Stevens Institute of Technology

Financial Engineering

Chooser Option Profitability:
Simulating Stock Prices with Jump-Diffusion

Author:
Riley Heiman
Junyu Lu
Dengyu Zhang

Advisor:
Dr. Khaldoun Khashanah

January 11, 2023

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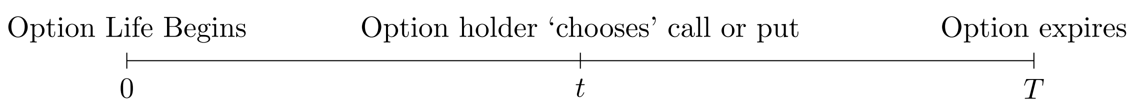
Abstract

News events can impact stock prices. Fama demonstrated this empirically in his paper, which showed stock prices adjust "rapidly" to new information (Fama et al., 1969). This is problematic to investors who are not diversified because a "bad" news event could move the stock price in the wrong direction. The chooser option is a financial derivative, which offers protection from extreme stock price jumps. This paper examines the profitability of the chooser option assuming unforeseen jumps in the underlying stock price.

Keywords: Chooser option, Hedging, Natural Language Processing, Jump-Diffusion

1 Definition and Pricing

The chooser option is an option contract that allows the investor to decide whether it is to be a call or a put prior the expiration date. The diagram below is a depiction of the Chooser option. (Rubinstein, 1991) (Whaley, 2006)



The goal of this paper is to study the chooser option, and it's practical application to mitigate idiosyncratic risk. The first step to understanding an exotic option is pricing it. The paper by Ďurica introduces an analytical solution to price a chooser option. (Ďurica and Švábová, 2014)

$$Chooser(S, X, t, T, q, r) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) - Se^{-qT}N(-d_1^*) + Xe^{-rT}N(-d_2^*) \tag{1}$$

$$d_1 = \frac{ln(\frac{S}{X}) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad , \quad d_1^* = \frac{ln(\frac{S}{X}) + (r - q)T + (\frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{2}$$

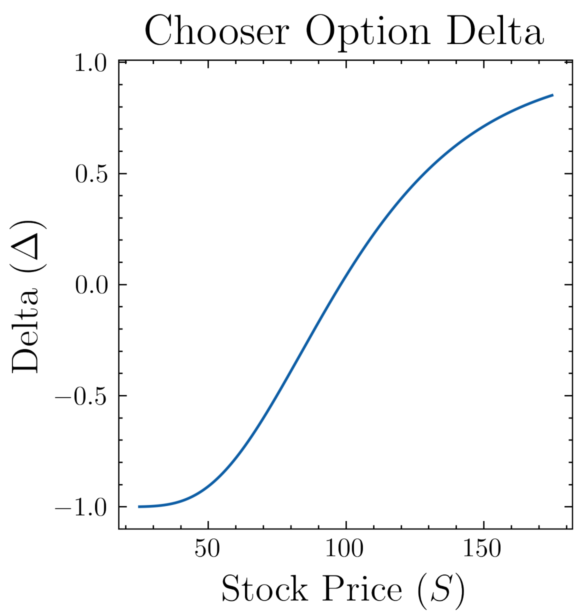
$$d_2 = d_1 - \sigma\sqrt{T} \quad , \quad d_2^* = d_1^* - \sigma\sqrt{t} \tag{3}$$

S = Stock Price, X = Strike Price, T = Time to maturity, r = risk-free rate, q = dividend yield, σ = volatility, t= time to choose option type.

1.1 Delta Hedge

From the perspective of the option seller, they need to understand how to delta-hedge a derivative after it's sold. The paper by Ďurica introduces an equation for delta

$$\Delta_{chooser} = e^{-qT} N(d_1) + e^{-qT} [N(d_1^*) - 1] \tag{4}$$



The plot above shows the Chooser option has a Δ ranges from -1 to +1.

