Lecture 27 (Ch. 7, 8, 9, 11)

We postponed two issues:

1) statistical vs. physical significance, and 2) power

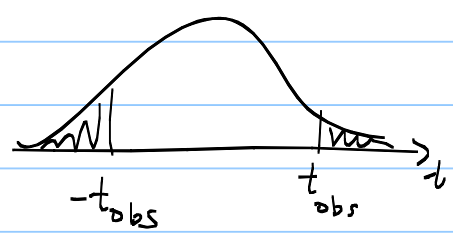
1) Suppose we are testing $\mu_1 = \mu_2$ against $\mu_1 \neq \mu_2$.

- C.I. $\quad (\bar{x}_2 - \bar{x}_1) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

- p-value $\quad t_{obs} = \dfrac{(\bar{x}_2 - \bar{x}_1) - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ ← Null param.

- p-value $= 2\,\text{prob}(t > t_{obs})$

  Compare with α



- Note that as n_1 & n_2 increase, They vary randomly.
  - The C.I. shrinks. [s_1 & s_2 don't shrink or expand]
  - t_{obs} grows, ie. p-value decreases.

As such, for sufficiently large n_1 & n_2,
we can always Reject H_0 in favor of H_1.
I.e. we can always find a difference between
μ_1 and μ_2, even if it is only a tiny difference!

Statistical significance is different from practical significance!

Statistics can help you with The former; nothing can help you with The latter; you need to decide based on $(\bar{x}_1 - \bar{x}_2)_{observed}$.

2) β and Power:

Suppose we're solving a problem like $\begin{cases} H_0: \mu \le \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$

we assume H_0 is True (ie. $\mu \le \mu_0$),

we compute a p-value from data,

Then compare it with α.

Therefore, $100\,\alpha\,\%$ of the time we do such tests,

we will commit a Type I error (ie. Reject H_0, when it's True).

$$\alpha = \text{prob(Type I)} = \text{prob(Data Reject } H_0 \text{ in favor of } H_1 \mid \underbrace{H_0 = T}_{H_1 = F})$$

As we said, all of That will lead to some Type II errors.

How often?

$$\beta = \text{prob(Type II)} = \text{prob(Data Cannot Reject } H_0 \text{ in} \ldots \mid \underbrace{H_0 = F}_{H_1 = T})$$

How often will we reject H_0, when $H_0 = F$?

$$\text{power} = 1 - \beta = \text{prob(Data Reject } H_0 \text{ in} \ldots \mid \underbrace{H_0 = F}_{H_1 = T})$$

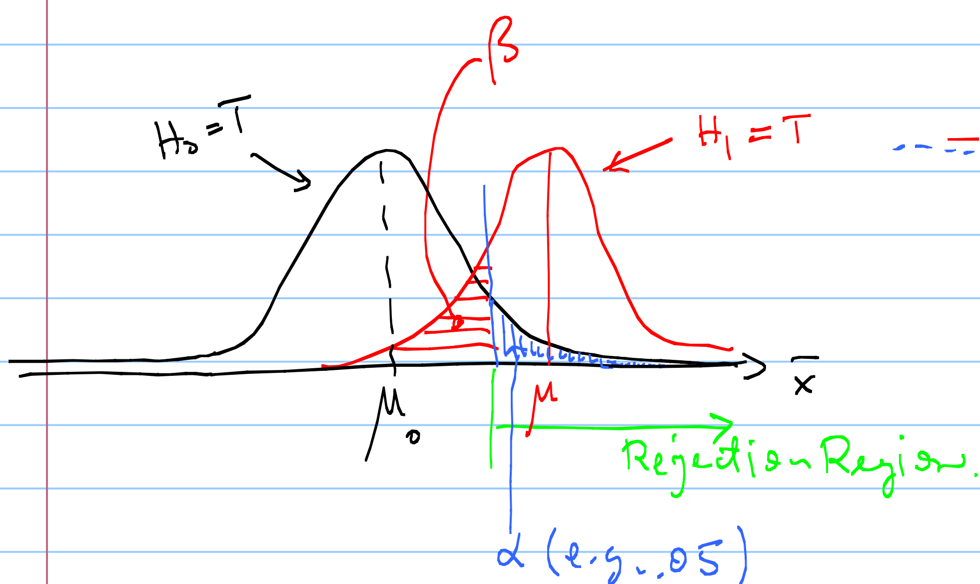
In short, when you use some test (z-test, t-test, chi-squared test, ...) it's important to also know what fraction of time you reject H_0, correctly!

