Chooser Option - Team Update

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Group 6

10/18/2022

Agenda

Chooser Option

The goal of our project is to study the chooser option and its application.

Research Question: How profitable is this option, assuming an unforeseen jump will occur in stock price?

There are three components to our analysis:

- Pricing a chooser option - Dengyu Zhang
  - Closed-form solution
  - Monte-Carlo method
- Simulate stock price with jumps - Riley Heiman
- Natural Language Processing (NLP) - Junyu Lu
  - Determine market sentiment using NLP. This will be an intelligent method to calibrate Jump-Diffusion

Chooser Option Pricing

- The paper by Ďurica provides an analytical solution for European-style chooser option.

$$C_{chooser}(S, X, t, T, q, r) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) - Se^{-qT}N(-d_1^*) + Xe^{-rT}N(-d_2^*)$$

- Chooser option is similar to a staddle.
  - Call option & Put combined together.

- Additionally, the following variables are defined as:
  - S = Stock Price
  - X = Strike Price
  - q = dividend yield
  - r = risk-free rate
  - T = time to maturity for *option*
  - t = time to choose option type (call or put)

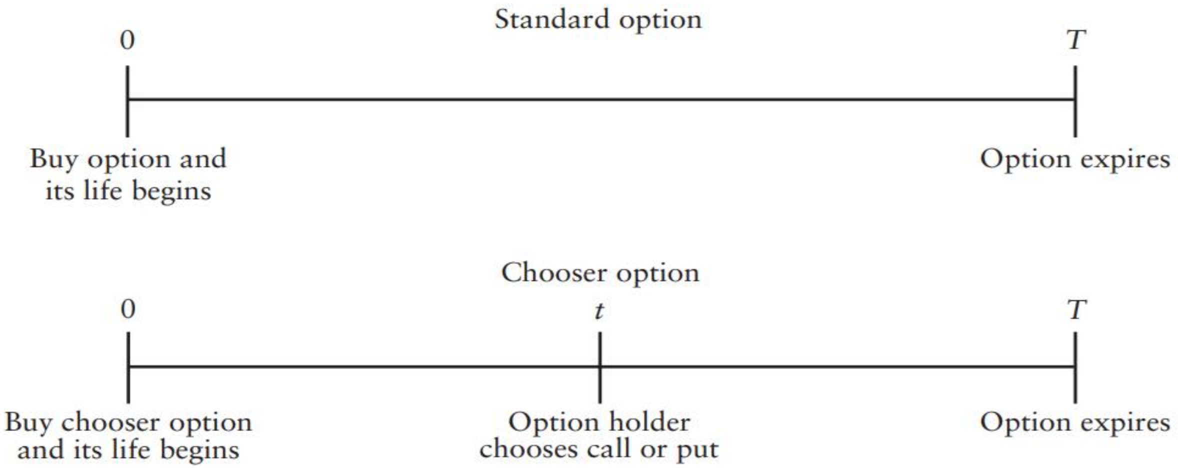


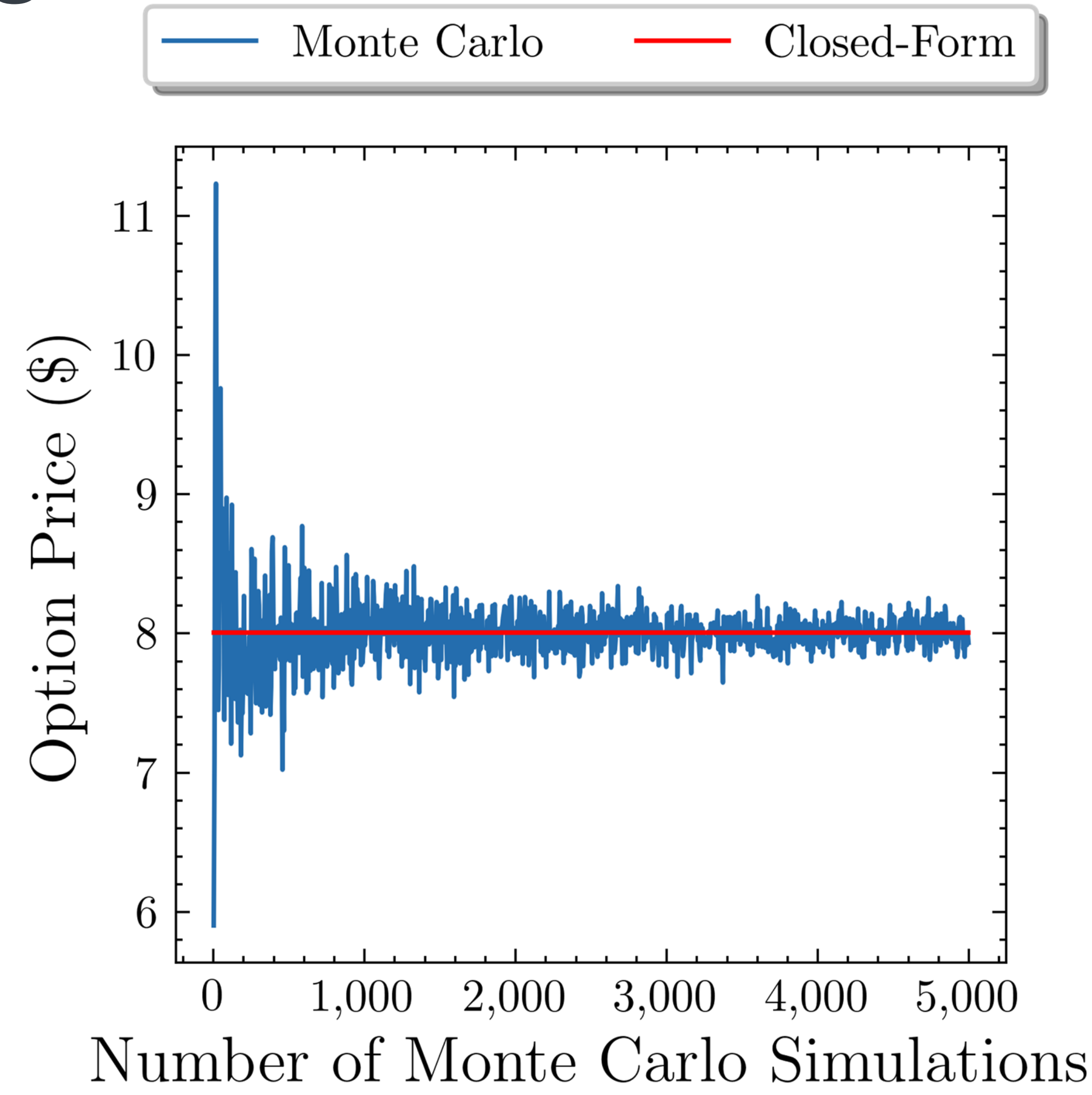
Figure 1: Diagram to compare chooser option with standard option (Whaley, 2006, 276)

Chooser Option Pricing

Price based upon the following inputs:
The plot on the right shows the Monte Carlo method converges to a closed-form solution.

S = \$100, X = \$99 σ = 20% , q = 0, **T = t = .25 years** , r = 2%

Method	Option Price
Monte Carlo *(1,000,000 simulations)*	\$ 8.00455
Analytical Solution *(Ďurica)*	\$ 7.91610
Black-Scholes (Call + Put)	\$ 8.00536



NLP

Methods and Data Source

- NLP sentiment classification model : RoBERTa[4]

```
text = "Covid cases are increasing fast!"
ranking = np.argsort(scores)
ranking = ranking[::-1]
for i in range(scores.shape[0]):
    l = config.id2label[ranking[i]]
    s = scores[ranking[i]]
    print(f"{i+1}) {l} {np.round(float(s), 4)}")
```

```
1) Negative 0.7236
2) Neutral 0.2287
3) Positive 0.0477
```

- Natural Language Processing can be used to determine how frequently good news and bad news events occur.
- This will be used to calibrate the Jump-Diffusion model discussed earlier.