2 Natural Language Processing (NLP)

2.1 Methodology

Traditional financial theory classifies securities markets into weakly efficient markets, and semi-strongly efficient markets. The information available to investors in the securities market determines the type of market. Behavioral finance theory breaks this traditional market division, the psychological factors of investors are added to the market analysis, and a model based on human psychological factors is established. The analysis model based on human psychological factors.

The first step will be data visualization, having an understanding of basic data characteristics then the basic idea using the sentiment analysis module called VADER to measure the sentiment. Then we test the causality between sentiment movement and stock returns by using Granger Causality Test.

Since this is a time series forecasting task, we will use LSTM, a type of Recurrent Neural Network, which has good performance in time series forecasting.

2.1.1 TD-IDF and Cross-entropy

TF-IDF (term frequency-inverse document frequency) is a statistical method used to measure the importance of words to a text. The importance of a word is proportional to its frequency of occurrence in the current text and inversely proportional to its frequency of occurrence in other texts in the corpus, so TD-IDF is also often used to extract features of texts(Wu et al., 2008).

$$w(d,t) = tf(d,t) * log\left(\frac{N}{df(t)}\right) = tf(d,t) * idf(t) \quad (5)$$

Cosine similarity measures the similarity between two vectors by measuring the cosine of the angle between them; the cosine of an angle of 0 degrees is 1, while the cosine of any other angle is not greater than 1; and its minimum value is -1. Thus, the cosine of the angle between two vectors determines whether the two vectors point approximately in the same direction. This result is independent of the length of the vectors and is only related to the direction in which the vectors are pointing. The cosine similarity is usually used for positive spaces, so the value given is between -1 and 1.

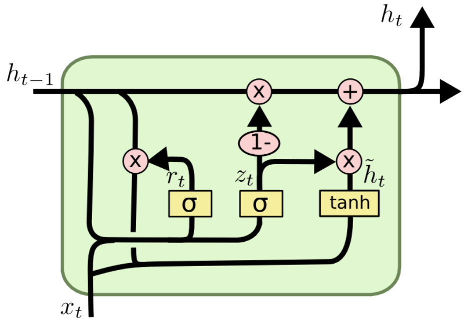
$$\begin{aligned} cos\theta &= \frac{\sum_{i=1}^{n}(A_i \times B_i)}{\sqrt{\sum_{i=1}^{n}(A_i)^2} \times \sqrt{\sum_{i=1}^{n}(B_i)^2}} \\ &= \frac{A \cdot B}{|A| \times |B|} \end{aligned} \quad (6)$$

2.1.2 Recurrent Neural Network

RNN(Recurrent Neural Network) are unfolded according to the time dimension, representing the transfer and accumulation of information from front to back in the time dimension, and the probability of the later information is built on the basis of the earlier information, which is expressed in the neural network structure as the input of the hidden layer of the later neural network is the output of the hidden layer of the earlier neural network.

Assuming that all inputs (outputs) are independent of each other, then this is difficult for many tasks. Also RNNs are called recurrent because they perform the same task for each element of the sequence and the output depends on the previous computation. Another way to consider RNNs is that they have a "memory" that captures the information computed so far.

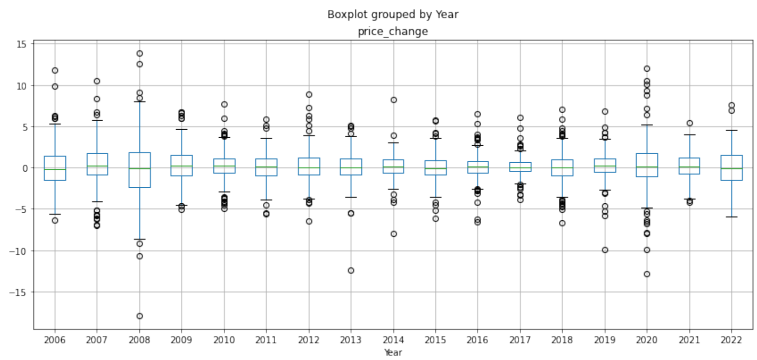
LSTM(Long Short-Term Memory networks) is the first proposed RNN gating algorithm with a corresponding loop unit, and the LSTM unit contains three gates: an input gate, an oblivion gate and an output gate. In contrast to the recursive computation established by RNN for the system state, the three gates establish a self-loop for the internal state of the LSTM cell. Specifically, the input gate determines the update of the internal state by the input of the current time step and the system state of the previous time step; the forgetting gate determines the update of the internal state of the previous time step to the internal state of the current time step; and the output gate determines the update of the internal state to the system state.

