Maths Exam Answers

answers: Here are the solutions to the exam paper, analyzed and presented within 5 steps for each sub-question.

Q1:

a.

1. Initial principal (P) = Rs 50000.

2. Annual interest rate (r) = 10% = 0.10.

3. Number of years (n) = 2.

4. Amount (A) after 2 years = P(1+r)^n = 50000(1+0.10)^2 = 50000 * (1.1)^2 = 50000 * 1.21 = Rs 60500.

5. Total interest = A - P = 60500 - 50000 = Rs 10500.

b.

1. Amount at the end of 1st year = 50000 * (1 + 0.10) = 55000.

2. Payment made = Rs 20000. Remaining balance = 55000 - 20000 = Rs 35000.

3. This remaining balance becomes the principal for the 2nd year.

4. Interest for 2nd year = 35000 * 0.10 = Rs 3500.

5. Total amount to be paid at the end of the 2nd year = 35000 + 3500 = Rs 38500.

Q2:

(i)

1. The graph passes through the points where y=0.

2. These points are A(2, 0) and B(6, 0).

3. These points represent the x-intercepts of the graph.

4. The x-intercepts are the roots of the equation f(x)=0.

5. Points A(2,0) and B(6,0) are on the graph.

(ii)

1. The roots of f(x) = 0 are the x-intercepts.

2. From part (i), the x-intercepts are 2 and 6.

3. Therefore, the roots of f(x) = 0 are x = 2 and x = 6.

(iii)

1. The x-coordinate of the turning point (vertex) of a parabola is the midpoint of its roots.

2. Roots are $x1 = 2$ and $x2 = 6$.

3. x-coordinate of turning point = $(x1 + x2) / 2$.

4. x-coordinate = $(2 + 6) / 2$.

5. x-coordinate = $8 / 2 = 4$.

(iv)

1. The x-coordinate of the turning point (a) is 4. So the form is $y = (x-4)^2 + b$.

2. The roots are 2 and 6, so the function can be written as $y = k(x-2)(x-6)$.

3. Expanding $(x-2)(x-6)$ gives $x^2 - 8x + 12$.

4. To get the vertex form $y = (x-a)^2 + b$, complete the square: $x^2 - 8x + 16 - 16 + 12 = (x-4)^2 - 4$.

5. Therefore, $y = (x-4)^2 - 4$. (This implies $k=1$).

(v)

1. The function is $y = (x-4)^2 - 4$.

2. The graph passes through (0, 12). Let's verify: $y = (0-4)^2 - 4 = (-4)^2 - 4 = 16 - 4 = 12$. This is consistent.

3. The minimum value of y occurs at the turning point (vertex).

4. The vertex is $(a, b) = (4, -4)$. So, the minimum value of y is -4.

5. The equation of the axis of symmetry is $x = a$, which is $x = 4$.

Q3:

1. The quadratic equation given is $x^2 - 6x + 4 = 0$.

2. Using the quadratic formula, $x = [-b \pm \mathrm{sqrt}(b^2 - 4ac)] / 2a$. Here, $a=1$, $b=-6$, $c=4$.

3. $x = [6 \pm \mathrm{sqrt}((-6)^2 - 4 * 1 * 4)] / (2 * 1) = [6 \pm \mathrm{sqrt}(36 - 16)] / 2$.

4. $x = [6 \pm \mathrm{sqrt}(20)] / 2 = [6 \pm 2*\mathrm{sqrt}(5)] / 2 = 3 \pm \mathrm{sqrt}(5)$.

5. Given $\mathrm{sqrt}(5) = 2.24$. So, $x = 3 + 2.24 = 5.24$ or $x = 3 - 2.24 = 0.76$.

Q4:

(i)

1. Let x be the number of Type 10 refrigerators and y be the number of Type 11 refrigerators.

2. Large compressors constraint: 6x (for Type 10) + 8y (for Type 11) <= 516 (total available).

3. Small compressors constraint: 3x (for Type 10) + 3y (for Type 11) <= 300 (total available).

4. Simplify the second inequality: x + y <= 100.

5. Also, the number of refrigerators cannot be negative: x >= 0 and y >= 0.

(ii)

1. Draw the lines: L1: 6x + 8y = 516 and L2: x + y = 100.

2. For L1: Intercepts are (0, 64.5) and (86, 0).

3. For L2: Intercepts are (0, 100) and (100, 0).

4. The feasible region is the area bounded by x>=0, y>=0, and below both lines. The intersection of L1 and L2 is (142, -42), which is outside the first quadrant, meaning L1 is the binding constraint for x,y >= 0.

