Term Project: STA 108

Riley Adams

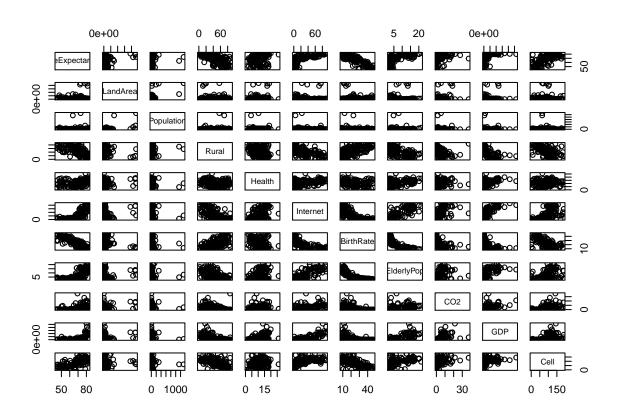
12/13/2020

[1] 186 13

[1] 149 13

#scatterplots (initial)

pairs(cbind(LifeExpectancy, LandArea, Population, Rural, Health, Internet, BirthRate, ElderlyPop, CO2,



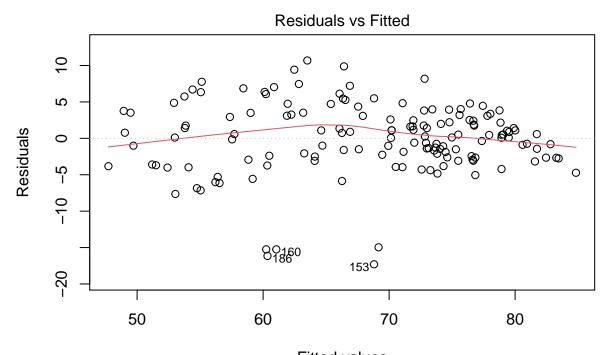
Life Expectancy appears to have no correlation with LandArea, Population.

There appears to be a negative linear trend with Rural, BirthRate.

There appears to be positive linear trend with Health, Cell.

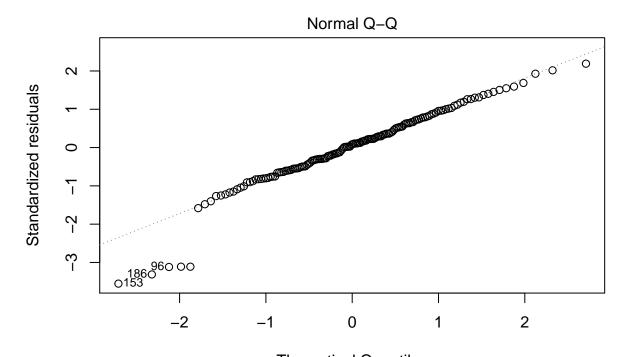
With internet, elderlypopulation, CO2 and GDP there is som kind of positive relationship that is not linear.

```
## fit model and summary output [1] -----
fit1 <- lm(LifeExpectancy ~ LandArea + Population + Rural + Health + Internet + BirthRate + ElderlyPop
summary(fit1)
##
## Call:
## lm(formula = LifeExpectancy ~ LandArea + Population + Rural +
##
      Health + Internet + BirthRate + ElderlyPop + CO2 + GDP +
##
      Cell, data = country80)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
                                           Max
## -17.3006 -2.6467
                      0.3558
                               3.1298 10.6910
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 8.464e+01 3.643e+00 23.236 < 2e-16 ***
## LandArea
              -1.655e-07 3.308e-07
                                    -0.500
                                              0.6176
## Population 4.588e-04 3.519e-03
                                     0.130
                                              0.8965
## Rural
              -5.196e-02 2.670e-02 -1.947
                                              0.0536 .
## Health
              1.942e-01 1.046e-01
                                     1.857
                                              0.0655 .
              5.611e-02 3.525e-02
## Internet
                                     1.592
                                              0.1137
## BirthRate -7.180e-01 7.847e-02 -9.150 7.07e-16 ***
## ElderlyPop -4.482e-01 1.716e-01 -2.612
                                              0.0100 *
              -6.835e-02 1.031e-01 -0.663
                                              0.5086
## CO2
## GDP
               7.031e-05 4.262e-05
                                     1.650
                                              0.1013
## Cell
              1.736e-02 1.412e-02
                                              0.2209
                                      1.230
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 4.959 on 137 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.7899, Adjusted R-squared: 0.7745
## F-statistic: 51.5 on 10 and 137 DF, p-value: < 2.2e-16
Initial model above.
# Residual analysis [1] ---
# Residual plots
plot(fit1, which = 1)
```



Fitted values Im(LifeExpectancy ~ LandArea + Population + Rural + Health + Internet + Bir ...

#Normal Probability Plot
plot(fit1, which = 2)



Theoretical Quantiles
Im(LifeExpectancy ~ LandArea + Population + Rural + Health + Internet + Bir ...

```
# Model Selection ----
library(leaps)
```

Warning: package 'leaps' was built under R version 4.0.3

```
## Forward selection
fit1_forward <- regsubsets(LifeExpectancy ~ LandArea + Population + Rural + Health + Internet + BirthRadata = country80, method = "forward")
cbind(summary(fit1_forward)$which, "adjusted r^2" = summary(fit1_forward)$adjr2)</pre>
```

```
(Intercept) LandArea Population Rural Health Internet BirthRate ElderlyPop
##
## 1
                 1
                            0
                                        0
                                                       0
                                                                  0
                                                                              1
## 2
                 1
                            0
                                        0
                                               0
                                                        0
                                                                  1
                                                                              1
                                                                                           0
                                                        0
                                                                              1
                                                                                          0
## 3
                 1
                            0
                                        0
                                               1
                                                                  1
                                                                              1
## 4
                 1
                            0
                                        0
                                               1
                                                       0
                                                                  1
## 5
                 1
                            0
                                        0
                                                        1
                                                                  1
                                                                              1
## 6
                 1
                            0
                                        0
                                               1
                                                        1
                                                                  1
                                                                              1
                                                                                           1
## 7
                 1
                            0
                                        0
                                               1
                                                        1
                                                                  1
                                                                              1
                                                                                           1
## 8
                 1
                            0
                                               1
                                                        1
                                                                  1
     CO2 GDP Cell adjusted r^2
## 1
        0
                  0
                        0.7327963
            0
## 2
        0
            0
                  0
                        0.7564876
## 3
            0
                        0.7649131
        0
                  0
## 4
        0
                        0.7699997
                        0.7751014
## 5
        0
            0
                  0
```

```
## 7
                      0.7777896
       0
           1
                 1
## 8
                      0.7773005
## Backward elimination
fit1_backward <- regsubsets(LifeExpectancy ~ LandArea + Population + Rural + Health + Internet + BirthR
                               data = country80, method = "backward")
cbind(summary(fit1_backward)$which, "adjusted r^2" = summary(fit1_backward)$adjr2)
     (Intercept) LandArea Population Rural Health Internet BirthRate ElderlyPop
##
## 1
                1
                                     0
## 2
                          0
                                      0
                                            0
                                                    0
                                                                        1
                                                                                    0
                1
                                                             1
## 3
                1
                          0
                                      0
                                            1
                                                    0
                                                             1
                                                                        1
                                                                                    0
                                                    0
## 4
                          0
                                      0
                                            1
                                                             1
                                                                        1
                1
                                                                                    1
## 5
                          0
                                      0
                                            1
                                                    1
                                                             1
                1
## 6
                          0
                                     0
                                            1
                                                    1
                                                             1
                                                                        1
                                                                                    1
                1
## 7
                                      0
                                                    1
                                                             1
                                                                        1
                1
                          0
                                            1
                                                                                    1
## 8
                1
                          0
                                                    1
                                                             1
                                                                        1
     CO2 GDP Cell adjusted r^2
## 1
       0
                      0.7327963
           0
                 0
## 2
                      0.7564876
       0
           0
                 0
## 3
           0
                      0.7649131
       0
                 0
## 4
       0
           0
                 0
                      0.7699997
## 5
                      0.7751014
       0
           0
                 0
## 6
       0
           1
                 0
                      0.7767840
## 7
           1
                      0.7777896
## 8
                      0.7773005
       1
            1
Backward Elimination suggests best model is: LifeExpectancy ~ Rural, Health, Internet, BirthRate, Elder-
lyPop, GDP, Cell
```

