + j = 0	\mathbb{C}						
i+j=2	\mathbb{C}	\mathbb{C}					
i+j=4	\mathbb{C}	\mathbb{C}	\mathbb{C}				
i + j = 6	\mathbb{C}	$\mathbb{C}L_{1,1,1}$	$_{1}$ $\mathbb{C}L_{1}$,1,1	\mathbb{C}		
i+j=8	\mathbb{C}	\mathbb{C}	$\mathbb{C}L_1$,1,1	\mathbb{C}	\mathbb{C}	
i+j=10	\mathbb{C}	\mathbb{C}	\mathbb{C}		\mathbb{C}	\mathbb{C}	\mathbb{C}
$h^{i,j}$	j-i=0	j - i = 2	j-i=	:4 j	j - i = 6	j-i=8	j - i = 10
i+j=0	1						
i+j=2	1	1					
i+j=4	1	1	1				
i + j = 6	1	8	8	1			
i + j = 8	1	1	8	1	1		
i + j = 10	1	1	1	1	1	1	
$h^{i,j}$	_						

module	multiplicity	dimension
all		42
\mathbb{C}	21	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	3	7

type B3, s=0, subset=[3]

```
\mathbb{C}
 i + j = 0
                  \mathbb{C}^2
 i + j = 2
                  \mathbb{C}^3
                                 \mathbb{C}^2
 i + j = 4
                  \mathbb{C}^4
                                  \mathbb{C}^4
 i + j = 6
                  \mathbb{C}^4
                                 \mathbb{C}^6L_{1,1,1}
 i + j = 8
                                                        \mathbb{C}^6L^2_{1,1,1}
                 \mathbb{C}^4
i + j = 10
                  \mathbb{C}^3
                                  \mathbb{C}^6 L^{2^{''}}_{1,1,1}
                                                        \mathbb{C}^7 L_{1,1,1}^4
i + j = 12
                                                        \mathbb{C}^6 L_{1,1,1}^{\overset{1}{2},1,1}
                                                                                                                                              \mathbb{C}^2
                  \mathbb{C}^2
                                                                               \mathbb{C}^7 L_{1,1,1}^{2^{7,1,1}}
                                                                                                     \mathbb{C}^6L_{1,1,1}
                                  \mathbb{C}^4
i + j = 14
                                  \mathbb{C}^2
                                                                                                     \mathbb{C}^4
                                                                                                                            \mathbb{C}^4
                                                                                                                                              \mathbb{C}^3
                                                                                                                                                                \mathbb{C}^2
                                                                                                                                                                                  \mathbb{C}
i + j = 16
     h^{i,j}
                 j-i=0
                                 j-i=2
                                                                               j-i=6
                                                                                                                                             j - i = 12
                                                                                                                                                                j - i = 14
                                                                                                                                                                                  j - i = 16
                                                        j-i=4
                                                                                                     j-i=8
                                                                                                                           j - i = 10
 i + j = 0
                 2
 i + j = 2
                  3
 i+j=4
                                 4
 i+j=6
                  4
                                  13
 i + j = 8
                                  21
                                                  20
                                                                                                  1
                  4
i + j = 10
                  3
                                  20
                                                  35
                                                                  20
i + j = 12
                                 4
                                                  20
                                                                  21
                                                                                  13
                                                                                                  4
                                                                                                                     2
                                                                                                                                       1
i + j = 14
                                                  3
                                                                                                                                       2
                                                                                                                                                         1
i + j = 16
     h^{i,j}
                                 i-i=2
                                                  j-i=4
                                                                  j-i=6
                                                                                  j-i=8
                                                                                                  j - i = 10
                                                                                                                                                        j - i = 16
                                                                                                                    j-i=12
                                                                                                                                      j - i = 14
```

module	multiplicity	dimension
all		273
\mathbb{C}	147	1
$L\left(\alpha_1 + \alpha_2 + \alpha_3\right)$	18	7

