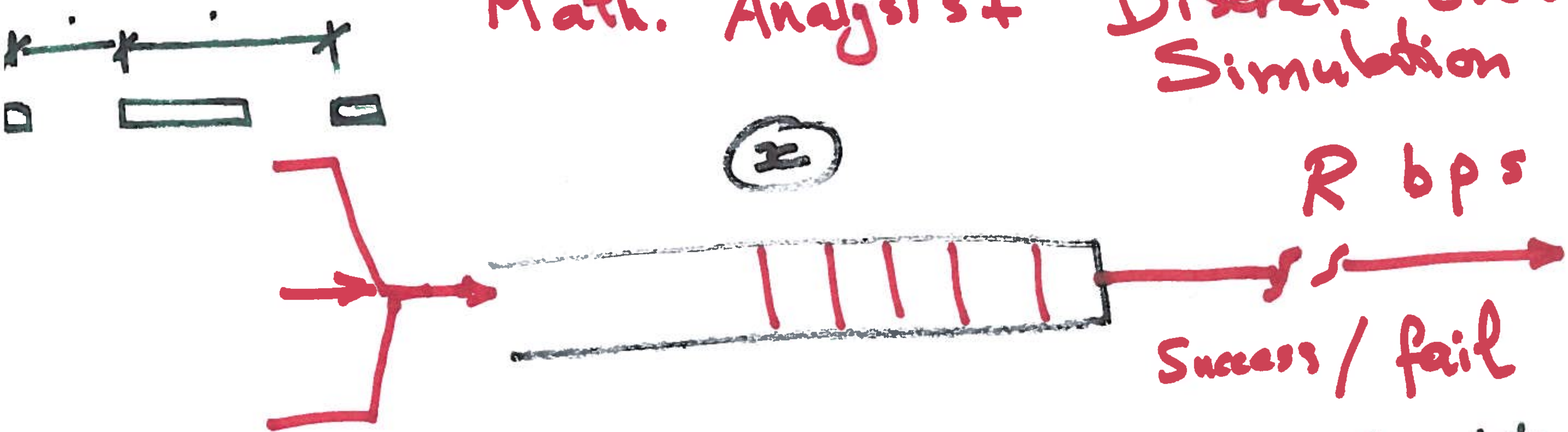
Tools of Analyzing Networks

Math. Analysis + Discrete Event Simulation



$\overline{N}$: the avg. number of pkts in the queue

Need to model

- pkt length
- Arrival Process
- Pkt inter-arrival time

Possible Model

discrete uniform
Poisson process
Exponential dist.

- $\dfrac{\text{Transmission}}{\text{likelihood}}$ success/fail — Bernoulli

- # of transmissions required to transmit one pkt successfully — Geometric

Ex. (events)

$\boxed{B}$ = # of pkts queued ≥ 10

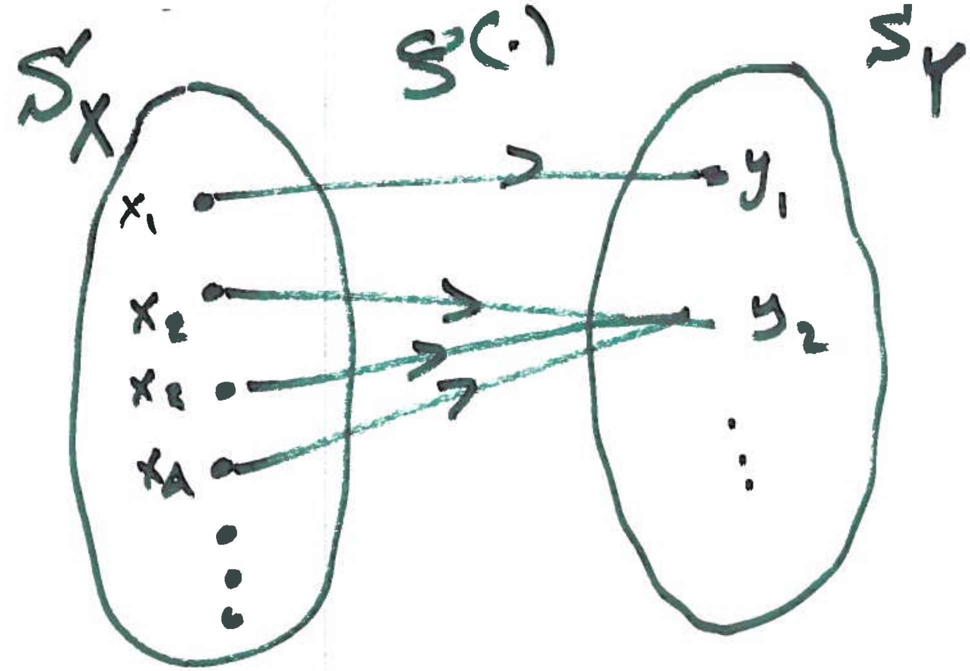
$$\text{Prob}[B] = \sum_{x=10,11,12,13,\ldots\ldots} P_X(x)$$