5. Shade the region in the first quadrant below the line 6x + 8y = 516. (This region is a polygon with vertices (0,0), (86,0), and (0, 64.5)).

(iii)

1. Profit function P = 2500x + 3000y.

2. Identify the vertices of the feasible region: (0,0), (86,0), and (0, 64.5).

3. Evaluate profit at integer vertices: P(0,0) = 0. P(86,0) = 2500 * 86 + 3000 * 0 = 215000.

4. For (0, 64.5), since y must be an integer, consider (0, 64). P(0,64) = 2500 * 0 + 3000 * 64 = 192000.

5. Comparing profits, the maximum is Rs 215000, achieved when x=86 and y=0. So, 86 Type 10 refrigerators and 0 Type 11 refrigerators.

Q5:

(i)

1. Angle AOB = 144 degrees is the central angle subtended by arc AB.

2. Angle ADB is an inscribed angle subtended by the same arc AB.

3. The inscribed angle is half the central angle subtending the same arc.

4. Angle ADB = (1/2) * Angle AOB.

5. Angle ADB = (1/2) * 144 degrees = 72 degrees.

(ii)

1. To prove AS is parallel to BC, we can show that alternate interior angles are equal, e.g., Angle ASB = Angle SBC (using transversal SB).

2. Angle ASB is an inscribed angle subtended by arc AB. Therefore, Angle ASB = (1/2) * Angle AOB = (1/2) * 144 = 72 degrees.

3. For AS || BC, we would need Angle SBC = 72 degrees.

4. Angle SBC is an inscribed angle subtended by arc SC. Thus, for parallelism, arc SC would need to be 2 * 72 = 144 degrees.

5. The provided information (Angle ABS = 54 degrees, which implies arc AS = 108 degrees) does not directly provide enough information to confirm that arc SC = 144 degrees, making a direct proof using the given values inconsistent or impossible without further clarification on point C's position relative to S, A, B on the circle.

(iii)

1. The radius of the circle (r) = 100 units.

2. The central angle subtended by arc AB (theta) = Angle AOB = 144 degrees.

3. Convert the angle to radians: theta_rad = 144 * (π / 180) = 4π / 5 radians.

4. The length of an arc (L) is given by L = r * theta_rad.

5. L = 100 * (4π / 5) = 80π units. Using $\pi \approx 3.14159$, L ≈ 80 * 3.14159 = 251.3272... ≈ 251.33 units (to 2 decimal places).

(iv)

1. The problem states "the angle subtended by arc AF at the center is 30 degrees", meaning Angle AOF = 30 degrees.

2. The question asks for Angle AFB. This is an inscribed angle with its vertex F on the circumference.

3. Angle AFB subtends arc AB.

4. The central angle subtending arc AB is Angle AOB = 144 degrees (given in the problem).

5. An inscribed angle is half the central angle subtending the same arc. So, Angle AFB = (1/2) * Angle AOB = (1/2) * 144 degrees = 72 degrees. (The information about arc AF is not needed for this part).

Q6:

(i)

1. Students in the 40-44 marks range: 4.

2. Students in the 44-48 marks range: 8.

3. Students in the 48-52 marks range: 12.

4. To find students scoring between 40 and 52 marks, sum the frequencies for these ranges.

5. Total students = 4 + 8 + 12 = 24 students.

(ii)

1. Calculate midpoints (x_i) for each class: 42, 46, 50, 54, 58, 62, 66.

2. Calculate f_i * x_i for each class: 4*42=168, 8*46=368, 12*50=600, 15*54=810, 9*58=522, 7*62=434, 5*66=330.

3. Sum of frequencies (Σf_i) = 4+8+12+15+9+7+5 = 60.

4. Sum of ($f_i * x_i$) = 168+368+600+810+522+434+330 = 3232.

5. Mean = ($\Sigma f_i * x_i$) / Σf_i = 3232 / 60 = 53.866... ≈ 53.87 marks.

(iii)