I'll fit that as the new model, fit2.

Health

Internet

BirthRate

6

1

0

0.7767840

```
## fit model and summary output [2] ----
fit2 <- lm(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP + Cell, data = cou
summary(fit2)
##
## Call:
## lm(formula = LifeExpectancy ~ Rural + Health + Internet + BirthRate +
       ElderlyPop + GDP + Cell, data = country80)
##
## Residuals:
                  1Q
                       Median
                                    3Q
                                            Max
## -17.6813 -2.4483
                       0.1573
                                3.1532 10.7907
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) 8.376e+01 3.469e+00 24.147
                                               <2e-16 ***
## Rural
               -5.063e-02 2.560e-02 -1.977
                                               0.0500 *
```

0.0524 .

<2e-16 ***

0.1367

1.956

1.497

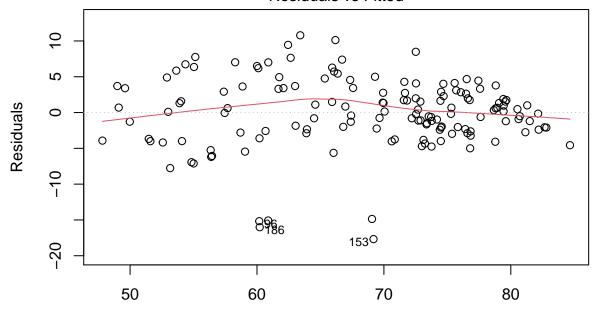
1.990e-01 1.017e-01

5.164e-02 3.450e-02

-7.031e-01 7.487e-02 -9.391

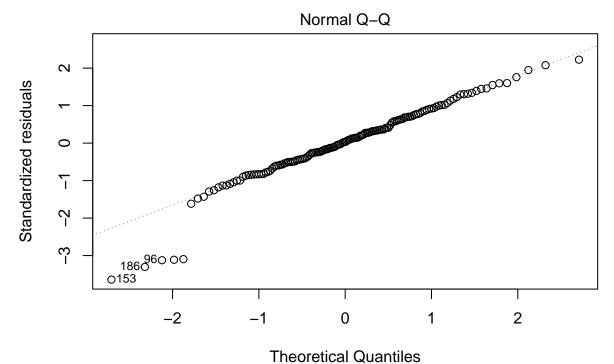
```
## ElderlyPop -4.078e-01 1.653e-01
                                     -2.467
                                               0.0148 *
## GDP
               5.893e-05 4.019e-05
                                       1.466
                                               0.1449
## Cell
                1.722e-02 1.346e-02
                                               0.2027
                                       1.280
## ---
                  0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 4.923 on 140 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.7884, Adjusted R-squared: 0.7778
## F-statistic: 74.51 on 7 and 140 DF, p-value: < 2.2e-16
# Residual analysis [2] ---
# Residual plots
plot(fit2, which = 1)
```

Residuals vs Fitted



Fitted values
Im(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop + GD ...

```
#Normal Probability Plot
plot(fit2, which = 2)
```



Im(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop + GD ...

Residual Plot:

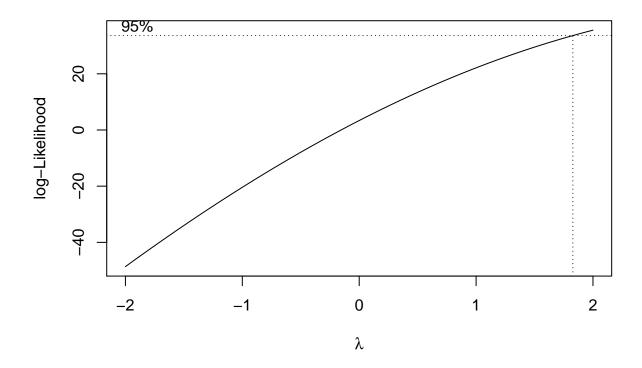
- Linearity: Residuals a little high in the middle. somewhat nonlinear.
- Variance: non-constant. Lower variance at high end.

QQ Plot:

• Fairly good. Values at lowest end are too low.

Good candidate for boxcox.

```
library(MASS)
# Boxcox for fit2
boxcox(fit2)
```

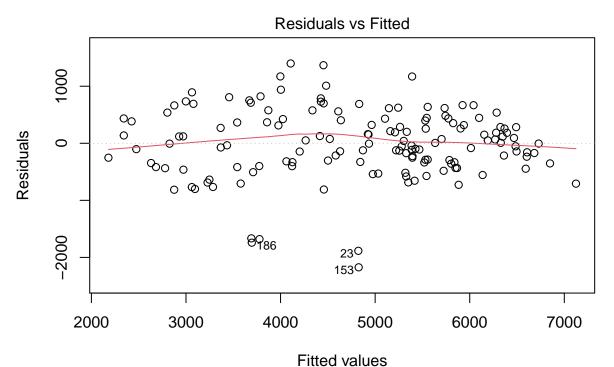


Box Cox suggests we raise Y to approx 1.5 or 2. Closer to 2. So we will try that.

```
## fit model and summary output [3] -----
LifeExpectSq <- (LifeExpectancy)^2</pre>
fit3 <- lm(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP + Cell, data = count
summary(fit3)
##
## Call:
  lm(formula = LifeExpectSq ~ Rural + Health + Internet + BirthRate +
       ElderlyPop + GDP + Cell, data = country80)
##
##
## Residuals:
        Min
                  1Q
                       Median
                                     3Q
                                              Max
                          0.37
## -2173.61 -338.56
                                 399.77
                                         1398.60
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                6.520e+03
                           4.286e+02
                                       15.213
                                               < 2e-16 ***
                                                 0.0305 *
## Rural
               -6.914e+00
                           3.164e+00
                                       -2.185
## Health
                2.682e+01
                            1.257e+01
                                        2.134
                                                 0.0346 *
## Internet
                                                 0.0780 .
                7.567e+00
                           4.262e+00
                                        1.775
## BirthRate
               -8.416e+01
                           9.252e+00
                                       -9.096 8.32e-16 ***
## ElderlyPop
               -4.355e+01
                            2.043e+01
                                       -2.132
                                                 0.0347 *
## GDP
                9.770e-03
                           4.967e-03
                                        1.967
                                                 0.0512 .
## Cell
                2.035e+00
                           1.663e+00
                                        1.224
                                                 0.2230
```

