(Quit the Kernel then run the cells)
Phase and Time-Plots of the different regions of the two-parameter bifurcation map illustrated in the paper “Dynamics of a SIR epidemic model with limited medical
resources, revisited again”(EcoEvo)

In[∘]:=
```
SetDirectory[NotebookDirectory[]];
AppendTo[$Path, Directory];
Clear["Global`*"]; << "def.m";
(*Contains dis, trE2,BP,BTP*)
R0 = β Λ / (μ (γ + δ + η + μ));
cp = {Λ > 0, δ > 0, γ > 0, β > 0, ξ > 0, ω > 0, μ > 0, α > 0, η > 0};

<< EcoEvo`
(*EcoEvoDocs;*)
ClearParameters; UnsetModel;
SetModel[{Pop[s] → {Equation :→ (-(i s β)/(1 + i ξ) + Λ - s μ), Color → Green},
  Pop[i] → {Equation :→ (-(i α)/(i + ω) + (i s β)/(1 + i ξ) - i (γ + δ + μ)), Color → Blue},
 Parameters :→ cp}]
(*Very long symbolic fixed points*)
equi = SolveEcoEq[];
```

Out[∘]=
```
EcoEvo Package Version 1.6.4 (November 5, 2021)
Christopher A. Klausmeier <christopher.klausmeier@gmail.com>
```

Phase plot of the region I:

In[ ]:=
```
ClearParameters;
T = 400;
xM = 140;
xm = 65;
yM = 12.5;
ym = 0;
```
$\Lambda = 16\,;\ \delta = \frac{1}{5};\ \gamma = \frac{3}{25};\ \beta = \frac{1}{100};\ \xi = \frac{1}{1000};\ \mu = \frac{3}{25};$

$\omega = \frac{7}{64};\ \alpha = \frac{179}{64};\ \eta = \alpha / \omega;$
```
Print["TrE2="]
trE2 // N
eq = N[equi, 19];
Print["Fixed points:", eq]

Print["Eigenvalues of FP:", EcoEigenvalues[eq]]

pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
E2n = RuleListTweak[eq[[3]], {s, i}, {0.08, 0.01}]
sol = EcoSim[E2n, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spRIE.pdf", sp]

Export["solTI.pdf", psol]
```

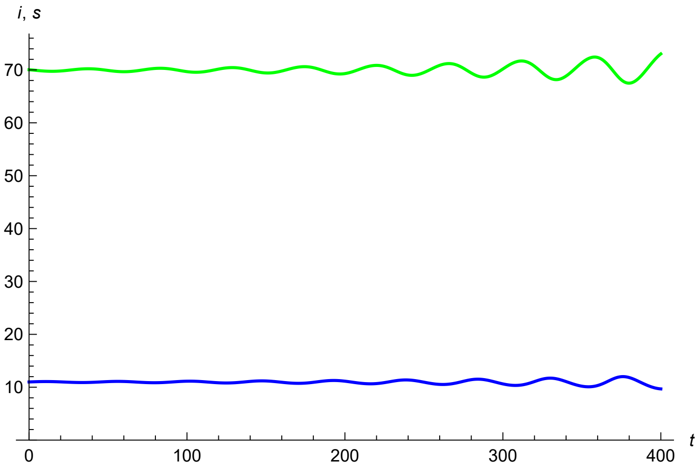
```
TrE2=
```

Out[ ]=
```
0.0132728
```

```
Fixed points:
 {{s → 133.3333333333333333, i → 0}, {s → 86.13605390716323460, i → 6.618784945020234440},
  {s → 69.95888114064486073, i → 10.99004545382992726}}

Eigenvalues of FP:{{-24.67809523809523810, -0.1200000000000000000},
  {0.2844363183923499985, -0.06687622684028318840},
  {0.006636380435495405640 + 0.1379093889778862190 i,
   0.006636380435495405640 - 0.1379093889778862190 i}}

Time plot around E2
```

Out[ ]=
```
{s → 70.0389, i → 11.}
```

Out[ ]=

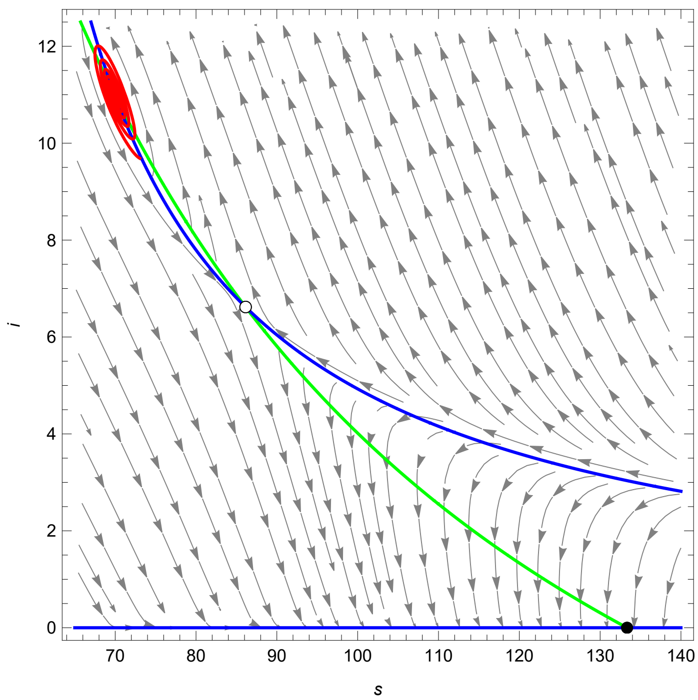


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 0.205}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 0.205}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 0.205297

