HWK 5 - Context-Free Grammars

rcrocke6 | Riley Crockett

① Produce a CFG for each of the following languages, assuming $\Sigma = \{0,1\}$.

(a) $\{w \mid w$ starts and ends with different symbols$\}$

$S \to 0R1 \mid 1R0$

$R \to 0R \mid 1R \mid 0 \mid 1 \mid \epsilon$

(b) $\{w \mid$ the length of w is an integer multiple of $3\}$

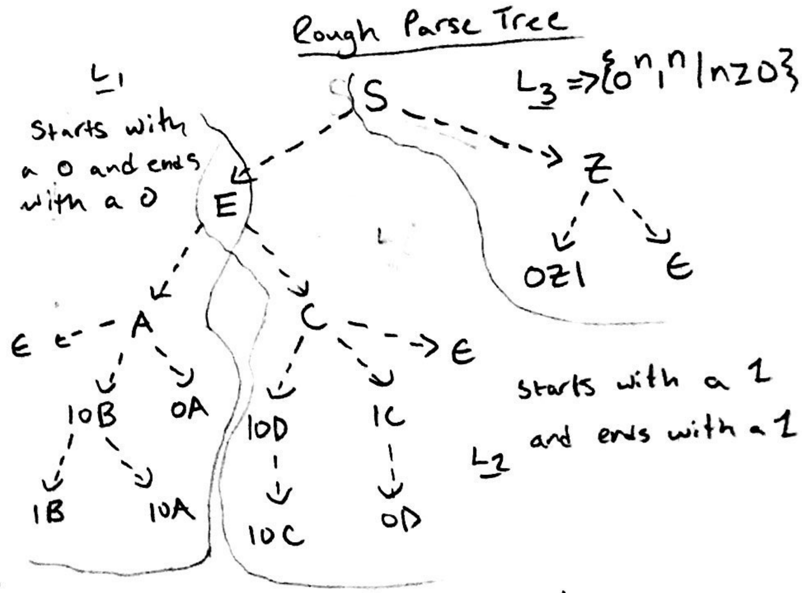
$S \to SRRR \mid \epsilon$

$R \to 0 \mid 1$

(c) $\{ww^R \mid$ i.e., a word followed by that word reversed$\}$ (Palindrome)

$S \to 0R0 \mid 1R1 \mid \epsilon$

② Let $G = (\{S, A, B, C, D, E, Z\}, (0,1), R, S)$,
where $R = \{S \to E \mid Z;\ E \to A \mid C;\ A \to 01B \mid 0A \mid \epsilon;\ B \to 1B \mid 10A;\ C \to 10D \mid 1C \mid \epsilon;$
$D \to 01C \mid 0D;\ Z \to 0Z1 \mid \epsilon\}$.



(a) $L = L_1 \cup L_2 \cup L_3$

$L_{12} = L_1 \cup L_2 = \{w \mid w$ starts and ends with the same symbol$\}$

$L = \{w \mid w$ starts and ends with the same symbol$\}$
$\cup \{0^n1^n \mid n \geq 0\}$

(b) Proof:
~~Assume~~ Assume L_3 is regular and let
p be the pumping length for L_3.
Choose the string $s = 0^p1^p \in L_3$ so that $|s| > p$.
By the PL s can be partitioned into 3 pieces $s = xyz$ such that for all $i \geq 0$, $xy^iz \in L_3$.
⇒ Consider the case that y consists of 0's and 1's. Then $s' = xy^2z$ can have the same number of 0's and 1's, but some of the 1's will come before some of the 0's and violate terms of the language L_3.
⇒ Thus we conclude that L_3 is not a regular language.

⇒ Regular languages are closed under union, and since the component L_3 in language L is not regular, we conclude that the language L is also not regular. □

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