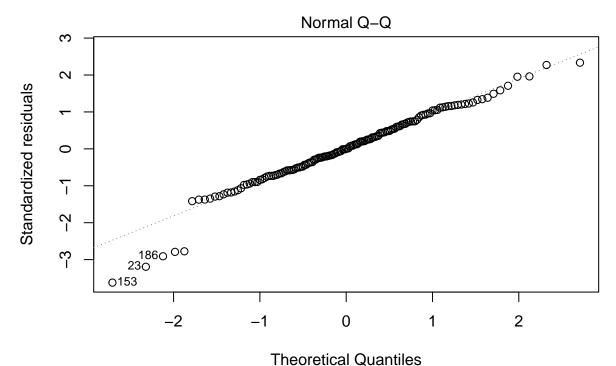
```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 608.3 on 140 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared: 0.8075, Adjusted R-squared: 0.7979
## F-statistic: 83.92 on 7 and 140 DF, p-value: < 2.2e-16

# Residual analysis [3] ---
# Residual plots
plot(fit3, which = 1)</pre>
```



Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP ...

#Normal Probability Plot
plot(fit3, which = 2)



Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP ...

Residuals are looking better. Variance has tightened up and linearity as well. Lower end of QQ plot tucked in a little and the rest almost perfectly normal.

We will run another model selection process to see if any predictors have become insignificant.

```
# Model Selection [3]
## Forward selection
fit3_forward <- regsubsets(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP + Ce
                            data = country80, method = "forward")
cbind(summary(fit3_forward) $which, "adjusted r^2" = summary(fit3_forward) $adjr2)
     (Intercept) Rural Health Internet BirthRate ElderlyPop GDP Cell adjusted r^2
##
## 1
                                                                             0.7364894
## 2
                1
                      0
                             0
                                       1
                                                                  0
                                                                       0
                                                                             0.7756859
                                                  1
## 3
                             0
                                                  1
                                                                            0.7852103
                                       1
                                                              0
## 4
                1
                             1
                                                  1
                                                                            0.7891219
                      1
                             1
                                       1
                                                                            0.7932579
## 5
                      1
                                                  1
                                                              1
## 6
                1
                      1
                             1
                                       1
                                                  1
                                                              1
                                                                       0
                                                                             0.7972095
## 7
                                                                             0.7979232
## Backward elimination
```

```
fit3_backward <- regsubsets(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP + C
data = country80, method = "backward")
cbind(summary(fit3_backward)$which, "adjusted r^2" = summary(fit3_backward)$adjr2)
```

(Intercept) Rural Health Internet BirthRate ElderlyPop GDP Cell adjusted r^2

```
## 1
                1
                                                                0
                                                                                0.7364894
                                                    1
## 2
                1
                       0
                               0
                                         1
                                                                0
                                                                     0
                                                    1
                                                                          0
                                                                                0.7756859
## 3
                1
                               0
                                         1
                                                    1
                                                                0
                                                                    0
                                                                                0.7852103
                                         1
                                                                0
                                                                    0
## 4
                1
                       1
                               1
                                                    1
                                                                          0
                                                                                0.7891219
## 5
                1
                       1
                               1
                                         1
                                                    1
                                                                1
                                                                     0
                                                                          0
                                                                                0.7932579
## 6
                                         1
                                                                     1
                                                                          0
                1
                       1
                               1
                                                    1
                                                                1
                                                                                0.7972095
## 7
                                                                                0.7979232
```

##		(Intercept)	Rural	Health	Internet	BirthRate	ElderlyPop	GDP	Cell	Mallows' Cp
##	1	1	0	0	0	1	0	0	0	46.385719
##	2	1	0	0	1	1	0	0	0	18.956288
##	3	1	1	0	0	1	0	1	0	12.743486
##	4	1	1	1	0	1	0	1	0	10.826770
##	5	1	1	1	1	1	1	0	0	9.278258
##	6	1	1	1	1	1	1	1	0	7.497954
##	7	1	1	1	1	1	1	1	1	8.000000

Step-wise method says to keep Cell, but not by a lot.

Mallows' Cp seems to fit the model better without cell.

Furthermore t-test for cell suggests it is not significant after other predictors accounted for.

Try building model without it and see if residuals tighten up.

```
## fit model and summary output [4] ----
fit4 <- lm(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP, data = country80)
summary(fit4)</pre>
```

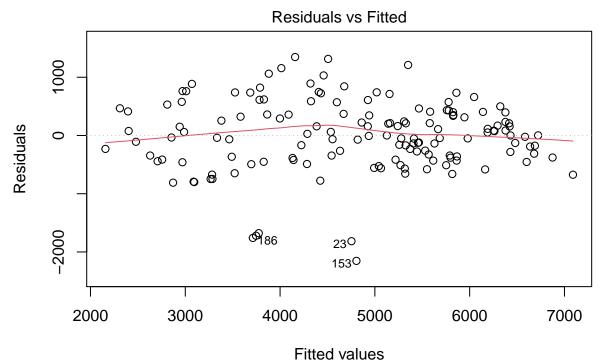
```
##
## Call:
## lm(formula = LifeExpectSq ~ Rural + Health + Internet + BirthRate +
       ElderlyPop + GDP, data = country80)
##
##
## Residuals:
##
       Min
                  1Q
                      Median
                                   3Q
                                           Max
## -2153.15 -379.01
                        0.57
                               397.42 1346.88
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.848e+03 3.352e+02 20.434
                                              <2e-16 ***
## Rural
               -7.796e+00 3.086e+00
                                     -2.526
                                              0.0126 *
               2.631e+01 1.259e+01
## Health
                                      2.090
                                              0.0384 *
## Internet
               8.367e+00 4.219e+00
                                      1.983
                                              0.0493 *
## BirthRate
              -8.841e+01 8.590e+00 -10.292
                                              <2e-16 ***
## ElderlyPop -4.715e+01 2.025e+01 -2.328
                                              0.0213 *
## GDP
               9.655e-03 4.975e-03
                                      1.941
                                              0.0543 .
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
```

```
##
## Residual standard error: 609.4 on 141 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared: 0.8055, Adjusted R-squared: 0.7972
## F-statistic: 97.31 on 6 and 141 DF, p-value: < 2.2e-16</pre>
```

adjR^2 dropped just a little bit. But t-tests for all variables are now significant.

