Tutorial 1 (Vectors: Vector Algebra)

1 Find the magnitude of the following vectors:
(a) $4\boldsymbol{i} - 3\boldsymbol{j}$ (b) $-7\boldsymbol{i} + 24\boldsymbol{j}$
(c) $\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ (d) $-4\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$

2 Find a vector of magnitude:
(a) 4 units in the direction to $\boldsymbol{i} + 2\boldsymbol{j}$.
(b) 6 units in the opposite direction to $-6\boldsymbol{i} + 4\mathbf{j}$
(c) 6 units in the direction to $\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$
(d) 3 units in the opposite direction to $-2\boldsymbol{i} - 4\mathbf{j} + 2\boldsymbol{k}$

3 If P is the point $(-2 , 4 , 1)$ find
(a) a unit vector parallel to OP
(b) the angle that the vector OP makes with the positive X-axis.

4 Given the point $Q(1 , -2 , 3)$, find
(a) a unit vector parallel to OQ
(b) the angle that the vector OQ makes with the positive Y-axis.

5 Given the point $R(-3 , 5 , -2)$, find
(a) a unit vector parallel to OR ,
(b) the angle that the vector OR makes with the positive Z-axis.

6 Given the two points $C(2 , -1 , 5)$ and $D(-3 , 4 , 2)$ find
(a) the vector from D to C
(b) the distance between the points C and D ,
(c) a unit vector parallel to CD ,
(d) the angle that the line CD makes with the positive Y-axis.

7 If $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = -3\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, then find the vectors
(a) $\boldsymbol{a} + \boldsymbol{b}$ (b) $\boldsymbol{a} - \boldsymbol{b}$ (c) $2\boldsymbol{a} + \boldsymbol{b}$

8 If $\boldsymbol{p} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$, then find the vectors
(a) $\boldsymbol{p} + \boldsymbol{q}$ (b) $\boldsymbol{p} - \boldsymbol{q}$ (c) $2\boldsymbol{p} - 3\boldsymbol{q}$

9 If $\boldsymbol{u} = 2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{v} = -\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$ find the value of $\boldsymbol{v}.\boldsymbol{u}$.

10 If $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j}$ find the value of $\boldsymbol{a}.\boldsymbol{b}$.

11 Given the vectors $\boldsymbol{a} = 2\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} - 4\boldsymbol{k}$ find:
(a) a unit vector parallel to $\boldsymbol{b}$,
(b) the scalar resolute of $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$,
(c) the vector resolute of $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$,
(d) the vector resolute of $\boldsymbol{a}$ in the perpendicular of $\boldsymbol{b}$,
(e) the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

12 If $\boldsymbol{p} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ find:
(a) a unit in the direction $\boldsymbol{q}$,
(b) the scalar resolute of $\boldsymbol{p}$ in the direction of $\boldsymbol{q}$,
(c) the vector resolute of $\boldsymbol{p}$ in the direction of $\boldsymbol{q}$,
(d) the vector resolute of $\boldsymbol{p}$ perpendicular to $\boldsymbol{q}$,
(e) the angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

13 If $\boldsymbol{p} = 2\boldsymbol{i} - \boldsymbol{j} - t\boldsymbol{k}$ and $\boldsymbol{q} = -6\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$ find the value of the scalar t so that $\boldsymbol{p}$ and $\boldsymbol{q}$ are:
(a) parallel (b) perpendicular.

14 If $\boldsymbol{p} = 6\boldsymbol{i} + y\boldsymbol{j} - 9\boldsymbol{k}$ and $\boldsymbol{q} = -4\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{k}$ find the values of the scalars y so that $\boldsymbol{p}$ and $\boldsymbol{q}$ are:
(a) parallel, (b) perpendicular

15 If A, B, C and D are four points in space with coordinates $A(2, 3, 2)$, $B(4, p, 0)$, $C(-1, -1, 0)$, $D(-2, 2, 1)$
(a) find the value of the scalar p if AB is parallel to DC,
(b) find the value of the scalar p if AB is perpendicular to DC,
(c) find in terms of p that angle that AB makes with the Z-axis.

16 If A, B, C are three points in space with coordinates $A(3, -2, 5)$, $B(-1, 0, 4)$, $C(2, -1, 3)$ and 0 is the origin $(0, 0, 0)$ find:
(a) a unit vector in the direction $\overline{BC}$,
(b) the scalar resolute of $\overline{AB}$ in the direction of $\overline{BC}$,
(c) the vector resolute of $\overline{AB}$ in the direction of $\overline{BC}$,
(d) the vector resolute of $\overline{AB}$ perpendicular to $\overline{BC}$,
(e) the angle between the vectors $\overline{AB}$ and $\overline{BC}$,
(f) the area of the triangle ABC.

17 Given the vectors $\boldsymbol{p} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + \boldsymbol{k}$, find:
(a) a unit vector parallel to $\boldsymbol{q}$,
(b) the scalar resolute of $\boldsymbol{p}$ in the direction of $\boldsymbol{q}$,
(c) the vector resolute of $\boldsymbol{p}$ in the direction of $\boldsymbol{q}$,
(d) the vector resolute of $\boldsymbol{p}$ in the perpendicular of $\boldsymbol{q}$,
(e) the angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$,
(f) a unit vector which is perpendicular to both $\boldsymbol{p}$ and $\boldsymbol{q}$.