1. Number of students in this class who scored above 60 marks: students in 60-64 (7) + students in 64-68 (5).

2. Total students above 60 marks = 7 + 5 = 12 students.

3. Proportion of students getting prizes in this sample = 12 / 60 = 1/5.

4. Total students in the school = 600.

5. Estimated number of students receiving prizes = (1/5) * 600 = 120 students.

(iv)

1. Plot the midpoints of each class interval against their corresponding frequencies.

2. Points to plot: (42, 4), (46, 8), (50, 12), (54, 15), (58, 9), (62, 7), (66, 5).

3. To close the polygon, add midpoints of adjacent zero-frequency classes: (38, 0) and (70, 0).

4. Connect these plotted points with straight line segments.

5. The resulting graph is the frequency polygon.

Q7:

(i)

1. Let the number of red balls (R) = 6.

2. Let the number of blue balls (B).

3. Total number of balls = R + B = 6 + B.

4. The probability of drawing a blue ball P(Blue) = B / (6 + B).

5. Given P(Blue) = 3/5. So, B / (6 + B) = 3/5. Cross-multiply: 5B = 3(6 + B) => 5B = 18 + 3B => 2B = 18 => B = 9.

(ii)

1. Initial red balls = 6. Initial blue balls = 9 (from part i).

2. 2 more red balls are added. New number of red balls = 6 + 2 = 8.

3. Number of blue balls remains 9.

4. New total number of balls = 8 + 9 = 17.

5. Probability of drawing a red ball = (New Red Balls) / (New Total Balls) = 8 / 17.

Q8:

(i)

1. Given P = [[2, -1], [3, 0]] and Q = [[1, 2], [-3, 4]].

2. Calculate 2P = 2 * [[2, -1], [3, 0]] = [[4, -2], [6, 0]].

3. Calculate 2P + Q = [[4, -2], [6, 0]] + [[1, 2], [-3, 4]] = [[4+1, -2+2], [6+(-3), 0+4]].

4. 2P + Q = [[5, 0], [3, 4]].

5. Given 2P + Q = [[x, 0], [3, 4]]. Comparing the elements, x = 5.

(ii)

1. Given P = [[2, -1], [3, 0]] and Q = [[1, 2], [-3, 4]].

2. To find PQ, multiply rows of P by columns of Q.

3. (Row 1 of P) * (Column 1 of Q) = (2*1) + (-1*-3) = 2 + 3 = 5. (Element (1,1) of PQ)

4. (Row 1 of P) * (Column 2 of Q) = (2*2) + (-1*4) = 4 - 4 = 0. (Element (1,2) of PQ)

5. (Row 2 of P) * (Column 1 of Q) = (3*1) + (0*-3) = 3 + 0 = 3. (Element (2,1) of PQ)

6. (Row 2 of P) * (Column 2 of Q) = (3*2) + (0*4) = 6 + 0 = 6. (Element (2,2) of PQ)

7. Therefore, PQ = [[5, 0], [3, 6]].

Q9:

(i)

1. Draw a vertical line segment AB representing the tower (A top, B base).

2. Draw a horizontal line segment BC representing the ground. C is the initial observation point.

3. Draw a line segment AC. Label Angle ACB as 60 degrees (angle of elevation).

4. Mark point D on BC such that CD = 10m and D is between B and C.

5. Draw a line segment AD. Label Angle ADB as 70 degrees (new angle of elevation).

(ii)

1. Let height of tower AB = h. Let BD = x. Then BC = x + 10.

2. In right triangle ABD, tan(70°) = AB/BD = h/x, so h = x * tan(70°).

3. In right triangle ABC, tan(60°) = AB/BC = h/(x+10), so h = (x+10) * tan(60°).

4. Equate the expressions for h: x * tan(70°) = (x+10) * tan(60°). Substitute tan(70°) ≈ 2.7475 and tan(60°) ≈ 1.7321.

5. 2.7475x = 1.7321x + 17.321 => (2.7475 - 1.7321)x = 17.321 => 1.0154x = 17.321 => x ≈ 17.058.

6. $BC = x + 10 = 17.058 + 10 = 27.058 \approx 27.06$ m (to 2 decimal places).

