1.

4) $y=\cos\frac{1-\sqrt{x}}{1+\sqrt{x}}=\cos\frac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}$

$y'=-\sin\frac{1-x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}\cdot\frac{\left(-\frac{1}{2}x^{-\frac{1}{2}}\right)\cdot\left(1+x^{\frac{1}{2}}\right)-\left(1-x^{\frac{1}{2}}\right)\cdot\frac{1}{2}x^{-\frac{1}{2}}}{\left(1+x^{\frac{1}{2}}\right)^2}=$

$=-\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\frac{\frac{1}{2}x^{-\frac{1}{2}}\left((-1)\cdot(1+\sqrt{x})\right)-(1-\sqrt{x})}{\left(1+x^{\frac{1}{2}}\right)^2}=$

$=-\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\frac{-1-\sqrt{x}-1+\sqrt{x}}{2\sqrt{x}\left(1+x^{\frac{1}{2}}\right)^2}=-\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}$

$\cdot\frac{-2}{2\sqrt{x}\left(1+x^{\frac{1}{2}}\right)^2}=+\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}$

6) $y=\ln\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}$

$u'=(x+3)^3+(x+1)\cdot 3(x+3)^2$

$v'=3(x+2)^2\cdot(x+4)+(x+2)^3$

$y'=\frac{\left((x+3)^3+(x+1)\cdot 3(x+2)^2\right)\cdot(x+2)^3(x+4)-\left((x+1)(x+3)^3\cdot\right.}{\left((x+2)^3(x+4)\right)^2}$

$\frac{\left.\cdot 3(x+2)^2\cdot(x+4)+(x+2)^3\right)}{}=\frac{(x+3)^2(x+2)^2\left(((x+3)+(x+1)\cdot 3)\cdot\right.}{}$

$\frac{\left.\cdot(x+2)(x+4)-\left((x+1)(x+3)\cdot 3(x+4)+(x+2)\right)\right)}{(x+2)^5(x+4)^2}=$

$$= \frac{x^3+6x^2+10x+10}{x^2+5x+4}$$

3.

1) $y = x^{\ln x}$

$\ln y = \ln^2 x$

$$y' = 2\ln x \cdot \frac{1}{x} \cdot x^{\ln x} = \frac{2\ln x \cdot x^{\ln x}}{x}$$

2) $$y = \frac{(x^3-2)\cdot\sqrt[3]{x-1}}{(x+5)^4}$$

$$\ln y = \frac{\ln(x^3-2)\cdot\frac{1}{3}\ln(x-1)}{4\ln(x+5)}$$

$$y' = \frac{\frac{1}{x^3-2}\cdot 3x^2\cdot\frac{1}{3}\cdot\frac{1}{x-1}}{4\,\frac{1}{x+5}} = \frac{3x^2\cdot(x+5)}{(x^3+2)\cdot 4\cdot 3\cdot(x-1)^{\frac{1}{2}}}\cdot\frac{(x^3-2)(x-1)^{\frac{1}{2}}}{(x+5)^4} =$$

$$= \frac{x^2}{4(x-1)^{\frac{1}{2}}(x+5)^3}$$

4.

1) $e^{xy} - \cos(x^2+y^2) = 0$

$e^{xy}\cdot y'x\cdot y + \sin(x^2+y^2)\cdot(2x+2y\cdot y') = 0$

$e^{xy}\cdot y'x\cdot y + 2x\sin(x^2+y^2) + 2y\sin(x^2+y^2)\cdot y' = 0$

$$y' = \frac{-2x\sin(x^2+y^2)}{e^{xy}\cdot x\cdot y + 2y\sin(x^2+y^2)}$$

2) $x \sin y + y \sin x = 0$

$\sin y + x \cdot \cos y \cdot y' + y' \cdot \sin x + y \cdot \cos x = 0$

$$y' = \frac{-\sin y - y \cdot \cos x}{x \cdot \cos y + \sin x}$$

5.

1) $x = t^3 + t, \ y = t^2 + t + 1$

$$y'(x) = \frac{y'(t)}{x'(t)} = \frac{(t^2+t+1)'}{(t^3+t)'} = \frac{2t}{3t^2} = \frac{2}{3t}$$

2) $x = e^t \sin t, \ y = e^t \cos t$

$$y'(x) = \frac{(e^t \cos t)'}{(e^t \sin t)'} = \frac{e^t \cdot \cos t + e^t \cdot (-\sin t)}{e^t \cdot \sin t + e^t \cdot \cos t} =$$

$$= \frac{\cos t - \sin t}{\sin t + \cos t}$$

6.

$y = e^x, \ x_0 = 0$

$y' = e^x$

$f'(0) = e^0 = 1$

$y = 1(x-0) + 1$

$y = x + 1$ – уравнение касательной

$y = -\frac{1}{1}(x-0) + 1 = -x + 1$ – уравнение нормали

7.

1) $y = -x \cdot \cos x$, $y'' = ?$

$y' = -\cos x + \sin x$

$y'' = \sin x + \sin x + x \cdot \cos x = 2\sin x + x\cos x$

2) $y = e^{2x}$, $y^{(v)} = ?$

$y' = e^{2x} \cdot 2$

$y'' = 2e^{2x} \cdot 2 = 4e^{2x}$

$y''' = 4e^{2x} \cdot 2 = 8e^{2x}$

$y^{iv} = 8e^{2x} \cdot 2 = 16e^{2x}$

$y^{v} = 32e^{2x}$

3) $y = \ln(1+x)$, $y^{(n)} = ?$

$y' = \frac{1}{1+x}$

$y'' = \frac{-1}{(1+x)^2}$

$y''' = \frac{2}{(1+x)^3}$

$y^{iv} = \frac{-6}{(1+x)^4}$

$y^{n} = \frac{(-1)^{n-1}(n-1)!}{(x+1)^n}$