<u>Averages</u>: Linearity

- $$E[\underbrace{\underset{=}{a}X + \underset{=}{b}}_{Y}] = a\,E[X] + b \quad \square$$
- $$E[\underbrace{X_1 + X_2 + \cdots + X_n}_{Y}] = \sum_{i=1}^{n} E[X_i]$$

Holds even if the X_is are <u>dependent</u>.

- $Y = g(X)$



Short cut theorem

$$E[Y] = \sum_{x \in S'_X} g(x) P_X(x) \equiv E[Y] = \sum_{y \in S'_Y} y P_Y(y)$$

Derived Random Variables

$$S_X = \{ 2, 4, 8 \}$$

$$Y_1 = X^2$$

$$\text{Prob}[Y_1 = x^2] \overset{?}{=} \text{Prob}[X = x]$$

E.g.

$$\text{Prob}[Y_1 = 4] = \text{Prob}[X = 2]$$

$$Y_2 = X_1 \cdot X_2$$

where X_1 & X_2 are $\underline{\text{i.i.d}}$ $\boxed{\text{indep., identically dist.}}$ r.v.s

$$\text{Prob}[Y = 16] \; ?$$

4.4 2.8 8.2

Variance: Why not

- $\sigma[X] \overset{?}{=} E[X - \mu_X]$ $= \overset{\mu_X}{E[X]} - \overset{\mu_X}{E[\mu_X]} = 0$

or

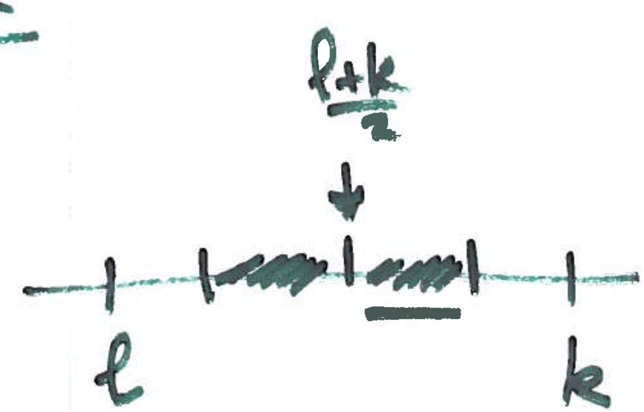
- $\sigma[X] \overset{?}{=} E[|X - \mu_X|]$