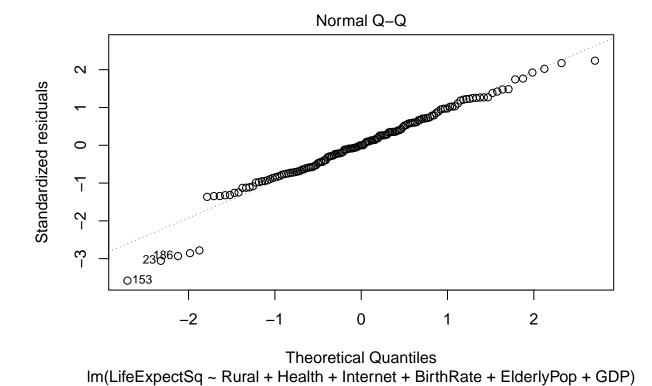
Let's carry out residual analysis, and see if it is any better.

```
# Residual analysis [4] ---
# Residual plots
plot(fit4, which = 1)
```



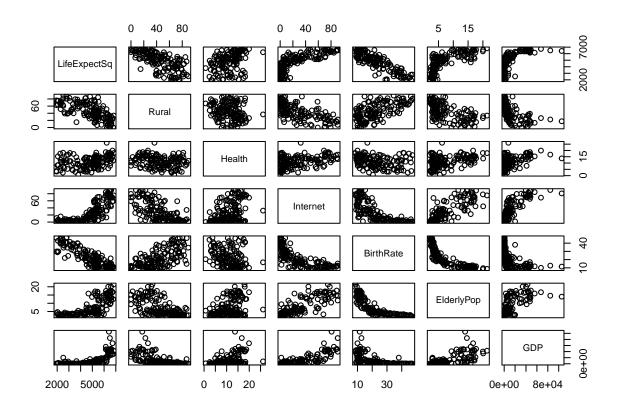
Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop + GDP)

```
#Normal Probability Plot
plot(fit4, which = 2)
```



hmmm QQ plot got worse. Let's roll with this for now though. Try some X transformations. We'll plot pairs() to see what could be more linear.

```
# Pairs scatterplot analysis [4]
pairs(cbind(LifeExpectSq, Rural, Health, Internet, BirthRate, ElderlyPop, GDP))
```



cor.test(Rural,BirthRate)

```
##
## Pearson's product-moment correlation
##
## data: Rural and BirthRate
## t = 9.4013, df = 146, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.5025229 0.7055204
## sample estimates:
## cor
## 0.6140779</pre>
```

I'll try transforming:

internet -> internet $^(1/2)$ elderlypop -> elderlypop $^(1/2)$ GDP -> GDP $^(1/2)$

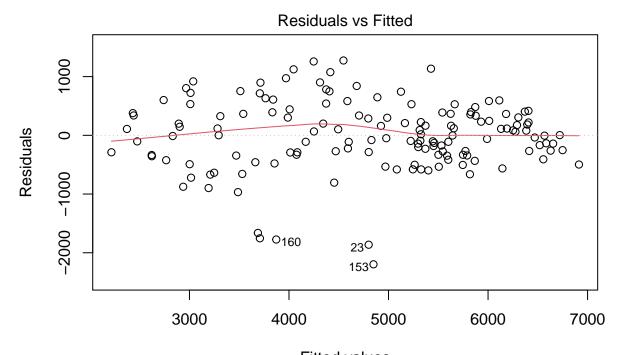
```
InternetSqrt <- Internet^(.5)
ElderSqrt <- ElderlyPop^(.5)
GDPsqrt <- GDP^(.5)

## fit model and summary output [5] -----
fit5 <- lm(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + GDPsqrt, data = countsummary(fit5)</pre>
```

```
##
## Call:
## lm(formula = LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate +
      ElderSqrt + GDPsqrt, data = country80)
##
## Residuals:
       Min
                 1Q
                      Median
                                   30
## -2196.67 -331.61
                        2.18
                               365.71 1274.34
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6699.619
                           509.300 13.155 < 2e-16 ***
## Rural
                 -5.916
                            3.205 -1.846
                                            0.0670 .
## Health
                 24.473
                                   1.954
                                             0.0527 .
                            12.526
                                             0.0396 *
## InternetSqrt 89.764
                            43.224
                                    2.077
## BirthRate
                -83.520
                             9.657 -8.649 1.05e-14 ***
## ElderSqrt
               -299.805
                           126.530 -2.369
                                             0.0192 *
## GDPsqrt
                  3.873
                             1.510
                                   2.565
                                             0.0113 *
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 602.5 on 141 degrees of freedom
    (1 observation deleted due to missingness)
## Multiple R-squared: 0.8099, Adjusted R-squared: 0.8018
## F-statistic: 100.1 on 6 and 141 DF, p-value: < 2.2e-16
```

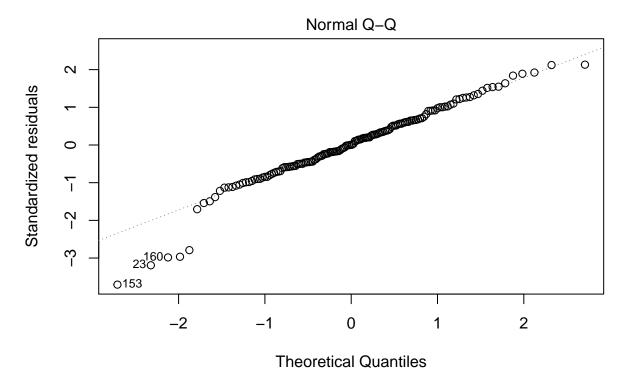
adjR2 remains the same as fit4. predictors all still significant.

```
# Residual analysis [5] ---
# Residual plots
plot(fit5, which = 1)
```



Fitted values Im(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + G ...

#Normal Probability Plot
plot(fit5, which = 2)



Im(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + G ...

Best looking QQ so far. Still have the same problem with non-constant variance.

Try model selection again.