18 If $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ find:
(a) $\underset{\sim}{a} \times \underset{\sim}{b}$ (b) $\left(\underset{\sim}{a} + \underset{\sim}{b}\right) \times \left(\underset{\sim}{a} - \underset{\sim}{b}\right)$ (c) $\underset{\sim}{a} \cdot \underset{\sim}{b}$

19 Find a unit vector perpendicular to the plane containing $\underset{\sim}{a} = 2\underset{\sim}{i} - 6\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$

20 If $\underset{\sim}{a} = 2\underset{\sim}{i} + y\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ find the value of y if,
(a) $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$
(b) $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$

21 If $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ find,

(a) the length of the vector $\underset{\sim}{a}$

(b) the length of the vector $\underset{\sim}{b}$

(c) $\underset{\sim}{a} \,.\, \underset{\sim}{b}$

(d) the angle between the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$

22 Find the area of a triangle with vertices $(1,1,2)$ $(2,3,5)$ and $(1,5,5)$.

23 If $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ $\underset{\sim}{b} = -\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{c} = 3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ find

a) $\underset{\sim}{a} \,.\, \underset{\sim}{b} \times \underset{\sim}{c}$ b) $\underset{\sim}{a} \times (\underset{\sim}{b} \times \underset{\sim}{c})$

c) verify that $\underset{\sim}{a} \times (\underset{\sim}{b} \times \underset{\sim}{c}) = (\underset{\sim}{a} \,.\, \underset{\sim}{c})\,\underset{\sim}{b} - (\underset{\sim}{a} \,.\, \underset{\sim}{b})\,\underset{\sim}{c}$

Answers

1 (a) 5 (b) 25 (c) $\sqrt{6}$ (d) $4\sqrt{3}$

2 (a) $\frac{4}{\sqrt{5}}(\boldsymbol{i} + 2\boldsymbol{j})$ (b) $\frac{6}{\sqrt{13}}(3\boldsymbol{i} - 2\boldsymbol{j})$

(c) $2(\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k})$ (d) $\frac{3}{\sqrt{6}}(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$

3 (a) $\frac{1}{\sqrt{21}}(-2\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k})$ (b) $115^0 53'$

4 (a) $\frac{1}{\sqrt{14}}(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$ (b) $122^0 19'$

5 (a) $\frac{1}{\sqrt{38}}(-3\boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k})$ (b) $108^0 56'$

6 (a) $5\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$ (b) $\sqrt{59}$

(c) $\frac{1}{\sqrt{59}}(-5\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k})$ (d) $49^0 23'$

7 (a) $-\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ (b) $5\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$ (c) $\boldsymbol{i} + 7\boldsymbol{j}$

8 (a) $5\boldsymbol{i} + 3\boldsymbol{j} - 7\boldsymbol{k}$ (b) $\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$ (c) $\boldsymbol{j} - 4\boldsymbol{k}$

9 -16 10 3

11 (a) $\frac{1}{\sqrt{26}}(3\boldsymbol{i} - \boldsymbol{j} - 4\boldsymbol{k})$ (b) $\frac{6}{\sqrt{26}}$

(c) $\frac{3}{13}(3\boldsymbol{i}-\boldsymbol{j}-4\boldsymbol{k})$ (d) $\frac{1}{13}(17\boldsymbol{i}-49\boldsymbol{j}+25\boldsymbol{k})$

(e) $75^0 7'$

12 (a) $\frac{1}{3}(2\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k})$ (b) 6

(c) $2(2\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k})$ (d) $-\boldsymbol{i}-\boldsymbol{k}$

(e) $13^0 16'$

13 (a) $\frac{5}{3}$ (b) -3

14 (a) -3 (b) 39

15 (a) -3 (b) $\frac{13}{3}$ (c) $\cos^{-1}\left(\frac{-2}{\sqrt{p^2-6p+17}}\right)$

16 (a) $\frac{1}{\sqrt{11}}(3\boldsymbol{i}-\boldsymbol{j}-\boldsymbol{k})$ (b) $-\frac{13}{\sqrt{11}}$

(c) $-\frac{13}{11}(3\boldsymbol{i}-\boldsymbol{j}-\boldsymbol{k})$ (d) $\frac{1}{11}(-5\boldsymbol{i}+9\boldsymbol{j}-24\boldsymbol{k})$

(e) $148^0 48'$ (f) 3.937 units^2

17 (a) $\frac{1}{\sqrt{5}}(2\boldsymbol{i}+\boldsymbol{k})$ (b) $\frac{8}{\sqrt{5}}$

(c) $\frac{8}{5}(2\boldsymbol{i}+\boldsymbol{k})$ (d) $\frac{3}{5}(-2\boldsymbol{i}-5\boldsymbol{j}+4\boldsymbol{k})$

(e) $48^0 22'$ (f) $\pm\frac{1}{3}(-\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k})$

18 (a) $10\underset{\sim}{i}+3\underset{\sim}{j}+11\underset{\sim}{k}$ (b) $-20\underset{\sim}{i}-6\underset{\sim}{j}-22\underset{\sim}{k}$ (c) -8

19 $\pm\frac{1}{7}\left(3\underset{\sim}{i}-2\underset{\sim}{j}+6\underset{\sim}{k}\right)$

20 (a) 5 (b) -1

21 (a) $\sqrt{14}$ (b) $\sqrt{21}$

(c) -8 (d) $117^0 48'$

22 $\frac{1}{2}\sqrt{61}$ 23 a) 8 b) $-16\underset{\sim}{i}+16\underset{\sim}{k}$