BU5526 - Portfolio Analysis
Lecture Notes

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Contents

Chapter 1

Introduction

1.1 Asset Pricing

1.1.1 Asset

Something valuable that an entity owns, benefits from, or has use of, in generating income.

1.1.2 Asset Pricing

Needed to buy/sell at a fair price.

- Obtained by discovery process:
  - Demand and supply forces.
- The price of an asset is simply the current value of its cash-flows.

$$PV = \frac{FV_1}{(1+r)^1} + \frac{FV_2}{(1+r)^2} + ... + \frac{FV_n}{(1+r)^n}$$

- What we need:
  - Prediction of cash flows;
  - Discount rate.

1.1.3 How to price a stock:

- Dividend Discount Model;
- FCF Model;
- Multipliers and Comparable approach.

Being able to estimate the required **rate of return** is everything you need to calculate the **value of any asset** - once you have predicted **future cash-flows**, including:

- Estimating the value of a stock;
- Estimating the value of a firm.

1.2 Portfolio Management

Prudent administration of **investable** (liquid) assets, aimed at achieving an **optimum risk-reward ratio**.

1.3 Mathematical Concepts

1.3.1 Mean

An average of different observations.

- Useful to describe a population;
- **Arithmetical**: sum of observations divided by the number of observations (if with equal weights).

$$\overline{X} = \frac{X_1 + X_2 + ... + X_n}{n}$$

  or

$$\overline{X} = W_1 \times X_1 + W_2 \times X_2 + ... + W_n \times X_n$$

- **Geometrical**: n root of the product of observations.

$$\overline{X}_G = \sqrt[n]{X_1 \times X_2 \times ... \times X_n}$$

  or

$$\overline{X}_G = \sqrt[1]{X_1^{W_1} \times X_2^{W_2} \times ... \times X_n^{W_n}}$$

There is a reason to use arithmetical or geometrical mean.

- **Equally weighted**: when all the observations have the same importance.
- **Unequally weighted**: different importance for different observations.

1.3.2 Variance

Seen as an extension of the mean.

- Dispersion to the mean (higher/lower):
  - Average of differences from the mean.

$$\sigma_x^2 = \frac{(X_1 - \mu_X)^2 + (X_2 - \mu_X)^2 + ... + (X_n - \mu_X)^2}{n}$$

1.3.3 Important Statistical Concept

- The formula is correct if we possess all the data on the population;
- If we only have a sample, we need to reflect "impreciseness" by removing "one degree of freedom".

$$\sigma_x^2 = \frac{(X_1 - \mu_X)^2 + (X_2 - \mu_X)^2 + ... + (X_n - \mu_X)^2}{n-1}$$

This will **always** be the case in finance.

1.3.4 Standard Deviation

$$\sigma_X = \sqrt[2]{\frac{(X_1 - \mu_n)^2 + (X_2 - \mu_n)^2 + ... + (X_n - \mu_n)^2}{(n-1)}}$$

1.3.5 Skewness

Brings back the sign. A positive skewness means more positive-value and reversely.

$$\sigma_X^3 = \frac{(X_1 - \mu_n)^3 + (X_2 - \mu_n)^3 + ... + (X_n - \mu_n)^3}{n}$$

1.3.6 Kurtosis

Outweighs extremes - dropping the sign.

$$\sigma_X^4 = \frac{(X_1 - \mu_n)^4 + (X_2 - \mu_n)^4 + ... + (X_n - \mu_n)^4}{n}$$

A large Kurtosis means a lot of extreme values.

- To get a **meaningful estimate: excess** kurtosis needs to be provided.
- Kurtosis of a normal distribution is 3.

Note: Unbiased equations of these two indicators are slightly more complex, but computer packages provide them automatically.

1.4 Assumptions of Mean-Variance Analysis

- Allows describing different assets simply.
- Assumes returns are normally distributed.

1.4.1 Normal distribution

- Its mean and median are equal;
- It's defined by two parameters, mean and variance;
- It's defined around its mean with:
  - 68% of observations within $\pm$ 1σ of the mean.
  - 95% of observations within $\pm$ 2σ of the mean.
  - 99% of observations within $\pm$ 3σ of the mean
- Returns are not normally distributed.
  - Skewed: not symmetric around the mean.
  - Characterized by high probability of extreme event.

1.5 Returns on Financial Assets

1.5.1 Holding period return

Return from holding an asset for a specific period of time.

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}}$$

Capital gain + Dividend yield
Holding period returns = compound returns

$$R = [(1 + r_1) \times (1 + r_2) \times (1 + r_3)] - 1$$

1.5.2 Geometric Mean Return

$$\overline{R} = \sqrt[T]{(1 + r_{i_1}) \times (1 + r_{i_2}) \times ... \times (1 + r_{i_T})} - 1$$

1.5.3 Annualized Return

These returns should be capitalized.

- Such as:
  $$r_{annual} = (1 + r_{period})^C - 1$$
  with C being the number of periods in a year.

1.5.4 Portfolio Return

- When several assets are combined into a portfolio, we can compute the portfolio return.
- Weighted average of the returns of individual assets

$$R_p = W_1 \times R_1 + W_2 \times R_2$$

1.5.5 Historical and Expected Returns

- Computed from historical data.
- What the investor expects to earn.
- Historical returns are different from expected returns.

1.5.6 Ways of Calculating Expected Returns

- Gut feeling;
- Modelling - calculated from a formula.

1.5.7 Standard deviation - Volatility

- Historical standard deviation;
- Defined as **risk** of equity return.

1.5.8 Portfolio Variance

- Cannot simply add up two variances.
- Covariance:

$$cov(X,Y) = E(XY) - E(X) \times E(Y)$$

  - The more the two assets move in the same way, the higher the covariance.
  - A negative covariance means the assets move in opposite directions.
- Variance:

$$\sigma_v^2 = wCw^T$$

1.5.9 Correlation and Portfolio Risk

The correlation among assets determine the portfolio's risk.

- Is a measure of tendency for N investments to act similarly;
- Can range from -1 and $+1$.

$$\rho_{ij} = \frac{cov(R_i, R_j)}{\sigma_i \sigma_j}$$

Chapter 2

The Optimal Portfolio

2.1 Portfolio

2.1.1 The Portfolio Perspective on Investing

One of the biggest challenges faced by individuals and institutions is to decide on how to invest for future needs. Should they invest in individual securities, or should they take a portfolio approach?
A portfolio and evaluating individual securities in relation to their contribution to the investment characteristics is important to achieve a "good" and "safe" return.

2.1.2 Diversification: Avoiding Disaster

Portfolio diversification helps investors avoiding disasters. It is of our utmost priority to balance investment weights as a way of having a good risk-return ratio.
On the other hand, it does not mean that, by reducing risk the portfolio will have reduced profits.

- The portfolio approach provides investors with a way to reduce the risk associated with their investment.
- Investors can choose the risk-return trade-off they prefer.
- A lower risk does not necessarily mean lower profits.

2.1.3 Reducing Risk

A Portfolio generally offers equivalent expected returns with lower overall volatility.
How to calculate its standard deviation?

- An interesting feature of combining assets is that:

- – While the combined return is the weighted average of individual returns, the combined variance is impacted by the covariance between securities.
- Covariance is how securities move together.

Variance of a Portfolio

$$\sigma_p{}^2 = wCw^T$$

or

$$\sigma_v{}^2 = w_1{}^2\sigma_1{}^2 + w_2{}^2\sigma_2{}^2 + 2w_1w_2c_{12}$$

Remember that using correlation instead of variance, we can reformulate the variance equation:

$$\sigma_v{}^2 = w_1{}^2\sigma_1{}^2 + w_2{}^2\sigma_2{}^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$$

This equation shows that as long as $\rho_{12} is < 1$ adding more assets will reduce the overall variance of the portfolio.