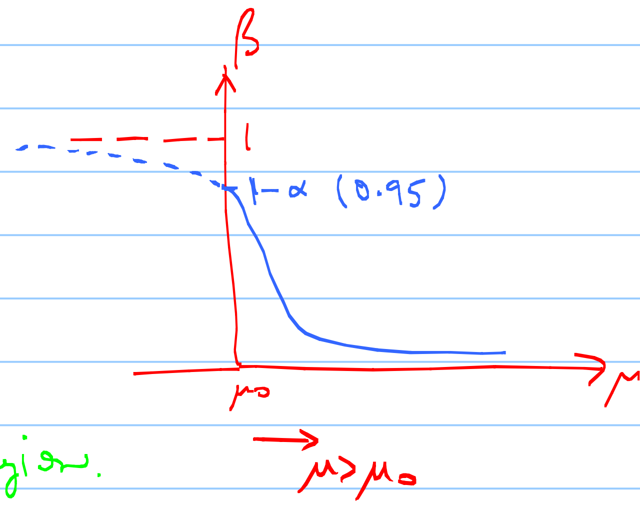
It can also be shown That paired tests have more power! So, do paired designs, if you can! If There is an effect, you'll be more likely to find it.

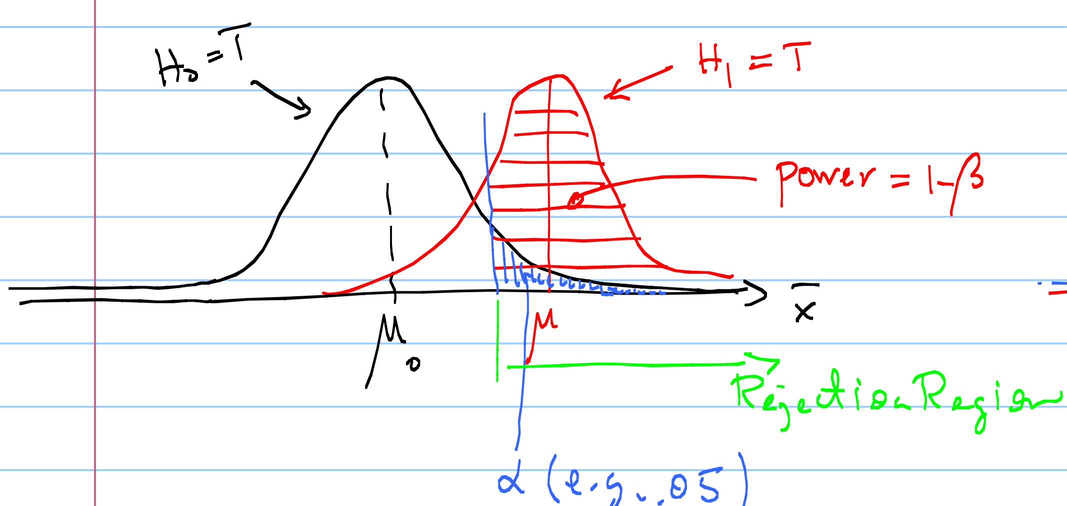
β = prob(Type II) = prob(Data Cannot reject H_0 ... | H_0 = F)

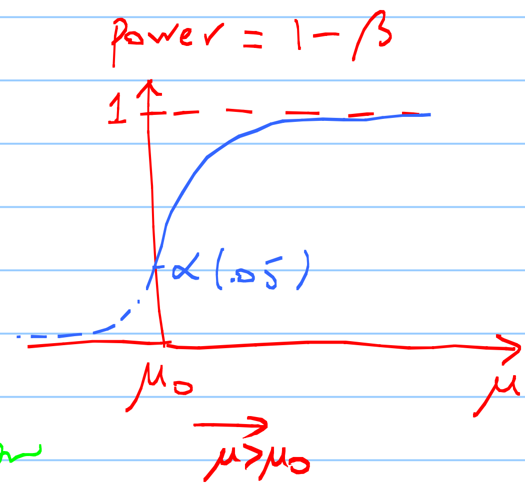
power = $1-\beta$ = prob(Data Reject H_0 ... | H_0 = F)

$\mu \le \mu_0$ $\qquad$ $\mu > \mu_0$









In the movie, $\Delta\mu$ is $\mu_0 - \mu$. That's why it looks like the "opposite" of what I've shown above.

Example

Suppose pop $\sim N(\mu, \sigma)$, i.e. $\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$.

For simplicity, fix $\sigma = 1$, $n = 121$

Suppose we are testing $H_0: \mu \le 0$ vs. $H_1: \mu > 0$, at $\alpha = .05$

a) If $H_0 = T$, what is the value of $\bar{x}$ with area 0.05 to its right?
I.e. what is the rejection region (RR)?