Discrete Uniform

$$Var[X] = \frac{(l-k)(l-k+2)}{12}$$

$$> \frac{(l-k)^2}{16}$$

$$\sigma = \sqrt{Var[X]} \geq \frac{l-k}{4}$$



X = # of times to do the road test

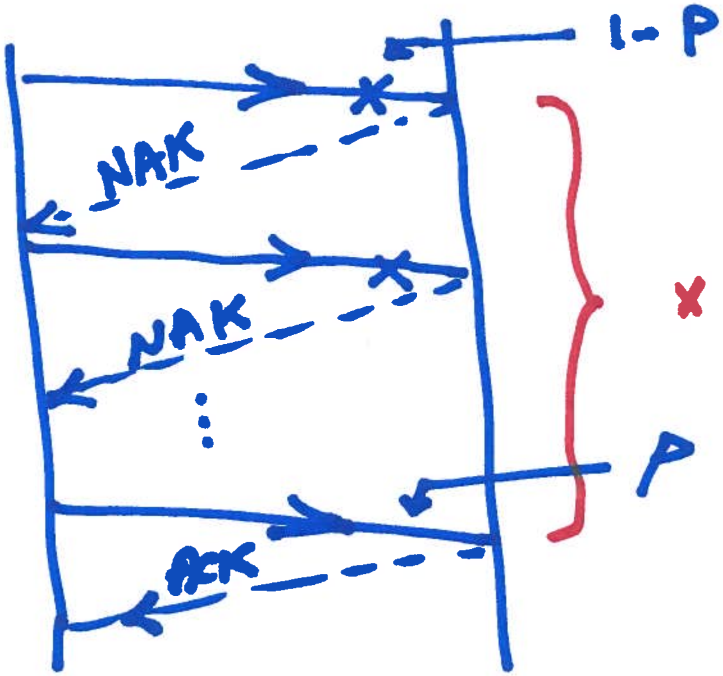
$C = \$100 \cdot X$

Thm

$Y = aX$

$\mathrm{Var}[Y] = a^2\,\mathrm{Var}[X]$

Geometric



X = # of transmissions until I get ACK

Binomial Dist.

$\longleftarrow$ $n = 12$ bits $\longrightarrow$

1	0	1	0	1	1	. . . . .

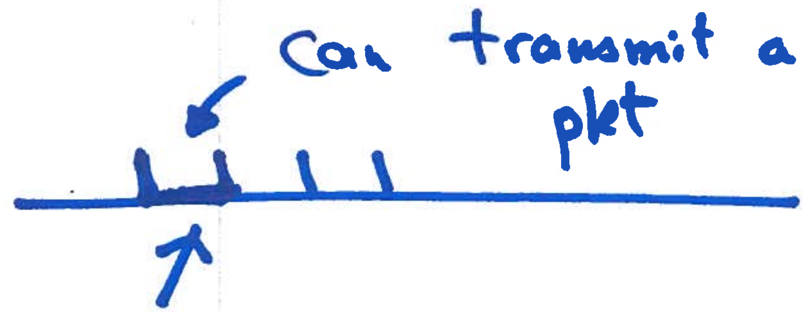
$P[\text{success}] = 0.7$

$P[\text{exactly 10 bits are sent successfully}] = \ldots$

Define $X = B_1 + B_2 + \cdots + B_n$

$\therefore \; E[X] = E[B_1] + \cdots + E[B_n]$
$= n.p$

$\therefore \; Var[X] = Var[B_1] + \cdots + Var[B_n]$
(only if the B_i's are i.i.d.)