type C3, s=0, subset=[]

```
i + j = 0
                  \mathbb{C}^3
  i + j = 2
                  \mathbb{C}^5
                                   \mathbb{C}^3
  i+j=4
                                   \mathbb{C}^6
                  \mathbb{C}^7
  i + j = 6
                  \mathbb{C}^8
                                   \mathbb{C}^{10}
                                                                                    \mathbb{C}^3
  i + j = 8
                                                            \mathbb{C}^{10}
                                                                                                             \mathbb{C}^3
i + j = 10
                                                           \mathbb{C}^{14}L^2_{1,2,1} \\ \mathbb{C}^{15}L^3_{1,2,1}
                                                                                     \mathbb{C}^{10}
                                                                                                              \mathbb{C}^6
                                                                                                                                       \mathbb{C}^3
                  \mathbb{C}^7
i + j = 12
                                   \mathbb{C}^{12}L_{1,2,1}
                                                                                    \mathbb{C}^{14}L_{1,2,1}^2
                  \mathbb{C}^5
                                                                                                              \mathbb{C}^{10}
                                                                                                                                       \mathbb{C}^6
                                                                                                                                                         \mathbb{C}^3
i + j = 14
                                                                                                             \mathbb{C}^{14}L^2_{1,2,1}
                                                           \mathbb{C}^{12}L_{1,2,1}
                                                                                    \mathbb{C}^{15}L_{1,2,1}^{3}
                                                                                                                                       \mathbb{C}^{10}
                                                                                                                                                         \mathbb{C}^6
                                                                                                                                                                            \mathbb{C}^3
i + j = 16
                                                                                                              \mathbb{C}^8
                                                                                                                                                                            \mathbb{C}^5
                                   \mathbb{C}^3
                                                           \mathbb{C}^5
                                                                                     \mathbb{C}^7
                                                                                                                                       \mathbb{C}^8
                                                                                                                                                         \mathbb{C}^7
                                                                                                                                                                                                                \mathbb{C}
i + j = 18
      h^{i,j}
                  j - i = 0
                                   j-i=2
                                                           j-i=4
                                                                                                                                      j - i = 10
                                                                                                                                                         j - i = 12
                                                                                                                                                                           j - i = 14
                                                                                                                                                                                                                j - i = 18
                                                                                    j-i=6
                                                                                                             j-i=8
                                                                                                                                                                                             j - i = 16
 i+j=0 \mid 1
  i + j = 2
  i+j=4
                                                    1
                                   6
                                                   3
                                                                    1
 i+j=6
                                                                    3
                                   10
  i + j = 8
                                   42
                                                    10
i + j = 10
                                   57
                                                   42
                                                                    10
                                                                                                     3
i + j = 12
                                   26
                                                   57
                                                                    42
                                                                                    10
                                                                                                                       3
i + j = 14
                                                                                                                                          1
i + j = 16
                                   8
                                                   26
                                                                    57
                                                                                    42
                                                                                                     10
                                                                                                                                          3
                                                                                                                                                             1
                                   3
                                                    5
                                                                                    8
                                                                                                     8
                                                                                                                                          5
                                                                                                                                                             3
                                                                                                                                                                               1
i + j = 18
      h^{i,j}
                                   j-i=2
                                                   j-i=4
                                                                   j-i=6
                                                                                    j-i=8
                                                                                                    j - i = 10
                                                                                                                       j - i = 12
                                                                                                                                         j - i = 14
                                                                                                                                                            j - i = 16
                                                                                                                                                                               j - i = 18
```

module	multiplicity	dimension
all		609
\mathbb{C}	343	1
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	19	14

type C3, s=0, subset=[1, 2]

```
i+j=0
 i+j=2
 i+j=4
                         \mathbb{C}^2 L_{1,2,1}
             \mathbb{C}^2
 i+j=6

\mathbb{C} \\
\mathbb{C}^2 L_{1,2,1}

                         \mathbb{C}^2 L_{1,2,1}
                                           \mathbb{C}^2L_{1,2,1}
 i+j=8
                                           \mathbb{C}^2L_{1,2,1}
i+j=10
                                                            \mathbb{C}^2
                                                                                                        \mathbb{C}
i + j = 12
    h^{i,j}
             j-i=0
                         j-i=2
                                           j-i=4
                                                            j-i=6
                                                                             j-i=8
                                                                                                        j - i = 12
                                                                                         j - i = 10
 i+j=0
 i+j=2
             1
 i+j=4
                          16
 i+j=6
             1
                          16
                                      16
 i+j=8
                                      16
                                                   16
i + j = 10
                                                   2
                                                                                         1
i + j = 12
             1
                                      1
    \overline{h^{i,j}}
             j - i = 0
                         j-i=2
                                      j-i=4
                                                  j-i=6
                                                                                         j - i = 12
                                                               j-i=8
                                                                           j - i = 10
```

module	multiplicity	dimension
all		105
\mathbb{C}	35	1
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	5	14