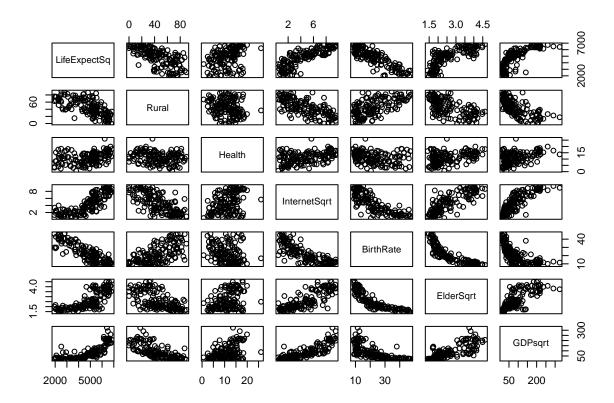
```
# Model Selection [5] ----
## Forward selection
fit5_forward <- regsubsets(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + GDPsq
                             data = country80, method = "forward")
cbind(summary(fit5_forward)$which, "adjusted r^2" = summary(fit5_forward)$adjr2)
     (Intercept) Rural Health InternetSqrt BirthRate ElderSqrt GDPsqrt
##
## 1
                1
                      0
                              0
                                                      1
                                                                 0
## 2
                1
                      0
                              0
                                            0
                                                      1
                                                                          1
## 3
                      0
                                                      1
                                                                 0
                1
                              0
                                            1
                                                                          1
                              0
                                            1
## 4
                1
                      0
                                                      1
                                                                 1
                                                                          1
## 5
                      0
                1
                              1
                                            1
                                                      1
                                                                 1
                                                                          1
## 6
                      1
                              1
                                            1
                                                                 1
                                                                          1
                1
##
     adjusted r^2
        0.7364894
## 1
## 2
        0.7842098
## 3
        0.7903413
## 4
        0.7954793
## 5
        0.7984503
## 6
        0.8018114
```

```
fit5_backward <- regsubsets(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + GDPs
                              data = country80, method = "backward")
cbind(summary(fit5_backward)$which, "adjusted r^2" = summary(fit5_backward)$adjr2)
     (Intercept) Rural Health InternetSqrt BirthRate ElderSqrt GDPsqrt
## 1
                     0
               1
                             0
## 2
               1
                     0
                             0
                                          0
                                                     1
                                                               0
                                                                        1
## 3
                     0
                             0
                                          1
                                                     1
                                                               0
                                                                        1
               1
## 4
               1
                     0
                             0
                                          1
                                                     1
                                                               1
## 5
                     0
                             1
                                          1
                                                     1
                                                               1
               1
## 6
               1
                     1
                             1
                                          1
                                                     1
                                                               1
                                                                        1
     adjusted r^2
## 1
        0.7364894
## 2
        0.7842098
## 3
        0.7903413
## 4
        0.7954793
## 5
        0.7984503
## 6
        0.8018114
# Mallow's Cp
fit5_subset <- regsubsets(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate + ElderSqrt + GDPsqr
                             data = country80, method = "exhaustive")
cbind(summary(fit5_subset)$which, "Mallows' Cp" = summary(fit5_subset)$cp)
##
     (Intercept) Rural Health InternetSqrt BirthRate ElderSqrt GDPsqrt Mallows' Cp
## 1
               1
                     0
                             0
                                          0
                                                     1
                                                                            50.120852
## 2
               1
                     0
                             0
                                          0
                                                     1
                                                               0
                                                                            15.877756
                                                                        1
## 3
                     0
                             0
                                                               0
                                                                            12.333948
               1
                                          1
                                                     1
                                                                        1
## 4
                     0
                             0
                                          1
                                                     1
                                                                            9.568842
               1
                                                               1
                                                                        1
## 5
               1
                     0
                             1
                                          1
                                                     1
                                                               1
                                                                        1
                                                                             8.408165
## 6
               1
                     1
                             1
                                          1
                                                     1
                                                               1
                                                                        1
                                                                             7.000000
```

All model selection methods confirm current model is best.

Backward elimination

```
pairs(cbind(LifeExpectSq, Rural, Health, InternetSqrt, BirthRate, ElderSqrt, GDPsqrt))
```



InternetSqrt looks much more linear now than internet did agains LifeExpectency.

Eldersqrt saw a bit of improvement.

InternetSqrt 105.793

GDP sqrt got closer to linear as well, but still not as much as the other two.

42.701

Rural and BirthRate are quite collinear, yet BirthRate has a stronger linear relationship with LifeExpectancy.

Drop Rural, build new model. Compare results.

```
## fit model and summary output [6] ----
fit6 <- lm(LifeExpectSq ~ Health + InternetSqrt + BirthRate + ElderSqrt + GDPsqrt, data = country80)
summary(fit6)
##
## Call:
## lm(formula = LifeExpectSq ~ Health + InternetSqrt + BirthRate +
       ElderSqrt + GDPsqrt, data = country80)
##
##
## Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                             Max
## -2140.58 -333.73
                       -20.19
                                372.70
                                        1295.78
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6390.883
                            485.122 13.174
                                            < 2e-16 ***
## Health
                  22.156
                             12.568
                                      1.763 0.08006 .
```

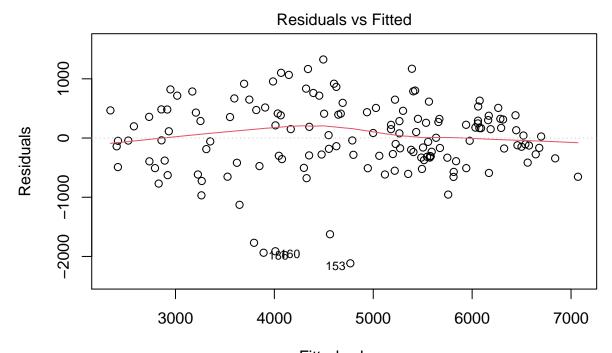
2.478 0.01440 *

```
## BirthRate
              -85.596
                            9.672 -8.850 3.16e-15 ***
## ElderSqrt
               -313.841
                           127.368 -2.464 0.01493 *
                             1.450 3.255 0.00142 **
## GDPsqrt
                  4.721
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 607.6 on 142 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.8053, Adjusted R-squared: 0.7985
## F-statistic: 117.5 on 5 and 142 DF, p-value: < 2.2e-16
All predictors are significant except Health. Drop health. New model
## fit model and summary output [6] ----
fit7 <- lm(LifeExpectSq ~ InternetSqrt + BirthRate + ElderSqrt + GDPsqrt, data = country80)
summary(fit7)
##
## Call:
## lm(formula = LifeExpectSq ~ InternetSqrt + BirthRate + ElderSqrt +
      GDPsqrt, data = country80)
##
##
## Residuals:
       \mathtt{Min}
                 1Q
                     Median
                                   3Q
## -2115.95 -330.82
                       15.31
                               398.55 1326.78
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6389.493 488.684 13.075 < 2e-16 ***
## InternetSqrt 113.700
                            42.776
                                    2.658 0.008756 **
## BirthRate
                            9.567 -8.610 1.21e-14 ***
                -82.371
## ElderSqrt
               -270.526
                           125.893 -2.149 0.033330 *
                                    3.427 0.000796 ***
## GDPsqrt
                  4.981
                             1.453
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 612 on 143 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.801, Adjusted R-squared: 0.7955
```

All predictors significant. AdjR2 has increased. Conduct model residual analysis.

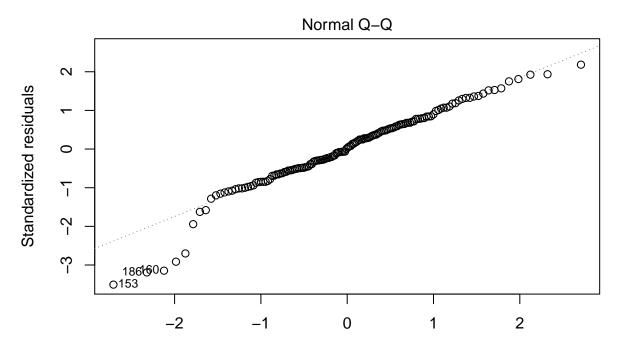
F-statistic: 143.9 on 4 and 143 DF, p-value: < 2.2e-16

```
# Residual analysis [7] ---
# Residual plots
plot(fit7, which = 1)
```



Fitted values
Im(LifeExpectSq ~ InternetSqrt + BirthRate + ElderSqrt + GDPsqrt)

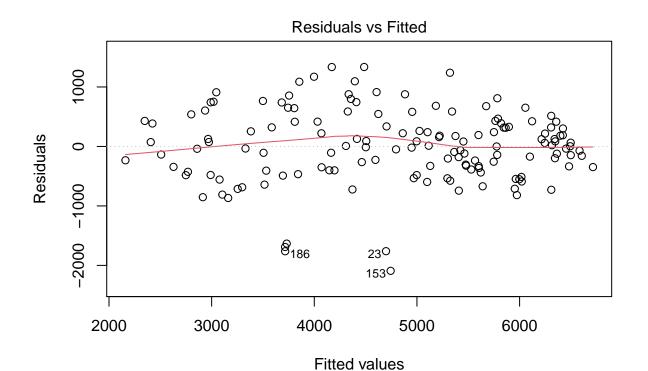
#Normal Probability Plot
plot(fit7, which = 2)



