Q5 Lucas & Kanade Derivation.

$$\epsilon(V_x, V_y) = \sum_{region} (I_x V_x + I_y V_y + I_t)^2$$

$$\frac{d\epsilon}{dV_x} = \sum_r 2(I_x V_x + I_y V_y + I_t)(I_x)$$

chain rule

$$0 = \sum_r 2(I_x V_x + I_y V_y + I_t)(I_x)$$

$$0 = \sum_r I_x^2 V_x + \sum_r I_x I_y V_y + \sum_r I_x I_t$$

split up sums

$$-\sum_r I_x I_t = V_x \sum_r I_x^2 + V_y \sum_r I_x I_y.$$

following a similar method we also get,

$$-\sum_r I_y I_t = V_y \sum_r I_y^2 + V_x \sum_r I_x I_y$$

then,

$$-\sum I_y I_t = \begin{bmatrix} \sum_r I_y^2 \\ \sum_r I_x I_y \end{bmatrix} \cdot \begin{bmatrix} V_y \\ V_x \end{bmatrix} = \begin{bmatrix} \sum_r I_y^2 \\ \sum_r I_x I_y \end{bmatrix}^T \begin{bmatrix} V_y \\ V_x \end{bmatrix}$$

$$= \begin{bmatrix} \sum I_x I_y \\ \sum_r I_y^2 \end{bmatrix}^T \begin{bmatrix} V_x \\ V_y \end{bmatrix}.$$

$$-\sum I_x I_t = \begin{bmatrix} \sum I_x^2 \\ \sum I_x I_y \end{bmatrix}^T \begin{bmatrix} V_x \\ V_y \end{bmatrix}.$$

$$\begin{bmatrix} \sum_r I_x^2 & \sum_r I_x I_y \\ \sum_r I_x I_y & \sum_r I_y^2 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} -\sum_r I_x I_t \\ -\sum_r I_y I_t \end{bmatrix}$$

$$\underbrace{\left(\sum_r \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}\right)}_{\substack{\shortparallel \\ A}} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \underbrace{\left(\sum_r \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix}\right)}_{\substack{\shortparallel \\ b}}$$

$$A \begin{bmatrix} v_x \\ v_y \end{bmatrix} = b$$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = A^{-1} b$$