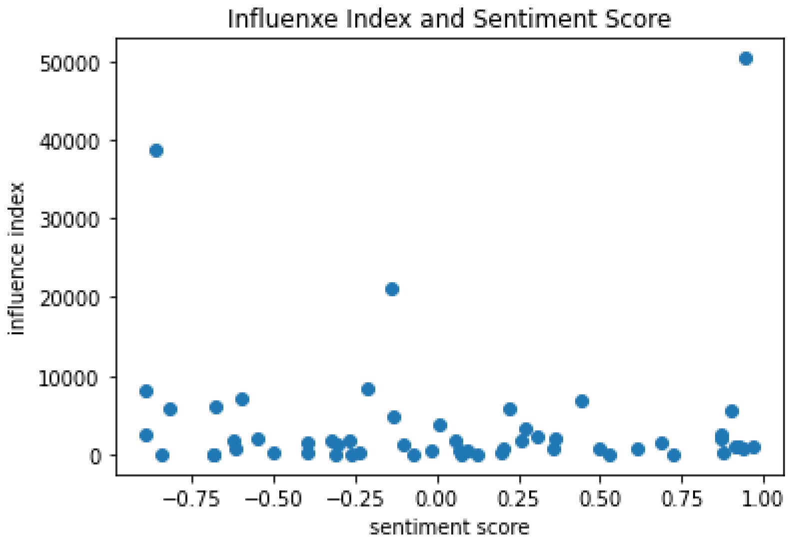
Text Data Source : Twitter API
Target Company : Apple

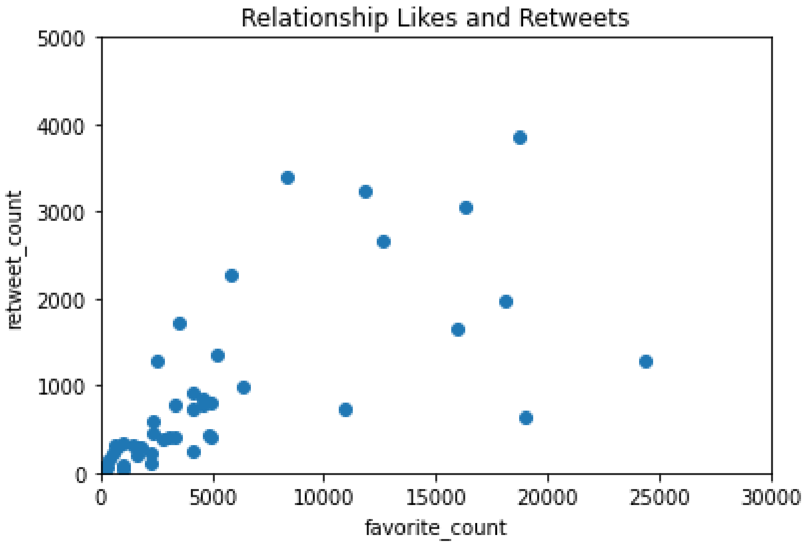
	tweet_id	text	favorite_count	retweet_count	created_at	source	reply_to_status	reply_to_user	retweets	favorites
0	1582074957206806528	Wireless headphones, like Apple' s popular AirP...	2521	1294	2022-10-17 18:23:57	Twitter Web App	NaN	None	1294	2521
1	1581967615253446656	U.S. technology companies founded by 1st and 2...	782	297	2022-10-17 11:17:25	Twitter for iPad	NaN	None	297	782
2	1581810143347417090	Get a copy from Apple Music !\n#CashOut ❤️ \n#...	3306	770	2022-10-17 00:51:41	Twitter for iPhone	NaN	None	770	3306
3	1581832785693532160	Why apple don' t got rolling ray GIFs -_-	24354	1286	2022-10-17 02:21:39	Twitter for iPhone	NaN	None	1286	24354
4	1581465816544858113	Lock and unlock your doors using your iPhone o...	18169	1964	2022-10-16 02:03:27	Twitter for iPhone	NaN	None	1964	18169
5	1581673328753790977	"At the end of the day what separates Marjorie...	997	352	2022-10-16 15:48:02	Twitter Web App	NaN	None	352	997
6	1581707447689183233	Ask yourself this simple question: Is your fam...	1602	292	2022-10-16 18:03:36	Twitter for iPhone	NaN	None	292	1602
7	1581961454735216641	[🇨🇦] Download (G)I-DLE's 5th Mini Album [I love...	5264	1362	2022-10-17 10:52:56	Twitter for iPhone	NaN	None	1362	5264
8	1581995344685985792	Today' s Democratic Party racializes everything...	6411	987	2022-10-17 13:07:36	Twitter Web App	NaN	None	987	6411