The floquet exponents are

Out[ ]= {-0.12, -24.6781}

Out[ ]= spRIE.pdf

Out[ ]= solTI.pdf

Phase plot of the region II:

In[ ]:=

```
ClearParameters;
T = 600;
xM = 140;
xm = 50;
yM = 20;
ym = 0;

RII = FindInstance[Join[{dis > 0, R0 > 1, (trE2) > 0}, cp, {η == α / ω}] //. cut, {ω, α, η}]
(*Λ=16 ;δ=1/5;γ=3/25;β=1/100;xi=1/1000;μ=3/25;w=51/8 ;α=43/8;*)
cII = Flatten[Join[cut, RII]]
Print["Fixed points:"]
eq = equi /. cII // N
Print["Eigenvalues of FP:"]
EcoEigenvalues[eq] /. cII // N
eqs = {s → IntegerPart[eq[[3, 1]][[2]]], i → IntegerPart[eq[[3, 2]][[2]]]}
```

Out[ ]= $\left\{\left\{\omega \to \frac{51}{8}, \alpha \to \frac{43}{8}, \eta \to \frac{43}{51}\right\}\right\}$

Out[ ]= $\left\{\Lambda \to 16, \delta \to \frac{1}{5}, \gamma \to \frac{3}{25}, \beta \to \frac{1}{100}, \xi \to \frac{1}{1000}, \mu \to \frac{3}{25}, v_2 \to \gamma + \delta + \mu, v_1 \to \beta + \mu\,\xi, V_2 \to \gamma + \delta + \eta + \mu, \omega \to \frac{51}{8}, \alpha \to \frac{43}{8}, \eta \to \frac{43}{51}\right\}$

Fixed points:

Out[ ]= {{s → 133.333, i → 0.}, {s → 149.218, i → -1.27578}, {s → 85.4965, i → 6.75961}}

Eigenvalues of FP:

Out[ ]= {{-0.12, 0.0501961}, {-0.3429, -0.0261396}, {0.00887972 - 0.13495 i, 0.00887972 + 0.13495 i}}

Out[ ]= {s → 85, i → 6}

In[∘]:=

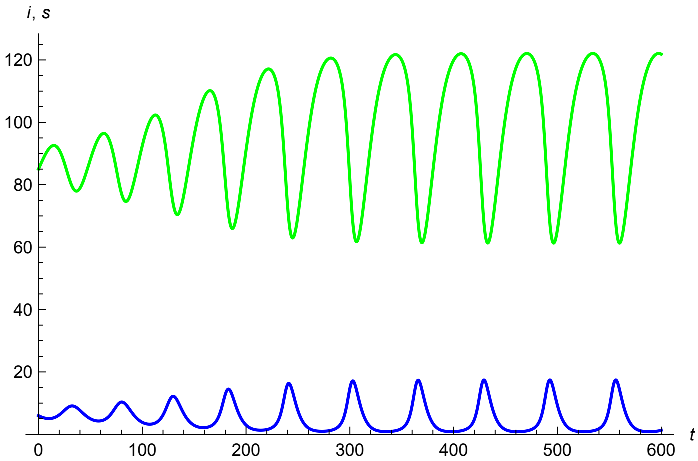
$$\Lambda = 16\;;\;\delta = \frac{1}{5}\;;\;\gamma = \frac{3}{25}\;;\;\beta = \frac{1}{100}\;;\;\xi = \frac{1}{1000}\;;\;\mu = \frac{3}{25}\;;\;\omega = \frac{51}{8}\;;\;\alpha = \frac{43}{8}\;;$$

```
sol = EcoSim[eqs, T]
psol = PlotDynamics[sol]
```

Out[∘]=

```
{s → InterpolatingFunction[ Domain: {{0., 600.}} Output: scalar ],
 i → InterpolatingFunction[ Domain: {{0., 600.}} Output: scalar ]}
```