Standard Deviation

$$\sigma_p = \sqrt{\sigma_p{}^2}$$

2.1.4 Correlation and Portfolio Risk

- A major reason that portfolios can effectively reduce risk is that combining securities whose returns do not move together provides diversification.
- The less correlated the assets, the better the risk-return trade-off obtained within a portfolio.

2.1.5 Return and Risk of a Portfolio

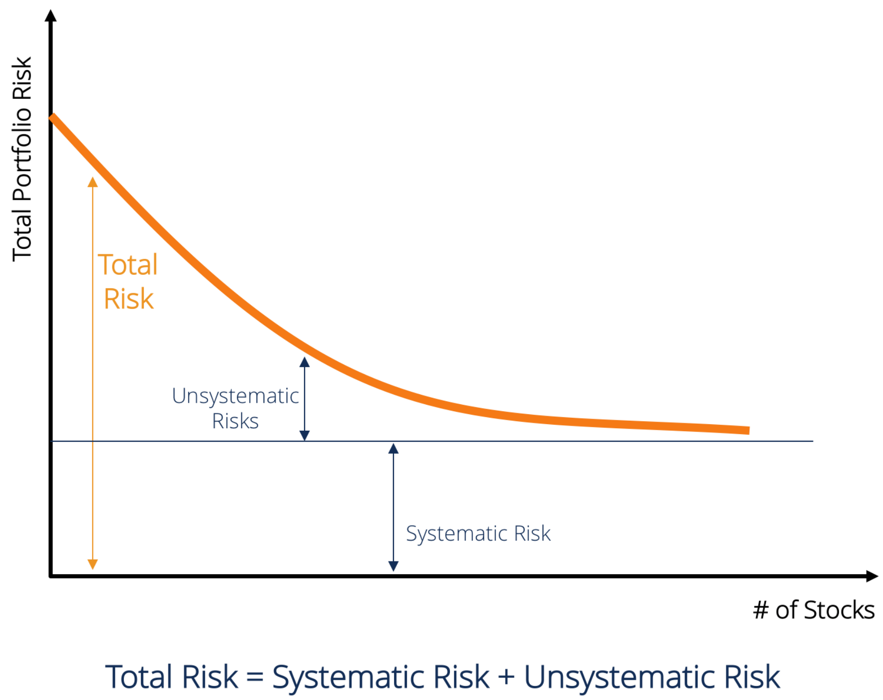
- This is the fundamental idea of **diversification**;
- Because assets do not move perfectly together (correlation = 1), combining several assets will reduce the overall variance;
- Assets don't need to be negatively correlated, just not perfectly correlated (correlation < 1)
- The more uncorrelated the assets, the greater the risk-return trade-off.

2.1.6 Avenues for Diversification

- Diversify with asset classes;
- Diversify with index funds;
- Diversify among countries;
- Buy insurance;
- Buy put options;
- Evaluate the assets.

2.1.7 Diversification: Not necessarily Downside Protection

- It does not mean that diversification eliminates the entire risk.
- It removes only unsystematic (idiosyncratic) risk.
  Total Risk = Systematic Risk + Unsystematic Risk



Unsystematic Risk Is the risk specific to a company or industry. It is also known as diversifiable risk. It can be reduced through diversification.

Systematic Risk Is the risk specific to the entire market. Also known as undiversifiable risk, affecting the overall market.

- The benefit of diversification varies over time.
  - Correlation among assets might change;
  - Diversification does not protect against a generalized decrease in returns and increase in risk.
- When all assets move together, like during a financial crisis, diversification benefits shrink.
  - Hedging against risk becomes more difficult.

2.1.8 The Emergence of Modern Portfolio Theory

- The concept and intuition of the benefit of diversification has been around for a long time.
- However, its modern concept is theorized more recently.
- The main conclusion is that: **investors should not only hold portfolios but also focus on the correlation among the securities included.**

2.2 Efficient Portfolio Frontier

2.2.1 Investment Opportunity Set

Include as many assets as possible, from different sectors.

2.2.2 Minimum Variance Frontier

- We create a portfolio of assets;
- We are interested in pushing the frontier onto the north-west:
  - Minimizing the variance for a given return.
- We calculate the combinations with different weights that minimize the variance for a given return.
- If we combine all the available assets and select the weights giving the lower variance for a given return, we end up with the **minimum variance frontier**.
- All portfolios on this frontier display a lower variance and a higher return than any individual asset.

Note: The frontier uses all the available assets. An efficient portfolio is *any* portfolio on the minimum variance frontier.

2.3 The Efficient Portfolio

2.3.1 Risk-free Asset

- To select the most efficient portfolio on the frontier, we need to add a risk-free asset.
- A risk-free asset will allow us to:
  - Find the most efficient portfolio;
  - Draw the capital allocation line (CAL);
  - Create any risk-return combination for investors.
- The efficient **investor portfolio** will always be a **combination** of:
  - The risk-free asset;
  - The efficient portfolio.

2.4 Capital Allocation Line

- A capital allocation line is a line that allocates capital between two assets;
- We will allocate between a risk-free and a risky asset;
- We will still think in terms of risk and return trade-off;
- We need two elements: an **intercept**, and a **slope**.

2.4.1 A Risk-free Asset

In our case, assume a portfolio of two assets, a risk-free asset and a risky asset. Expected return can be determined as:

$$E(R_p) = W_1 R_f + (1 - W_1) E(R_i)$$

Because the risk-free asset has 0 risk, its variance is equal to zero, hence, the variance of this portfolio can be calculated as:

$${\sigma_p}^2 = (1 - W_1)^2 {\sigma_i}^2$$

And volatility:

$$\sigma_p = \sqrt{(1 - W_1)^2 {\sigma_i}^2} = (1 - W_1)\sigma_i$$

If we combine the portfolio return and standard deviation formula, we can rewrite the expected return in terms of risk.
The expected return of a portfolio that mixes a risk-free asset and the optimal portfolio is based on two elements:

$$E[R_p] = R_f + \frac{E(R_i) - R_f}{\sigma_i}\sigma_i$$

- The risk-free rate: R_f
  - It is the Y-intercept;
  - You cannot earn less than that.
- The market price of risk: $\frac{E(R_i)-R_f}{\sigma_i}\sigma_p$
  - It is the slope of the capital allocation line (CAL);
  - The highest, the best: plus it is high, plus the risk is profitable.

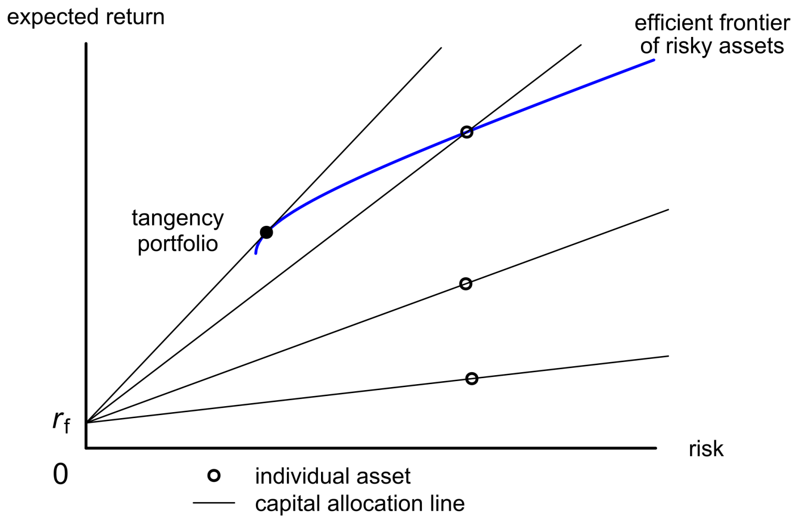
2.4.2 The Capital Allocation Line (CAL)

- The combination between a risky asset and a risk-free asset is called the **capital allocation line**;
- The risk-free rate does not change;
- Maximizing this angle maximizes your benefits for the risk you are taking.
- You do not control the market price of risk;
  - It is priced by the market.
- At an investor level, you can vary your risk, by changing your weights in the portfolio.