type C3, s=1, subset=[1, 2]

```
i+j=1 \mid \mathbb{C}L_{2,2,1}
                   \mathbb{C}L_{2,2,1}
 i+j=3 \mid \mathbb{C}L_{2,2,1}
 _{i+j=5} \mid \mathbb{C}^2 L_{1,2,1} L_{2,2,1} \quad \mathbb{C} L_{1,2,1} L_{2,2,1} L_{2,3,2} \quad \mathbb{C} L_{2,2,1}
 _{i+j=7} \mid \mathbb{C}^2L_{1,2,1}L_{2,2,1} \quad \mathbb{C}^3L_{1,2,1}^3L_{2,2,1}^2L_{2,3,2} \quad \mathbb{C}L_{1,2,1}L_{2,2,1}^2L_{2,3,2}L_{2,4,3} \quad \mathbb{C}L_{2,2,1}
i-i=3
                                                     i-i=5
                                                                                   i-i=7
                                                                                                           i - i = 9
 i+j=1 | 22
 i+j=3 \mid 22 22
 i+j=5 | 37  106
                           22
 i+j=7 | 37 	 157
                           211
                                  22
 i+j=9 \mid 1 121
                           157
                                   106
i+j=11 | 1  1
                           37 37
```

i - i = 11

module	multiplicity	dimension
all		1185
\mathbb{C}	30	1
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	21	21
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	15	14
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	6	70
$L\left(2\alpha_1 + 4\alpha_2 + 3\alpha_3\right)$	1	84

 $h^{i,j}$ j-i=1 j-i=3 j-i=5 j-i=7 j-i=9

```
type C3, s=2, subset=[1, 2]
```

```
i+j=0 \mid L_{2,2,1}
       i+j=2 \mid L_{2,2,1} \quad L_{2,2,1}^3 L_{3,4,2}
       i+j=4 L_{2,2,1} L_{1,2,1}L_{2,2,1}^3L_{2,3,2}^2L_{3,4,2} L_{2,2,1}^3L_{2,3,2}L_{3,4,2}
                                              \begin{bmatrix} L_{2,2,1} & \mathbb{C}L_{1,2,1}^{2}L_{2,2,1}^{4}L_{2,2,1}L_{2,3,2}L_{3,4,2} & \mathbb{C}L_{1,2,1}^{3}L_{2,2,1}^{5}L_{2,3,2}L_{3,4,2} \\ 0 & L_{1,2,1}L_{2,2,1}^{2}L_{2,3,2}L_{3,4,2} & \mathbb{C}L_{1,2,1}^{3}L_{2,2,1}^{5}L_{2,3,2}L_{3,4,2} \\ \end{bmatrix} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}^{2}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}^{2}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}^{3}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,1}L_{2,3,2}L_{3,4,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,1}L_{2,2,2}L_{2,3,2}L_{3,4,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \mathbb{C}L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2} \\ \end{array}} \underbrace{ \begin{array}{c} L_{1,2,1}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2}L_{2,2,2}L_{2,2,2}L_{2,2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L_{2,2}L
       i+j=6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \mathbb{C}L_{1,2,1}^{3^3}L_{2,2,1}^5L_{2,3,2}^4L_{2,3,2}L_{2,4,2}L_{3,4,2}^2L_{2,4,3}L_{3,5,3} L_{2,2,1}^3L_{2,3,2}L_{3,4,2}
      i+j=8 \mid 0
                                                                                                                                                                                                                                                    L_{1,2,1}L_{2,2,1}^2L_{2,3,2}L_{3,4,2}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               \mathbb{C}L_{1,2,1}^{2,2,1}L_{2,2,1}^{4,2,2,1}L_{2,3,2}L_{3,4,2}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        L_{1,2,1}L_{2,2,1}^3L_{2,3,2}^2L_{3,4,2} L_{2,2,1}^3L_{3,4,2}
 i+j=10 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        L_{2,2,1}
i+j=12 | 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                L_{2,2,1}
              h^{i,j}
                                                                                         i-i=2
                                                                                                                                                                                                                                                       i-i=4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                i-i=6
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        i-i=8
       i+j=0 | 21
       i+j=2 \mid 21
                                                                                              252
      i+j=4 | 21
                                                                                             406
                                                                                                                                           322
      i+j=6 | 21
                                                                                              372
                                                                                                                                          1492
                                                                                                                                                                                     322
                                                                                                                                           736
                                                                                                                                                                                     1492
                                                                                                                                                                                                                                  322
       i+j=8
  i+j=10 | 0
                                                                                                                                           315
                                                                                                                                                                                     372
                                                                                                                                                                                                                                  406
                                                                                                                                                                                                                                                                               252
```

