Oregonians for Rationality

Underground Pi

Digging Out from Under Pseudoscience

By Mark Cowan

"How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics..."

In the Washington Park Station of the Westside Light Rail Tunnel, in the 16-million year recitation of history that runs alongside a 260-foot core sample of Portland's West Hills, you will find, etched in granite, the first 107 digits of the transcendental constant pi (figure 1).

```
3.1415926535
821480865144
288109757245
870066330572
703698336733
620005681271
420199561160
244592555982
534904897932
```

Figure 1. Pi as it appears in the Washington Park Station of the Westside MAX Light Rail.

Pi, as every schoolchild knows, is the ratio of the circumference of a circle to its diameter - one of the fundamental ratios of the space that fills our universe. Its digits never repeat, and they follow no pattern. Other than its role in geometry, and the fact that it pops up in many diverse branches of mathematics, pi has no special significance. It does not mean anything...so far as we know.

Yet pi has exerted a steady pull on the human imagination. The Babylonians and Egyptians knew its value to within a half a percent some 4000 years ago. By the 3rd century BC, Archimedes had rectified the circle, nearly invented the calculus, and established pi's value to about one part in 100,000 by the use of regular polygons. And by the 5th century AD a Chinese father and son, using a variation of this method, pinned down eight digits-a precision unequaled in Europe until the 16th century. Their laborious extraction of square roots was aided by the early Chinese introduction of a blank for zero.

By diligent use of Archimedes' method, in a 1596 paper entitled "On the Circle," Dutch mathematician Ludolph van Ceulen single-handedly delivered the first 20 digits - then challenged anybody to top it. None did, but he soon extended his claim on history first to 32, and then to 35 digits (figure 2), of which the last three were engraved on his tombstone. To this day pi remains "the Ludolphine number" in Germany.

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3.1415926535
897932384626
433832795028
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Figure 2. Pi as per van Ceulen (1596).

But the big guns were ready to fire. The methods known since Archimedes' time could, theoretically at least, calculate pi to any desired degree of accuracy, the only limits being the calculator's fortitude. When European mathematics began to flourish, the methods themselves were improved.

In 1665 and 1666, during the Plague, Issac Newton developed the calculus-and offhandedly produced an efficient infinite series for calculating pi . Evaluating only 22 terms of the series yielded 16 digits. He saw no practical value to this effort, however, and later apologized for how far he had carried his computation "Having," as he wrote, "no other business at the time."

But if a giant like Newton could fall under pi's transcendental spell just for lack of anything better to do, was anyone truly immune?

Heedless of such reservations, the hunt continued. With various modifications to improve efficiency, by 1719 the French mathematician de Lagny had sweated his way through 127 decimal digits (figure 3), a record that would stand for 75 years.

Further progress required more efficient tools. Around 1755 Leonhard Euler, perhaps the greatest mathematician of all time, discovered the fastest converging series yet known. Using it, he worked out the first 20 decimal places of pi in a single hour. But, doubtless mindful of the limited value of this pursuit, like Newton he went no further. Others, of course, were more than willing to extend the tally-and naturally they used his methods.

But one wonders: if any of these early pi hunters were somehow to wander down the Washington Park MAX Station today, what might they think of those 107 digits etched in cold granite? Would Newton sneer? Would Euler wince?

```
3.1415926535
897932384626
433832795028
841971693993
751058209749
445923078164
062862089986
280348253421
170679821480
865132823066
470938446
```

Figure 3. Pi as per de Lagny (1719).

For the physical accuracy implied by 106 decimal places of pi has no counterpart in reality. If you inscribed a circle the diameter of the known universe (which has varied recently, but we'll use 24-billion light-years), and then calculated its circumference by use of those 106 digits, the error due to truncation would be 1/1061 of the width of an atomic nucleus!

Still the hunt went on. Calculating prodigy Johann Dase produced 200 decimal digits of pi in just two months in 1844, with others ringing up slightly larger tallies - until finally William Shanks published 707 digits of pi in 1873-74. This record stood until 1945-when he was shown to have gotten the last 180 digits wrong.

But now the electronic computer was on the horizon, and by 1949 ENIAC had churned out 2037 places in 70 hours. The digital floodgates opened. Pi's current world record now stands at 51,539,600,000 decimal digits set in June of 1997 by Kanada and Takahashi (1) at the Tokyo Computer Centre after 29 hours on a machine with 1024 processors. That's 61 million times faster than ENIAC per digit. Interestingly enough, the two digits beginning at position 49,999,999,999 in both pi and 1/pi are 42.

Ivars Peterson's online *Mathland* delivers more information on pi mania - which, of course, continues unabated - with Internet links to get you started (2). There you can marvel at people who memorize great hunks of pi . You can also learn of a new formula that delivers specific hexadecimal digits of pi -without knowing any of the preceding ones! This completely unanticipated result is being put to use to calculate, via an Internet network (3), both the 5 and 40 trillionth binary digit of pi . No equivalent formula yet exists for decimal digits. Seems like there's an argument against creation in there somewhere...

If, by now, you just can't live without your own big piece of pi, running Piw131 (4) overnight on a decent PC with 32 megabytes or more of memory will get you a cool million digits by morning. If that's not enough, you can search (5) the first 50 million digits for any string of numbers up to 127 digits. But consider this: in about an hour I wrote, from scratch, a simple program (6) that computes the circumference of a unit circle using nothing more advanced than

square roots. Run under QBASIC it delivers Newton's 16 digits after only 26 iterations.