2.4.3 CAL and Optimal Portfolio

- Using the CAL, we will now find **the** optimal portfolio;
- It is one of the portfolios on the efficient frontier;
- By using the risk-free asset, you maximize your risk-return ration.
- Mathematically, it is the point that is tangent between the efficient frontier and the CAL.
- You cannot achieve a point better than one on the CAL;
  - This would only be by changing the set of assets.

- By holding a risk-free asset, an investor can achieve a point of higher return than the Markowitz Efficient Frontier with the same risk-level.
  - Negative weight on the risk-free asset (Y).
- The CAL and the optimum portfolio evolve over time.
  - If the risk-free asset return changes, it will affect the CAL Y-intercept;
  - Hence, it will modify the CAL, and which portfolio is optimum.



2.5 Optimal Investor Portfolio

All investors will choose a **combination** of the risk-free and optimum portfolio. They will just change their weightings. But how do we know which portfolio will be chosen by a **specific investor**?

2.5.1 The concept of Risk Aversion

The choice of portfolio will differ across individuals because each individual has a different risk aversion.

- Risk-seeking: utility increase with uncertainty.

- The individual will preferred a gamble with an expected value of £45 than a certain £50;
- Lottery and casinos.

- Risk-neutral
  - The individual is indifferent between the gamble or a £50 guaranteed income;
  - A billionaire may be indifferent in this case.

- Risk-adverse
  - The investor will prefer a certain value of £50 than an expected value of £45;
  - The risk-return trade-off is an indicator of risk-aversion;
  - Historical data supports risk-aversion: higher returns, come with higher risks.

2.5.2 Utility Theory

- The **utility** he derives from the guaranteed income of £50 greater than the **utility** he derives from the alternative.
- Individuals are different in their preferences.
  - All risk-averse investors will not rank their investments in the same manner;
  - With a guaranteed outcome of £40, some may find it inadequate.
- We can calculate the utility of an investor.
- This is not its return: it combines return and risk.
  - Utility is a function of return for each individual;
  - Individuals prefer higher utility.
- We usually assume that investors are adverse to risk.
- There are plenty of ways to **modelize** utility.

The formula

$$U = E(r) - \frac{1}{2}A\sigma^2$$

- U = Utility of an investment;
- E(r) = Expected Return;

- A = Measure of risk tolerance or risk aversion - can be either negative or positive;
  - Risk adverse investor: $A > 0$
  - Risk neutral investor: $A = 0$
  - Risk seeking investor: $A < 0$
- σ^2 = Variance or risk.
- Higher returns = higher utility;
- Higher variance reduce/increase the utility, depending on A;
- A risk-free asset generates the same utility for all individuals.

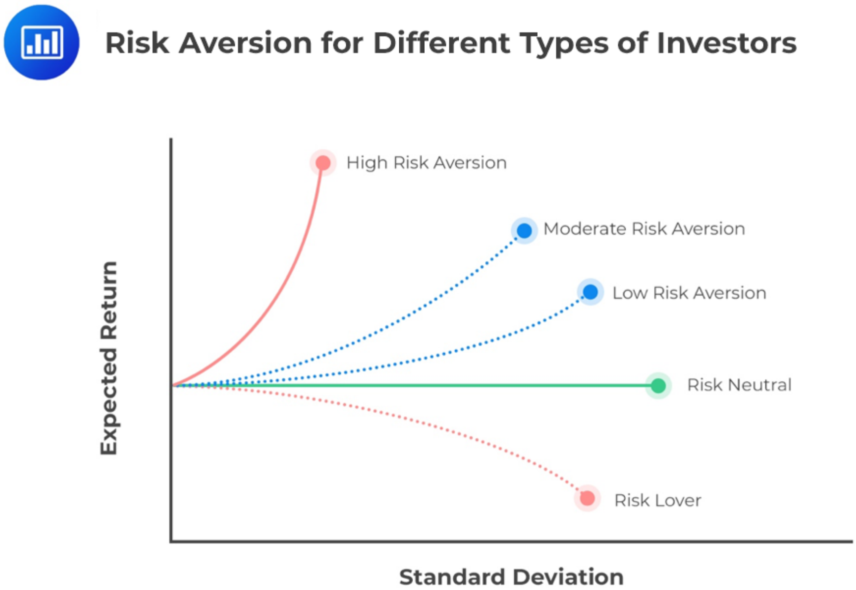
2.6 Indifference Curve

An indifference curve plots the combination of risk-return pairs that an investor would accept to maintain a given level of utility. For each investor there is an infinity of indifference curves, but each indifference curve for each investor never cross over each other.

- Represent the trade-off between risk and returns, with the *same utility.*
- All risk-adverse investors will prefer curves on the north-west.
- Risk-adverse curves are convex because as risk increases, an investor needs even higher returns to compensate.
- The **slope** of curves is the **same for one investor** and different among investors.

2.7 Optimal Investor Portfolio

- To know which combination is optimal for a given investor, we will use its indifference curve.
- The optimal investor's portfolio is the one which is tangent with the indifference curve.
  - It gives them the highest **achievable** return.
  - Lower or higher curbs, respectively, give:
    * A lower utility;
    * An unachievable utility.



- There is a different optimal portfolio for each investor - depending on its risk-aversion.
  - All the points on CAL(P) are achievable;
  - The selection will depend on the investor's preference.

2.8 The Two-Fund Separation Theorem

- Efficient Frontier;
- The Capital Allocation Line;
- Indifference Curve;

These concepts can be synthesized as a key theorem of modern portfolio theory.

- **The two-fund separation theorem:** we can divide an investor's problem into two distinct steps.
  - The investment decision;
  - The financing decision.
- For the investment decision: all investors, regardless of taste, risk preferences and initial wealth, will hold a combination of two funds:

  - A risk-free asset;
  - The optimal portfolio of risky assets.

- **Note:** All investors should lie on CAL(P).
- For the financing decision: each investor chooses the appropriate weight of risk-free and risky portfolio (P).
- The utility indifference curve of each investor will determine the investor's allocation to risky assets.
  - Portfolios before the optimal risky portfolio are obtained by lending at the risk-free rate.
  - Portfolios beyond the optimal risky portfolio are obtained by borrowing at the risk-free rate.

2.9 Summary of the Optimal Portfolio Choice

- Build the minimum variance frontier using all the assets available;
- Calculate and draw the capital allocation line (P) that is tangent to the minimum variance frontier.
- Use the investor's utility curves to obtain the portfolio that lies on the utility curve and on the capital allocation line (P).