 $L_{2,2,1}$

i - i = 10

 $L_{2,2,1}$

i - i = 12

module	multiplicity	dimension
all		7544
$L\left(2\alpha_1+2\alpha_2+\alpha_3\right)$	56	21
$L\left(3\alpha_1+4\alpha_2+2\alpha_3\right)$	17	189
$L\left(\alpha_1+2\alpha_2+\alpha_3\right)$	17	14
$L\left(2\alpha_1+3\alpha_2+2\alpha_3\right)$	22	70
\mathbb{C}	5	1
$L\left(2\alpha_1+4\alpha_2+2\alpha_3\right)$	2	90
$L\left(2\alpha_1+4\alpha_2+3\alpha_3\right)$	2	84
$L\left(3\alpha_1+5\alpha_2+3\alpha_3\right)$	2	512

j-i=0 j-i=2 j-i=4 j-i=6

21

21

j-i=8 j-i=10 j-i=12

 $i+j=12 \ \ 0$

type D4, s=0, subset=[]

```
\mathbb{C}
 i+j=0
              \mathbb{C}^4
                          \mathbb{C}
 i+j=2
              \mathbb{C}^9
                          \mathbb{C}^4
                                      \mathbb{C}
 i+j=4
              \mathbb{C}^{16}
                          \mathbb{C}^{11}
                                      \mathbb{C}^4
                                                   \mathbb{C}
 i + j = 6
              \mathbb{C}^{23}
                          \mathbb{C}^{24}
                                      \mathbb{C}^{11}
                                                   ???
                                                               ???
 i + j = 8
              \mathbb{C}^{28}
                          \mathbb{C}^{42}
                                       \mathbb{C}^{24}
                                                   ???
                                                               ???
                                                                            ???
i + j = 10
                          \mathbb{C}^{60}
              \mathbb{C}^{30}
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                          ???
i + j = 12
              \mathbb{C}^{28}
                          ???
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                          ???
                                                                                                       ???
i + j = 14
              ???
                          ???
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                          ???
                                                                                                       ???
                                                                                                                      ???
i + j = 16
                          ???
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                                       ???
                                                                                                                      ???
              ???
                                                                                          ???
                                                                                                                                   \mathbb{C}
i + j = 18
              ???
                          ???
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                          ???
                                                                                                       \mathbb{C}^{24}
                                                                                                                      \mathbb{C}^{11}
                                                                                                                                   \mathbb{C}^4
                                                                                                                                                 \mathbb{C}
i + j = 20
              ???
                          ???
                                      ???
                                                   ???
                                                               ???
                                                                            ???
                                                                                          \mathbb{C}^{60}
                                                                                                        \mathbb{C}^{42}
                                                                                                                      \mathbb{C}^{24}
                                                                                                                                   \mathbb{C}^{11}
                                                                                                                                                 \mathbb{C}^4
                                                                                                                                                               \mathbb{C}
i + j = 22
              ???
                          ???
                                       ???
                                                   ???
                                                               ???
                                                                            \mathbb{C}^{28}
                                                                                          \mathbb{C}^{30}
                                                                                                        \mathbb{C}^{28}
                                                                                                                      \mathbb{C}^{23}
                                                                                                                                   \mathbb{C}^{16}
                                                                                                                                                 \mathbb{C}^9
                                                                                                                                                               \mathbb{C}^4
                                                                                                                                                                             \mathbb{C}
i + j = 24
    h^{i,j}
             j-i=0
                          i-i=2
                                                                           i - i = 10
                                                                                                       i - i = 14
                                                                                                                     j - i = 16
                                                   i-i=6
                                                               i-i=8
                                                                                         i - i = 12
                                                                                                                                                 i - i = 20
                                                                                                                                                               i - i = 22
                                                                                                                                                                             i - i = 24
                                      i-i=4
                                                                                                                                   i - i = 18
 i+j=0
             1
              4
                          1
 i+j=2
                          4
             9
                                      1
 i+j=4
                          11
                                       4
                                                   1
             16
 i+j=6
              23
                          24
                                      11
                                                   -1
                                                               -1
 i+j=8
                          42
                                       24
                                                                            -1
              28
                                                   -1
                                                               -1
i + j = 10
                          60
                                                                            -1
i + j = 12
              30
                                       -1
                                                   -1
                                                               -1
                                                                                          -1
              28
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            -1
                                                                                          -1
                                                                                                        -1
i + j = 14
              -1
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            -1
                                                                                          -1
                                                                                                        -1
                                                                                                                      -1
i + j = 16
                                                                                                                      -1
              -1
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            -1
                                                                                          -1
                                                                                                        -1
                                                                                                                                   1
i+j=18
                                                                                                        24
                                                                                                                      11
              -1
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            -1
                                                                                          -1
                                                                                                                                   4
                                                                                                                                                 1
i + j = 20
                                                                                                        42
                                                                                                                      24
i + j = 22
              -1
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            -1
                                                                                          60
                                                                                                                                   11
                                                                                                                                                 4
                                                                                                                                                               1
              -1
                          -1
                                       -1
                                                   -1
                                                               -1
                                                                            28
                                                                                          30
                                                                                                        28
                                                                                                                      23
                                                                                                                                   16
                                                                                                                                                 9
                                                                                                                                                               4
                                                                                                                                                                             1
i + j = 24
    h^{i,j}
                          j-i=2
                                                   j-i=6
                                                               j-i=8
                                                                           j - i = 10
                                                                                         j - i = 12
                                                                                                       j - i = 14
                                                                                                                                   j-i=18
                                                                                                                                                 j - i = 20
                                                                                                                                                               j - i = 22
                                                                                                                                                                             j - i = 24
             j-i=0
                                       j-i=4
                                                                                                                     j - i = 16
```