Out[∘]=



In[∘]:=

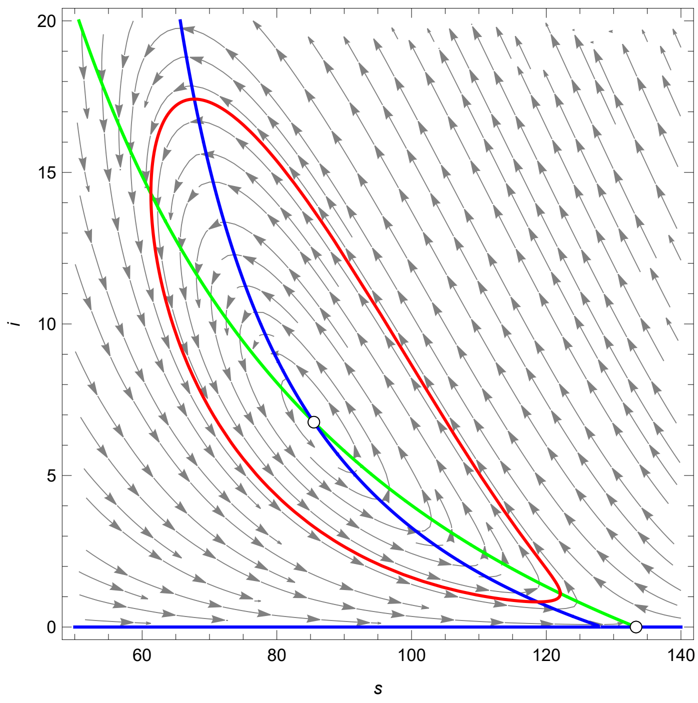
```
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]

Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[lc, {s, i}, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spR2E.pdf", sp]

Export["solT2.pdf", psol]
```

```
Time plot around E2

Finding the cycle using time slice
```

Out[∘]=

```
{i → InterpolatingFunction[ Domain: {{0., 63.5}} Output: scalar ],
 s → InterpolatingFunction[ Domain: {{0., 63.5}} Output: scalar ]}
```

Out[ ]=




```
Final time slice is
```

Out[ ]= 63.5035

```
The floquet exponents are
```

Out[ ]= $\{-3.20788\times10^{-8}, -0.0228667\}$

Out[ ]= spR2E.pdf

Out[ ]= solT2.pdf

Phase plot of the region III:

```
ClearParameters;
T = 380;
xM = 140;
xm = 25;
yM = 30;
ym = 0;
```

$\Lambda = 16\,;\ \delta = \frac{1}{5}\,;\ \gamma = \frac{3}{25}\,;\ \beta = \frac{1}{100}\,;\ \xi = \frac{1}{1000}\,;\ \mu = \frac{3}{25}\,;$

$\omega = 6\,;\ \alpha = \frac{5}{32}\,;$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
EcoEigenvalues[eq]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → 55, i → 10}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spR3E.pdf", sp]