Chapter 3

Capital Asset Pricing Model

3.1 The Capital Asset Pricing Model

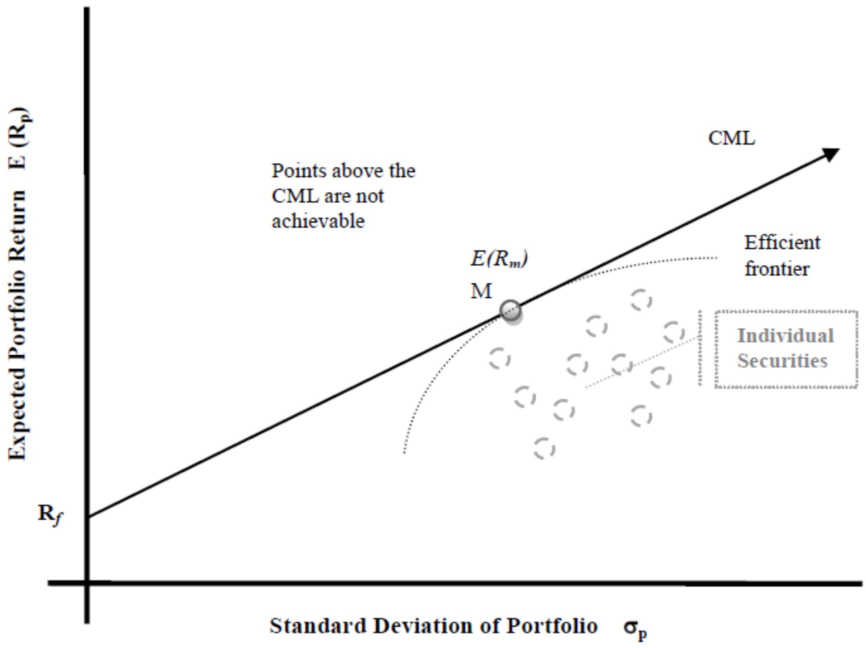
- In the previous lectures, we demonstrated the diversification effect and the two-fund theorem;
- We know that all investors should possess the optimal portfolio;
- We will draw upon these results to estimate the expected return of one asset.
- The Capital Asset Pricing Model draws upon the key results of the portfolio approach;
- It gives the expected return of an asset, based on the hypothesis that all the agents hold the optimal portfolio;
- This expected return can then be used to value the asset.
- In the portfolio theory, we show that investors should lie on the CAL;
- In practice, investors should hold the market portfolio - Capital Market Line (CML).

3.2 The Capital Market Line

What is the **Market Portfolio**?

- Theoretically, the *market portfolio* includes all risky assets or anything that has value;
- But it is practically limited to assets that are tradable and investable;
- Typically, a local or regional stock market index is used as a proxy, because of its visibility to local investors;

- The S&P 500 Index is commonly used by analysts as a benchmark for market performance throughout the US;
- The *capital market line* is a special case of the capital allocation line in which the risky portfolio is the market portfolio.



- Using the *same* equations as before, we can calculate the expected return and standard deviation of an investor lying on the CML;
- We find the same results, but they are now based on a real market portfolio.
- The expected return of a portfolio on the CML is:

$$E(R_p) = w_1 R_f + (1 - w_1)E(R_m)$$

$$\sigma_p = (1 - w_1)\sigma_m$$

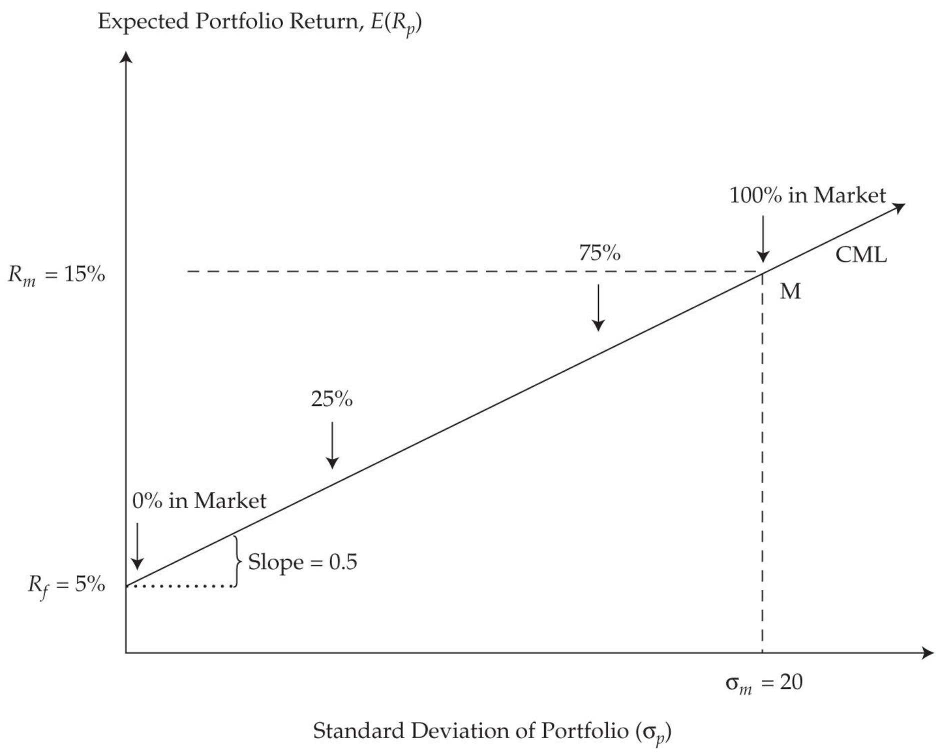
- By substitution, $E(R_p)$ can be expressed in terms of σ_p, and this yield the equation for the CML:

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \times \sigma_p$$

3.3 Derivation of the CAPM

- We now assume that all investors have access to the same market portfolio on the CML;
- The market portfolio is an efficient portfolio and contains all risky assets in the conomy with optimal weights;
- Each investor holds an optimal portfolio of risk-free assets and market portfolio that reflects his risk attitude.
  - Graphically:

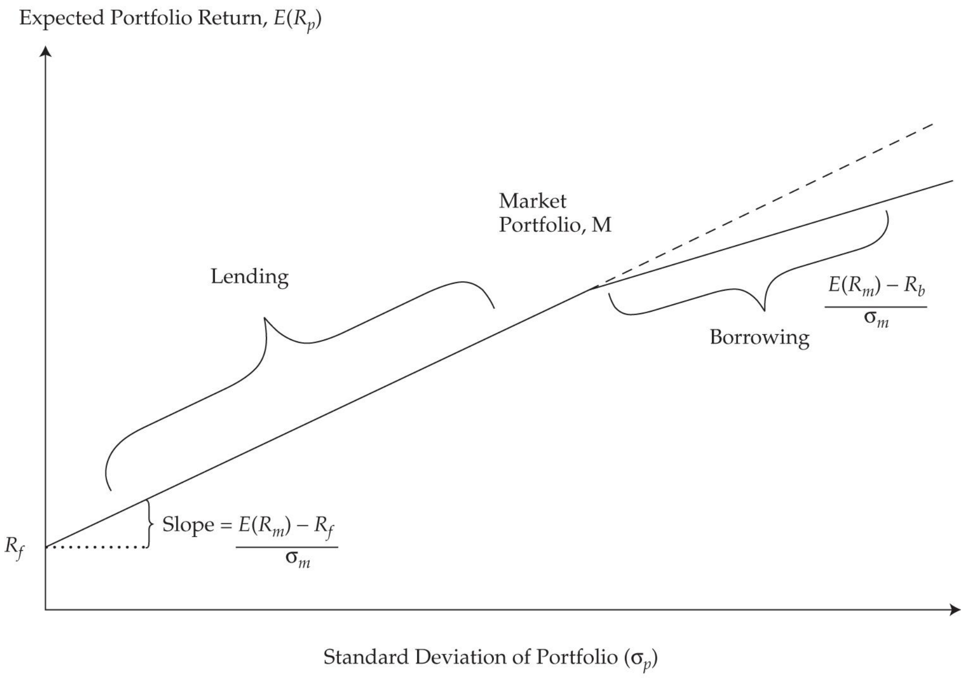
EXHIBIT 6-4 Risk and Return on the CML



3.3.1 Leveraged Portfolios

An investor can go above 100% in the market portfolio by borrowing. Meaning that there is a negative investiment in the risk-free asset. The lending and borrowing rate can be different.