NLP

Some analysis on twitter data

date	tweet_id	text	favorite_count	retweet_count	Sentiment Score	Influence Index
2022-10-17	1582074957206806528	Wireless headphones, like Apple’ s popular AirP...	2521	1294	-0.624309	1662.1
2022-10-17	1581967615253446656	U.S. technology companies founded by 1st and 2...	782	297	0.091430	442.5
2022-10-17	1581810143347417090	Get a copy from Apple Music !\n#CashOut ❤️ 🙏 \n#...	3306	770	0.690745	1530.8
2022-10-17	1581832785693532160	Why apple don’ t got rolling ray GIFs -__-	24354	1286	-0.894796	8206.4
2022-10-16	1581465816544858113	Lock and unlock your doors using your iPhone o...	18169	1964	0.441314	6825.5





Jump-Diffusion

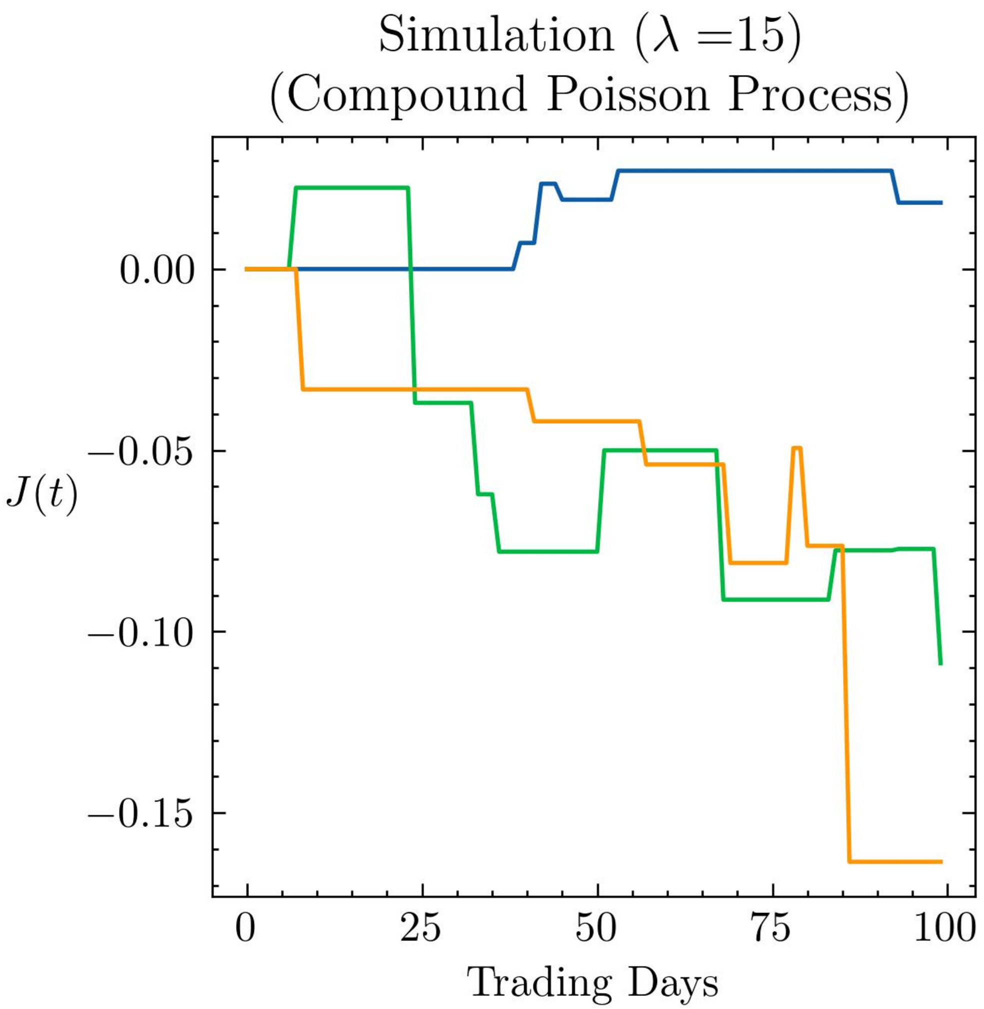
Mertons model can be used to simulate jumps assuming a set of paraments: λ, σ, μ