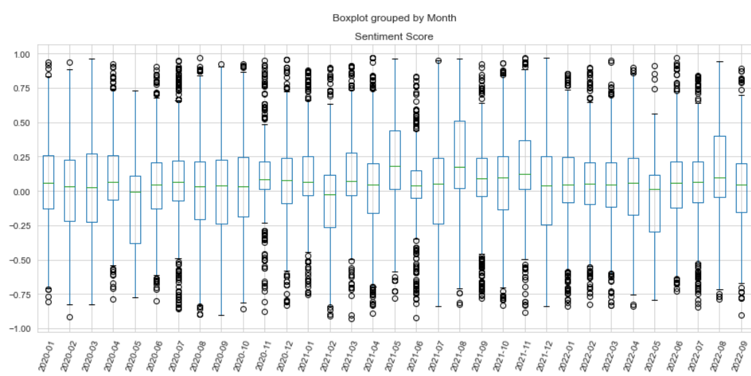


$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

2.2 Data Prepossessing and Visualization

In this part, we will clean the news data first and there are two tasks remain: Define and find price jumps and run econometrics test.





Jumping behavior mainly refers to large swings in stock prices, often containing important market information and market sentiment. The stock market is influenced by policy factors and irrational investment by a large number of retail investors. Emotional factors and information effects are important drivers of market movements, and the advantage of incorporating jumping behavior into the model is that market sentiment and information effects can be well absorbed to improve prediction efficiency.

Essentially, the price jump is the large stock return in a certain time range. So we use log return rate to define jumps, there is a rule in statistics named the 68–95–99.7 rule, also known as the empirical rule.

$$P(\mu - 1\sigma \leqslant X \leqslant \mu + 1\sigma) \approx 68.27\% \tag{7}$$

$$P(\mu - 2\sigma \leqslant X \leqslant \mu + 2\sigma) \approx 95.45\% \tag{8}$$

$$P(\mu - 3\sigma \leqslant X \leqslant \mu + 3\sigma) \approx 99.73\% \tag{9}$$

According to Eq.(8), if the variable X follows a normal distribution, then 95% of the values of X will fall in the $[\mu - 2\sigma, \mu + 2\sigma]$ interval, and the value out of this interval will be considered as a jump, then we count how many price jumps there are in a given time period