module	multiplicity	dimension
all		644
\mathbb{C}	644	1

type G2, s=0, subset=[]

```
i+j=0
                 \mathbb{C}^2
  i+j=2
                                 \mathbb{C}^2
\mathbb{C}^2
                 \mathbb{C}^2
  i+j=4
                 \mathbb{C}^2
                                                                       \begin{array}{l} \mathbb{C} \\ \mathbb{C}^2 \\ \mathbb{C}^2 \end{array}
 i+j=6
                                \mathbb{C}^2L_{2,1}^2 \mathbb{C}^2L_{2,1}
                 \mathbb{C}^2
  i+j=8
                                \mathbb{C}^3L_{2,1}
                                                                                       \mathbb{C}^2
                                                   \mathbb{C}^2L_{2,1}^2
                 \mathbb{C}^2
i + j = 10
                                                    \mathbb{C}^2
                                                                       \mathbb{C}^2
                                                                                       \mathbb{C}^2
                                 \mathbb{C}^2
i + j = 12
     h^{i,j}
                 j-i=0
                                i-i=2
                                                    i-i=4
                                                                       i-i=6
                                                                                       i-i=8
                                                                                                       i - i = 10
                                                                                                                         i - i = 12
 i + j = 0
  i+j=2
  i+j=4
                                 2
                                                 2
                  2
                                                                 1
 i+j=6
                 2
                                 16
                                                 9
                                                                                 1
 i+j=8
i + j = 10
                                 10
                                                 16
                                                                 2
                  1
                                 2
                                                 2
                                                                 2
                                                                                 2
                                                                                                                   1
i + j = 12
     h^{i,j}
                 j-i=0
                                 j-i=2
                                                 j-i=4
                                                                 j-i=6
                                                                                j-i=8
                                                                                                j-i=10
                                                                                                                  j - i = 12
```

module	multiplicity	dimension
all		91
\mathbb{C}	49	1
$L\left(2\alpha_1+\alpha_2\right)$	6	7

type G2, s=1, subset=[]

```
i+j=1 \mid \mathbb{C}L_{3,2}
 _{i+j=3} \mid \mathbb{C}^2 L_{2,1} L_{3,2}^2 \quad \mathbb{C} L_{3,2}
 _{i+j=5} \mid \mathbb{C}^{2}L_{2,1}^{2}L_{3,2}^{2,7} \quad \mathbb{C}^{2}L_{2,1}L_{3,2}^{2} \qquad \mathbb{C}L_{3,2}
i+j=1 | 15
 i+j=3 | 37  15
        44 37 15
 i+j=5
                    37 	 15
        30 100
 i+j=7
                          37 15
        10 114
                    100
 i+j=9
i+j=11 | 2 10 30 44 37 15
 h^{i,j} j-i=1 j-i=3 j-i=5 j-i=7 j-i=9 j-i=11
```