EXHIBIT 6-5 CML with Different Lending and Borrowing Rates



3.3.2 Derivation of the CAPM

- Accessing the market portfolio means that we have included a very large number of securities;
- A portfolio's total risk can be categorized into two elemetens;
  - Systematic Risk;
  - Idiosyncratic Risk;

3.3.3 Portfolio of Many Risky Assets

- As long as we introduce assets with a correlation < 1 we reduce the portfolio's risk;
- To see the share of specific and systematic risk, we can rewrite the variance equation of an equally-weighted protfolio, such as:

$$\sigma_{p^2} = \sum_{i=1}^{n} w_{i^2}\sigma_{i^2} + \sum_{i,j=1,i\neq j}^{n} w_i w_j Cov(i,j)$$

- As n becomes larger, the first term on the right side, becomes smaller and smaller;

    - The contribution of one asset's variance to the portfolio variance gradually becomes negligible.

- The second term, however, approaches the average covariance as n increases.
- For portfolios with a alrge number of assets, covariance among the assets accounts for almost all of the portfolio risk;
    - Only the market, or **systematic** risk, remains.

The access to the market protfolio menas that:

- all non-systematic risk is diversified and they only endorse the systematic risk;
    - Systematic risk = market risk;
    - Idiosyncratic or specific risk = the security risk;
- In finance, you are only paid for the risk you take;
    - Investors are compensated only for systematic risk.

3.3.4 Derivation of the CAPM

- What if an asset would yield more than its sensitivity to the systematic risk?
- From the CAPM perspective, all investors hold the market portfolio, therefore are not affected by that specific risk;
- Investors can have return by holding non-diversified portfolios;
    - However, total risk of the portfolio will increase.
- As the diversification is a matter of investors' choice, the portfolio should be assessed by their sensibility to systematic risk;
    - How much they move with the market - called **beta**.
- Flip side: any investor should hold the optimal portfolio;
    - Otherwise, they are exposed to the specific risk;
    - Investors **must** diversify their portfolio to get a fair return.
- So, we now know that an asset only yield its **sensitivity to the market risk**;
- This is its **beta**: β_i

- The CAPM derives the formula of the beta.
- To estimate the price of an additional securirty, we should know how this secuirty modifies the risk of the portfolio **P**;
- When a bit more of an asset x is added to the market portfolio, investors only care how much risk asset x adds to the risk of the market portfolio.
  - Investors care about the change in the variance of the whole portfolio rather than individual variance.
- The contribution of an asset to the riskiness of the market portfolio, is the new measure of riskiness for each component of an efficient portfolio.
- The risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios;
- For any individual asset x, we can calculate the reward-to-risk ratio:

$$\frac{E(r_x) - r_f}{\sigma_{x,M}}$$

- The numerator is the contribution of asset x to the expected risk premium of the market portfolio.
  - It measures the amount of risk increased y adding an asset.
- The denominator is the contribution of asset x to the risk of the market portfolio.
  - There is no idiosyncratic risk.
- For the market portfolio, we have:

$$\frac{E(r_M) - r_f}{\sigma_{M^2}}$$

- This specific ratio is known as the market price of risk;
  - Numerator - market risk premium;
  - Denominator - risk of the market;
- The ratio shows amount of market risk premium at the market risk level;
  - It scales the net return of the portfolio by its risk.

- At equilibrium, all assets must offer the same risk-return trade-off;
- This reflects the market price of risk - constant across securities;
- Re-writing the equation for any security x:

$$E(R_x) = r_f + \frac{\sigma_{x,M}}{\sigma_{M^2}} \times (E(r_M) - r_f)$$

- This gives us the expected return of the security x;
- It depends on **three elements**:
  - The level of risk-free rate;
  - The covariance of the asset x with the market, scaled by the overall market variance: **sensibility to market risk**;
  - The expected return of the market portfolio (net of the risk-free rate);
- It does **not** depend on the security's own risk.

3.4 Calculation and Interpretation of the Beta

- Beta is the sensitivity to market risk;
  - It is the second term of the equation as is calculated as:

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_{m^2}}$$

- It captures an assets' systematic risk, or the portion of an asset risk that cannot be eliminated by diversification;
  - The formula of the beta directly stems from portfolio theory - thus its hypotheses.
- Everything else being constant (market return, market variance and risk-free rate), the return of a security only depends on its **covariance** with the market portfolio;
- The variance of an asset x does not enter the expected return formula;
- This demonstrates that **only systematic risk** matters.
- As variance is always positive, we can easily interpret the **sign**of the beta;
- A **positive** beta indicates that the return of an asset follows the general market trend;

- A **negative** beta shows that the return of an asset generally follows a trend that is opposite to the market.
- Rewriting the beta in terms of correlation, we can deepen its interpretation;
- The beta is the correlation multiplied by the ratio of the security and the market volatitlity;
- It is not the sole correlation;
- All things being equal, the higher the correlation, the more β tends to 1.
- A risk-free asset beta is zero;
- The beta of the market is one;
  - The average beta of stocks in the market is also one;
- Beta is not bounded;
- Even if a stock has a weak correlation with the market, if its volatility is high, compared to the market, the beta will be high;
- The sign of the beta, does not predict the sign of the return;
  - Even with a negative beta, both the market and the asset can have positive returns;
- A beta < -1 means that the asset moves in an opposite direction **and** in a greater way than the market index;
- A beta between -1 and 0 means that the asset moves in an opposite direction, **but** in a lesser way than the market;
- A beta between 0 and 1 means that the asset moves in the same direction, **but** in a less extent than the market;
- Beta > 1 means that the asset moves in the same direction **and** in a greater way than the market;
- **Conclusion** - The CAPM shows that the primary determinant of expected return for a security is its beta or systematic risk.
- The CAPM is one of the most significant innovations in portfolio theory;
- The model is simple, yet powerful; intuitive, yet profound; and uses only one factor, yet is broadly applicable;
- It resolves the fundamental question of the cost of equity;

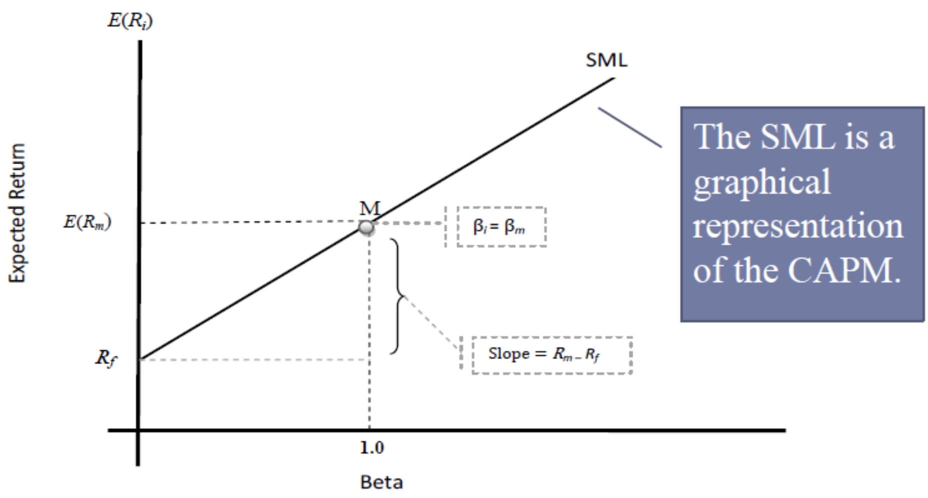
3.5 The Security Market Line

We can represent the equation of the CAPM graphically.

$$E(R_i) = R_f + \beta_i[E(R_m) - Rf]$$

- This is called the secuirty market line (SML):
  - Beta on the x-axis;
  - Expected return on the y-axis;
  - Rf is the y-intercept;

  This line represents the relation between the expected return and the beta of an asset.



3.6 Portfolio Beta

It can be calculated from the expected return equation.

