Tutorial 3: (Complex Numbers)

1 Simplify each of the following

i) $(3+2i)+(7-4i)$ ii) $(3+4i)-(3-2i)$

iii) $(3+2i)(6-5i)$ iv) $i(3-4i)$

v) $(2+3i)(2-3i)$ vi) $(3-4i)^2$

vii) $i^5\left(8i^2-7i^3\right)$

2 Given that $z_1=4+3i$ and $z_2=3-2i$ then find each of the following giving your answer in rectangular form.

i) $z_1 z_2$ ii) $2z_1+3z_2$

iii) $3z_1-2z_2$ iv) $\dfrac{z_1}{z_2}$

v) $\dfrac{z_2}{z_1}$ vi) $\dfrac{z_1-1}{z_2+i}$

vii) $\overline{z}_1\overline{z}_2$ viii) $\dfrac{\overline{z}_1}{\overline{z}_2}$

ix) $(z_1-z_2)^2$ x) $z_1^{-1}z_2^{-1}$

3 Find each of the following

i) $\mathrm{Re}\left(\dfrac{1}{3+2i}\right)$ ii) $\mathrm{Im}\left(\dfrac{1}{3-2i}\right)$

iii) $\mathrm{Re}\left(\dfrac{3-4i}{3+4i}\right)$ iv) $\mathrm{Im}\left(\dfrac{3-5i}{7+6i}\right)$

4 Find the values of x and y if

i) $(x+yi)(3+4i)=-2i$

ii) $(x+yi)(2+3i)=6$

iii) $(x+yi)(2-5i)=7-4i$

iv) $(1+2i)x-(3+3i)y+5=0$

v) $(3-4i)x+(5-4i)y=7-3i$

5 Show that for $z_1, z_2 \in C$

a) $\overline{z}_1+\overline{z}_2=\overline{z_1+z_2}$ b) $\overline{z}_1\times\overline{z}_2=\overline{z_1\times z_2}$

c) $\dfrac{\overline{z}_1}{\overline{z}_2}=\overline{\left(\dfrac{z_1}{z_2}\right)}$

6 Express each of the following in polar form (without using calculators)

a) $3+3i$ b) $-\sqrt{3}+i$ c) $-\sqrt{2}-\sqrt{2}i$

d) $1-\sqrt{3}i$ e) -5 f) $-2i$

7 Express each of the following in rectangular form (without using calculators)

a) $7\sqrt{2}\,\text{cis}\left(-45^0\right)$ b) $3\,\text{cis}\left(\pi\right)$

c) $4\,\text{cis}\left(-\frac{\pi}{2}\right)$ d) $2\,\text{cis}\left(\frac{\pi}{6}\right)$

e) $4\,\text{cis}\left(-150^0\right)$ f) $2\,\text{cis}\left(\frac{2\pi}{3}\right)$

8 Given that $z_1 = 5\sqrt{2}\,\text{cis}\left(-\frac{3\pi}{4}\right)$ and $z_2 = 2-3i$ then find each of the following, giving your answer in rectangular form

i) $\overline{z}_1$ ii) $z_1 z_2$

iii) $z_1 + z_2$ iv) $2z_1 - 3z_2$

v) $\frac{z_1}{z_2}$ vi) $\overline{z}_1\overline{z}_2$

9 a) Express $2+3i$ in polar form, hence find $\left(2+3i\right)^5$ giving your answer in rectangular form.

b) Find $\left(7-4i\right)^4$ giving your answer in rectangular form.

c) If $\left(-\sqrt{3}+i\right)^7 = 64\left(x-yi\right)$ find the values of x and y

10 Simplify each of the following, giving your answers in rectangular form

a) $\left(1-i\right)^3\left(2+2i\right)^4$ b) $\frac{\left(\sqrt{3}+i\right)^6}{\left(1-i\right)^8}$

11 If $z = -3\sqrt{3}-3i$ and $w = 1-i$ then find each of the following (showing all working) giving exact simplified answers.

i) $\left|z\right|$ and Arg z, hence find z^6 ii) $\text{Arg}\left(\frac{z^5}{w^8}\right)$

12 If $z = -2-2i$ and $w = \sqrt{3}-i$ then find each of the following (showing all working) giving exact simplified answers.

i) $\left|z\right|$ and Arg z, hence find z^{12} ii) $\text{Arg}\left(z^5 w^8\right)$

Answers

1	i)	$10-2i$	ii)	$6i$
	iii)	$28-3i$	iv)	$4+3i$
	v)	13	vi)	$-7-24i$
	vii)	$-7-8i$		
2	i)	$18+i$	ii)	17
	iii)	$6+13i$	iv)	$\frac{1}{13}(6+17i)$
	v)	$\frac{1}{25}(6-17i)$	vi)	$\frac{3}{5}(1+2i)$
	vii)	$18-i$	viii)	$\frac{1}{13}(6-17i)$
	ix)	$-24+10i$	x)	$\frac{1}{325}(18-i)$
3	i)	$\frac{3}{13}$	ii)	$\frac{2}{13}$
	iii)	$-\frac{7}{25}$	iv)	$-\frac{53}{85}$
4	i)	$-\frac{8}{25}$, $-\frac{6}{25}$	ii)	$\frac{12}{13}$, $-\frac{18}{13}$
	iii)	$\frac{34}{29}$, $\frac{27}{29}$	iv)	5 , $\frac{10}{3}$
	v)	$-\frac{13}{8}$, $\frac{19}{8}$		
6	a)	$3\sqrt{2}\,\mathrm{cis}(45^0)$	b)	$2\,\mathrm{cis}(150^0)$
	c)	$2\,\mathrm{cis}(-135^0)$	d)	$2\,\mathrm{cis}(-60^0)$
	e)	$5\,\mathrm{cis}(180^0)$	f)	$2\,\mathrm{cis}(-90^0)$
7	a)	$7-7i$	b)	-3
	c)	$-4i$	d)	$\sqrt{3}+i$
	e)	$-2\sqrt{3}-2i$	f)	$-1+\sqrt{3}\,i$
8	i)	$-5+5i$	ii)	$-25+5i$
	iii)	$-3-8i$	iv)	$-16-i$
	v)	$\frac{1}{13}(5-25i)$	vi)	$-25-5i$
9	i)	$122-597i$	ii)	$-2047-3696i$
	iii)	$\sqrt{3}$, 1		
10	i)	$128(1+i)$	ii)	-4
11	i)	6 , $-\frac{5\pi}{6}$ -46656	ii)	$-\frac{\pi}{6}$
12	i)	$2\sqrt{2}$, $-\frac{3\pi}{4}$ -262144	ii)	$\frac{11\pi}{12}$