Step 1: Generate a Poisson Process, $N(t)$

Step 2: Generate a random sample $Y \sim Normal$

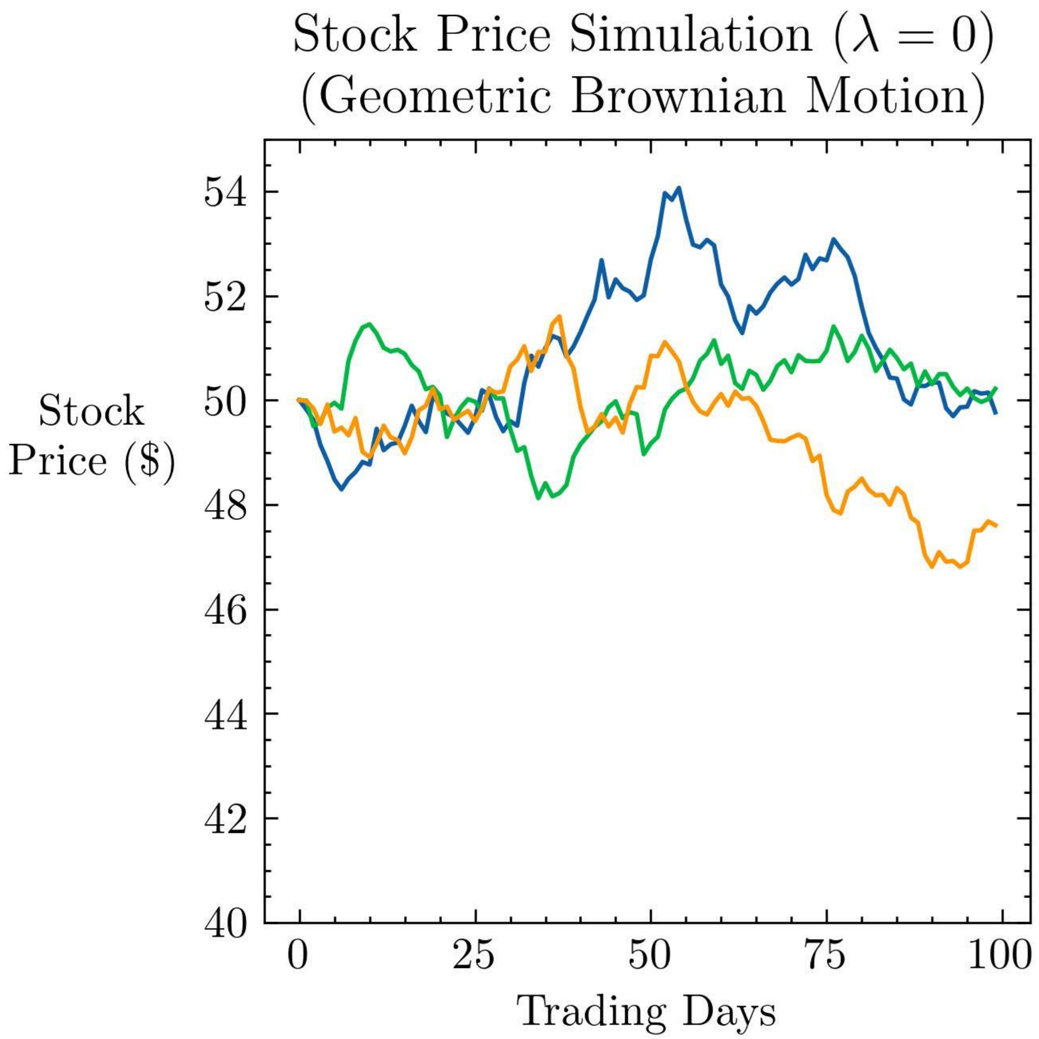
Step 3: Calculate $J(t) = \sum_{i=1}^{N_t} Y_i$

