FE-680 – Assignment 1

Submit your detailed solutions as a single PDF. If you wrote codes to solve the problems (C++, R, MATLAB, Mathematica, Python, etc) please attach these files as well. No late submissions will be accepted after the due date. This is an individual assessment; no collaboration is allowed.

Problem 1

		Inputs	discount curve	zero curve	forward curve	par curve		Bond Cash Flow
Overnight	0	**1.900%**						
Cash	1	**2.200%**						**4**
	2	**2.400%**						**4**
Forwards	3	**2.500%**						**4**
	4	**2.750%**						**4**
	5	**2.800%**						**4**
Swaps	6	**2.850%**						**4**
	7	**3.100%**						**4**
	8	**3.150%**						**4**
	9	**3.300%**						**104**
	10	**3.450%**						

a) Fill in discount curve, zero curve, forward curve
b) Compute the PV of the bond cashflows
c) Change the forward curve by +0.5% (at each maturity one at a time) and compute the discount factors and PV, DV01, duration of the bond for each case. Which forward change has the highest DV01?
d) Compute the PV of the bond when increasing simultaneously all the forward rates by 1%,2%, and 3%
e) What is the forward price of the bond 18 months from today?

Problem 2

Consider an eight-month European put option on a Treasury bond that currently has 14.25 years to maturity. The current cash bond price is \$908, the exercise price is \$900, and the volatility for the bond price is 10% per annum. A coupon of \$25 will be paid by the bond in three months. The risk-free interest rate is 1.5% for all maturities up to one year. Use Black's model to determine the price of the option. Consider both the case where the strike price corresponds to the cash price of the bond and the case where it corresponds to the quoted price.

Problem 3

Consider the following table with Treasury Yield Rates:

Time to Maturity	Yield Rate (continuously compounded)
3 months	1.20%
6 months	1.23%
1 year	1.30%
2 years	1.50%
3 years	1.77%
5 years	2.40%
7 years	3.14%
10 years	3.45%
20 years	3.80%

Use the data provided to build the yield curve using the cubic spline model. Report the value for the estimated coefficients and write the final expression for the rate as

$$R(0,t) = a + b(t - t_1) + c(t - t_1)^2 + \sum_{k=1}^{n-1} d_k (t - t_k)_+^3$$

where $(t_1 - t_k)_+ = \max\{t - t_k, 0\}$

a) Plot the fitted model and the original yield rates on the same graph. Compare the results.
b) Calculate the yield rate for $t = 4$ years.

Problem 4

The European Central Bank reports the Euro yield curve by providing the Nelson-Siegel parameters. Use the functional form of Nelson-Siegel model to estimate the parameters $\beta_0, \beta_1, \beta_2, \tau_1$ for the Treasury Coupon Bonds provided below. The Nelson-Siegel model assumes that:

$$R(0,t) = \beta_0 + \beta_1 \left(\frac{1 - e^{-t/\tau_1}}{t/\tau_1} \right) + \beta_2 \left(\frac{1 - e^{-t/\tau_1}}{t/\tau_1} - e^{-t/\tau_1} \right)$$

Treasury Coupon Bonds

Time to next payment	Payment frequency	Time to maturity	Coupon rate	Clean Price
0.4356	2	0.4356	0.78%	100.30
0.2644	2	0.7644	0.78%	100.48
0.2658	2	1.2658	0.65%	100.50
0.4342	2	1.9342	0.53%	100.31
0.0192	2	2.0192	0.28%	99.78
0.4753	2	2.9753	0.65%	100.16
0.3534	2	3.3534	1.40%	102.34
0.1000	2	3.6000	1.65%	103.08
0.2685	2	4.2685	2.03%	104.19
0.4342	2	4.9342	1.65%	102.06
0.2274	2	5.2274	4.40%	115.91
0.1027	2	5.6027	2.28%	104.36
0.2712	2	6.2712	2.65%	105.86
0.4370	2	6.9370	2.28%	102.97
0.4822	2	7.4822	3.40%	110.53
0.2260	2	7.7260	3.78%	113.09
0.4822	2	8.4822	2.65%	103.98
0.2260	2	8.7260	3.03%	106.50
0.2301	2	9.2301	3.28%	108.00
0.4808	2	9.9808	2.53%	101.19
0.4932	2	25.4932	4.40%	117.58
0.4959	2	26.4959	4.65%	122.28
0.2397	2	26.7397	4.90%	126.97
0.4959	2	27.4959	4.28%	115.19
0.2397	2	27.7397	4.40%	117.47
0.4959	2	28.4959	3.40%	98.98
0.2397	2	28.7397	4.15%	112.44
0.2438	2	29.2438	4.28%	114.67
0.4945	2	29.9945	3.78%	105.75