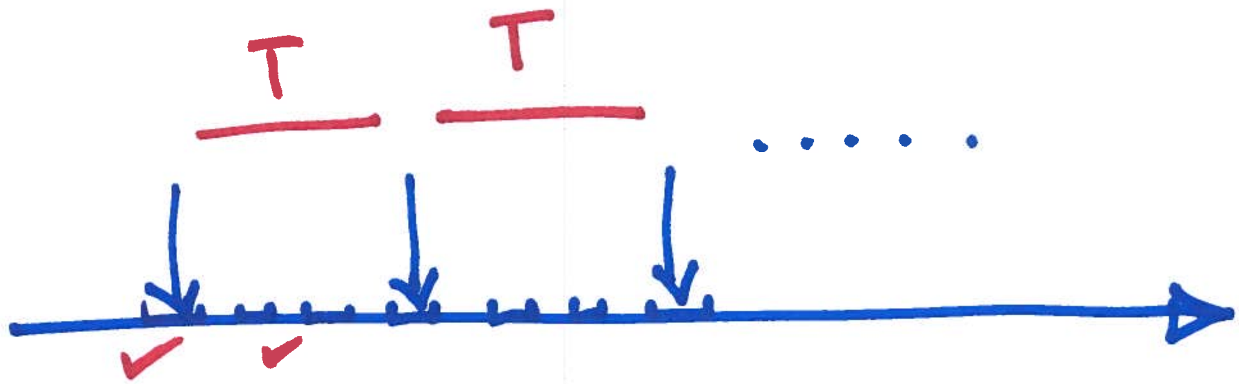
10% active →

P[a user has a pkt to transmit] = 0.1

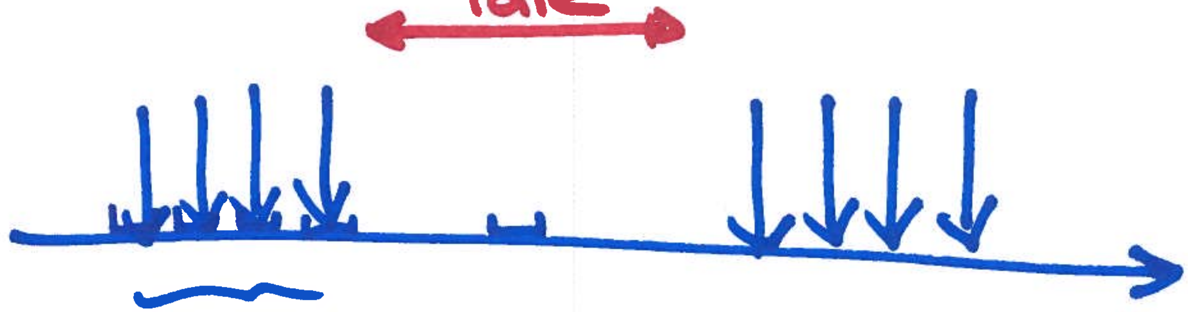
Poisson Process