- Portfolio beta is the weighted sum of the betas of the component secuirties:
- $$\beta_p = \sum_{i=1}^{N} W_i \beta_i$$
- The portfolio's expected return is given by the CAPM;
- The return of any portfolio is now easily computable;

3.7 Application of the CAPM

- The CAPM is used extensively in practice;
- There are three main applications:
  - To give an estimate of expected return;
  - To evaluate the performance of a portfolio;
  - To select securities.

3.7.1 Estimate of Expected Return

- The price of an asset is the sum of all the future cash-flows, discounted at the required rate of return;
- The CAPM provides an estimate of the required rate of return;
- For instance, for the FCF model
- $$V_0 = \sum_{t=1}^{\infty} \frac{CF_t}{(1+r)^t}$$

3.7.2 Portfolio Performance Evaluation

- How to evaluate the performance of a portfolio (manager)?
- Active management is more costly: does it yield a better perfomance?
- Four ratios related to the CAPM are generally used in performance evaluation.

Sharpe Ratio

Sharpe Ratio focuses on total risk:

$$\frac{R_p - R_f}{\sigma_p}$$

- It scales the excess return of the portfolio on its standard deviation;
- How much premium extracted for the risk taken;
  - The higher, the better!

M-Squared

M^2 is an extension of the Sharpe ratio:

$$(R_p - R_f)\frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

- M^2 gives rankings that are identical to those of the Sharpe ratio;
- A portfolio that matches the performance of the market will have an M^2 of zero, whereas a portfolio that outperforms the market will have an M^2 that is positive;
- They are easier to interpret, however, because they are in percentage terms;
- We can see whether the portfolio beats the market on a risk-adjusted basis.

Treynor Ratio

The Tryenor ratio draws on the CAPM beta:

$$\frac{R_p - R_f}{\beta_p}$$

- It gives the excess return (risk premium), scaled by the systematic risk taken;
- The idea is that only the systematic risk should be taken into account, as idiosyncratic risk is already diversified.

Jensen's Alpha

- Jensen's alpha also draws on the CAPM beta;
- It gives the excess return obtained with this portoflio, that is not predicted by the CAPM.

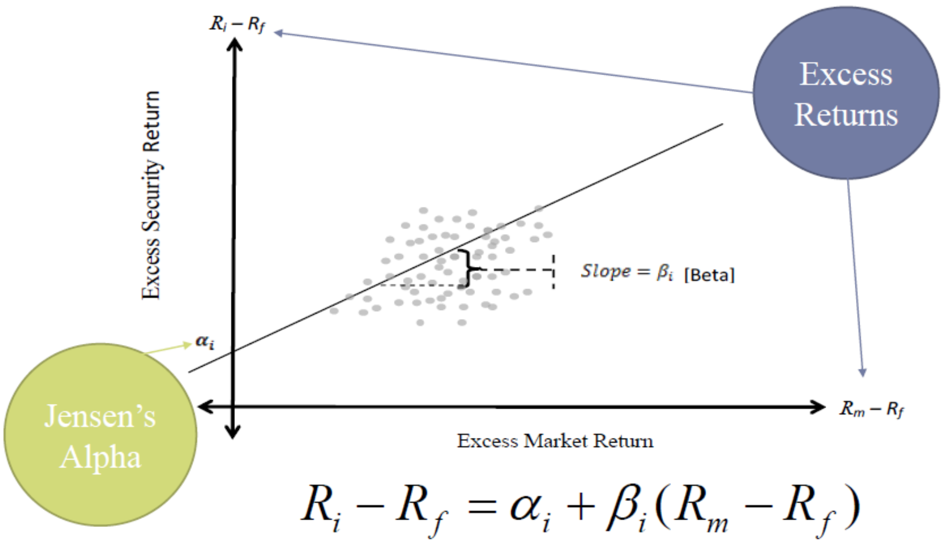
$$\alpha_p = R_p - E[R_p]$$

$$\alpha_p = R_p - [R_f + \beta_p(R_m - R_f)]$$

- "Abnormal return" - can be attributed to the performance of the manager.
- Jensen's alpha can be represented graphically:

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

- This is called the **Security Characteristic Line (SCL)**
  - On the y-axis: the security excess return;
  - On the x-axis: the market excess return.
- Alpha is the y-intercept and beta the slope;
- While the SML is for the market portfolio, the SCL gives the relative performance of a portfolio.



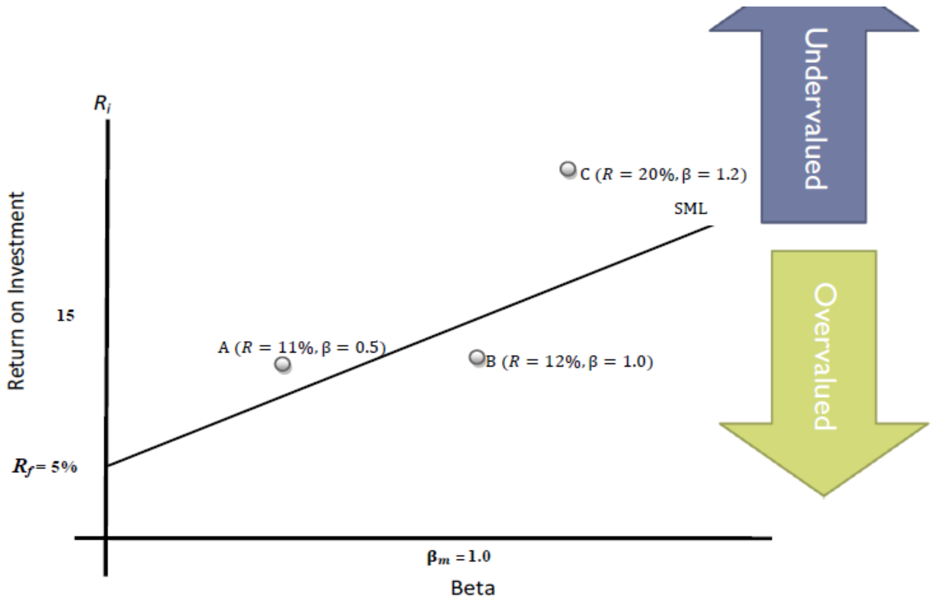
Measures of Portfolio Performance

- We now have 4 measures of portfolio performance;
- Two of them draw on the total risk, while two draw on the portfolio beta;
- They offer different perspectives on the performance of a fund:
  - They are useful to synthesise different infromation.

3.7.3 Security Selection

- When we built the CAPM, we assume that investors hold common beliefs and expectations;
- Here, we introduce heterogeneity in beliefs of investors;
  - Tehy do not have similar expectations.

- Investors are price-takers, it is assumed, that such heterogeneity does not significantly affect the market price of an asset;
- Differences in beliefs could result in an **investor-estimated** return that is different from the CAPM calculated return;
- Jensen's alpha can be used for security selection.
- If alpha is different from zero, it means that there is a residual (positive or negative) return which is not priced in the security by the CAPM;
- This is due to different expectations between the investor and the overall market.
- From this perspective, an investor shoud:
  - **Buy** an asset with a positive alpha, because it is undervalued;
  - **Sell** an asset with a negative alpha, because it is overvalued.
- This is done using the **Security Market Line**;
- Investors can then plot their own expectations on the SML graph:
  - Above the SML - higher returns for the same β - undervalued (**buy**)
  - Below the SML - lower returns for the same β - overvalued (**sell**)



3.7.4 Constructing a Portfolio

- The more practical way to start is to hold a broad protfolio security;
  - The S&P500 is available for trade and offers the largest US corporations.
- Any security not included in the S&P500 can be evaluated to determine whether it should be included.
- To make the decision, one computes the alpha of the security.
- A security with a positive alpha means additional returns in comparison with the market portfolio - should be included.
  - A security with a negative α can be short sold.
- Similarly, one can compute the alpha of the securities within the S&P500 and decide which to keep;
- If α indicates higher returns compared to the market portfolio, which wheight should be given to the security?
- We aim to maximize **risk-adjusted returns**:
  - Securities with a higher alpha should have a higher weight;
  - Securities with greater nonsystematic risk should be given less weight in the portfolio.
- The information ratio:

$$\frac{R_i - E(R_i) = \alpha_i}{\sigma_{\alpha_i}}$$

- It measures the abnormal return per unit of risk (risk-adjusted returns) added by the security to a well-diversified portfolio;
- The larger the information rato is, the more valuable is the security to be in the portfolio.

