Tutorial 2 (Vectors: Lines and Planes)

1 Find the vector equation and parametric equation of the line:

(i) through the point $(2,-3)$ parallel to $\underset{\sim}{i}+2\underset{\sim}{j}$

find the points corresponding to $t=0,1,2$

(ii) through the point $(2,-3,4)$ parallel to $\underset{\sim}{i}+2\underset{\sim}{j}-3\underset{\sim}{k}$

find the points corresponding to $t=0,1,2$

2 Find the equations of the line in Cartesian form passing through the points:

(i) $(2,-3,4)$ and $(5,2,-2)$

(ii) $(1,0,-3)$ and $(-3,2,5)$

3 Given the following lines, if they intersect find the point of intersection and the angle between the lines.

(i) $\dfrac{x-14}{-3}=\dfrac{y+5}{1}=\dfrac{z-11}{-2}$ and $x=-6-4t$, $y=-7-3t$, $z=7+2t$

(ii) $\dfrac{x-2}{3}=\dfrac{y+3}{-4}=\dfrac{z-5}{2}$ and $x=-8-4t$, $y=1+3t$, $z=7+2t$

(iii) $\dfrac{x-13}{3}=\dfrac{y-4}{2}=\dfrac{z+7}{-2}$ and $x=-1+t$, $y=8-2t$, $z=14-3t$

4 Find the equation of the plane passing through the point P and normal to the vector $\underset{\sim}{n}$ where;

(i) $P(2,-3,4)$ and $\underset{\sim}{n}=\underset{\sim}{i}+2\underset{\sim}{j}-3\underset{\sim}{k}$

(ii) $P(1,0,3)$ and $\underset{\sim}{n}=-2\underset{\sim}{j}+5\underset{\sim}{k}$

5 Find the equation of the plane passing through the following three points and find the area of the triangle.

(i) $P(2,-3,3)$ $Q(-2,5,0)$ $R(-1,2,-3)$

(ii) $A(2,-4,3)$ $B(3,6,-2)$ $C(-5,3,4)$

6 Find the line of intersection between the planes and the angle between the planes.

(i)
$$x+2y+3z=1$$
$$2x-y+4z=3$$

(ii)
$$-x+y+3z=1$$
$$2x-2y-6z=3$$

(iii)
$$2x-3y+z=11$$
$$3x+y+2z=7$$

Answers

1 (i) $\underset{\sim}{r} = (2+t)\underset{\sim}{i} + (-3+2t)\underset{\sim}{j}$ $t = \dfrac{x-2}{1} = \dfrac{y+3}{2}$

$(2,-3)$ $(3,-1)$ $(4,1)$

(ii) $\underset{\sim}{r} = (2+t)\underset{\sim}{i} + (-3+2t)\underset{\sim}{j} + (4-3t)\underset{\sim}{k}$ $t = \dfrac{x-2}{1} = \dfrac{y+3}{2} = \dfrac{z-4}{-3}$

$(2,-3,4)$ $(3,-1,1)$ $(4,1,-2)$

2 (i) $t = \dfrac{x-2}{3} = \dfrac{y+3}{5} = \dfrac{z-4}{-6}$ (ii) $t = \dfrac{x-1}{-4} = \dfrac{y}{2} = \dfrac{z+3}{8}$

3 (i) $(2,-1,3)$ $75^0 38'$ (ii) do not intersect (iii) $(4,-2,-1)$ $71^0 5'$

4 (i) $x + 2y - 3z = -16$ (ii) $-2y + 5z = 15$

5 (i) $33x + 15y - 4z = 9$ $\frac{1}{2}\sqrt{1330}$

(ii) $45x + 34y + 77z = 185$ $\frac{1}{2}\sqrt{9110}$

6 (i) $t = \dfrac{5x-7}{-11} = \dfrac{5y+1}{-2} = z$ $45^0 35'$ (ii) no solution

(iii) $t = \dfrac{-11x+33}{7} = -(19+11y) = z$ $38^0 13'$