module	munipheny	difficusion
all		759
\mathbb{C}	42	1
$L\left(3\alpha_1+2\alpha_2\right)$	27	14
$L\left(2\alpha_1+\alpha_2\right)$	33	7
$L\left(4\alpha_1+2\alpha_2\right)$	4	27

modulo multiplicity dimension

type G2, s=2, subset=[]

```
i+j=0 \mid L_{3,2}
         L_{2,1}L_{3,2}^2 L_{3,2}^3L_{6,3}
       i+j=6
                                                                                                                 \begin{array}{ccc} L_{3,2}^3L_{6,3} & & & \\ L_{2,1}^5L_{3,2}^7L_{4,2}L_{5,3}L_{6,3}^2 & L_{3,2}^3L_{6,3} \\ L_{2,1}^2L_{3,2}^2 & & L_{2,1}L_{3,2}^2 \end{array}
i+j=8
i + j = 10
i + j = 12
   h^{i,j}
                                                                                                                   i-i=8
         j-i=0
                       i-i=2
                                                     i-i=4
                                                                                    i-i=6
i+j=0
          35
                   119
i+j=2
                   378
                             119
i+j=4
                   511
                             378
                                      119
i+j=6
                                      378
i+j=8
                   190
                             889
                                               119
                             190
                                               378
                                                         119
i + j = 10
                                      14
                                                         35
                                                                   14
```

i - i = 12

i - i = 12

module	multiplicity	dimension
all		4616
$L\left(3\alpha_1+2\alpha_2\right)$	78	14
$L\left(2\alpha_1+\alpha_2\right)$	67	7
$L\left(6\alpha_1+3\alpha_2\right)$	23	77
$L\left(4\alpha_1+2\alpha_2\right)$	21	27
$L\left(5\alpha_1+3\alpha_2\right)$	11	64
\mathbb{C}	13	1

i-i=4

i-i=6

i-i=8

i - i = 10

 $\frac{i+j=12}{h^{i,j}}$

type G2, s=0, subset=[1]

i+j=0	\mathbb{C}	<i>C</i>				
i+j=2	\mathbb{C}	\mathbb{C}	~			
i+j=4	\mathbb{C}	\mathbb{C}	$\mathbb C$			
i + j = 6	\mathbb{C}	$\mathbb{C}L_{2,1}$	\mathbb{C}	$\mathbb C$		
i + j = 8	\mathbb{C}	$\mathbb{C}L_{2,1}$	$\mathbb{C}L_{2,1}$	\mathbb{C}	\mathbb{C}	
i+j=10	\mathbb{C}	\mathbb{C}	\mathbb{C}	$\mathbb C$	\mathbb{C}	\mathbb{C}
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10
i+j=0	1					
i+j=2	1	1				
i+j=4	1	1	1			
i + j = 6	1	8	1	1		
i+j=8	1	8	8	1	1	
i+j=10	1	1	1	1	1	1
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10

module	multiplicity	dimension
all		42
\mathbb{C}	21	1
$L\left(2\alpha_1+\alpha_2\right)$	3	7

type G2, s=1, subset=[1]

$h^{i,j}$	j-i=1	j - i = 3	j - i = 5	j - i = 7	j-i=9
i+j=1 $i+j=3$ $i+j=5$ $i+j=7$ $i+j=9$	1			22	15
i+j=7	8		29	15	
i + j = 5	22	29	15		
i+j=3	22	15			
i+j=1	15				

module	multiplicity	dimension
all		302
\mathbb{C}	16	1
$L\left(3\alpha_1+2\alpha_2\right)$	12	14
$L\left(2\alpha_1+\alpha_2\right)$	13	7
$L\left(4\alpha_1+2\alpha_2\right)$	1	27

type G2, s=2, subset=[1]

```
i+j=0 \mid L_{3,2}
i+j=2 L_{2,1}L_{3,2} L_{2,1}L_{3,2}^3L_{6,3}
i+j=4 L_{3,2} L_{2,1}^{4}L_{3,2}^{3}L_{4,2}L_{5,3}L_{6,3} L_{2,1}L_{3,2}^{3}L_{6,3}
L_{3,2}
i+j=10 | 0
                 0
                                     0
                                                            L_{3,2}
                                                                                L_{2,1}L_{3,2}
  h^{i,j} \mid j-i=0
                                     j-i=4
                                                            j-i=6
                                                                                j - i = 8
                i-i=2
                                                                                              i - i = 10
i+j=0 | 14
i+j=2 | 21
              126
              238
```

