1 Topic 1

Classification of integer binary quadratic forms up to change of variables.

1.1 Possible Questions

1. Prove If $p \equiv 1(mod 4)$, then $\exists x, y \in \mathbb{Z}$ such that $x^2 + y^2 = p$
2. Prove the spectral theorem for symmetric matrices.
3. Show how the discriminant of a binary quadratic function can be computed from the cells of a topograph.

2 Topic 4

Find all integer solutions to $x^2 + xy + y^2 = n$, or a similar equation.

2.1 Possible Questions

1. Show that Mordell's equation ($y^2 = x^3 + k$) for certain values of k have no integer solution. For example, k = 7, -5, -6.
2. Show how to determine if a Mordell's equation has an integer solution.
3. Given n, how can we determine the number of ways in which it can be written as the sum of two squares?
4. Show that n is properly representable as a sum of two squares if and only if the prime factors of n are all of the form 4k+1 (except for the prime 2).

3 Topic 5

Study recurrence sequences, such as the Fibonacci sequence, in modular arithmetic.

3.1 Possible Questions

1. Suppose $F_b \equiv 0$ and $F_{b+1} \equiv 1 mod(p)$. Prove $F_{b+i} \equiv F_i mod(p)$
2. Characterize/investigate the Pisano period, and prove that (with one exception) it is always even.