Export["solT3.pdf", psol]
```

Fixed points:

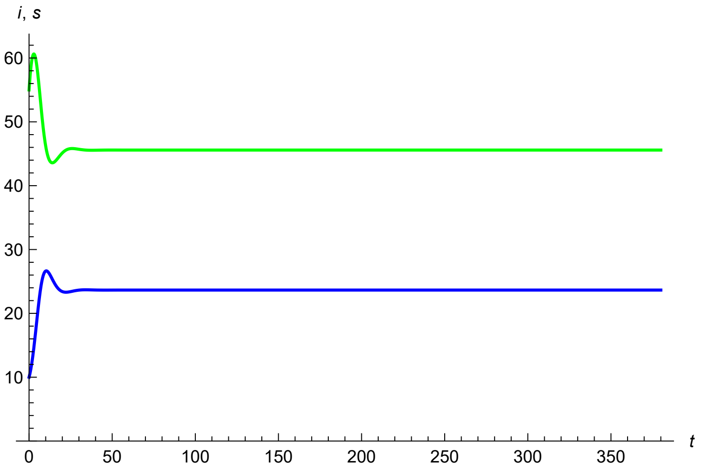
Out[]= {{s → 133.333, i → 0.}, {s → 45.58, i → 23.6495}}

Eigenvalues of FP:

Out[]= {{0.867292, -0.12}, {-0.178557 + 0.265984 i, -0.178557 - 0.265984 i}}

Time plot around E2

Out[ ]=

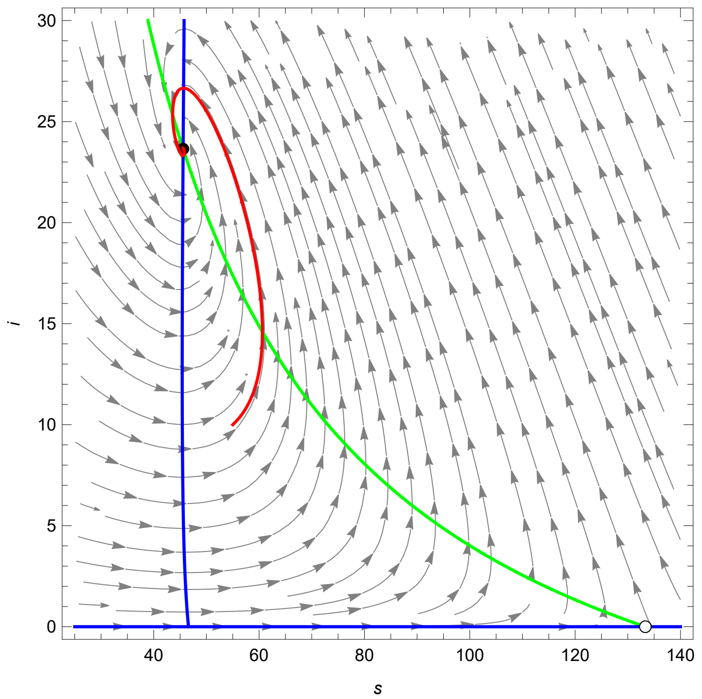


Finding the cycle using time slice

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[ ]= $Failed

Out[ ]=



Final time slice is

Out[ ]= FinalTime[$Failed]

The floquet exponents are

Out[ ]= EcoEigenvalues[$Failed]

Out[ ]= spR3E.pdf

Out[ ]= solT3.pdf

Phase plot of the region IV:

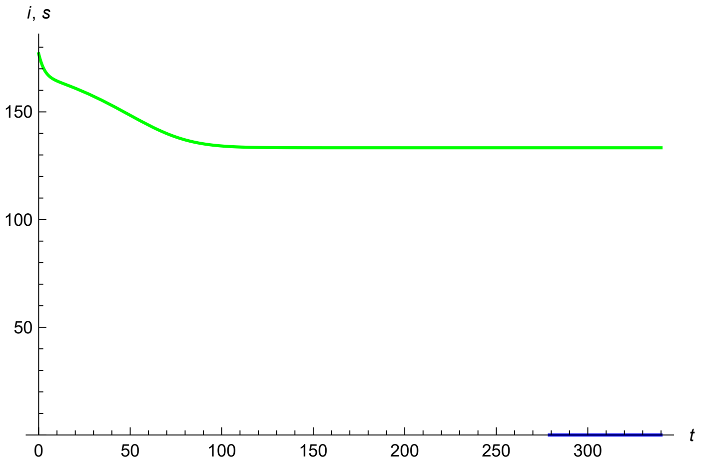
In[ ]:=
```
ClearParameters;
T = 340;
xM = 140;
xm = 25;
yM = 14;
ym = 0;
Λ = 16 ; δ = 1/5 ; γ = 3/25 ; β = 1/100 ; ξ = 1/1000 ; μ = 3/25 ;
ω = 47/4 ; α = 47/4 ; η = α / ω;
Print["TrE2="]
trE2 // N
eq = N[equi, 19];
Print["Fixed points:", eq]

Print["Eigenvalues of FP:", EcoEigenvalues[eq]]

pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[3, 1]][[2]]], i → IntegerPart[eq[[3, 2]][[2]]] + 1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spRIVE.pdf", sp]

Export["solTIV.pdf", psol]
```

```
TrE2=
```

Out[ ]=
```
-0.534435
```

```
Fixed points:
 {{s → 133.3333333333333333, i → 0}, {s → 3594.562322769512524, i → -11.42289302686083784},
  {s → 177.1024703551928460, i → -2.956912937925364096}}
Eigenvalues of FP:{{-0.1200000000000000000, -0.1066666666666666667},
  {-1253.97840825243832, -0.00106193059039627809},
  {-0.557250711059899373, 0.0228161854446970380}}
Time plot around E2
```

Out[•]=

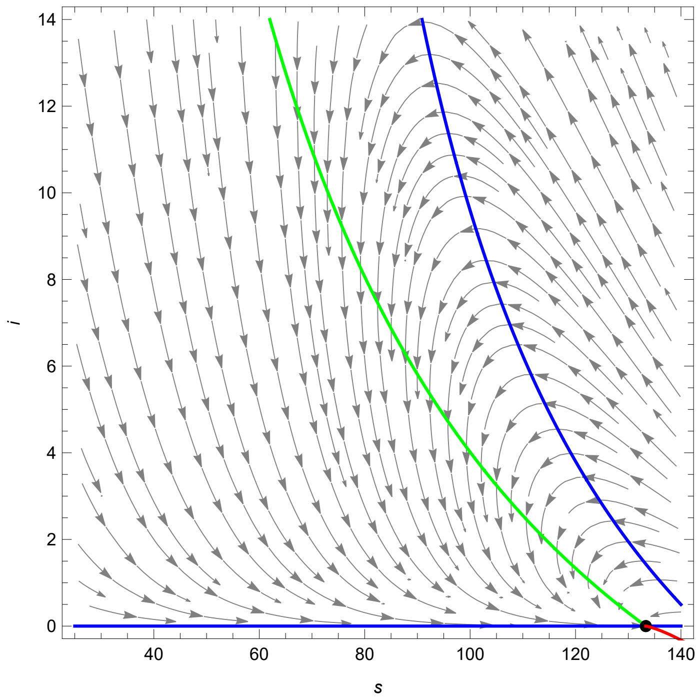


Finding the cycle using time slice

Out[•]= {i → InterpolatingFunction[Domain: {{0., 1.00 × 10^3}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 1.00 × 10^3}} Output: scalar]}

Out[•]=



Final time slice is

Out[•]= 1000.

The floquet exponents are

Out[•]= {−∞, −∞}

Out[•]= spRIVE.pdf

Out[•]= solTIV.pdf

Phase plot of Region VIa :

In[ ]:=
```
ClearParameters;
T = 10^4;
xM = 137;
xm = 100;
yM = 5;
ym = 0;
```
$\omega = \frac{502}{67}$; $\alpha = \frac{41102}{6131}$; $\Lambda = 16$; $\delta = \frac{1}{5}$; $\gamma = \frac{3}{25}$; $\beta = \frac{1}{100}$; $\xi = \frac{1}{1000}$; $\mu = \frac{3}{25}$;

$\frac{502}{67}$ // N
```
α // N
eq = N[equi, 19];
Print["Fixed points:", eq]