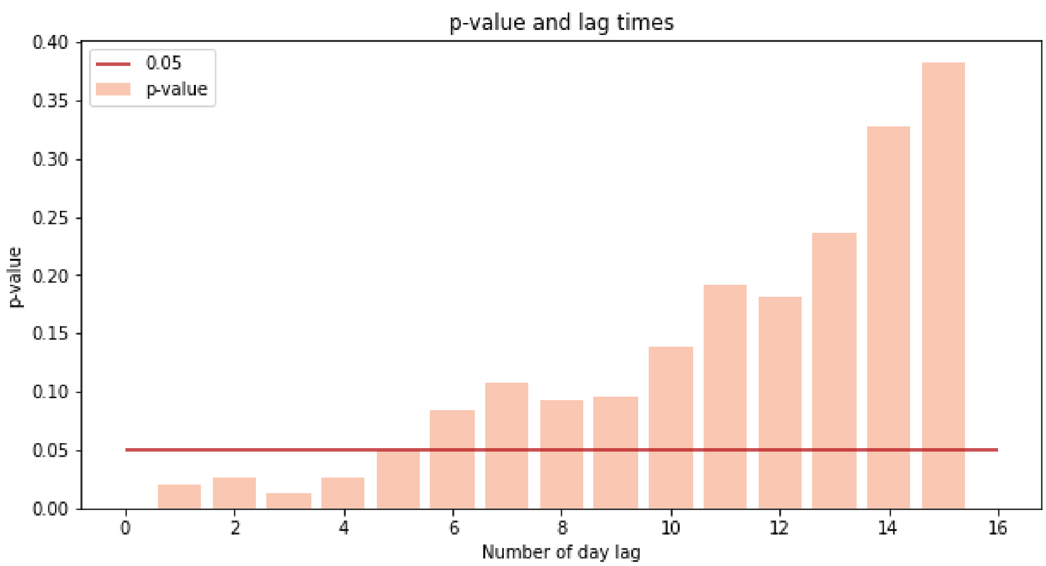
2.3 Econometric Test

ADF test: Augmented Dickey Fuller test (ADF Test) is a common statistical test used to test whether a given Time series is stationary or not.

the Null Hypothesis (H0): Series data is not stationary
the Alternative Hypothesis (H1): Series data is stationary

The Granger causality test is a statistical hypothesis test for determining whether one time series is useful in forecasting another

the Null Hypothesis (H0): Time series X does not cause time series Y
the Alternative Hypothesis (H1): Time series X cause time series Y



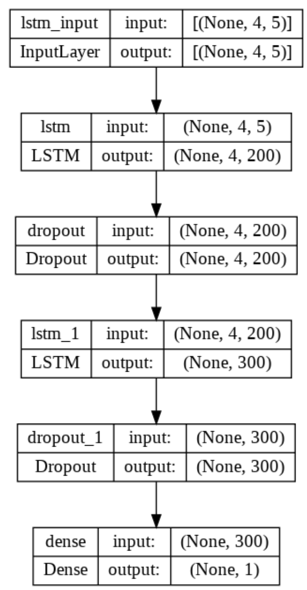
Granger's original hypothesis is that, with a lag of N periods, changes in the sentiment series do not predict changes in the underlying stock returns. The results of Granger causality test are shown in the figure above. When the lag is 3 days, the change in the time series of sentiment tendency predicts the change in the return of the underlying stock The p-value of the original hypothesis is less than 5%. Therefore, the original hypothesis is rejected at 95% confidence level, and it is concluded that there is a causal relationship between the change in sentiment and the change in return of the underlying stock.

Analysis in terms of economic implications. Excluding public holidays and double holidays, the actual number of trading days in a week in the stock market is about 5 days, and the results of Granger test show that the change in the sentiment series 3-5 days ago can predict the change stock price change. The results of this test can be interpreted that people's life is always repeated in a week cycle due to their habits and occupations, i.e., people are always used to accomplish the same things at the same time of the week and have the same emotional tendencies or psychological feelings. And people with highly positive emotional tendencies can be considered as not being able to invest rationally.

2.4 Modeling

We use the LSTM model to predict stock price jumps, returns and volatility over time.

The following is the basic framework of the machine learning model and the performance of the model forecasts, we will use the time series results of these three forecasts as the three parameters of Jump-diffusion for Monte Carlo simulation



Outputs	Jumps	Return	Volatility
MSE	1.6406250000	0.0000925593	0.0000336751
MAE	1.0312500000	0.0078922452	0.0045919166

2.5 Data Source

There are two data sources in this Natural Language Processing analysis, which are news data and stock price data.

For news data, we download the Apple Inc. news from Hanlon Lab database. The data source of this database is provided and maintained by Refinitiv, which is a world-leading provider of news content to the financial community. It is an American-British global provider of financial market data and infrastructure. Refinitiv Real-Time Machine Readable News is the only low-latency, structured textual news service powered by Reuters.

