ДЗ №3

1. Найти области определения функций:

а) $f(x) = \ln(x+2)$

$x+2>0$

$x>-2$

$D(f) = (-2;+\infty)$

б) $f(x) = 2^{\frac{1}{x}} + \arcsin\frac{x+2}{3}$

$x \neq 0$

$1 \geq \frac{x+2}{3} \geq -1$

$3 \geq x+2 \geq -3$

$1 \geq x \geq -5$

$D(f) = [-5;0) \cup (0;1]$

2. Найти множества значений функции

а) $f(x) = x^2+4x+1$

$x_0 = -\frac{b}{2a}$ - вершина параболы

$x_0 = -\frac{4}{2} = -2$

$y_0 = a(x_0)^2 + bx_0 + c$

$y_0 = 4+(-8)+1 = -3$

$(f) = [-3;+\infty)$

б) $f(x) = 2^{x^2}$

$E(x^2) \geq 0$

$E(2^0) \geq 1$

$E(f) = [1; +\infty)$

в) $f(x) = 3 - 5\cos x$

$-1 \leq \cos x \leq 1$

$5 \geq -5\cos x \geq -5$

$8 \geq 3 - 5\cos x \geq -2$

$E(f) = [-2; 8]$

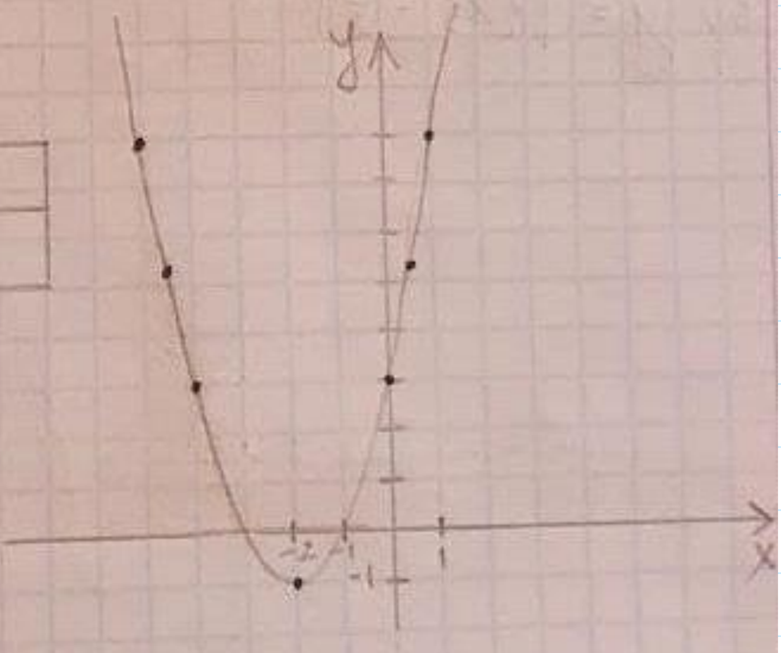
3. Построить график функции

а) $y = x^2 + 4x + 3$

$x_0 = -\frac{4}{2} = -2$

