1
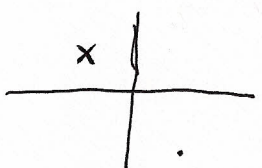
a) $|z| = \sqrt{1^2 + \sqrt{3}^2} = 2$; $\text{arctg}\,\theta = -\sqrt{3} \Rightarrow$ ~~$\theta = -\pi/3$~~ ~~$\theta = -\pi/3 + \pi/3$~~

$\theta = \Big\langle$ $\begin{matrix} \nearrow & -\pi/3 = 5\pi/3 \\ \boxed{2\pi/3} \longrightarrow \end{matrix}$ porque: está en el 2º cuadrante

$z = 2_{2\pi/3}$

b) $5 - 5i$ $\qquad |z| = \sqrt{50} = 5\sqrt{2}$ $\qquad$ $\text{arctg}\,\theta = -1$

$\theta = \Big\langle$ $\begin{matrix} \nearrow & \boxed{7\pi/4} \\ \searrow & 3\pi/4 \end{matrix}$ porque ~~está~~ está en el 4º cuadrante.

$z = 5\sqrt{2}_{7\pi/4}$

c) ~~$\cancel{}$~~ $2\sqrt{3}_{\pi/3}$ $\qquad$ d) 5_π $\qquad$ e) $6_{3\pi/2}$ $\qquad$ f) $5\sqrt{2}_{3\pi/4}$

g) $2_{5\pi/3}$ $\qquad$ h) $6_{\pi/6}$ $\qquad$ i) $2_{4\pi/3}$ $\qquad$ j) 2_π

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② a) $x = 2\sqrt{3}\cos\frac{3\pi}{4} = 2\sqrt{3}\cdot\left(\frac{-\sqrt{2}}{2}\right) = -\sqrt{6}$ $\Big\}$ $-\sqrt{6} + i\sqrt{6}$ ó $\sqrt{6}(-1 + i)$

$y = 2\sqrt{3}\,\text{sen}\frac{3\pi}{4} = 2\sqrt{3}\,\frac{\sqrt{2}}{2} = \sqrt{6}$

b) $-\frac{3\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ $\qquad$ c) $-\frac{5\sqrt{3}}{2} - \frac{5}{2}i$ $\qquad$ d) $-\frac{5}{2} + \frac{5\sqrt{3}}{2}i$

e) $\sqrt{2} - \sqrt{2}i$ $\qquad$ f) -2 $\qquad$ g) $-2\sqrt{3} - 2i$ $\qquad$ h) $-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

i) $e^2 \cdot e^{-i} = e^2_{-1} = e^2\big(\cos(-1) + i\,\text{sen}(-1)\big)$ $\qquad$ j) $-\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$

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③ a) $-2i$ $\qquad$ b) $\frac{53}{6} + \frac{5}{2}i$ $\qquad$ c) $2 + i$ $\qquad$ d) $\frac{1}{2}i$

e) $\frac{16}{25} + \frac{63}{25}i$ $\qquad$ f) ~~$128\sqrt{2}$~~ $128\sqrt{2}_{3\pi/4}$ $\qquad$ g) $\frac{18}{29} - \frac{16}{29}i$

h) $10\,e^{i\pi/6}$ $\qquad$ i) $\frac{5}{2}e^{i\pi/2}$ $\qquad$ j) ~~$5\sqrt{2}_{-\pi/12} = \frac{5}{2}\big[(1+\sqrt{3}) + i(1-\sqrt{3})\big]$~~

$\left(\frac{5}{2} + \sqrt{3}\right) + i\left(\frac{5\sqrt{3}}{2} - 1\right)$ ~~$\cancel{}$~~

k) $6_{\frac{\pi}{2}} = 6i$ l) $\frac{\sqrt{2}}{2}$

$\frac{1}{2}\sqrt{6}$

4) a) $7i$ b) $3\sqrt{2}_{\frac{-\pi}{12}}$

5) a) ~~(-1+i)~~ $-1+i = \sqrt{2}_{3\pi/4}$ b) $4\sqrt{2}_{3\pi/4}$ c) $128_{4\pi/3}$ d) $3''_{3\pi/4}$

~~$\left(3\frac{\pi}{4}\right)'' = 3'' _{11\pi/4}$~~

e) $16_{2\pi/3}$ 8) $1_{\frac{\pi}{2}} = ~~(0,1)~~ = i$

6) a) $\sqrt[3]{8_{\pi/6}} = $
$\nearrow 2_{\frac{13}{18}\pi}$
$\to 2_{\frac{\pi}{18}}$
$\searrow 2_{\frac{25\pi}{18}}$