3.8 Beyond the CAPM

3.8.1 Limitations of the CAPM

The CAPM is a powerful tool to estimate an expected return for any asset. However it has its limits:

- Underlying assumptions;
- Theoretical limitations;
- Practical limitations;
- We can identify 8.

Single-factor model

- Only systematic risk or beta risk is priced in the CAPM;
- Thus, the CAPM states that no other investment characteristics should be considered in estimating returns;
- As a consequence, it is prescriptive and easy to understand and apply, although it is very restrictive and inflexible.

Single-period model

- The CAPM does not consider multi-period implications or investment objectives of future periods, which can lead to myopic and suboptimal investment decisiions;
- The only horizon is the one of expected returns;
- A single-period model like the CAPM is unable to capture factors that vary over time adn span several periods.

Homogeneity in investor expectations

- The CAPM assumes that homogeneity exists in investor expectations for the model to generate a single optimal risky portfolio (the market) and a single securirty market line;
- Without this assumption, there will be numerous risky portfolios and numerous security market lines;
- Clearly, investors can process the same information in a rational manner and arrive at different optimal risky portfolios. This is assumed in secuirty selection.

Market portfolio

- The true market portfolio according to the CAPM includes all assets, financial and non-financial, which means that it also includes many assets that are not investable, such as human capital and assets in closed economies;
- Richard Roll noted that one reason the CAPM is not testable is that true market portfolio is unobservable.

Proxy for a market portfolio

- In the absence of a true market portfolio, market participants generally use proxies;
- These proxies, however, vary among analysts, the country of the investor - and so on - generating different return estimates for the same asset, which is impermissible in the CAPM;

Estimation of beta risk

- There is no consensus on the length of the estimation period;
- Using different periods for estimation results in different estimates of beta;
- Companies may not have enough historical data;
- Thus, we are likely to estimate different returns for the same asset, depending on the estimate of beta risk used in the model.

Efficient Markets

- Observed returns lies on the assumption that markets are efficient - investors possess all the information and the means to trade assets:
  - Cost efficiency;
  - Information efficiency;
  - Allocation efficiency.
- This is rarely the case, and there are different levels of market efficiency;
- This impacts the expected return and CAPM predictions.

Normal Distribution

- The portfolio approach relies on a mean-variance analysis, makign the assumption that returns are normally distributed;
- Using only mean and variance would be appropriate to evaluate instruments if returns were normally distributed;
- However, returns are not normally distributed. They are:
  - Skewed: they are not symmetric around the mean;
  - Characterized by a high probability of extreme events.

3.8.2 Limitations of the CAPM

Overall, these limitations make the CAPM a poor predictor of returns:

- If the CAPM is a good model, its estimate of asset returns should be closely associated with realized returns;
- However, empirical support for the CAPM is weak. In other workds, tests of the APM show that asset returns are not determined only by systematic risk;
- Poor predicatability of returns when using the CAPM is a serious limitation because return-generating models are used to estimate future returns.

3.8.3 Extensions of the CAPM

New models have been suggested to overcome the limitations of the CAPM, creating their own limitations.

X-Factors CAPM

- Historical analysis shows that the coefficient on market return is not significantly different from zero, which implies that sotck returns are unrelated to the market.
- In 1992, Fama and French offer a three factor model:
  - size (smaller companies outperform larger companies) - Small-Minus-Big;
  - Book-to-Market ratio (value companies with low BM ratio outperform glamour companies) - High-Minus-low.
- In 1997, Carhart add a momentum factors:
  - Momentum (past winners outperform past losers) - Winner-Minus-Losers.

X-Factor Models

- The underlying concept behind the model is that the return generated by portfolio managers are due in part to factors that are beyond the managers' control;
- Specifically, value stock have historically outperformed growth stock on average, while smaller companies have outperformed larger ones;
- These factors are at the market-level - each stock will have a specific sensitivity to it.

3-Factors CAPM

You then obtain "factors":

- These are numerical values;
- They are at the market-level;
- People use the values provided by Fama and French.

  $E(R_{it} - R_f = \alpha_i + \beta_{i,MKT} MKT_t + \beta i, SMBSMB_t + \beta i, , HMLHML_t)$

  , MKT = Systematic Risk, SMB = Size Anomaly, HML = Value Anomaly;
- You then employ an OLS-regression to esmitate the sinsibility of a given secuirty to the market wide factor.

4-Factors CAPM

- There is an improvement in the R-square compared with the one-factor model;

  $E(R_{it} - R_f = \alpha_i + \beta_{i,MKT} MKT_t + \beta i, SMBSMB_t + \beta i, , HMLHML_t + \beta_{i,WML} WML_t)$

  , MKT = Systematic Risk, SMB = Size Anomaly, HML = Value Anomaly, WML = Momentum Anomaly

Practical Models

- The three and four-factor models have been found to predict asset returns much better than the CAPM:
  - It is extensively used in estimating returns for U.S stocks;
- Recent work expand the number of factors:
  - In 2015 Fama and French offer a five factor model;
  - They add profitability and investment.

Factor Models

- Factor models try to estimate the expected returns based on a list of variables;
- The best example is the **Arbitrage Pricing Theory (APT)**;
- Like the CAPM, APT proposes a linear relationship between expected return and risk;
- It can include many factors, without considering theoretical framework of the CAPM.

APT Model

- The APT model does not make specific assumptions like the CAPM;
- The market can misprice an asset, and arbitrageurs can take the opportunity to make profits;
- Which factors to use is really dependent on empirical research, and the investor's gut.

  $$E(R_p) = R_F + \lambda_1 \beta_{p,1} + ... + \lambda_K \beta_{p,K}$$

  , λ = Risk Premium for Factor 1, $\beta_{p,1}$ = Sensitivity of the Portfolio to Factor 1
- Unlike the CAPM, APT allows for numerous risk factors (K) - as many as relevant to a particular asset;
- Other than the risk-free rate, the risk factors do not need to be common and may vary from one asset to antoher;
- The coefficient of each factor is estimated using econometric models;
- Common factors are GDP growth, inflation, M3 growth, S&P return, VIX, etc;
- APT is not often used, because it is more time-consuming to implement;
- Though, it can be developed internally to offer a valuation tool to traders;
- It does rely a lot on the decision to include certain factors, but modern big data approaches can allow to better select them, and outperform the CAPM.

Chapter 4

Behavioural Finance - Portfolio Management Industry

4.1 Efficient Market Hypothesis

- There are many investors out there doing market analysis;
- If investors are not analysing the stcok price, the market wouldn't be efficient;
- An efficient market means that you will earn a return that is appropriate for the risk undertaken and that there are no bias to be exploited to eran an excess return;
- Market efficiency will not protect you from wrong choices, if you do not diversify.

4.1.1 Forms of Market Efficiency

- Weak Form Efficiency - prices reflect **all historical market information** such as price and volume;
  - Invesors cannot earn abnormal returns by trading on past market data.
- Semi-strong Form Efficiency - prices reflect all **publicly available information** including trading information, annual reports, press releases, etc;
  - investors cannot earn abnormal returns by trading on public information.
- Strong Form Efficiency - reflect **all information, including public and private**;

– investors could not earn abnormal returns regardless of the information they possessed.

