HW 3 – Problem 2

Page 742 (Hull)

$$\Delta R = \sigma\sqrt{3\Delta t}$$

$$-p_u \Delta R - p_d \Delta R = -a j \Delta R \Delta t$$

$$-p_u \Delta R^2 + p_d \Delta R^2 = \sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2$$

$$p_d \Delta R^2 = \sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - p_u \Delta R^2$$

$$p_d = \frac{\sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - p_u \Delta R^2}{\Delta R^2}$$

$$p_u \Delta R = p_d \Delta R - a j \Delta R \Delta t$$

$$p_u = \frac{p_d \Delta R - a j \Delta R \Delta t}{\Delta R} \Rightarrow$$

$$p_u = p_d - a j \Delta t \leftarrow$$

1 equation
2 unknowns.
Plug in p_d

$$p_u = \left(\frac{\sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - p_u \Delta R^2}{\Delta R^2} \right) - aj\Delta t$$

(Multiply by ΔR^2)

$$p_u = \frac{\sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - p_u \Delta R^2}{\Delta R^2} - aj\Delta t$$

$$p_u \Delta R^2 = \sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - p_u \Delta R^2 - aj\Delta t \Delta R^2$$

add to both sides!

$$2 p_u \Delta R^2 = \sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - aj\Delta t \Delta R^2$$

(divide by $2\Delta R^2$)

$$\boxed{p_u = \frac{\sigma^2 \Delta t + a^2 j^2 \Delta R^2 \Delta t^2 - aj\Delta t \Delta R^2}{2\Delta R^2}}$$

Step 1 calculate p_u

Step 2 calculate p_d

Step 3 calculate $p_m = 1 - p_u - p_d$