That’s really cool!

I haven’t used Mathematica before, but the equations are clearly outlined in your screenshot.

Problem 2=binomial tree Vasicek
-Look through class example.

Problem 4= trinomial tree Hull-White.
-use equations from Matematica.

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From: Dragos Bozdog <dbozdog@stevens.edu>
Sent: Tuesday, October 25, 2022 12:27 PM
To: Riley Heiman <rheiman@stevens.edu>
Subject: Re: FE 680 Advanced Derivatives - Fall Semester Questions

Hello Riley,

Problem 2 in HW3 is related to the construction of the recombining binomial tree for Vasicek model, not the Hull-White trinomial tree. The formulas in your email are related to the calculation of the probabilities for branches in Hull-White model. For Vasicek, I provided an example in class with respect to the steps required to construct the binomial tree.

Please see below symbolic computations in Mathematica. The solutions match the solutions in the textbook for Hull-White model central branch probabilities.

```
In[7]:= sol = Solve[{
          pu * ΔR - pd * ΔR == -a * j * ΔR * Δt,
          pu * ΔR^2 + pd * ΔR^2 == σ^2 * Δt + a^2 * j^2 * ΔR^2 * Δt^2,
          pu + pm + pd == 1},
         {pu, pm, pd}]
```

Out[7]= $\left\{\left\{p_u \to -\frac{a\, j\, \Delta R^2\, \Delta t - a^2\, j^2\, \Delta R^2\, \Delta t^2 - \Delta t\, \sigma^2}{2\, \Delta R^2},\ p_m \to -\frac{-\Delta R^2 + a^2\, j^2\, \Delta R^2\, \Delta t^2 + \Delta t\, \sigma^2}{\Delta R^2},\ p_d \to -\frac{-a\, j\, \Delta R^2\, \Delta t - a^2\, j^2\, \Delta R^2\, \Delta t^2 - \Delta t\, \sigma^2}{2\, \Delta R^2}\right\}\right\}$

```
In[9]:= Simplify[sol /. ΔR → σ * Sqrt[3 * Δt]]
```

Out[9]= $\left\{\left\{p_u \to \frac{1}{6}\left(1 - 3\, a\, j\, \Delta t + 3\, a^2\, j^2\, \Delta t^2\right),\ p_m \to \frac{2}{3} - a^2\, j^2\, \Delta t^2,\ p_d \to \frac{1}{6}\left(1 + 3\, a\, j\, \Delta t + 3\, a^2\, j^2\, \Delta t^2\right)\right\}\right\}$

Best Regards,
Dragos

From: Riley Heiman <rheiman@stevens.edu>
Sent: Monday, October 24, 2022 2:16 PM
To: Dragos Bozdog <dbozdog@stevens.edu>
Subject: RE: FE 680 Advanced Derivatives - Fall Semester Questions

Hi Professor,

I'm working on HW 3 problem 2, and could use your wisdom once again. :)

It's ***unclear*** from the textbook how $p_u, p_d, and\ p_m$ are being calculated.

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$p_u, p_m,$ and p_d as the probabilities of the highest, middle, and lowest branches emanating from the node. The probabilities are chosen to match the expected change and variance of the change in R^* over the next time interval Δt. The probabilities must also sum to unity. This leads to three equations in the three probabilities.

As already mentioned, the mean change in R^* in time Δt is $-aR^*\Delta t$ and the variance of the change is $\sigma^2\Delta t$. At node (i, j), $R^* = j\Delta R$. If the branching has the form shown in Figure 32.5a, the $p_u, p_m,$ and p_d at node (i, j) must satisfy the following three equations to match the mean and standard deviation:

$$p_u\Delta R - p_d\Delta R = -aj\Delta R\Delta t$$
$$p_u\Delta R^2 + p_d\Delta R^2 = \sigma^2\Delta t + a^2j^2\Delta R^2\Delta t^2$$
$$p_u + p_m + p_d = 1$$

Using $\Delta R = \sigma\sqrt{3\Delta t}$, the solution to these equations is

$$p_u = \tfrac{1}{6} + \tfrac{1}{2}(a^2j^2\Delta t^2 - aj\Delta t)$$
$$p_m = \tfrac{2}{3} - a^2j^2\Delta t^2$$
$$p_d = \tfrac{1}{6} + \tfrac{1}{2}(a^2j^2\Delta t^2 + aj\Delta t)$$

I rearranged the equations next to the Blue star to have an explicit formula for the probabilities. Do you believe these equations are correct? (.PDF of proof attached)

$$p_u = \frac{\sigma^2\,\Delta t + a^2j^2\Delta R^2\Delta t^2 - aj\,\Delta t\,\Delta R^2}{2\,\Delta R^2}$$

$$p_d = p_u - aj\Delta t$$

$$p_m = 1 - p_u - p_d$$

Are we approaching the problem the correct way? I'm skeptical, because problem 2 doesn't provide a value for a.