Tutorial 4: (Complex Numbers)

1 For each of the following, find all the solutions and plot all the roots on an Argand diagram, in each case comment on the position of the roots.

a) $z^2 - 9 = 0$ b) $z^2 + 4 = 0$

c) $z^2 = -5 + 12i$ d) $z^2 + 7 = 24i$

e) $z^3 + 8i = 0$ f) $z^3 + 27 = 0$

g) $z^4 + 64 = 0$ h) $z^4 - 4 = 0$

2 Solve each of the following equations over C:

a) $z^2 + 36 = 0$ b) $z^2 + 8 = 0$

c) $z^2 + 6z + 58 = 0$ d) $z^2 - 10z + 29 = 0$

e) $z^2 - 2\sqrt{3}z + 7 = 0$ f) $z^2 + 6z + 14 = 0$

3 Form a quadratic with real coefficients having amongst its roots

a) $2i$ b) $-2\ ,\ 3$

c) $-2 + 5i$ d) $3 - 4i$

e) $-\frac{1}{2}\ ,\ \frac{3}{4}$ f) $2 - \sqrt{7}i$

4 Find the roots of each of the following

a) $z^3 + 3z^2 + 7z + 5 = 0$

b) $z^3 + 3z^2 + 49z + 147 = 0$

c) $z^3 + 6z^2 + 21z + 26 = 0$

d) $z^3 - 8z^2 + 46z - 68 = 0$

e) $z^4 - 7z^3 + 11z^2 + 123z = 0$

f) $z^4 + 9z^3 + 49z^2 + 41z = 0$

5 Form a cubic with real coefficients having amongst its roots

a) $-3\ ,\ 2i$ b) $2\ ,\ -3i$

c) $3\ ,\ 2 + 3i$ d) $-2\ ,\ 4 - 5i$

e) $\frac{3}{4}\ ,\ -3 + 7i$ f) $-\frac{2}{3}\ ,\ -5 - 2i$

6 Reduce the following to linear factors, hence solve

a) $z^4 - 3z^2 - 4 = 0$ b) $z^4 - 5z^2 - 36 = 0$

c) $z^4 - 9z^2 - 400 = 0$ d) $z^4 + 13z^2 + 36 = 0$

7 a) Find the value of a for ai is a root of the equation
$z^4 - 2z^3 + 7z^2 - 4z + 10 = 0$

b) Show that $-2 + 3i$ is a root of the equation
$2z^4 + 3z^3 + 3z^2 + bz - 39 = 0$ find the real number b and all the roots.

c) Show that $-3-2i$ is a root of the equation
$z^4+2z^3+cz^2-22z+65=0$ find the real number c and all the roots.

d) Show that $5-4i$ is a root of the equation
$z^4+dz^3+114z^2-376z+533=0$ find the real number d and all the roots.

Answers

1 a) ± 3 b) $\pm 2i$
c) $\pm(2+3i)$ d) $\pm(3+4i)$
e) $\pm\sqrt{3}-i\ ,\ 2i$ f) $\frac{3}{2}\left(1\pm\sqrt{3}i\right),\ -3$
g) $2(1\pm i)\ ,\ -2(1\pm i)$ h) $\pm\sqrt{2}\ ,\ \pm\sqrt{2}i$

2 a) $\pm 6i$ b) $\pm 2\sqrt{2}i$ c) $-3\pm 7i$
d) $5\pm 2i$ e) $\sqrt{3}\pm 2i$ f) $-3\pm\sqrt{5}i$

3 a) $z^2+4=0$ b) $z^2-z-6=0$ c) $z^2+4z+29=0$
d) $z^2-6z+25=0$ e) $8z^2-2z-3=0$ f) $z^2-4z+11=0$

4 a) $-1\ ,\ -1\pm 2i$ b) $-3\ ,\ \pm 7i$ c) $-2\ ,\ -2\pm 3i$
d) $2\ ,\ 3\pm 5i$ e) $0\ ,\ -3\ ,\ 5\pm 4i$ f) $0\ ,\ -1\ ,\ -4\pm 5i$

5 a) $z^3+3z^2+4z+12=0$ b) $z^3-2z^2+9z-18=0$
c) $z^3-7z^2+25z-39=0$ d) $z^3-6z^2+25z+82=0$
e) $4z^3+21z^2+214z-174=0$ f) $3z^3+32z^2+107z+58=0$

6 a) $\pm 2\ ,\ \pm i$ b) $\pm 3\ ,\ \pm 2i$ c) $\pm 5\ ,\ \pm 4i$ d) $\pm 2i\ ,\ \pm 3i$

7 a) $a=\sqrt{2}\quad \pm\sqrt{2}i\ ,\ 1\pm 2i$
b) $b=-77\quad 3\ ,\ -\frac{1}{2}\ ,\ -2\pm 3i$
c) $c=-6\quad 2\pm i\ ,\ -3\pm 2i$
d) $d=-16\quad 5\pm 4i\ ,\ 3\pm 2i$