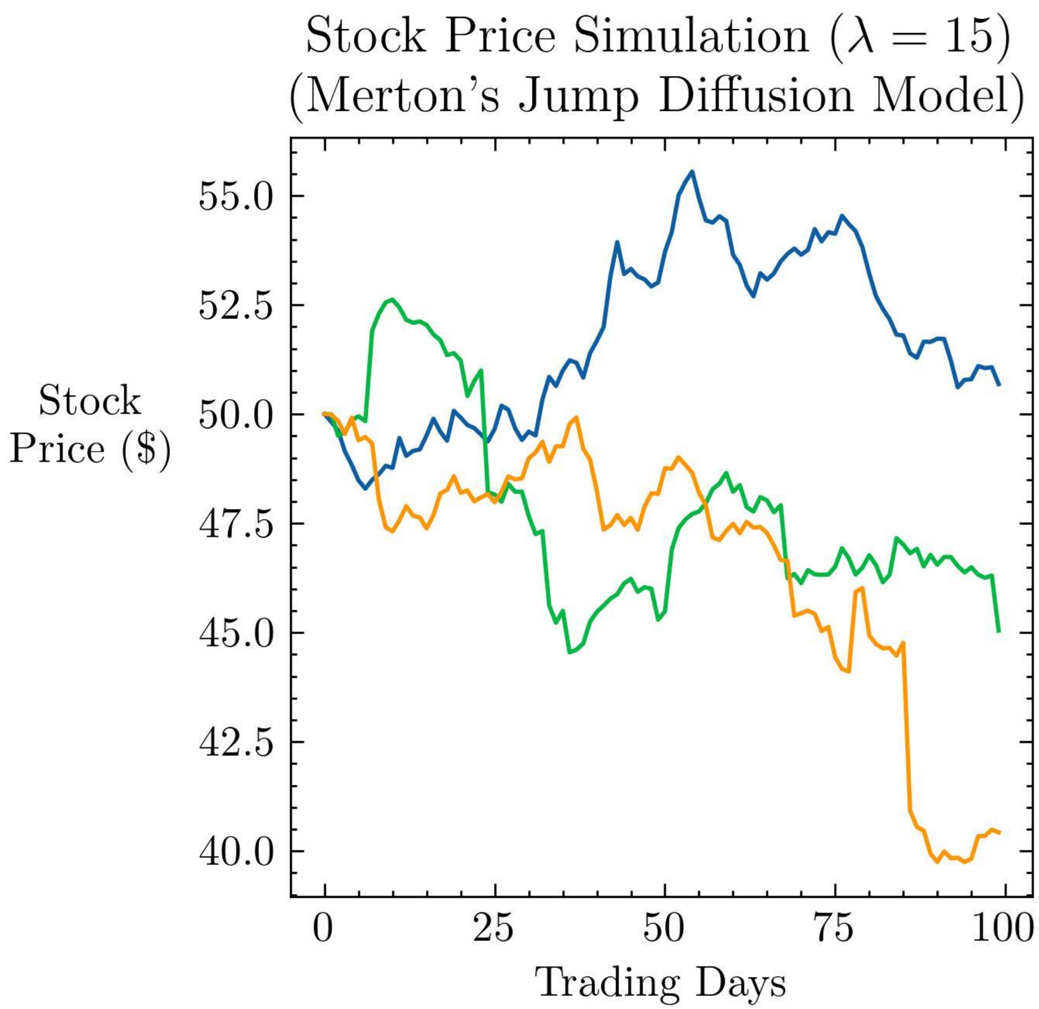
$N(t)$	Y	$J(t)$
1	0	0
1	0	0
2	.12	.12
2	0	.12
3	-.01	.11
⋮	⋮	⋮