For stock price data, we download Apple's stock price from yfinance api, yfinance offers a way to download market data from Yahoo finance.

3 Jump-Diffusion

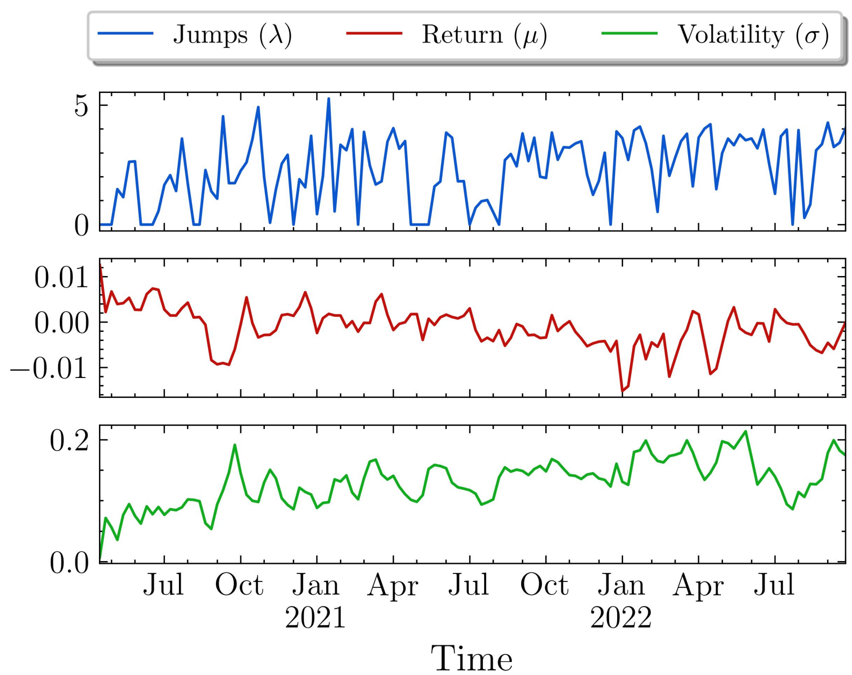
News event can create unexpected jumps in stock price, which will increase volatility. This is motivation to purchase a chooser option. If an unforeseen jump occurs, this option will provide a payoff. Merton's Jump-diffusion model is used to simulate these jumps. It's defined below (Cont and Tankov, 2004)

$$S_t = S_0 \, exp[\mu t + \sigma W_t + J(t)] \qquad (10.2) \tag{10}$$

$$J(t) = \sum_{i=1}^{N_t} Y_i \tag{11}$$

In this model, μ is the drift factor, σ is the diffusion term, and $J(t)$ is the jump component (compound Poisson process). We assume $Y_i \sim N(\mu, \sigma)$, and $N(t)$ is a Poisson process.

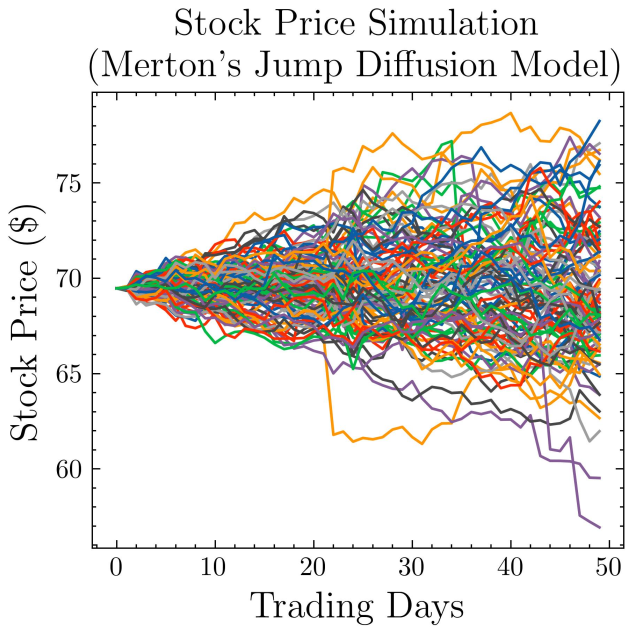
The NLP analysis provides predictions for volatility (σ) , drift (μ), and jump components (λ) . The plot below shows the future predictions.



