27.02.2021

1. Найти производные указанных ф-ций

1) $y = x^3 \log_2 x$

$y' = 3x^2 \log_2 x + x^3 \frac{1}{x \ln 2} = 3x^2 \log_2 x + \frac{x^2}{\ln 2}$

2) $y = -10 \operatorname{arctg} x + 7 \cdot e^x$

$y' = -10 \frac{1}{1+x^2} + 7 \cdot e^x$

3) $y = \frac{1}{\sqrt[3]{x^2}} - \frac{2}{x^3} + \sqrt{7} \cdot x$

$y' = (x^2)^{-\frac{1}{3}} - 2 \cdot \left(-\frac{3x^2}{x^6}\right) + \sqrt{7} = x^{-\frac{2}{3}} + \frac{6}{x^4} + \sqrt{7} =$

$= x^{-\frac{2}{3}} + 6x^{-4} + \sqrt{7}$

4) $y = \cos \frac{1-\sqrt{x}}{1+\sqrt{x}} = -\sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right) \cdot \left(-\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}x^{-\frac{1}{2}}}\right) =$

$= \sin\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

5) $y = e^{sh^2 5x}$

$y' = e^{sh^2 5x} \cdot (sh^2 5x)' \cdot 5 = e^{sh^2 5x} \cdot 2 ch 5x \cdot sh 5x \cdot 5 =$

$= 10 e^{sh^2 5x} \cdot ch 5x \cdot sh 5x$

6) $y = \ln \frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}$

$y' = \frac{1}{\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}} \cdot \frac{((x+1)(x+3)^3)' - (x+2)^3(x+4)'}{((x+2)^3(x+4))^2} =$

$$= \frac{(x+2)^3(x+4)}{(x+1)(x+3)^3} \cdot \frac{3(x+3)^2 - (x+2)^3(x+4)}{(x+2)^5(x+4)^2} =$$

$$= 12\,\frac{(x+3)^2 - (x+2)^3(x+4)}{(x+2)^2(x+4)(x+1)(x+3)^3}$$

7) $y = \frac{\sin^2 x}{ctg x+1} + \frac{\cos^2 x}{tg x+1}$

$$y' = \frac{2\sin x \cdot (ctg x+1) + \sin^2 x \cdot \frac{1}{\sin^2 x}}{(ctg x+1)^2} +$$

$$+ \frac{2\cos x \cdot (tg x+1) - \cos^2 x \cdot \frac{1}{\cos^2 x}}{(tg x+1)^2} = \frac{2\sin x \cdot (ctg x+1)+1}{(ctg x+1)^2}$$

$$+ \frac{2\cos x\,(tg x+1) - 1}{(tg x+1)^2} = 2\sin x\left(\frac{\cos x}{\sin x}+1\right)+1 =$$

$$= 2\cos x + 2\sin x + 1$$

2. Найти производную данной ф-ции в точке

1) $y = \frac{\ln x}{x}$, $x_0 = e$

$$y' = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = -\frac{\ln x}{x^2}$$

$$f(x_0) = -\frac{\ln e}{e^2} = -\frac{1}{e^2}$$

2) $y = \frac{\sqrt{x}}{\sqrt{x}+1}, x_0 = 9$

$$y' = \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = 1$$

3. Используя логарифм. производную, найти производные:

1) $y = x^{\ln x}$

$\ln y = \ln^2 x$

$$\frac{y'}{y} = \frac{1}{x^2}$$

$$y' = \frac{x^{\ln x}}{x^2}$$

2) $y = \frac{(x^3-2)\cdot\sqrt[3]{(x-1)}}{(x+5)^4}$

$$\ln y = \ln \frac{(x^3-2)\cdot(x-1)^{\frac{1}{3}}}{(x+5)^4} =$$

$$\ln y = \frac{\ln(x^3-2)\cdot\frac{1}{3}\ln(x-1)}{4\ln(x+5)}$$

$$\frac{y'}{y} = \frac{\frac{1}{x}\cdot 3x^2\cdot\frac{1}{3}\ln(x-1) + \ln(x^3-2)\cdot\frac{1}{3}\frac{1}{x-1}}{4\cdot\frac{1}{x+5}}$$

$$y' = \frac{3x \cdot \frac{1}{3}\ln(x-1) + \ln(x^3-2)\cdot\frac{1}{3(x-1)}}{\frac{4}{x+5}} \cdot$$

$$\cdot \frac{(x^3-2)\cdot(x-1)^{\frac{1}{3}}}{(x+5)^4} = \frac{\left(3x\cdot\frac{1}{3}\ln(x-1) + \ln(x^3-2)\cdot\frac{1}{3(x-1)}\right)\cdot}{4(x+5)^3}$$

$$\cdot (x^3-2)\cdot(x-1)^{\frac{1}{3}}$$