FE-680 – Assignment 3

Submit complete solutions to all problems in a single PDF. If you wrote codes to solve the problems (C++, R, MATLAB, Mathematica, Python, etc.) please upload these files as well. No late submissions will be accepted after the due date. This is an individual assessment; no collaboration is allowed.

Problem 1

In the Hull–White model, a $= 0.08$ and $\sigma = 0.02$. Calculate the price of a one-year European call option on a zero-coupon bond that will mature in five years when the term structure is flat at 5%, the principal of the bond is $100, and the strike price is $70.

Problem 2

Consider the example in Lecture Notes: Vasicek Model Tree Construction and implement an algorithm to calculate the interest rates for 6 months ($\Delta t = 1$ month). The risk-neutral dynamics of the Vasicek model is given by the following stochastic process:

$$dr = k(\theta - r)dt + \sigma dW$$

where $k = 0.025$, $\sigma = 126$ basis points per year, $r_0 = 5.121\%$, and $\theta = 15.339\%$.

a) Report the interest rate values on the terminal nodes at $\Delta t = 6$ months.
b) Calculate the price of a zero-coupon bond with maturity in 6 months with face value 100.
c) Calculate the price of a European call option on the zero-coupon bond that expires at t = 6 months and has strike $90.
d) Calculate the price of an American call option on the zero-coupon bond that expires at t = 6 months and has strike $90.

Problem 3

Risk-free zero rates are flat at 5% in the U.S and flat at 9% in Australia (both rates are annually compounded). In a 4-year differential swap Australian floating risk-free rate is received and 8% is paid with both being applied to a USD principal of $10 million. Payments are exchanged annually. The volatility of all 1-year forward rates in Australia is estimated to be 30%, the volatility of the forward USD/AUD exchange rate (AUD per USD) is 20% for all maturities, and the correlation between the two is 0.3. What is the value of the swap?

Problem 4

Calculate the alternative duration measure for a 2-year bond with principal of $100 paying coupon semiannually at the rate of $3 per year when Vasicek's model is used with $a = 0.13$, $b = 0.012$, $\sigma = 0.01$, and $r = 1\%$. Show that it correctly predicts the effect of an increase in r to 1.05%.