Theoretical Quantiles
Im(LifeExpectSq ~ InternetSqrt + BirthRate + ElderSqrt + GDPsqrt)

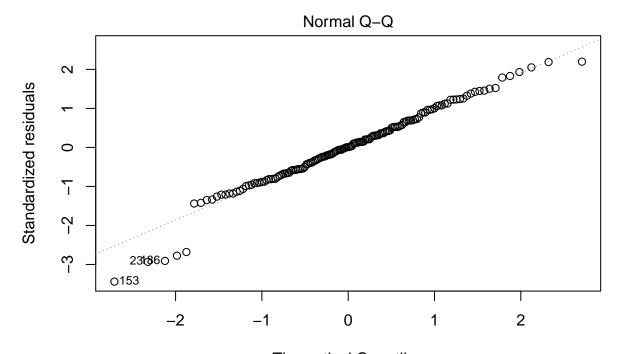
```
shapiro.test(resid(fit7))
##
##
    Shapiro-Wilk normality test
##
## data: resid(fit7)
## W = 0.9584, p-value = 0.0001952
qqplot sucks. try untransforming X vars.
Back up to fit4. notice that GDP had t-test pval > .05
## fit model and summary output [8] ----
fit8 <- lm(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop, data = country80)</pre>
summary(fit8)
##
   lm(formula = LifeExpectSq ~ Rural + Health + Internet + BirthRate +
##
##
       ElderlyPop, data = country80)
##
## Residuals:
        Min
##
                   1Q
                        Median
                                      3Q
                                              Max
##
   -2091.47 -368.27
                          5.91
                                  396.47
                                          1335.88
##
```

```
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 6709.962
                           330.653
                                    20.293
## Rural
                 -8.714
                             3.079
                                    -2.830 0.00533 **
## Health
                 28.800
                            12.642
                                     2.278
                                            0.02421 *
## Internet
                 12.664
                             3.627
                                     3.492 0.00064 ***
## BirthRate
                -84.497
                             8.432 -10.021
                                            < 2e-16 ***
                -39.381
                            20.042
                                   -1.965 0.05138 .
## ElderlyPop
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 615.3 on 142 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.8003, Adjusted R-squared: 0.7933
## F-statistic: 113.8 on 5 and 142 DF, p-value: < 2.2e-16
# Residual analysis [8] ---
# Residual plots
plot(fit8, which = 1)
```



#Normal Probability Plot
plot(fit8, which = 2)

Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop)



Theoretical Quantiles
Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate + ElderlyPop)

```
shapiro.test(resid(fit8))
```

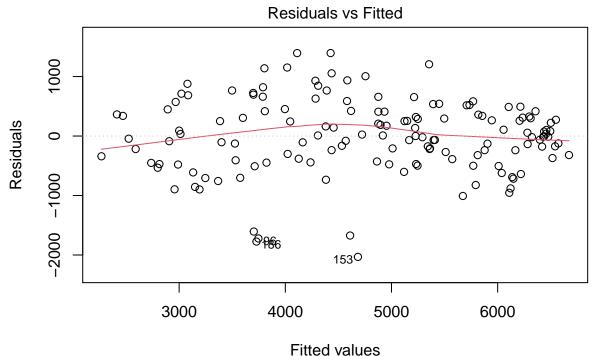
```
##
## Shapiro-Wilk normality test
##
## data: resid(fit8)
## W = 0.97112, p-value = 0.003265
```

model looking pretty good, however, elderlypop has pval > .05 and collinear with internet. drop elderlypop.

```
## fit model and summary output [9] -----
fit9 <- lm(LifeExpectSq ~ Rural + Health + Internet + BirthRate, data = country80)
summary(fit9)</pre>
```

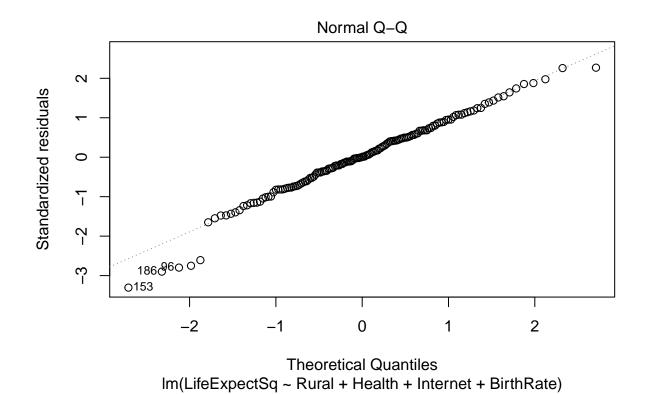
```
##
  lm(formula = LifeExpectSq ~ Rural + Health + Internet + BirthRate,
##
##
       data = country80)
##
## Residuals:
        Min
##
                   1Q
                        Median
                                     ЗQ
                                              Max
##
   -2031.67 -382.93
                          4.78
                                 409.11
                                         1394.98
##
```

```
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 6377.839
                           287.014
                                    22.221
                                    -2.857
                                            0.00491 **
## Rural
                 -8.883
                             3.109
## Health
                 24.001
                            12.527
                                     1.916
                                            0.05736
## Internet
                  9.325
                             3.236
                                     2.882
                                           0.00457 **
## BirthRate
                -76.120
                             7.347 -10.361
                                            < 2e-16 ***
## ---
## Signif. codes:
                  0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 621.5 on 143 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.7949, Adjusted R-squared: 0.7891
## F-statistic: 138.5 on 4 and 143 DF, p-value: < 2.2e-16
# Residual analysis [9] ---
# Residual plots
plot(fit9, which = 1)
```



Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate)

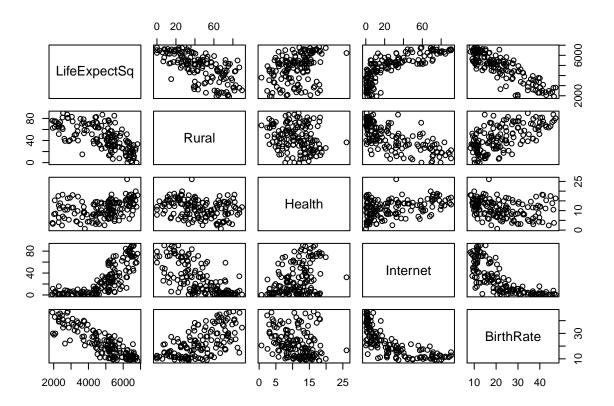
```
#Normal Probability Plot
plot(fit9, which = 2)
```



shapiro.test(resid(fit9))

```
##
## Shapiro-Wilk normality test
##
## data: resid(fit9)
## W = 0.97807, p-value = 0.01806

pairs(cbind(LifeExpectSq, Rural, Health, Internet, BirthRate))
```