Sure, it was fun to do - but is there any real point to any of this, after all?

Well, that's where it gets interesting. The distribution of digits in the first 50 billion digits of pi is statistically normal (6). But a recent study (7) has found that the distribution of repeating strings of digits is not. And nobody knows why that should be so! So pi , it would seem, still contains some curious implications for number theory. And the digital expression of it is the source of a new kind of mathematical analysis.

Which brings us, uhm, full circle-and back to "Pi Underground."

According to Rebecca Banyas of Tri-Met's Westside Light Rail, the artist, Bill Will, "got his information on pi from a reference book called *The History of Pi* (8). The numbers that appear on the wall are the same as those in the book."

Well, sort of. You may have already noticed, however, a slight discrepancy between the values carved into the tunnel and those worked out nearly 300 years earlier by de Lagny. This discrepancy was first spotted by a MAX engineer who had memorized pi to 12 places as a child. But the reason for the error remained obscure. Was it Art? A bad job of typesetting? Deliberate? Just to see if anybody was paying attention?

After I searched strings of the Washington Park Station digits against the half-million pi digits on my computer, the source of the error became clearer. And checking out a copy of *A History of Pi* made it obvious. Artist Bill Will wasn't taking liberties with a constant of the universe. He was just unfamiliar with the format of mathematical tables (figure 4).

(Widen your browser...)

```
PI = 3.+
1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825
3421170679
8214808651 3282306647 0938446095 5058223172 5359408128 4811174502 8410270193 8521105559 6446229489
5493038196
4428810975 6659334461 2847564823 3786783165 2712019091 4564856692 3460348610 4543266482 1339360726
0249141273
7245870066 0631558817 4881520920 9628292540 9171536436 7892590360 0113305305 4882046652 1384146951
9415116094
3305727036 5759591953 0921861173 8193261179 3105118548 0744623799 6274956735 1885752724 8912279381
8301194912
9833673362 4406566430 8602139494 6395224737 1907021798 6094370277 0539217176 2931767523 8467481846
7669405132
0005681271 4526356082 7785771342 7577896091 7363717872 1468440901 2249534301 4654958537 1050792279
6892589235
4201995611 2129021960 8640344181 5981362977 4771309960 5187072113 4999999837 2978049951 0597317328
1609631859
5024459455 3469083026 4252230825 3344685035 2619311881 7101000313 7838752886 5875332083 8142061717
7669147303
5982534904 2875546873 1159562863 8823537875 9375195778 1857780532 1712268066 1300192787 6611195909
2164201989

3809525720 1065485863 2788659361 5338182796 8230301952 0353018529 6899577362 2599413891 2497217752
8347913151
5574857242 4541506959 5082953311 6861727855 8890750983 8175463746 4939319255 0604009277 0167113900
```

```
9848824012
8583616035 6370766010 4710181942 9555961989 4676783744 9448255379 7747268471 0404753464 6208046684
2590694912
9331367702 8989152104 7521620569 6602405803 8150193511 2533824300 3558764024 7496473263 9141992726
0426992279
6782354781 6360093417 2164121992 4586315030 2861829745 5570674983 8505494588 5869269956 9092721079
7509302955
3211653449 8720275596 0236480665 4991198818 3479775356 6369807426 5425278625 5181841757 4672890977
7727938000
8164706001 6145249192 1732172147 7235014144 1973568548 1613611573 5255213347 5741849468 4385233239
0739414333
4547762416 8625189835 6948556209 9219222184 2725502542 5688767179 0494601653 4668049886 2723279178
6085784383
8279679766 8145410095 3883786360 9506800642 2512520511 7392984896 0841284886 2694560424 1965285022
2106611863
0674427862 2039194945 0471237137 8696095636 4371917287 4677646575 7396241389 0865832645 9958133904
7802759009
```

Figure 4. The first 2000 of the 10,000 decimal places of pi as printed in A History of Pi. *Note that the artist used the digits going down the columns rather than across the rows.*

Of course, most people aren't going to notice - or probably much care. But if you spot somebody standing around in the tunnel, reciting something that begins "How I need a drink...," at least you'll be able to chime in, "alcoholic of course..."(9).

And they'll think you're both loonies, and won't let either of you on the train.

REFERENCES AND NOTES

1. Details (not the result, though!). Some statistical analysis of the first 50 billion digits is provided.

2. Search here on "pi" in the Mathland columns.

3. More info.

4. Harry J. Smith has all kinds of stuff to help you have "Fun With Mathematics!"

5. Pi served daily. Observe the ubiquitous nature of 42 yet again.

6. A QBASIC program to compute pi :

```
REM CIRCLE
H# = 2
FOR I = 1 TO 26
L# = 1 - SQR(1 - H# * H# / 4)
H# = SQR(L# * L# + H# * H# / 4)
PRINT "H="; H#, "I="; I, "Value="; H# * 2 ^I
```

NEXT I

7. Unfortunately I've been unable to find where I saw this; you'll have to trust me. :)

8. Beckmann, Petr. 1971. *A History of Pi*. The Golem Press. I owe a great debt to this excellent book and have drawn much from it for this article.

9. "How I need a drink, alcoholic of course, after the heavy lectures involving quantum mechanics" gives you 15 digits. Other mnemonics exist, including a poem somewhat reminiscent of Poe's *The Raven* that delivers 740 digits. See (2).

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