Print["Eigenvalues of FP:", EcoEigenvalues[eq]]

pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["We will evaluate near the point "]

Print["Time plot around E2"]
E2n = RuleListTweak[eq⟦3⟧, {s, i}, {0.5, 0.5}]
sol = EcoSim[E2n, T];
E1n = RuleListTweak[eq⟦2⟧, {s, i}, {0.01, 0.02}]
sol1 = EcoSim[E1n, T];
psol2 = PlotDynamics[sol, PlotPoints → 400];
psol1 = PlotDynamics[sol1, PlotPoints → 400, PlotStyle → {Yellow, Pink}];
psol = Show[psol2]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp2 = Show[pp, RuleListPlot[sol, PlotStyle → Red, PlotPoints → 400]];
sp1 = Show[pp, RuleListPlot[sol1, PlotStyle → Magenta, PlotPoints → 400]];
Show[sp2]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spVIa.pdf", sp2]
Export["solVIa.pdf", psol]
```

Out[ ]= 7.49254

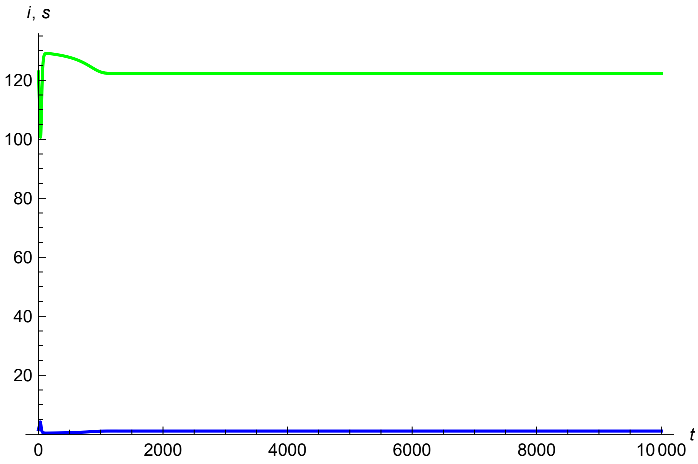
Out[ ]= 6.70396

```
Fixed points:
{{s → 133.3333333333333333, i → 0}, {s → 130.4528532406476235, i → 0.2650376858970212231},
 {s → 122.3268071810354282, i → 1.080883889607936805}}
Eigenvalues of FP:{{-0.1200000000000000, -0.001418762507735599},
 {-0.094779579177624718, 0.0013091616317688633l},
 {-0.0167667251579691182 + 0.0132809395724372326 ⅈ,
  -0.0167667251579691182 - 0.0132809395724372326 ⅈ}}
We will evaluate near the point
Time plot around E2
```

Out[∘]= {s → 122.827, i → 1.58088}

Out[∘]= {s → 130.463, i → 0.285038}

Out[∘]=

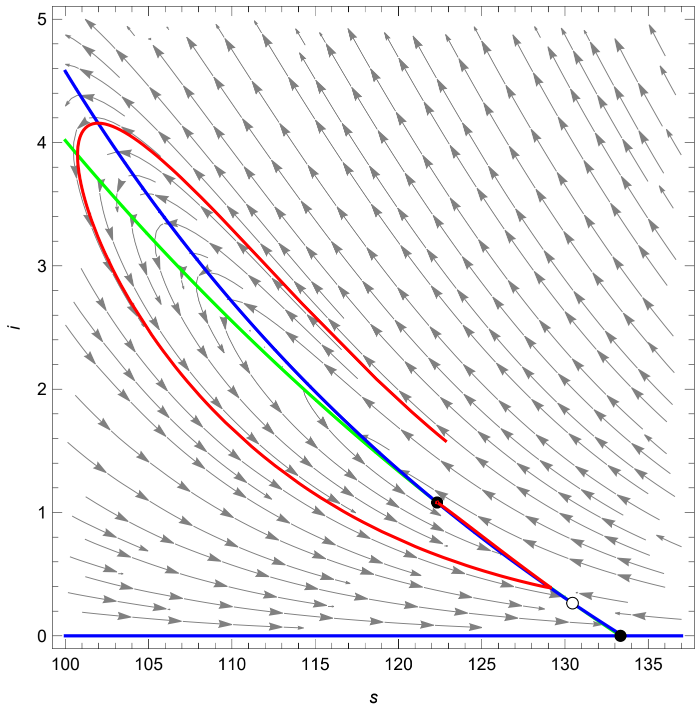



