No: MTH Date:

(1) $x_1 + x_2 - x_3 = 0$

$-2x_1 - x_2 + 2x_3 = 0$

$-x_1 + x_3 = 0$

$$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{matrix} \\ +2 \\ -1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \begin{matrix} -1 \\ \\ +1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \frac{1}{2} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

basis = {}

Dimensi

$n = 3$

$\text{null}(A) = 1$

$\text{rank}(A) = 2$

(2) $3x_1 + x_2 + x_3 + x_4 = 0$

$5x_1 - x_2 + x_3 - x_4 = 0$

$$\begin{bmatrix} 3 & 1 & 1 & 1 \\ 5 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 \\ 5 & -1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 \\ 0 & -8/3 & -2/3 & -8/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 \\ 0 & 1 & 1/4 & 1 \end{bmatrix} \begin{matrix} 0 \\ 0 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1/4 & 0 \\ 0 & 1 & 1/4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 + \frac{1}{4}x_3 = 0 \quad x_1 = -\frac{1}{4}p$

$x_2 + \frac{1}{4}x_3 = 0 \quad x_2 = \frac{1}{4}p$

$x_3 = p$

$x_4 = q$

No.: basis ↓ Date.:

$$\left\{ P = \begin{bmatrix} -1/4 \\ -1/4 \\ 1 \\ 0 \end{bmatrix} q \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Dimensi

A = 4

null = 2

rank = 2

3, $2x_1 + x_2 + 3x_3$

$x_1 + 5x_3 = 0$

$x_2 + x_3 = 0$

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \quad \frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} -2$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -7 \\ 0 & 1 & 1 \end{bmatrix} -i$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -7 \\ 0 & 0 & 8 \end{bmatrix} \frac{1}{8} \rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} -5 \; -7$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

basis = {}

dimensi

A = 3

null(A) = 0

rank(A) = 3

4, $x_1 - 4x_2 + 3x_3 - x_4 = 0$

$2x_p - 8x_2 + 6x_3 - 2x_4 = 0$

$$\begin{bmatrix} 1 & -4 & 3 & -1 \\ 2 & -8 & 6 & -2 \end{bmatrix} -2$$

$$\begin{bmatrix} 1 & -4 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 4x_2 + 3x_3 - x_4 = 0$

$x_1 = 4p - 3q + r$

$x_2 = p$

$x_3 = q$

$x_4 = r$

basis

$$\left\{\begin{bmatrix}4\\1\\0\\0\end{bmatrix}\begin{bmatrix}-3\\0\\1\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}\right\}$$

Dimensi = 4

null(A) = 3

rank(A) = 1

5, $x_1 - 3x_2 + x_3 = 0$

$2x_1 - 6x_2 + 2x_3 = 0$

$3x_1 - 9x_2 + 3x_3 = 0$

$$\begin{bmatrix}1 & -3 & 1\\2 & -6 & 2\\3 & -9 & 3\end{bmatrix}\begin{matrix}-2\\-3\end{matrix}$$

$$\begin{bmatrix}1 & -3 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

$x_1 = 3x_2 + x_3 = 3p - q$

$p \; x_2 = p$

$x_3 = q$

basis

$$\left\{p\begin{bmatrix}3\\1\\0\end{bmatrix}\begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}$$

6 $x + y + z = 0$

$3x + 2y - 2z = 0$

$4x + 3y - z = 0$

$6x + 5y + z = 0$

$$\begin{bmatrix}1 & 1 & 1\\3 & 2 & -2\\4 & 3 & -1\\6 & 5 & 1\end{bmatrix}\begin{matrix}-3\\-4\\-6\end{matrix}$$

$$\begin{bmatrix}1 & 1 & 1\\1 & -1 & -5\\6 & -3 & -5\\6 & -1 & -5\\0 & & \end{bmatrix}\cdot -1$$

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 5\\0 & -4 & -5\\0 & -1 & -5\end{bmatrix}\begin{matrix}+1\\p\\+1\\+1\end{matrix}$$

$$\begin{bmatrix}1 & 0 & -4\\0 & 1 & 5\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x = 4z = 4p$

$y = 5z = 5p$

$z = p$

$$\text{Basis} = \left\{ \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} \right\}$$