Jump-Diffusion

$\lambda = 0 \;\Rightarrow\;$ Standard Geometric Brownian Motion
$\lambda > 0 \;\Rightarrow\;$ Meton′s Jump Diffusion Model





What's the plan for the rest of the semester?

Next Steps

- Pricing a chooser option
  - Simulate a Delta hedge scenario
  - Assess the accuracy of the analytical solution by comparing the price with chooser option data

- Jump-Diffusion
  - Calibrate Jump-Diffusion using empirical stock data.

- Natural Language Processing (NLP).
  - Conduct data cleaning on twitter data
  - Generate analytics using twitter data with RoBERTa predictions.

THANK **YOU**

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References

[1] Whaley, Robert E. *Derivatives: markets, valuation, and risk management*. Vol. 345. John Wiley & Sons, 2006.

[2] Ďurica, Marek and Lucia. Švábová. "Delta and Gamma for Chooser Options." In *International Scientific Conference Applications of Mathematics and Statistics in Economics AMSE 2015 Full paper proceedings*, pp. 75-84. 2014.

[3] Cont, Rama and Tankov, Peter. *Financial modelling with jump processes*. Chapman and Hall/CRC, 2004.

[4] Loureiro, Daniel, et al. "Timelms: Diachronic language models from twitter." arXiv preprint arXiv:2202.03829 (2022).

Appendix - Chooser Option Pricing

Option price, V(t), is calculated using the expected value

risk-neutral pricing formula

$$\boxed{\begin{aligned} D(t)V(t) &= \tilde{E}[D(T)V(T)] \\ V(t) &= \tilde{E}[\frac{D(T)}{D(t)}V(T)] \end{aligned}}$$

Monte Carlo method, can be used to estimate the expected value below.

The academic literature agrees V(T) is the following.