```
Finding the cycle using time slice
```

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[∘]= $Failed

Out[∘]=



Final time slice is

Out[•]= FinalTime[$Failed]

The floquet exponents are

Out[•]= EcoEigenvalues[$Failed]

Out[•]= spVIa.pdf

Out[•]= solVIa.pdf

Phase plot of the boundary TrE2=0 between Regions I and VIa:

In[ ]:=
```
ClearParameters;
T = 10^3;
xM = 140;
xm = 60;
ym = 12;
ym = 0;

Λ = 16 ;
δ = 2/10;
γ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω = 390/133;
α = 3.70… ;
(* 251/71 ;α= 3.93… ;*)
ω // N
α // N

Print["Fixed points:"]
eq = N[equi, 4]
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-12]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
E2n = RuleListTweak[eq⟦3⟧, {s}, {0.8}]
sol = EcoSim[E2n, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[lc, {s, i}, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["sp16a.pdf", sp]
Export["solT16a.pdf", psol]
```

Out[ ]= 2.93233

Out[∘]= 3.69658

Fixed points:

Out[∘]= {{s → 133.3, i → 0}, {s → 106.9, i → 2.971}, {s → 73.82, i → 9.770}}

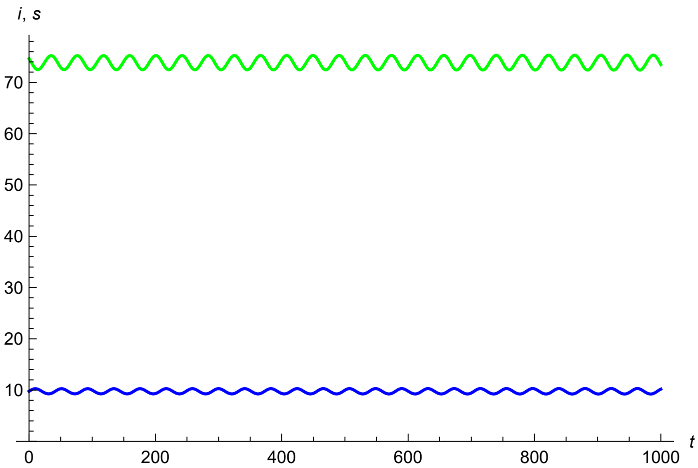
Eigenvalues of FP:

Out[∘]= {{-0.367, -0.1200}, {0.229, -0.0664}, {0.152 i, -0.152 i}}

Time plot around E2

Out[∘]= {i → 9.770, s → 74.6151}

Out[∘]=



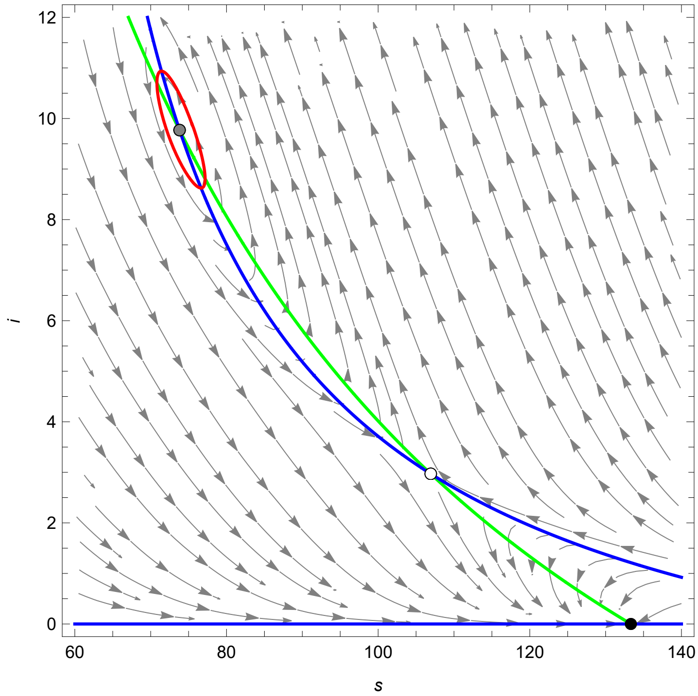
Finding the cycle using time slice

FindEcoCycle: Failed to converge to the requested accuracy within 100 iterations.

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 41.8}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 41.8}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 41.7589

The floquet exponents are

Out[ ]= {0.00149016, 0.000030097}

Out[ ]= sp16a.pdf

Out[ ]= solT16a.pdf

Phase plot of the boundary TrE2=0 between Regions II and III:

In[ ]:=
```
ClearParameters;
T = 10^4;
xM = 140;
xm = 60;
yM = 12;
ym = 0;

