BU5526 - Portfolio Analysis
Lecture Notes

Rodrigo Miguel

January 28, 2023

Contents

Chapter 1

Lecture 1

1.1 Asset Pricing

Asset Something valuable that an entity owns, benefits from, or has use of, in generating income.

Asset Pricing Needed to buy/sell at a fair price.

- Obtained by discovery process:
  - Demand and supply forces.
- The price of an asset is simply the current value of its cash-flows.

$$PV = \frac{FV_1}{(1+r)^1} + \frac{FV_2}{(1+r)^2} + ... + \frac{FV_n}{(1+r)^n}$$

- What we need:
  - Prediction of cash flows;
  - Discount rate.

How to price a stock:

- Dividend Discount Model;
- FCF Model;
- Multipliers and Comparables approach.

Being able to estimate the required **rate of return** is everything you need to calculate the **value of any asset** - once you have predicted **future cash-flows**, including:

- Estimating the value of a stock;
- Estimating the value of a firm.

1.2 Portfolio Management

Prudent administration of **investable** (liquid) assets, aimed at achieving an **optimum risk-reward ratio**.

1.3 Mathematical Concepts

Mean an average of different observations.

- Useful to describe a population;
- **Arithmetical**: sum of observations divided by the number of observations (if with equal weights).

$$\overline{X} = \frac{X_1 + X_2 + ... + X_n}{n}$$

or

$$\overline{X} = W_1 \times X_1 + W_2 \times X_2 + ... + W_n \times X_n$$

- **Geometrical**: product of observations divided by the number of observations.

$$\overline{X}_G = \sqrt[n]{X_1 \times X_2 \times ... \times X_n}$$

or

$$\overline{X}_G = \sqrt[n]{X_1^{W_1} \times X_2^{W_2} \times ... \times X_n^{W_n}}$$

There is a reason to use arithmetical or geometrical mean.

- **Equally weighted**: when all the observations have the same importance.
- **Unequally weighted**: different importance for different observations.

Variance seen as an extension of the mean.

- Dispersion to the mean (higher/lower):
  - Average of differences from the mean.

$$\sigma_x^2 = \frac{(X_1 - \mu_X)^2 + (X_2 - \mu_X)^2 + ... + (X_n - \mu_X)^2}{n}$$

Important Statistical Concept

- The formula is correct if we possess all the data on the population;
- If we only have a sample, we need to reflect "impreciseness" by removing "one degree of freedom".

$$\sigma_x^2 = \frac{(X_1 - \mu_X)^2 + (X_2 - \mu_X)^2 + ... + (X_n - \mu_X)^2}{n-1}$$

This will **always** be the case in finance.

Standard Deviation

$$\sigma_X = \sqrt[2]{\frac{(X_1 - \mu_n)^2 + (X_2 - \mu_n)^2 + ... + (X_n - \mu_n)^2}{(n-1)}}$$

Skewness Brings back the sign. A positive skewness means more positive-value and reversely.

$$\sigma_X^3 = \frac{(X_1 - \mu_n)^3 + (X_2 - \mu_n)^3 + ... + (X_n - \mu_n)^3}{n}$$

Kurtosis Outweighs extremes - dropping the sign.

$$\sigma_X^4 = \frac{(X_1 - \mu_n)^4 + (X_2 - \mu_n)^4 + ... + (X_n - \mu_n)^4}{n}$$

A large Kurtosis means a lot of extreme values.

- To get a **Meaningful estimate**: needs to provide **excess** kurtosis.
- Kurtosis of a normal distribution is 3.

Note: Unbiased equations of these two indicators are slightly more complex, but computer packages provide them automatically.

1.4 Assumptions of Mean-Variance Analysis

- Allows to describe different assets simply.
- Assumes returns are normally distributed.

Normal distribution

- Its mean and median are equal;
- It's defined by two parameters, mean and variance;
- It's defined around its mean with:
  - 68% of observations within $\pm$ 1σ of the mean.
  - 95% of observations within $\pm$ 2σ of the mean.
  - 99% of observations within $\pm$ 3σ of the mean
- Returns are not normally distributed.
  - Skewed: not symmetric around the mean.
  - Characterized by high probability of extreme event.

1.5 Returns on Financial Assets

Holding period return: Return from holding an asset for a specific period of time.

$$R = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}}$$

Capital gain + Dividend yield
Holding period returns = compound returns

$$R = [(1 + r_1) \times (1 + r_2) \times (1 + r_3)] - 1$$

Geometric Mean Return

$$\overline{R} = \sqrt[T]{(1 + r_{i_1}) \times (1 + r_{i_2}) \times ... \times (1 + r_{i_T})} - 1$$

Annualized Return These returns should be capitalized.

- Such as:

  $$r_{annual} = (1 + r_{period})^C - 1$$

  with C being number of periods in a year.

Portfolio Return

- When several assets are combined into a portfolio, we can compute the portfolio return.
- Weighted average of the returns of individual assets

  $$R_p = W_1 \times R_1 + W_2 \times R_2$$

Historical and Expected Returns

- Computed from historical data.
- What the investor expects to earn.
- Historical returns are different from expected returns.

Ways of Calculating Expected Returns

- Gut feeling;
- Modelling - calculated from a formula.

Standard deviation - Volatility

- Historical standard deviation;
- Defined as **risk** of equity return.

Portfolio Variance

- Cannot simply add up two variances.
- Covariance:
$$cov(X, Y) = E(XY) - E(X) \times E(Y)$$
  - The more the two assets move in the same way, the higher the covariance.
  - A negative covariance means the assets move in opposite directions.
- Variance:
$$\sigma_v^2 = wCw^T$$

Correlation and Portfolio Risk The correlation among assets determine the porfolio's risk.

- Is a measure of tendency for N investments to act similarly;
- Can range from -1 and $+1$.

$$\rho_{ij} = \frac{cov(R_i, R_j)}{\sigma_i \sigma_j}$$

Chapter 2

Lecture 2