1.

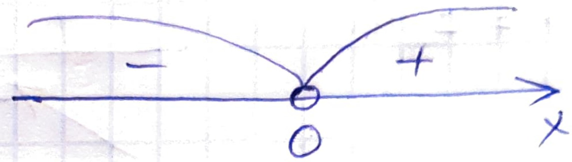
1) $f(x) = x + e^{-x}$

$f'(x) = 1 + e^{-x} \cdot (-1) = 1 - e^{-x}$

$1 - e^{-x} = 0$

$e^{-x} = 1$

$x = 0$



$f'(x) < 0: x \in (-\infty; 0)$

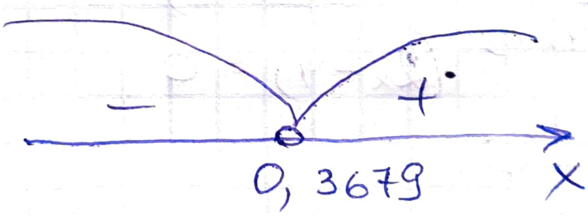
$f'(x) > 0: x \in (0; +\infty)$

2) $f(x) = x \ln x$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$\ln x = -1$

$x = \frac{1}{e}$

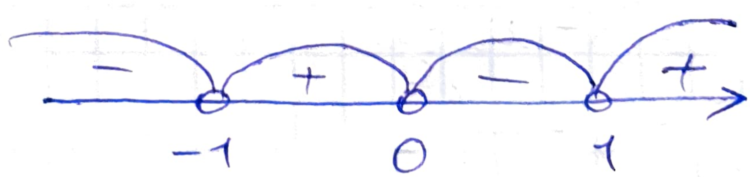


$f'(x) < 0: x \in (-\infty; 0{,}3679)$

$f'(x) > 0: x \in (0{,}3679; +\infty)$

3) $y = \frac{1}{1-x^2}$

$f'(x) = \frac{2x}{(1-x^2)^2}$



$x = 0$

$x \neq 1$

$x \neq -1$

$f'(x) < 0: x \in (-\infty; -1) \cup (0; 1)$

$f'(x) > 0: x \in (-1; 0) \cup (1; +\infty)$

2.

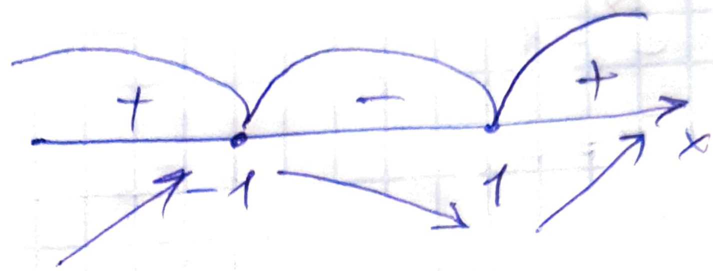
1) $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$



$f(-1) = -1 + 3 + 1 = 3$ – максимум

$f(1) = -1$ – минимум

2) $y = e^{x^2 - 4x + 5}$

$y' = e^{x^2 - 4x + 5} \cdot (2x - 4)$

$e^{x^2 - 4x + 5} \cdot (2x - 4) = 0$

$$\begin{cases} 2x - 4 = 0 \\ e^{x^2 - 4x + 5} = 0 \end{cases}$$

$x = 2$

$f(2) = e$ – минимум

3) $y = x - \operatorname{arctg} x$

$y' = 1 - \frac{1}{1 + x^2}$

$\frac{1}{1 + x^2} = 1$

$x^2 = 0$

$f(0) = 0$ – минимум

3.

1) $f(x) = e^{-x^2}$

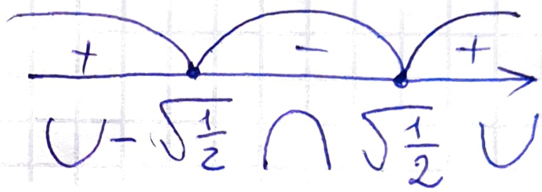
$f'(x) = e^{-x^2} \cdot (-2x)$

$f''(x) = (e^{-x^2})' \cdot (-2x) + e^{-x^2} \cdot (-2) = 2e^{-x^2}(2x^2-1)$

$2e^{-x^2}(2x^2-1) = 0$

$2x^2 - 1 = 0$

$x = \pm\sqrt{\frac{1}{2}}$



$\sqrt{\frac{1}{2}}$ и $-\sqrt{\frac{1}{2}}$ – точки перегиба

и. выпуклости: $\left(-\sqrt{\frac{1}{2}}; \sqrt{\frac{1}{2}}\right)$; и. вогнутости: $\left(-\infty; -\sqrt{\frac{1}{2}}\right) \cup \left(\sqrt{\frac{1}{2}}; +\infty\right)$

2) $y = \cos x$

$y' = -\sin x$

$y'' = -\cos x$

$-\cos x = 0$

$x = \frac{\pi}{2} + \pi k$ – точка перегиба; и. вогнутости $\left(\frac{\pi}{2} + \pi k; +\infty\right)$

и. выпуклости: $\left(-\infty; \frac{\pi}{2} + \pi k\right)$

3) $y = x^5 - 10x^2 + 7x$

$y' = 5x^4 - 20x + 7$

$y'' = 20x^3 - 20$

$x = \pm 1$

интервал выпуклости $(-\infty; -1)$

инт. вогнутости $(-1; 1) \cup (1; +\infty)$