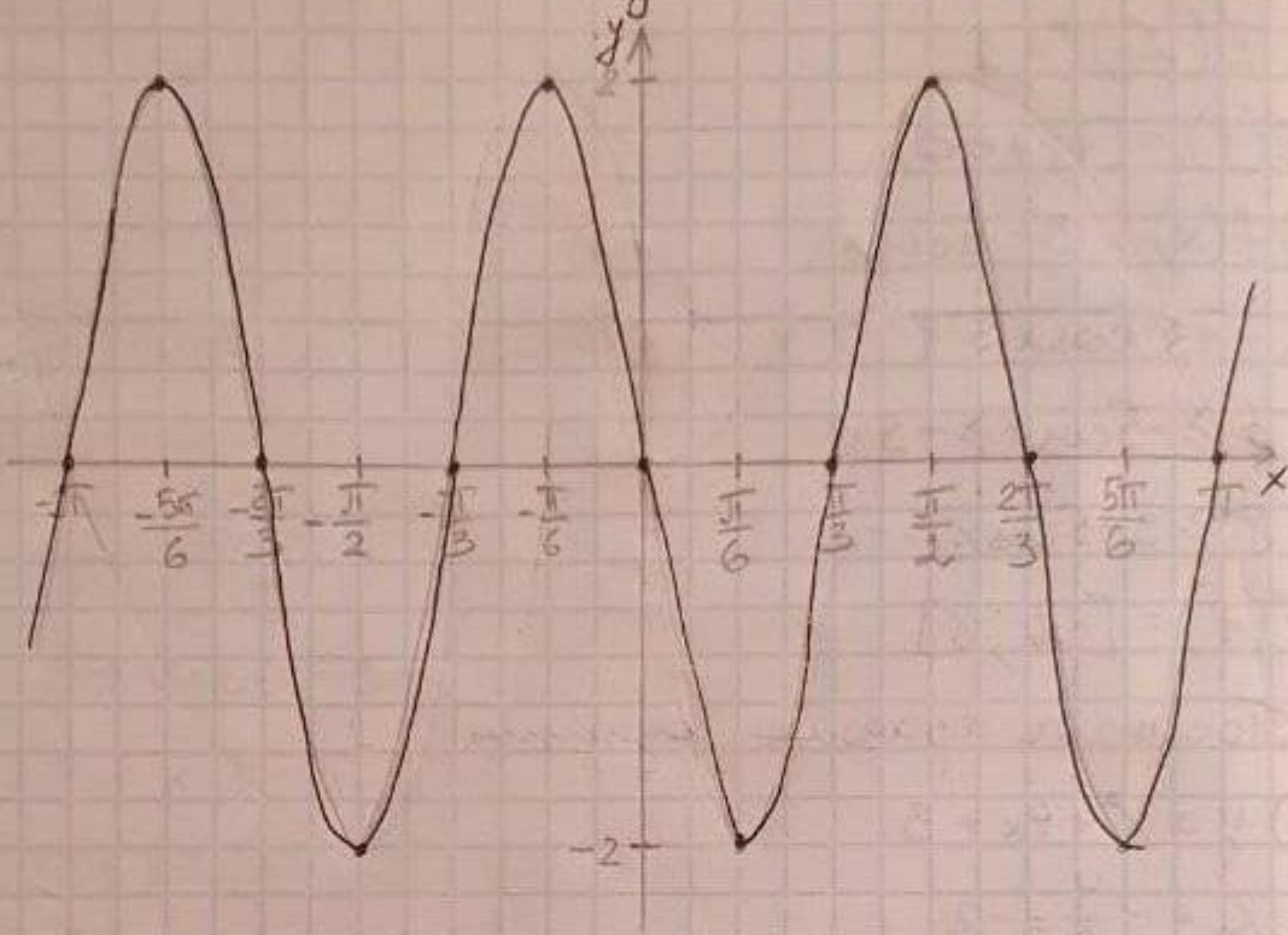
$y_0 = 4 + (-8) + 3 = -1$

| x | 0 | 0,5 | 1 |
|---|---|---|---|
| y | 3 | 5,25 | 8 |



б) $y = -2\sin 3x$

| $x$ | $0$ | $\pi$ | $\frac{\pi}{2}$ | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{5\pi}{6}$ | $\frac{2\pi}{3}$ |
|---|---|---|---|---|---|---|---|
| $y$ | $0$ | $0$ | $2$ | $-2$ | $0$ | $-2$ | $0$ |




в) $y = \left|\{x\} - \frac{1}{2}\right|$

4. Найти обратную функцию

а) $y = x - 1$

$x = y + 1$

$y = x + 1$

б) $y = \sqrt{x}$

$x = y^2$

$$x = y^2$$
$$y = x^2$$

5. Найти пределы

1) $\lim\limits_{x\to -2} (5x^2+2x-1) = 20-4-1=15$

2) $\lim\limits_{x\to 0} \frac{x}{x^2-x} = \lim\limits_{x\to 0} \frac{1}{x-1} = -1$

3) $\lim\limits_{x\to 5} \frac{x^2-6x+5}{x^2-25} = \lim\limits_{x\to 5} \frac{(x-5)(x-1)}{(x-5)(x+5)} = \frac{4}{10} = 0{,}4$