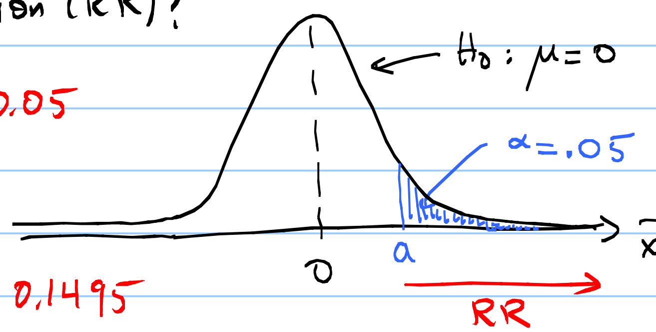
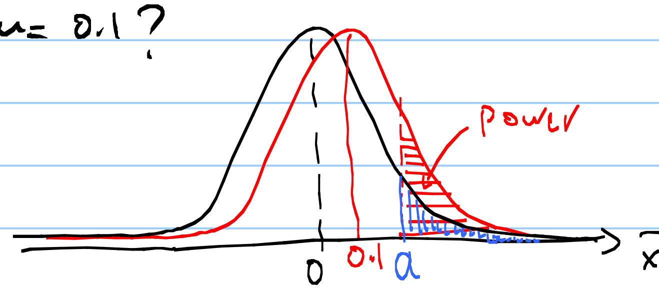


Table 1: $z = 1.645$ has area 0.05 to its right.

$$\frac{a - 0}{1/\sqrt{121}} = 1.645 \Rightarrow \underline{a = \frac{1.645}{11} = 0.1495}$$

$\therefore$ $\underline{RR: \ \bar{x} > 0.1495}$

b) If $\mu = 0.1$, what is the area to the right of the answer in part a)?
I.e. what is the power if $\mu = 0.1$?



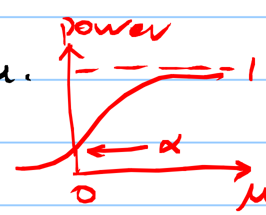
$$\text{power} = \text{prob}(\bar{x} > a \mid \mu = 0.1)$$

$$= \text{prob}\left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} > \frac{a - .1}{1/\sqrt{121}}\right)$$

$$= \text{prob}\left(z > \frac{0.1495 - 0.1}{1/11}\right) = \text{prob}(z > 0.5445) = 1 - \text{pr}(z < .54)$$

$$= 1 - 0.7054 = \boxed{0.2946}$$

c) Repeat for "all" $\mu > \mu_0$, and plot power vs. μ.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\mu_0 =$ here, 0.



Notes:

1) It can be shown that power increases with sample size, n.
2) However, data does <u>not</u> enter the computation of power.

hw. lect 27-1 In class

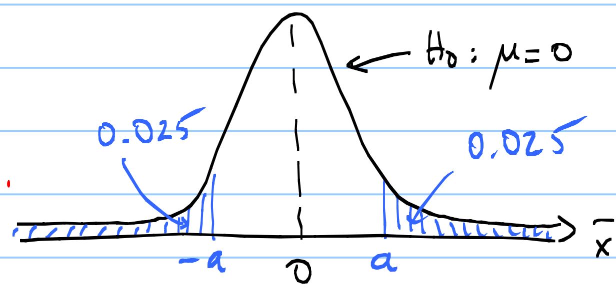
Suppose pop $\sim N(\mu, \sigma^2)$, i.e. $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$.

For simplicity, fix $\sigma = 1$ and $n = 121$.

We are testing $H_0: \mu = 0$, $H_1: \mu \neq 0$ at $\alpha = 0.05$

a) Where is the rejection region (RR)?

Table I $\Rightarrow$ $z = -1.96$ has 0.025 area to its left. Find the value of a.

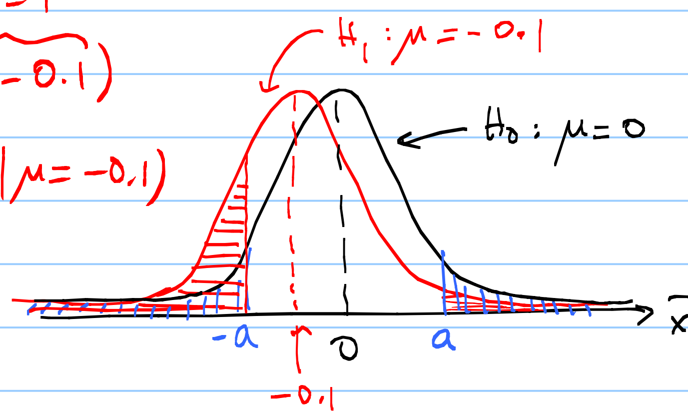


RR: ______

b) What is the power if $\mu = -0.1$? Hint: you need to add two areas.

$$\text{power} = pr(\underbrace{\bar{x}\ \_\_\_ \text{ and } \bar{x}\ \_\_\_}_{RR} \mid \underbrace{\mu = -0.1}_{H_1 = T})$$

$$= pr(\bar{x}\ \_\_\_ \mid \mu = -0.1) + pr(\bar{x}\ \_\_\_ \mid \mu = -0.1)$$



$$= pr(z < -0.86) + pr(z > 3.06)$$

$$= 0.1949 + 0.0011 = 0.196$$

c) What is the power if $\mu = +0.1$?