$\sqrt[5]{2}\, e^{\pi/5\, i}$

b) $\sqrt[5]{2}\, e^{\frac{3\pi}{5}i}$

$\sqrt[5]{2}\, e^{\pi i}$

$\sqrt[5]{2}\, e^{\frac{7\pi i}{5}}$

$\sqrt[5]{2}\, e^{\frac{9\pi i}{5}}$

c) 3_0

$3_{\frac{\pi}{2}}$

3_π

$3_{\frac{3\pi}{2}}$

d) $\sqrt{2}_{\pi/4}$

$\sqrt{2}_{19\pi/12}$

$\sqrt{2}_{11\pi/12}$

e) $\sqrt[3]{\frac{\sqrt{2}}{3}}\, e^{\frac{5\pi i}{12}}$

$\sqrt[3]{\frac{\sqrt{2}}{3}}\, e^{\frac{13\pi i}{12}}$

$\sqrt[3]{\frac{\sqrt{2}}{3}}\, e^{7\pi i/4}$

7) a) $z^6 - 1 = 0 \rightarrow z^6 = 1 \rightarrow z = \sqrt[6]{1} = \sqrt[6]{1_0} = \sqrt[6]{z_k} = 1_{\theta_k}$

$$\theta_k = \frac{\theta + 2K\pi}{n} = \frac{0 + 2K\pi}{6}$$

$$\theta_0 = 1 \ ; \ \theta_1 = \frac{2\pi}{6} = \frac{\pi}{3} \ ; \ \theta_2 = \frac{4\pi}{6} = \frac{2\pi}{3} \ ; \ \theta_3 = \frac{6\pi}{6} = \pi$$

$$\theta_4 = \frac{8\pi}{6} = \frac{4\pi}{3} \qquad \theta_5 = \frac{10\pi}{6} = \frac{5\pi}{3}$$

$$z = \begin{cases} 1_0 = 1 \\ 1_{2\pi/3} \\ 1_\pi = -1 \\ 1_{4\pi/3} \\ 1_{5\pi/3} \end{cases}$$

b) $z = \dfrac{-2 \pm \sqrt{4 - 24}}{2} = \dfrac{-2 \pm \sqrt{-20}}{2} = \dfrac{-2 \pm \sqrt{20}\,i}{2} =$

$= \dfrac{-2 \pm 2\sqrt{5}\,i}{2} = \dfrac{-1 \pm \sqrt{5}\,i}{2} = \begin{cases} \dfrac{-1}{2} + \dfrac{\sqrt{5}}{2}i \\ \dfrac{-1}{2} - \dfrac{\sqrt{5}}{2}i \end{cases}$

c) $z = \dfrac{6 + 8i \pm \sqrt{-36 + 64 - 96i - 4 - 120i}}{2} = \dfrac{6 + 8i \pm \sqrt{24 - 216i}}{2}$

$\dfrac{6 + 8i \pm \sqrt{36 - 64 + 96i - 4 - 120i}}{2} = \dfrac{6 + 8i \pm \sqrt{-32 - 24i}}{2}$

Llamamos: $\sqrt{-32 - 24i} = x + yi \Rightarrow -32 - 24i = (x + yi)^2 = x^2 - y^2 + 2xyi$

$$\left. \begin{array}{l} x^2 - y^2 = -32 \\ 2xy = -24 \end{array} \right\} \quad \begin{array}{l} x^2 - y^2 = -32 \\ x^2 + y^2 = 40 \\ \hline 2x^2 = 8 \rightarrow x^2 = 4 \rightarrow \boxed{x = \pm 2} \\ y^2 = 40 - 4 = 36 \quad \boxed{y = \pm 6} \end{array}$$

además $|x + yi| = \sqrt{x^2 + y^2}$

entonces $|-32 - 24i| = x^2 + y^2 = 40$

Para que $2xy = -24 \rightarrow xy = -12$ entonces $\begin{array}{l} x = 2, \ y = -6 \\ x = -2, \ y = 6 \end{array}$

$z = \dfrac{6 + 8i \pm (2 - 6i)}{2} = 3 + 4i \pm (1 - 3i) = \begin{cases} \boxed{4 + i} \\ \boxed{2 + 7i} \end{cases}$

8) a)

$$\Delta = \begin{vmatrix} 1-i & -\sqrt{3}\,i \\ 2i & (1+i) \end{vmatrix} = 2 - 2\sqrt{3}$$

Pero $\Delta_x = \cancel{\Delta} y = 0$ el sistema es compatible con la solución trivial

$$x=0, \quad y=0.$$

b)

$$\Delta = \begin{vmatrix} 3+4i & -2 \\ -1-i & -1-2i \end{vmatrix} = 5 - 10i - 2 + 2i = \cancel{3-12i} \; \cancel{3-12i}$$

$$x = \cancel{\frac{3-12i}{-5-2i}} =$$

$$= \cancel{\frac{(3-12i)(-5+2i)}{29}} =$$

$$\Delta_x = \begin{vmatrix} 1 & -2 \\ -2 & -1-2i \end{vmatrix} = -1-2i - 4 = -5-2i$$

$$\cancel{\frac{-11}{29} + \frac{10}{29}i} \qquad \cancel{\cancel{\frac{9+66i}{29}} = \cancel{\frac{9}{29} + \frac{66}{29}i}}$$

$$x = \frac{-5-2i}{3-12i} = \frac{9-66i}{153}$$

$$\Delta_y = \begin{vmatrix} 3+4i & 1 \\ -1-i & -2 \end{vmatrix} = -6-8i +1+i = -5-7i$$

$$\boxed{y = \frac{-5-7i}{3-12i} = \frac{(-5-7i)(3+12i)}{153} = \frac{69-81i}{153}}$$

$$\boxed{\begin{aligned} x &= \frac{1}{17} - \frac{22}{51}i \\[2mm] y &= \frac{23}{51} - \frac{9}{17}i \end{aligned}}$$