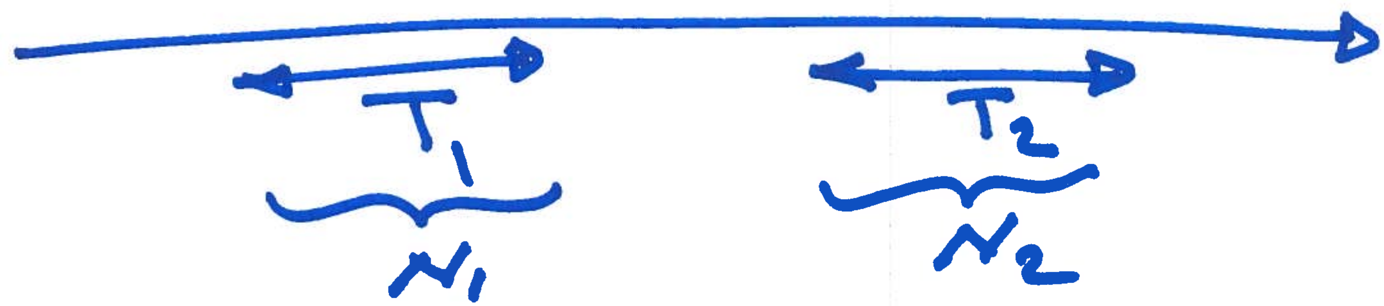
Prob [1 event] = $\lambda\Delta$

Regular Traffic



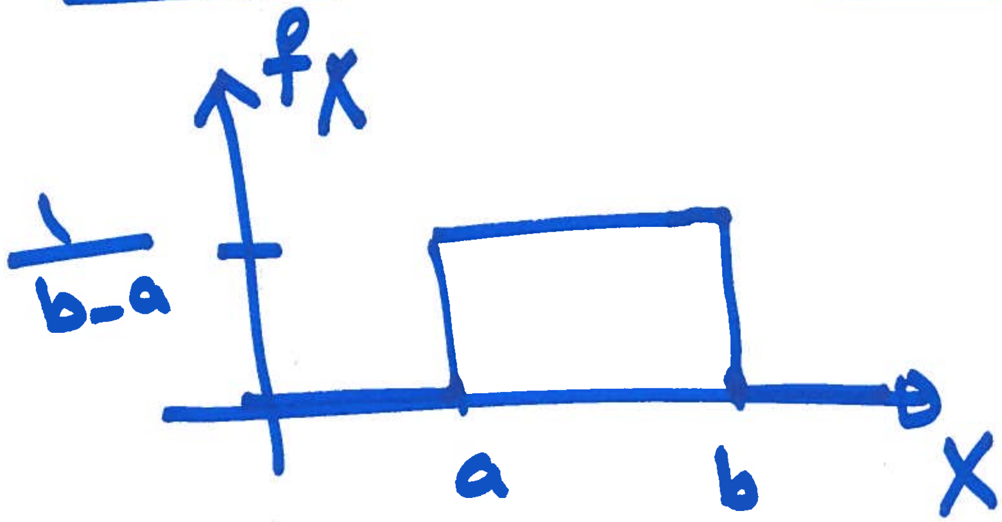
Bursty Traffic