- Empirical evidence indicates that **markets are generally weka (or semi-strong) form effic**

4.1.2 What is Behavioural Finance?

There are three sub fields to modern financial research.

- *Theoretical finance* is the study of logical relationships among assets;
- *Empirical finance* deals with the study of data in order to infer relationships;
- *Behavioural finance* integrates psychology into the investment process.

- Behavioural finance research focuses on:
  - how investors make decisions to buy and sell secuirties;
  - how they choose between alternatives, and
  - how **reasoning erors** influence investor decisions and market prices.
- Much of behavioural finance research stems from the research in the are of cognitive psychology;
- Some people believe that cognitive errors made by investors will cause market inefficiencies.

4.2 Standard vs. Behavioural Finance

4.2.1 Standard Finance

- People are rational;
- People construct portfolios as described by mean-variance portfolio theory, where they want to include only high expected returns and low risk;
- People save and spend as described by standard life-cycle theory;
- Expected returns of investments are accounted for by standard asset pricing theory, where differences in expected returns are determined only by differences in risk;
- Markets are efficient, in the sense that prices equal values in them and in the sense that they are hard to beat;

4.2.2 Behavioural Finance

- People are normal;
- People construct portfolios described by behavioural portfolio theory, where they want to extend beyond high expected returns and low risk, such as for social responsibility and social status;
- People save and spend as described by behavioural lifecycle theory, where impediments, such as weak self-control, make it difficult to find and follow the right way to save and spend;
- Expected returns of investments are accounted for by behavioural asset pricing theory, where different in expected returns are determined by more than differences in risk, such as by levels of social responsibility and social status;
- Markets are not efficient in the sense that prices equal values in them, but they are hard to beat.

4.3 Prospect Theory

- Prospect theory provides an alternative to classical, rational economic decision-making;
- Risk averse investors get increasing utility from higher levels of wealth, but at a decreasing rate;
- Research shows that while risk aversion may accurately describe investor behaviour with gains, investors often show risk seeking behaviour when they face a loss;
- The foundation of prospect theory: investors are much more distressed by prospective losses than they are happy about prospective gains.
-
- There are well-know judgement errors consistent with the predictions of prospect theory.
  - Frame dependence - tendency that people make decision depends on how situation is framed;
  - Mental accounting - tendency to put things in different imaginary account;
  - The House Money effect - willing to take big risks with money wom from gambling;

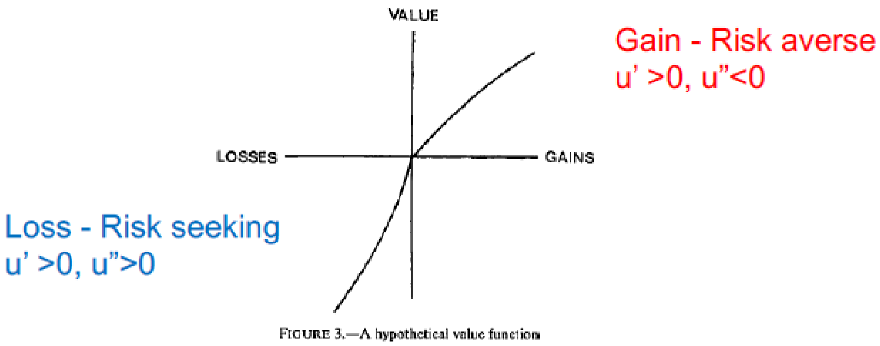


Figure 1: Value Function in Kahneman and Tversky (1979)

4.3.1 Frame Dependence

If an investment problem is presented in two different ways (with identical results), investors often make inconsistent choices.

4.3.2 Mental Accounting

- Mental accounting refers to our tendency to "put things in boxes" and track them individually;
- Transaction utility;
  - The perceived value of the "deal" - difference between amount paid and reference price;
- Opening and closing accounts;
  - Realized vs. paper gain (or loss);
  - Realized gain (or loss) is more delightful (painful);
  - Mental accounting favours selling the winner;
- Advance purchase, sunk costs and payment depreciation;
- Payment decoupling;
  - Credit card: later payment & installment;

Choice Bracketing and Dynamic Mental Accounting

- Narrow framing and myopic loss aversion;
  - Loss-averse people are willing to take risk if they combine many bets together;

- Diversification heuristic
  - Variety-seeking: people tend to diversify, when they are asked to make several choices at once.

4.3.3 The House Money Effect

- Thinking this way means that you are guilty of playing with house money;
- Whether you lose money from your original investment (your money) or lose money from your investment gains (house money) is irrelevant;
- There are two important investment lessons:
  - There are no "paper profits". Your profits are yours;
  - All your money is your money. You should not separate your money into bundles labelled "my money" and "house money".

4.4 Overconfidence

- A serious error in judgement you can make as an investor is to be overconfident;
- We are all overconfident about our abilities in many areas.

4.4.1 Overconfidence and Portfolio Diversification

- Investors tend to invest too heavily in shares of the company for which tehy work;
- This loyalty can be very bad financially:
  - Your earning power (income) depends on this company;
  - Your retirement nest-egg also depends on this company.
- Another example of the lack of diversification is investing too heavily in the stocks of local companies:
  - Perhaps you know someone personally who works there;
  - Perhaps you read about them in your local paper;
  - Basically, you are unduly confident that you have a high degree of knowledge about local companies.

4.4.2 Overconfidence and Trading Frequency

- If you are overconfident about your investment skill, it is likely that you will trade too much;
- Researchers have found that investors who make relatively more trades havelower returns than investors who trade less frequently;
- The moral is clear: **Excessive trading is hazardous to your wealth**.
- Psychologists have found that mean are more overconfident than women in the area of finance.
- Men trade about 45% more than women;
- Accounting for the effects of marital status, age and income, researchers also show that men invest in riskier positions.

4.5 Regret Avoidance

- Did I screw up or am I just unlucky?
- "Courageous" to invest in these firms. I need a bigger return. I will discount the cashflows of these firms at a higher rate. The price is lower and my reutrn is higher.

4.6 Herd Behaviour

- A phenomenon in which individuals act collectively as a group;
- Often, individual investors mimic the actions of a larger group or markt, rather thean following their own analysis;
- This happens when there is: lack of experience, social pressure;
- Herd behaviour is a significant driver of extreme volaility and asset bubbles in financial markets - e.g. 2008 financial crisis, bitcoin bubble, etc;

4.7 Emotional Gap

- Refers to decision making based on extreme emotions or emotional strains such as anxiety, anger, fear or excitement;

4.8 Anchoring

- Bias that a certain price they choose for some reason would work as an individual is given a pice of information to which they refer subsequent judgements during decision making.
  - Market anomalies/inefficiencies and Technical analysis (support/resistance) can be explained by anchoring;
  - Investors are slow to adjust their expectations to the present price;

4.9 Self-attribution

- Self-attribtion bias: individuals' tendency to attribute successes to personal skills and failures to factors beyond their control;
  - Individuals tend to overlook their mistakes or external factors;

4.10 Culture and Investment Behaviour

- Cultural differences matter for financial decisions;
- Cultural differences lead to systematic deviations from rational decision making in:
  - risk taking;
  - negotiations for direct investment, mergers & acquistions;
  - returns of bonds and stocks at local markets;

4.11 Application of Behavioural Finance - Technical Analysis

- **Technical analysis** attempts to exploit **recurring and predictable patterns in stock prices** to generate superior investment performance;
- This is different from the **fundamental analysis** which attempts to measure **the security's intrinsic value by reviewing financial and non-financial factors**;
- Technicians do not deny the value of fundamental information. But they believe that prices only gradually close in on intrinsic value;