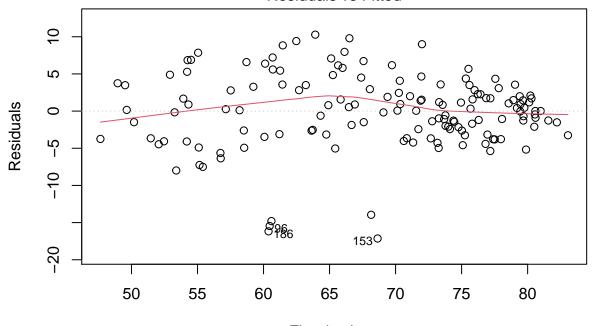
looks quite good. see about untransforming Y, for comparison to above model.

```
## fit model and summary output [10] ----
fit10 <- lm(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop, data = country80)
summary(fit10)
##
## lm(formula = LifeExpectancy ~ Rural + Health + Internet + BirthRate +
       ElderlyPop, data = country80)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                            Max
## -17.1379 -2.7523
                       0.1361
                                2.9832 10.2826
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 85.70067
                           2.66149
                                   32.200 < 2e-16 ***
## Rural
               -0.06360
                           0.02478
                                    -2.566 0.01132 *
## Health
               0.20962
                          0.10175
                                     2.060 0.04122 *
## Internet
               0.08420
                          0.02919
                                     2.884 0.00453 **
## BirthRate
             -0.71564
                          0.06787 -10.545 < 2e-16 ***
## ElderlyPop -0.39165
                          0.16133 -2.428 0.01645 *
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 4.953 on 142 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared: 0.7828, Adjusted R-squared: 0.7751
## F-statistic: 102.3 on 5 and 142 DF, p-value: < 2.2e-16

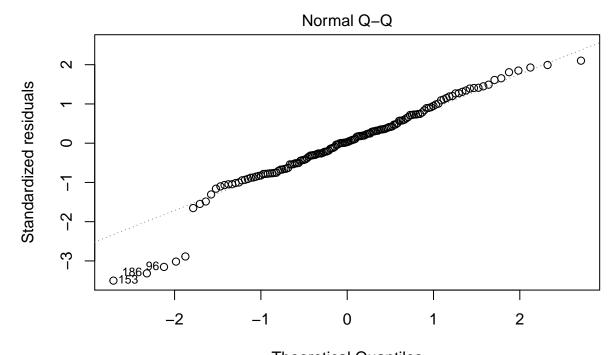
# Residual analysis [10] ---
# Residual plots
plot(fit10, which = 1)</pre>
```

Residuals vs Fitted



Fitted values
Im(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop)

#Normal Probability Plot
plot(fit10, which = 2)



Theoretical Quantiles
Im(LifeExpectancy ~ Rural + Health + Internet + BirthRate + ElderlyPop)

```
shapiro.test(resid(fit10))
```

```
##
## Shapiro-Wilk normality test
##
## data: resid(fit10)
## W = 0.95109, p-value = 4.55e-05
```

Yea fit9 is best. but health still has borderline pval. try dropping to compare fit11 with fit9 and make judgement call about significance of Health predictor.

```
## fit model and summary output [11] -----
fit11 <- lm(LifeExpectSq ~ Rural + Internet + BirthRate, data = country80)
summary(fit11)</pre>
```

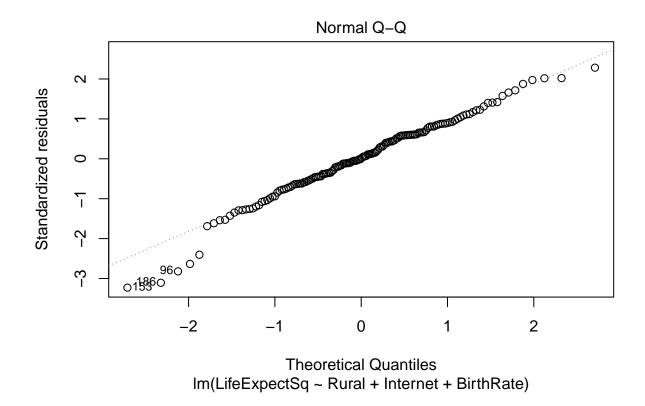
```
##
  lm(formula = LifeExpectSq ~ Rural + Internet + BirthRate, data = country80)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
##
  -2003.80 -366.99
                         5.13
                                403.27
                                         1423.65
##
## Coefficients:
```

```
Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6555.016
                           274.218 23.904 < 2e-16 ***
## Rural
                 -8.537
                             3.132
                                   -2.726 0.007212 **
                 11.327
                             3.091
                                     3.664 0.000348 ***
## Internet
## BirthRate
                -75.494
                             7.407 -10.192 < 2e-16 ***
##
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 627.2 on 144 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.7896, Adjusted R-squared: 0.7852
## F-statistic: 180.1 on 3 and 144 DF, p-value: < 2.2e-16
# Residual analysis [11] ---
# Residual plots
plot(fit11, which = 1)
```

Residuals vs Fitted 0 0 0 0 000 00 000 0 Residuals 000 0 00 00 -2000 -1000 0 0 0 153^O 3000 4000 5000 6000 Fitted values

```
#Normal Probability Plot
plot(fit11, which = 2)
```

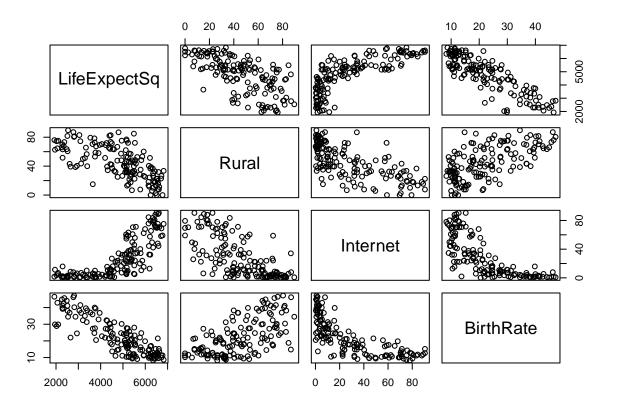
Im(LifeExpectSq ~ Rural + Internet + BirthRate)



shapiro.test(resid(fit11))

```
##
## Shapiro-Wilk normality test
##
## data: resid(fit11)
## W = 0.97794, p-value = 0.01743

pairs(cbind(LifeExpectSq, Rural, Internet, BirthRate))
```



WORSE.