At every time step we use these variables as inputs to Merton's model. This creates a more realistic stock price simulation of apple.

These values are used to generate the final results of profit and loss. Here we assume three points in time:

- t_0 - Initial starting point (today)
- $t = \frac{T}{2}$ - Time to choose option (Call or Put)
- T - Terminal stock price

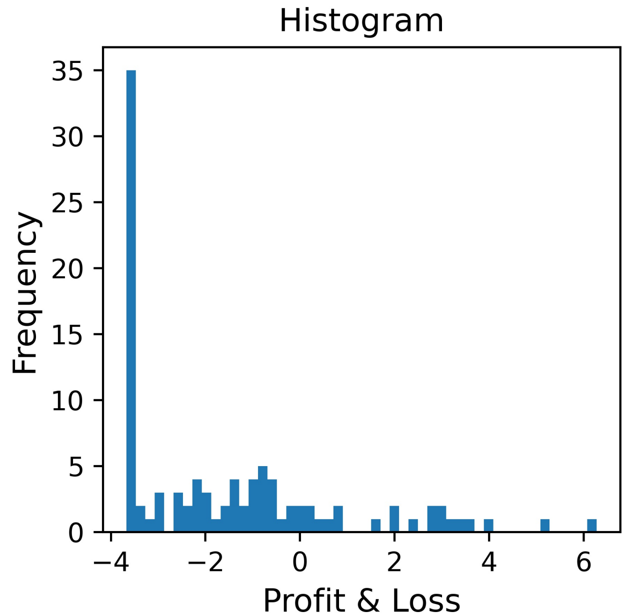


We also assume the strike price, K, is equal to the initial stock price S_0 (at-the-money). The stock price at these three points in time are used to determine the profitability of the option from the option buyer.

$$\begin{aligned} \text{Profit} &= \mathrm{V(T)}_{\text{call or put}} - \text{ Option Price} \\ \mathrm{V(T)}_{\text{call}} &= \max(\mathrm{S_T} - \mathrm{K}, 0) \\ \mathrm{V(T)}_{\text{put}} &= \max(\mathrm{K} - \mathrm{S_T}, 0) \end{aligned} \tag{12}$$

4 Final Results

The histogram below is the Profit and Loss for the option buyer. The spike at −$4 represents the situation where the terminal stock price approaches the strike price. The option holder does not exercise his/her right, but still has to pay a premium. The remaining distribution is centered around −$1 , which means our option holder did not receive a payout under this simulation.



5 Future Work

The simulation assumes a time to maturity of 50 days. If the time to maturity is increased to 252 days, then this will allow more time for the underlying stock price to experience jumps, and provide a payoff. It would be beneficial to see the relationship between a larger increase in volatility and profitability of this option.

6 Acknowledgment

We want to thank our advisor Dr. Khashanah for the support throughout the semester. This project has provided a practical experience conducting research and communicating our results to peers.

Thank you to Dr. Pirjol for the feedback on our project proposal. It's clear that pricing and hedging is an essential skill for financial engineers, so we are happy to apply our learnings from our previous class (Financial Engineering – 620 Pricing and Hedging)

Thank you to Dr. Florseco and Mr. Loeser from the Hanlon lab. The news data from the internal PostgreSQL database saved our project. Without this data, it would have been impossible to calibrate our model. We are fortunate to have rich datasets here at Stevens Institute of Technology.

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