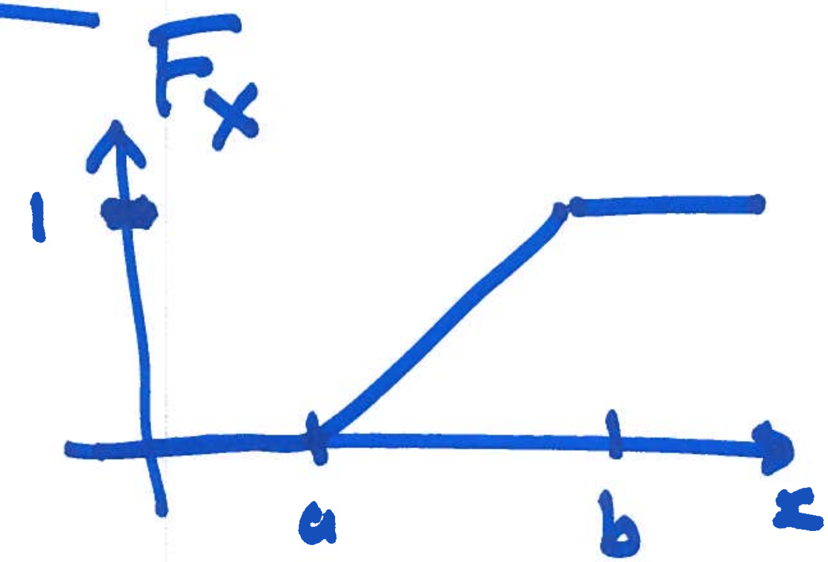




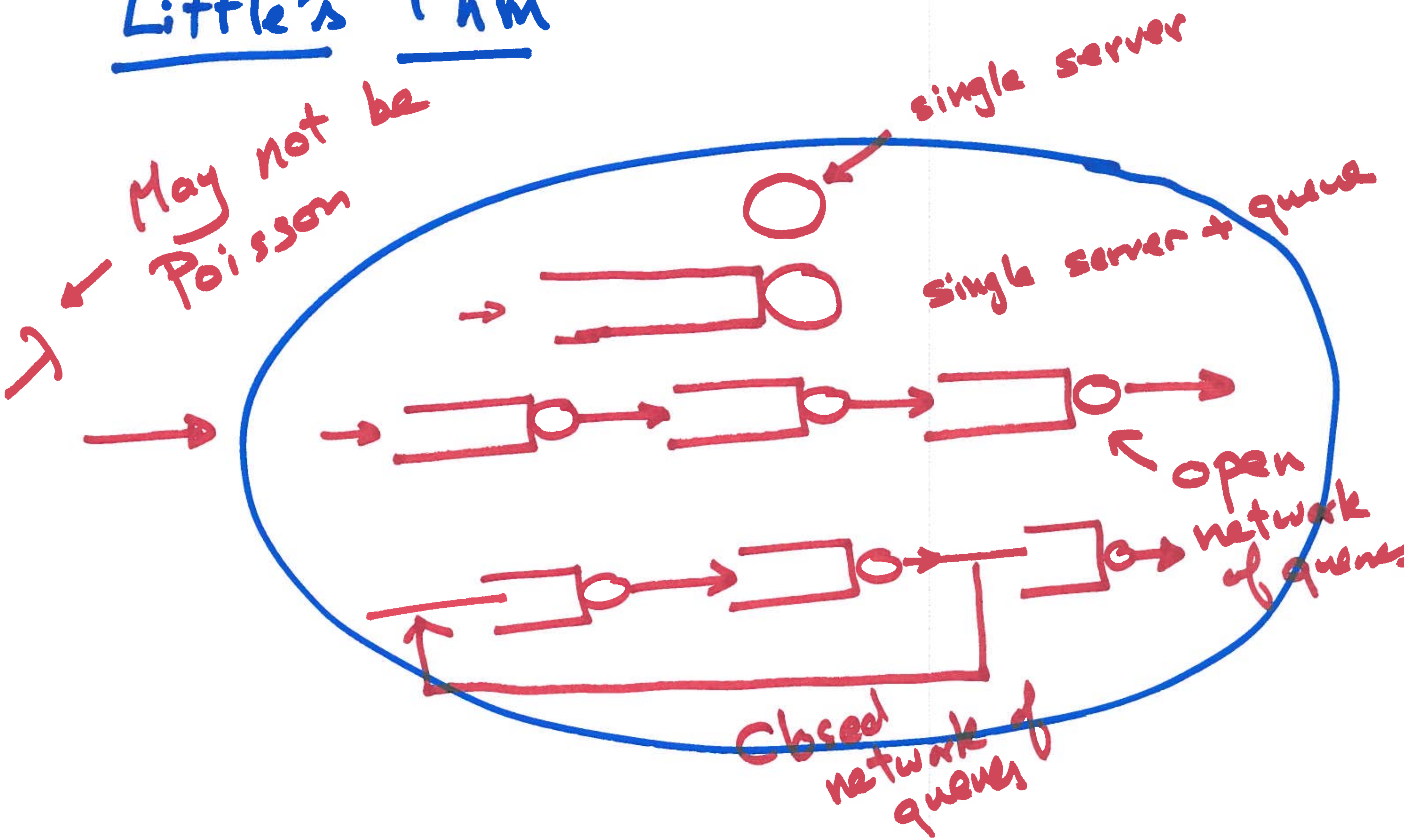
T_1
N_1
T_2
N_2

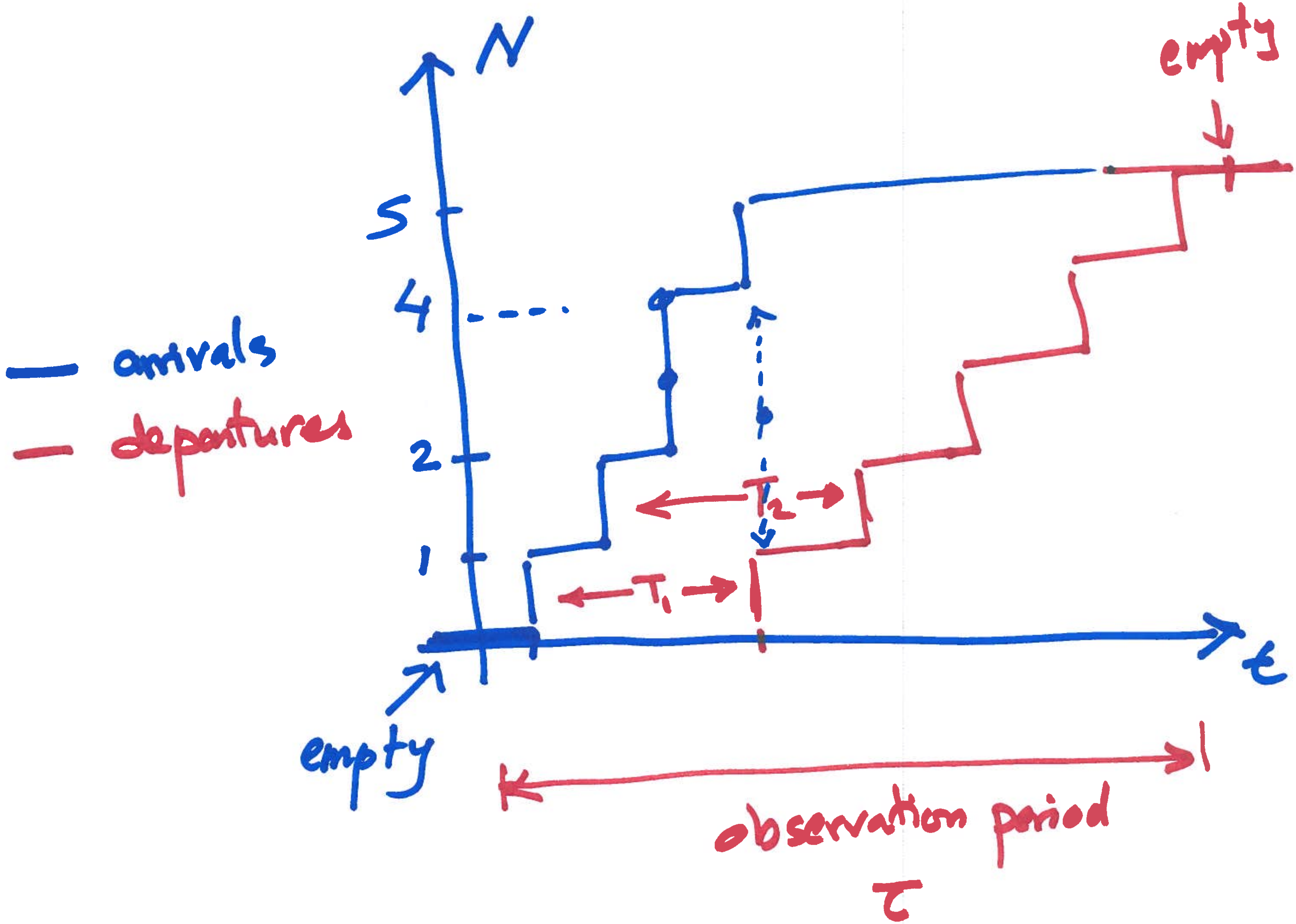
Continuous Uniform Dist.