Back to fit9 try one more with internet transformed to sqrt(internet)

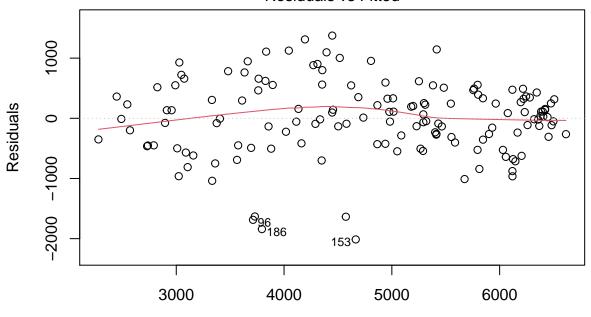
```
## fit model and summary output [12] -----
fit12 <- lm(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate, data = country80)
summary(fit12)</pre>
```

```
##
## Call:
## lm(formula = LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate,
      data = country80)
##
##
## Residuals:
       Min
                      Median
##
                 1Q
                       15.86
## -2012.86 -366.63
                               362.24
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                           375.762 15.853 < 2e-16 ***
## (Intercept) 5956.840
## Rural
                 -8.263
                             3.130 -2.640 0.00921 **
## Health
                 23.601
                            12.394
                                    1.904 0.05888
## InternetSqrt 117.630
                            37.245
                                     3.158 0.00194 **
## BirthRate
                -70.791
                             8.131 -8.706 6.95e-15 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
```

```
##
## Residual standard error: 618.1 on 143 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared: 0.7971, Adjusted R-squared: 0.7914
## F-statistic: 140.4 on 4 and 143 DF, p-value: < 2.2e-16

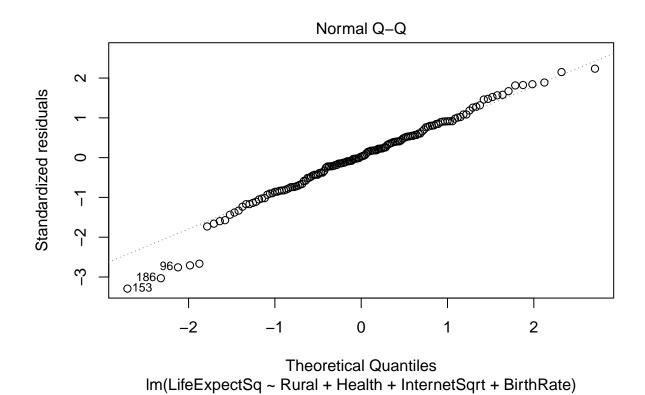
# Residual analysis [12] ---
# Residual plots
plot(fit12, which = 1)</pre>
```

Residuals vs Fitted



Fitted values Im(LifeExpectSq ~ Rural + Health + InternetSqrt + BirthRate)

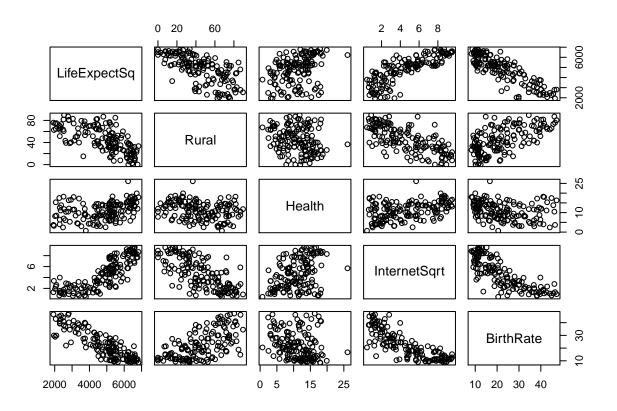
#Normal Probability Plot
plot(fit12, which = 2)



shapiro.test(resid(fit12))

```
##
## Shapiro-Wilk normality test
##
## data: resid(fit12)
## W = 0.97583, p-value = 0.01026

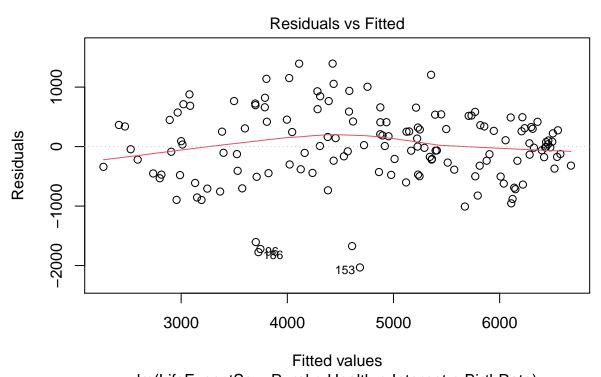
pairs(cbind(LifeExpectSq, Rural, Health, InternetSqrt, BirthRate))
```



fit 9 for comparison side-by-side -----summary(fit9)

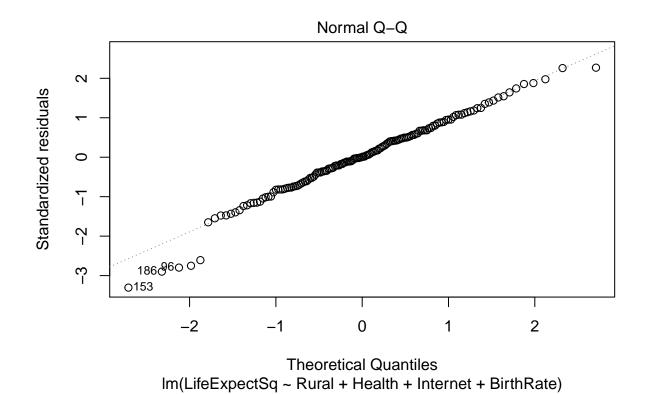
```
##
## Call:
## lm(formula = LifeExpectSq ~ Rural + Health + Internet + BirthRate,
      data = country80)
##
##
## Residuals:
       Min
                 1Q
                      Median
                                   30
## -2031.67 -382.93
                        4.78
                              409.11 1394.98
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                          287.014 22.221 < 2e-16 ***
## (Intercept) 6377.839
                            3.109 -2.857 0.00491 **
## Rural
                -8.883
                24.001
                           12.527
                                    1.916 0.05736 .
## Health
                            3.236
## Internet
                 9.325
                                    2.882 0.00457 **
## BirthRate
               -76.120
                           7.347 -10.361 < 2e-16 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 621.5 on 143 degrees of freedom
     (1 observation deleted due to missingness)
## Multiple R-squared: 0.7949, Adjusted R-squared: 0.7891
## F-statistic: 138.5 on 4 and 143 DF, p-value: < 2.2e-16
```

```
# Residual analysis [9] ---
# Residual plots
plot(fit9, which = 1)
```



Im(LifeExpectSq ~ Rural + Health + Internet + BirthRate)

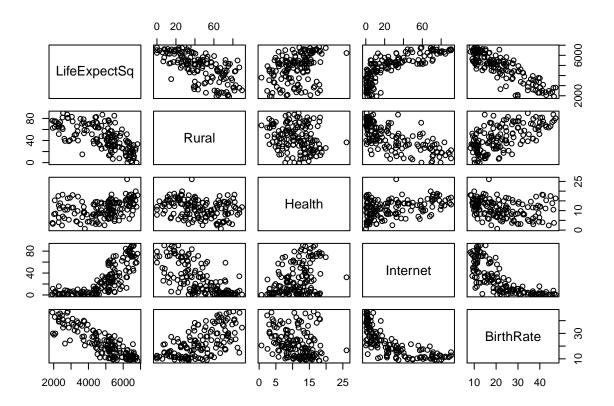
#Normal Probability Plot
plot(fit9, which = 2)



shapiro.test(resid(fit9))

```
##
## Shapiro-Wilk normality test
##
## data: resid(fit9)
## W = 0.97807, p-value = 0.01806

pairs(cbind(LifeExpectSq, Rural, Health, Internet, BirthRate))
```



Although fit12 has higher adjR2 and all predictors highly significant, fit9 has what could be considered normality in qq plot. best of all models, the only variable of concern is health, but this is borderline and seems to be important in model.

fit9 is best model.