Λ = 16 ;
δ = 2/10;
γ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω = 6 ;
α = (*Rationalize[5.00625]*)Root[{-686 757 738 052 533 + 1 647 448 543 030 550 * #1 -
     1 077 591 212 548 125 * #1^2 + 107 367 096 375 000 * #1^3 &, 6 * #1 - #2 &}, {1, 1}];

Print["Fixed points:"]
eq = N[equi, 19]
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-12]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
E2n = RuleListTweak[eq[[3]], i, 0.5]
sol = EcoSim[E2n, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[lc, {s, i}, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB23E.pdf", sp]
Export["solT23.pdf", psol]
```

```
Fixed points:
```

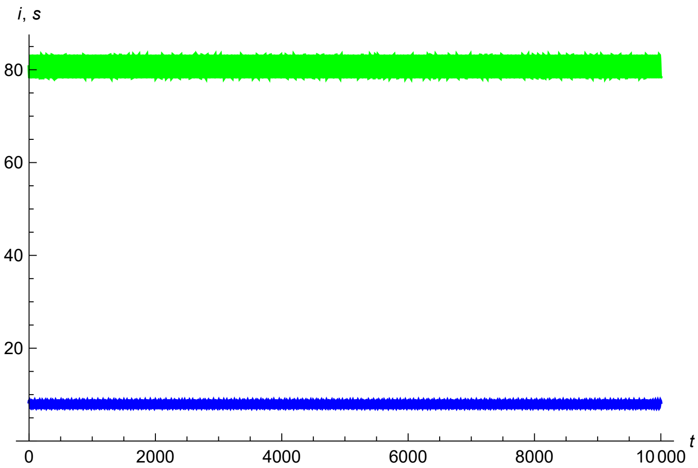
Out[ ]=
```
{{s → 133.3333333333333333, i → 0},
 {s → 148.2540278357179640, i → -1.206256305190749303},
 {s → 80.64038208876516802, i → 7.903145785444163751}}
```

```
Eigenvalues of FP:
```

Out[∘]= {{-0.120000000000000000, 0.058957665969571868},
{-0.338749521159635600, -0.0301670848201448014},
{0.151250071483287974 i, -0.151250071483287974 i}}

Time plot around E2

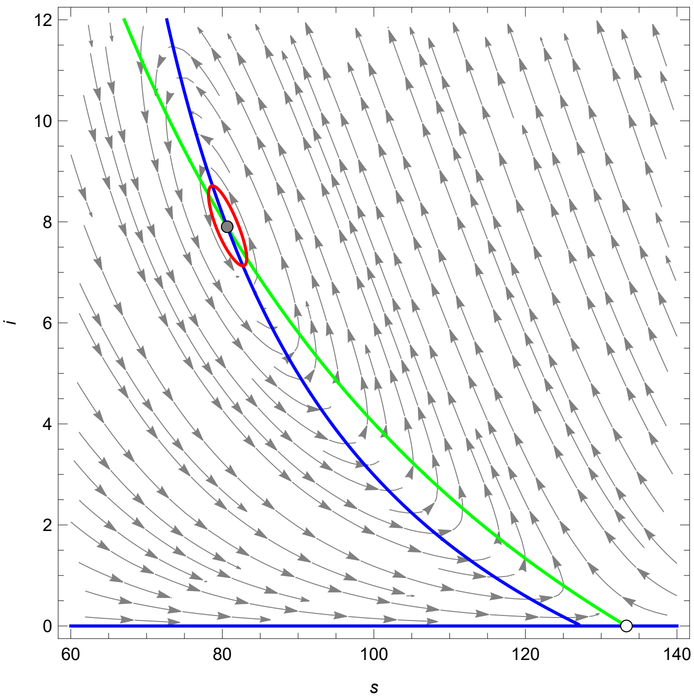
Out[∘]= {s → 80.64038208876516802, i → 8.40315}

Out[∘]=


Finding the cycle using time slice

FindEcoCycle: Failed to converge to the requested accuracy within 100 iterations.

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 41.6}} Output: scalar],
s → InterpolatingFunction[Domain: {{0., 41.6}} Output: scalar]}

Out[∘]=


Final time slice is

Out[ ]= 41.6159

The floquet exponents are

Out[ ]= $\{-3.18843\times10^{-7}+0.0000102417\,\mathrm{i},\ -3.18843\times10^{-7}-0.0000102417\,\mathrm{i}\}$

Out[ ]= spB23E.pdf

Out[ ]= solT23.pdf

In[ ]:=
```
ClearParameters;
T = 200;
xM = 136;
xm = 75;
yM = 9;
ym = 0;