4) $\lim\limits_{x\to -1} \frac{x^3+x+2}{x^3+1} = \lim\limits_{x\to -1} \frac{x^3-x^2+x^2+2x-x+2}{(x+1)(x^2-x+1)} =$

$= \lim\limits_{x\to -1} \frac{x(x^2-x+2)+(x^2-x+2)}{(x+1)(x^2-x+1)} = \lim\limits_{x\to -1} \frac{(x+1)(x^2-x+2)}{(x+1)(x^2-x+1)} = \frac{4}{3}$

5) $\lim\limits_{x\to 3} \frac{\sqrt{2x+3}-3}{\sqrt{x-2}-1} = \lim\limits_{x\to 3} \frac{(2x+3-9)(\sqrt{x-2}+1)}{(x-2-1)(\sqrt{2x+3}+3)} =$

$= \lim\limits_{x\to 3} \frac{2(x-3)(\sqrt{x-2}+1)}{(x-3)(\sqrt{2x+3}+3)} = \lim\limits_{x\to 3} \frac{2(\sqrt{x-2}+1)}{\sqrt{2x+3}+3} = \frac{4}{6} = \frac{2}{3}$

6) $\lim\limits_{x\to +\infty} (\sqrt{x^2+4}-x) = [\infty-\infty] = \lim\limits_{x\to +\infty} \frac{x^2+4-x^2}{\sqrt{x^2+4}+x} =$

$= \frac{4}{\infty} = 0$

7) $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} =$

$= \lim \frac{1-\cos^2 x}{\ldots} = \lim \frac{\sin^2 x}{\ldots} = \lim \frac{1}{\ldots} = \frac{1}{\ldots}$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{1}{1+\cos x} = \frac{1}{2}$$

8) $$\lim_{x\to 0} x \cdot \operatorname{ctg} x = \lim_{x\to 0} \frac{x \cdot \cos x}{\sin x} = \lim_{x\to 0} \cos x = 1$$

9) $$\lim_{x\to 0} \frac{\cos 5x - \cos 3x}{x^2} = \lim_{x\to 0} \frac{-2\sin\frac{5x+3x}{2} \cdot \sin\frac{5x-3x}{2}}{x^2} =$$

$$= \lim_{x\to 0} \frac{-2\sin 4x \cdot \sin x}{x^2} = \lim_{x\to 0} \frac{-2\sin 4x}{x} = \left[\begin{matrix} y = 4x \\ x\to 0,\ y\to 0 \end{matrix}\right] =$$

$$= -2\lim_{y\to 0} \frac{\sin y}{\frac{y}{4}} = -2\lim_{y\to 0} \frac{4\sin y}{y} = -8$$

10) $$\lim_{x\to 0} \sqrt[2x]{1+3x} = \lim_{x\to 0} (1+3x)^{\frac{1}{2x}} = [1^\infty] = \left[\begin{matrix} y = 3x \\ x\to 0,\ y\to 0 \end{matrix}\right] =$$

$$= \lim_{y\to 0} (1+y)^{\frac{3}{2y}} = \left[\lim_{y\to 0} (1+y)^{\frac{1}{y}}\right]^{\frac{3}{2}} = e^{\frac{3}{2}}$$

11) $$\lim_{x\to 0} \left(\frac{3+5x}{3+2x}\right)^{\frac{1}{x}} = \lim_{x\to 0} \left(\frac{1+\frac{5x}{3}}{1+\frac{2x}{3}}\right)^{\frac{1}{x}} = \lim_{x\to 0} \frac{\left(\left(1+\frac{1}{\frac{5x}{3}}\right)^{\frac{5x}{3}}\right)^{\frac{3}{5x}\cdot\frac{1}{x}}}{\left(\left(1+\frac{1}{\frac{2x}{3}}\right)^{\frac{2x}{3}}\right)^{\frac{3}{2x}\cdot\frac{1}{x}}} =$$

$$= \frac{e^{\lim_{x\to 0}\frac{3}{5x^2}}}{e^{\lim_{x\to 0}\frac{3}{2x^2}}} = \frac{e^\infty}{e^\infty} = 1$$