i+j=4 $i+j=6$ $i+j=8$ $i+j=10$	14	238	126			
i + j = 6	0	147	350	126		
i + j = 8	0	7	147	238	126	
i+j=10	0	0	0	14	21	14
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10

module	multiplicity	dimension
all		1729
$L\left(3\alpha_1+2\alpha_2\right)$	31	14
$L\left(2\alpha_1+\alpha_2\right)$	24	7
$L\left(6\alpha_1+3\alpha_2\right)$	10	77
$L\left(4\alpha_1+2\alpha_2\right)$	6	27
$L\left(5\alpha_1+3\alpha_2\right)$	3	64
\mathbb{C}	3	1

type G2, s=0, subset=[2]

i+j=0 i+j=2 i+j=4 i+j=6		\mathbb{C} \mathbb{C} $\mathbb{C}L_{2,1}$ \mathbb{C}	\mathbb{C} $\mathbb{C}L_{2,1}$ $\mathbb{C}L_{2,1}$	\mathbb{C}	$\mathbb C$	
i+j=8 $i+j=10$	C	\mathbb{C}	$\mathbb{C}^{L_{2,1}}$	\mathbb{C}	\mathbb{C}	\mathbb{C}
1 1 1	j-i=0	j-i=2	j-i=4	j-i=6	j-i=8	j-i=10
i+j=0	1					
i+j=2	1	1				
i+j=4	1	1	1			
i + j = 6	1	8	8	1		
i + j = 8	1	1	8	1	1	
i + j = 10	1	1	1	1	1	1
$h^{i,j}$	j-i=0	j-i=2	j-i=4	j - i = 6	j-i=8	j-i=10

module	multiplicity	dimension
all		42
\mathbb{C}	21	1
$L\left(2\alpha_1+\alpha_2\right)$	3	7

type G2, s=1, subset=[2]

$h^{i,j}$	j-i=1	j - i = 3	j - i = 5	j - i = 7	j - i = 9
i+j=3 $i+j=7$ $i+j=9$		-			22
i + j = 7	1	50	71	22	
i+j=5	0	1 1	44		

module	multiplicity	dimension
all		365
\mathbb{C}	18	1
$L\left(2\alpha_1+\alpha_2\right)$	20	7
$L\left(3\alpha_1+2\alpha_2\right)$	9	14
$L\left(4\alpha_1+2\alpha_2\right)$	3	27

type G2, s=2, subset=[2]

```
i+j=0 \mid L_{2,1}L_{3,2}
          L_{2,1}L_{3,2} L_{2,1}^4L_{3,2}^4L_{4,2}L_{5,3}L_{6,3}
                   \mathbb{C}L_{2,1}^{\overline{5}}L_{3,2}^{\overline{3}}L_{4,2}^{2}L_{5,3}L_{6,3} L_{2,1}^{4}L_{3,2}^{4}L_{4,2}L_{5,3}L_{6,3}
 i+j=4
                                                    \mathbb{C}L_{2,1}^2L_{4,2}
 i+j=6
                                                  \mathbb{C}L_{2,1}^2L_{4,2}
 i+j=8
                                                     0
                                                                                                                  L_{2,1}L_{3,2}
i + j = 10
   h^{i,j}
         j-i=0
                                                     j-i=4
                                                                                    j-i=6
                                                                                                                   j - i = 8
                       i-i=2
 i+j=0
                   252
 i+j=2
                   273
                             252
 i+j=4
                   42
                             539
                                      252
 i+j=6
                             42
                                      273
 i+j=8
                                                252
                                      0
                                                21
                                                         21
```

i - i = 10

 $L_{2,1}L_{3,2}$

j - i = 10

module	multiplicity	dimension
all		2261
$L\left(2\alpha_1+\alpha_2\right)$	43	7
$L\left(3\alpha_1+2\alpha_2\right)$	30	14
$L\left(4\alpha_1+2\alpha_2\right)$	15	27
$L\left(5\alpha_1+3\alpha_2\right)$	8	64
$L\left(6\alpha_1+3\alpha_2\right)$	8	77
\mathbb{C}	7	1

j-i=2

j-i=4

j-i=6

j-i=8

 $\frac{i+j=10}{h^{i,j}}$