$$V(T) = max[\ V(T)_{call}\ ,\ V(T)_{put}\ ]$$

$$V(T) = max[\ max(S_T - K, 0)\ ,\ max(K - S_T, 0)\ ]$$

Appendix - Chooser Option Pricing

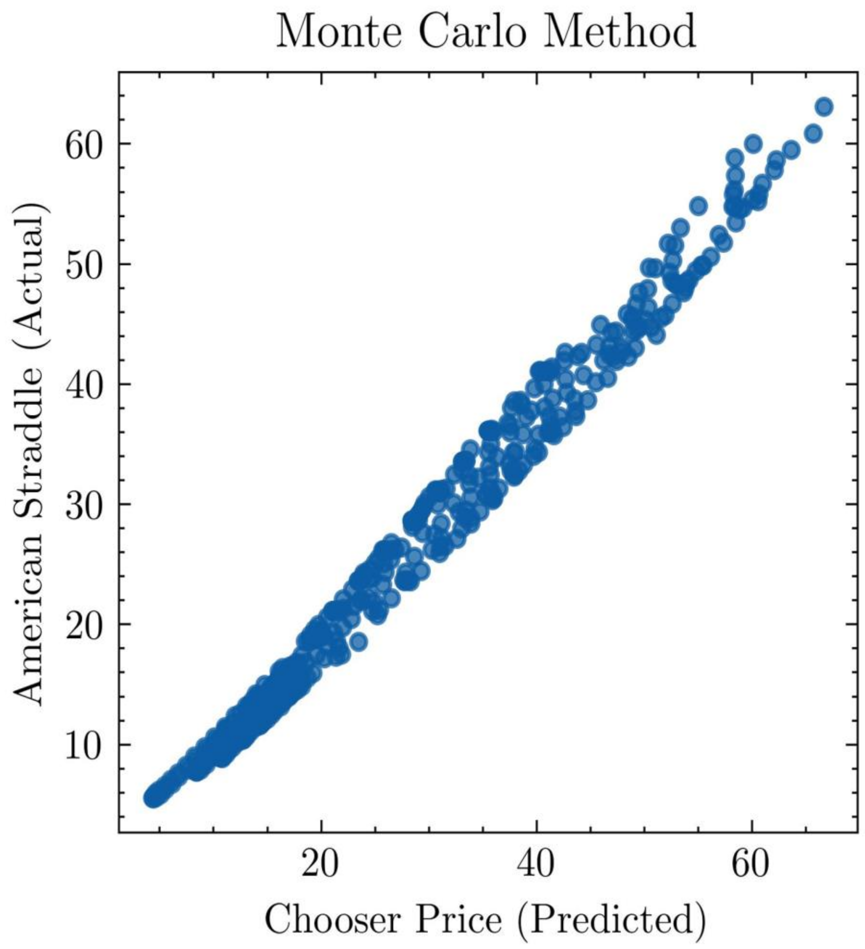
Collected American option chain data (Yahoo Finance)

- Single snapshot date: 11-7-2022

- Varying Strike Price
  - K = [$ 110, $ 115, ..., $ 180]

- Varying time to maturities
  - T = [4 days , 11 days , .. , 3 years]

Strike Price	Time to Maturity	American Straddle *(Actual)*	Monte Carlo *(Predicted)*
138.92	0.015873	68.96	67
138.92	0.015873	63.96	62.61
⋮	⋮	⋮	⋮



S(0) = $138.92, sigma = 36% , r= 4.25%, q = .65%

The plot above shows the Monte Carlo method can predict prices of American straddles.

Appendix

Jump-Diffusion

- Equation (10.2) is Merton's Jump diffusion model
- $J(t)$ represents the compound poisson process
- The equation comes from the textbook by Rama Cont & Peter Tankov (Cont and Tankov, 2004, 326)

$$S_t = S_0\, exp[\mu t + \sigma W_t + J(t)] \qquad (10.2)$$

$$J(t) = \sum_{i=1}^{N_t} Y_i$$

Appendix

Analytical Solution - Chooser Option

The equation below comes from the paper titled “Delta and Gamma for chooser option” (Ďurica and Švábová, 2014, 4)

$$C_{chooser}(S, X, t, T, q, r) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) - Se^{-qT}N(-d_1^*) + Xe^{-rT}N(-d_2^*)$$

$$d_1 = \frac{ln(\frac{S}{X}) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1^* = \frac{ln(\frac{S}{X}) + (r - q)T + (\frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2^* = d_1^* - \sigma\sqrt{t}$$