Q10:

(i)

1. First term (a) = 3.

2. Common ratio (r) = 2.

3. First term = a = 3.

4. Second term = $ar = 3 * 2 = 6$.

5. Third term = $ar^2 = 3 * 2^2 = 12$.

6. Fourth term = $ar^3 = 3 * 2^3 = 24$.

The first 4 terms are 3, 6, 12, 24.

(ii)

1. Formula for the sum of the first n terms of a geometric series: $S_n = a(r^n - 1) / (r - 1)$.

2. Given a=3, r=2, n=8.

3. $S_8 = 3 * (2^8 - 1) / (2 - 1)$.

4. Calculate $2^8 = 256$.

5. $S_8 = 3 * (256 - 1) / 1 = 3 * 255$.

6. $S_8 = 765$.

Q11:

a.

(i)

1. Draw a line segment RS.

2. At point R, construct a line perpendicular to RS (using compass and straightedge).

3. Measure the length of RS with a compass. Mark point T on the perpendicular line from R such that RT = RS.

4. Triangle RST is a right-angled triangle at R. The hypotenuse is ST.

5. Find the midpoint of ST by constructing its perpendicular bisector. Let the midpoint be O. With O as center and OS (or OR or OT) as radius, draw the circle.

(ii)

1. The line RT is constructed perpendicular to RS, so Angle TRS = 90 degrees.

2. By construction, RT = RS.

3. A triangle with one right angle and two equal sides is a right-angled isosceles triangle.

4. Therefore, triangle RST is a right-angled isosceles triangle.

(iii)

1. Triangle RST is a right-angled isosceles triangle with legs RS and RT (where $RT=RS$).

2. The hypotenuse ST can be found using Pythagoras theorem: $ST^2 = RS^2 + RT^2 = RS^2 + RS^2 = 2 * RS^2$.

3. So, $ST = \sqrt{2} * RS$.

4. The circumcenter of a right-angled triangle is the midpoint of its hypotenuse. The radius of the circumcircle is half the length of the hypotenuse.

5. Radius $= ST / 2 = (\sqrt{2} * RS) / 2$.

b.

Note: The problem statement for Q11(b) contains significant ambiguities and contradictions (e.g., undefined point 'D' in part (i), and the definition of point 'S' in the text "CX intersects BX at S" appears to contradict the diagram and also leads to degenerate triangles for subsequent parts). Due to these critical ambiguities, a definitive solution within 5 steps is not possible without further clarification or correction of the problem statement. Therefore, I must state that this section cannot be confidently answered as presented.

Q12:

(i)

1. The sector P represents students who walk to school.

2. The angle subtended by sector P at the center is 60 degrees.

3. The number of students corresponding to sector P is 63.

4. The fraction of total students represented by P is $60/360 = 1/6$.

5. Total number of students $= 63 / (1/6) = 63 * 6 = 378$ students. (Assuming the initial "108 students" is a typo, as it's inconsistent with the given data for P).

(ii)

1. Total number of students = 378 (from part i).

2. Number of students who travel by bus = 23.

3. Fraction of students traveling by bus $= 23 / 378$.

4. Angle of the sector for bus travel = (Fraction of students for bus travel) * 360 degrees.

5. Angle $= (23 / 378) * 360 \approx 21.904$ degrees ≈ 21.9 degrees (to one decimal place).

(iii)

1. Total students = 378.

2. Students who walk = 63.

3. Students who travel by bus = 23.

4. Students who travel by bicycle = 99.

5. Students who travel by train = Total - (Walk + Bus + Bicycle) = 378 - (63 + 23 + 99) = 378 - 185 = 193 students.

(iv)

1. Number of students who walk = 63.

2. Number of students who travel by bus = 23.

3. Number of students who travel by bicycle = 99.

4. Number of students who travel by train = 193 (from part iii).

5. Comparing the numbers, 193 is the highest. So, the most popular mode of transport is Train.

(v)

1. The number of students for each transport mode are: Walk (63), Bus (23), Bicycle (99), Train (193).

2. There are 4 transport modes given.

3. Sum of students across all modes = 63 + 23 + 99 + 193 = 378.

4. Mean number of students per transport mode = (Sum of students) / (Number of modes).

5. Mean = 378 / 4 = 94.5 students.

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