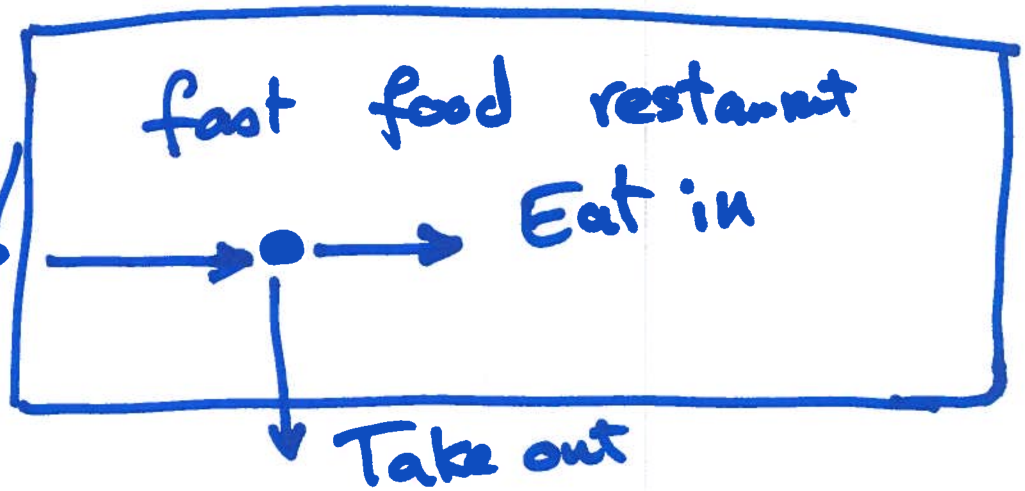
Little's Thm


May not be Poisson
λ
single server
single server + queue
open network of queues
Closed network of queues


arrivals
departures
N
5
4
2
1
empty
T_1
T_2
t
empty
observation period
τ

Problem

$\lambda = 5$ customers/min



- Avg. wait time to get order = 5 min.
- Subsequently, either take out with prob. 0.5

  or, [eat in with prob. 0.5
  Avg. eating time in restaurant = 20 min.

- Find, N: avg. # of costumers in restaurant.

- **Lower Bound on N:**

  Every one eats out:

  $$N_{lower} = (\lambda = 5) * (5 \text{ min}) = 25 \text{ costumers}$$

- **Upper bound on N:**

  Every one eats in:

  $$N_{upper} = (\lambda = 5) * (25 \text{ min}) = 125 \text{ costumers}$$

Queuing Theory

arrival process / service process / # of servers / capacity / service discipline

arrival process	service process	# of servers	capacity	service discipline
Poisson	Exponential service time	1	omitted	omitted
M = "memoryless"	M = "memoryless"		(∞)	(FCFS)

$N = \dfrac{\lambda}{\mu - \lambda}$ pkts

Little's Thm

$N = \lambda T$

$T = \dfrac{1}{\mu - \lambda}$ sec.

$$\overline{\text{Delay}} = \frac{1}{\mu - \lambda}$$