

# Deri

No.:

Date.:

## 1. Refleksi

terhadap  $R_2$

- Sumbu  $x$

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

matrix:

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$= x = 0$$

$$-y = 0$$

$$\text{Basis} = \{ \} \text{ kosong}$$

Dimensi

$$n = 2$$

$$\text{null}(A) = 0$$

$$\text{Rank}(A) = 2$$

- terhadap sumbu  $y$

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-x = 0$$

$$y = 0$$

$$\text{Basis} = \{ \}$$

Dimensi

$$n = 2$$

$$\text{null}(A) = 0$$

$$\text{Rank}(A) = 2$$

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Pada  $R_3$ 

- terhadap sumbu X

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = T\left(\begin{pmatrix} x \\ -y \\ -z \end{pmatrix}\right)$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x = 0$$

$$-y = 0$$

$$-z = 0$$

$$\text{Basis} = \{ \}$$

Dimensi

$$n = 3$$

$$\text{null}(A) = 0$$

$$\text{rank}(A) = 3$$

terhadap Sumbu Y

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} -x \\ y \\ -z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x = 0$$

$$y = 0$$

$$-z = 0$$

$$\text{Basis} = \{ \}$$

Dimensi

$$n = 3$$

$$\text{null}(A) = 0$$

$$\text{rank}(A) = 3$$



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terhadap sumbu z

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x=0, -y=0, z=0$$

$$\text{Basis} = \{\}$$

Dimensi:

$$n=3$$

$$\text{null}(A)=0$$

$$\text{Rank}(A)=3$$

terhadap xy

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x=0$$

$$y=0$$

$$-z=0$$

$$\text{Basis} = \{\}$$

Dimensi:

$$n=3$$

$$\text{null}(A)=0$$

$$\text{rank}(A)=3$$

- terhadap  $xz$ 

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -y \\ z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x=0, -y=0, z=0$$

$$\text{Basis} = \{ \}$$

dimensi

$$n=3$$

$$\text{Null}(A) = 0$$

$$\text{rank}(A) = 3$$

- terhadap  $yz$ 

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ y \\ z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x=0$$

$$y=0$$

$$z=0$$

$$\text{Basis} = \{ \}$$

Dimensi

$$n=3$$

$$\text{Null}(A) = 0$$

$$\text{rank}(A) = 3$$



## 2. Proyeksi

Date: .

↳ mengubah Dimensi

$$3D \rightarrow 2D$$

$$R^2 \rightarrow R$$

terhadap  $P$   $X$

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x = 0, y = P$$

$$\text{Basis} = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

Dimensi:

$$n = 2$$

$$\text{null}(A) = 1$$

$$\text{rank} = 1$$

terhadap  $P$   $Y$

$$T \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ y \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x = P, y = 0$$

$$\text{basis} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

Dimensi:

$$n = 2, \text{null} = 1, \text{rank} = 1$$

$$R^3 \rightarrow R^2$$

terhadap  $P$   $XZ$

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 0 \\ z \end{pmatrix}$$

$$e_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x = 0, z = 0, y = P$$

$$\text{basis} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Dimensi:

$$n = 3, \text{null}(A) = 1$$

$$\text{rank} = 2$$

terhadap bidang  $YZ$ 

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}$$

$$x=0 \quad y=0 \quad z=0$$

$$\text{basis} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Dimensi:

$$N=3, \text{ null} = 1, \text{ rank} = 2$$

terhadap  $XZ$ 

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ z \end{pmatrix}$$

$$e_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$x=0 \quad y=0 \quad z=p$$

Dimensi

$$n=3 \quad \text{rank}=2$$

$$n=1$$

3 Rotasi

↳ perputaran

 $R^2$  terjadi secara  
clock wise

$$\Theta = \pi$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix} \text{ pada } 0$$

$$+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin \Theta \\ -\cos \Theta \end{bmatrix} \text{ pada } 180$$

$$A = \begin{bmatrix} \cos \Theta & \sin \Theta \\ \sin \Theta & -\cos \Theta \end{bmatrix}$$



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## 4. Dilatasi dan Kontraksi

Dilatasi = membesar

Kontraksi = mengecil

Proses ini terjadi dengan memperbanyak matriksnya

### R2

$$T \begin{pmatrix} x \\ y \end{pmatrix} = T \begin{pmatrix} kx \\ ky \end{pmatrix}$$

$$e_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} k \\ 0 \end{pmatrix}$$

$$e_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ k \end{pmatrix}$$

$$A = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

$$A \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x = k \quad y = k$$

$$\text{basis} = \{ \}$$

$$n = 2$$

$$\text{Null} = 0$$

$$\text{rank} = 2$$

### R3

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} kx \\ ky \\ kz \end{pmatrix}$$

$$e_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$$

$$e_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ k \\ 0 \end{pmatrix}$$

$$e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$$

$$A = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x = k \quad y = k \quad z = k$$

$$\text{basis} = \{ \}$$

Dimensi

$$n = 3$$

$$\text{Null} = 0$$

$$\text{rank} = 3$$

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# Kompresi dan Ekspansi

↳ pengecilan dan pembesaran

terhadap  $X$

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \rightarrow T\left(\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$e1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ k \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

$$\left\{ \begin{aligned} A \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ x &= 0 \quad y = k \\ n &= 2 \quad \text{null } A = 0 \\ \text{rank } (A) &= 2 \\ \text{basis} &= \{ \} \end{aligned} \right.$$

terhadap  $Y$

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) \rightarrow \begin{pmatrix} x \\ ky \end{pmatrix}$$

$$e1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$e2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ k \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

$$\left\{ \begin{aligned} A \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ x &= 1 \quad y = k \\ \text{basis} &= \{ \} \\ \text{Dimensi} & \\ n &= 2 \\ \text{null } (A) &= 0 \\ \text{rank } (A) &= 2 \end{aligned} \right.$$