Λ = 16 ;
δ = 2/10;
γ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω = 6 ;
α = (*Rationalize[5.00625]*)Root[{-686757738052533 + 1647448543030550 * #1 -
     1077591212548125 * #1^2 + 107367096375000 * #1^3 &, 6 * #1 - #2 &}, {1, 1}];

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-12]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]] + 1, i → IntegerPart[eq[[2, 2]][[2]]] + 1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[lc, {s, i}, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB23Eb.pdf", sp]
Export["solT23b.pdf", psol]
```

Fixed points:

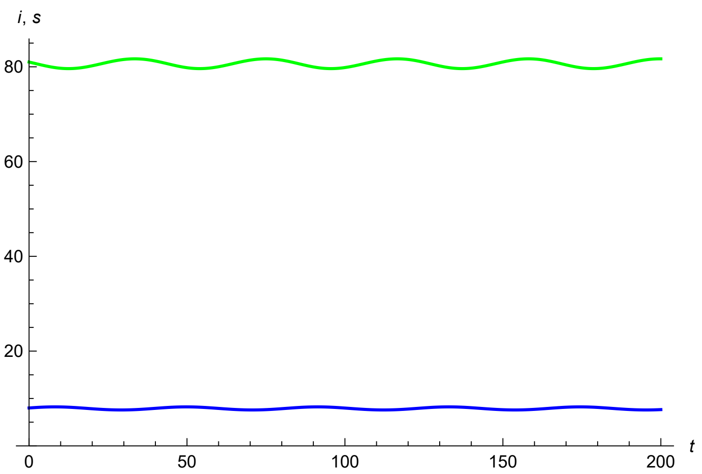
Out[•]= {{s → 133.333, i → 0.}, {s → 80.6404, i → 7.90315}}

Eigenvalues of FP:

Out[•]= {{-0.12, 0.0589577}, {0. + 0.15125 ⅈ, 0. - 0.15125 ⅈ}}

Time plot around E2

Out[•]=



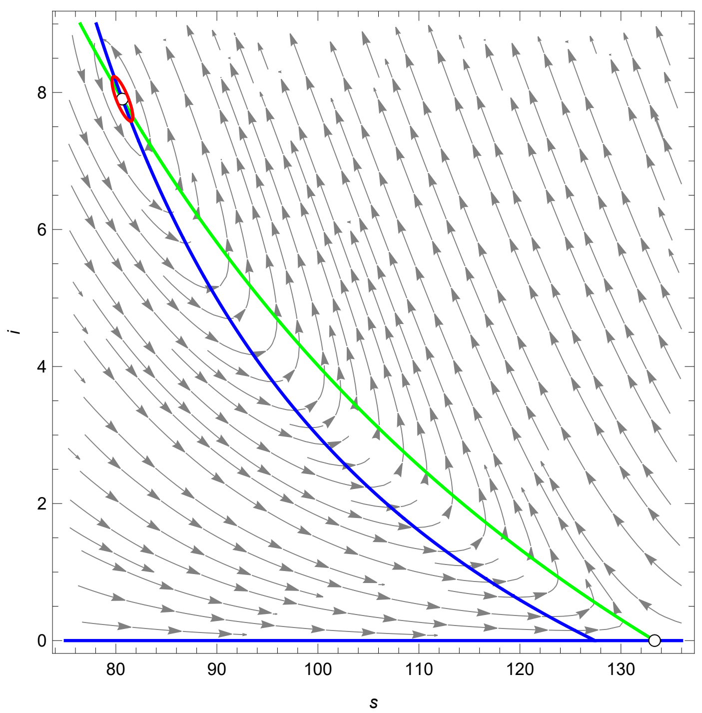
Finding the cycle using time slice

FindEcoCycle: Failed to converge to the requested accuracy within 100 iterations.

Out[•]= {i → InterpolatingFunction[Domain: {{0., 41.6}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 41.6}} Output: scalar]}

Out[•]=



Final time slice is

Out[•]= 41.5543

The floquet exponents are

Out[∘]= $\{7.65401\times10^{-6}, -7.57769\times10^{-6}\}$

Out[∘]= spB23Eb.pdf

Out[∘]= solT23b.pdf

Phase plot of the boundary R0=1 between Regions VI and III (between 0 and B1):

In[∘]:=
```
ClearParameters;
T = 600;
xM = 137;
xm = 65;
yM = 12;
ym = 0;
Λ = 16 ; δ = 2/10; γ = 12/100; β = 1/100; ξ = 1/1000; μ = 12/100;
ω = 901/195;
α = 60367/14625;
eq = N[equi, 19];
Print["Fixed points:", eq]

Print["Eigenvalues of FP:", EcoEigenvalues[eq]]

pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
E2n = RuleListTweak[eq[[3]], {s, i}, {0.01, 0.8}]
sol = EcoSim[E2n, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
(*Print["The floquet exponents are"]
 Chop[Evaluate[EcoEigenvalues[lc]]]*)
Export["sp0B1.pdf", sp]
Export["sol0B1.pdf", psol]
```

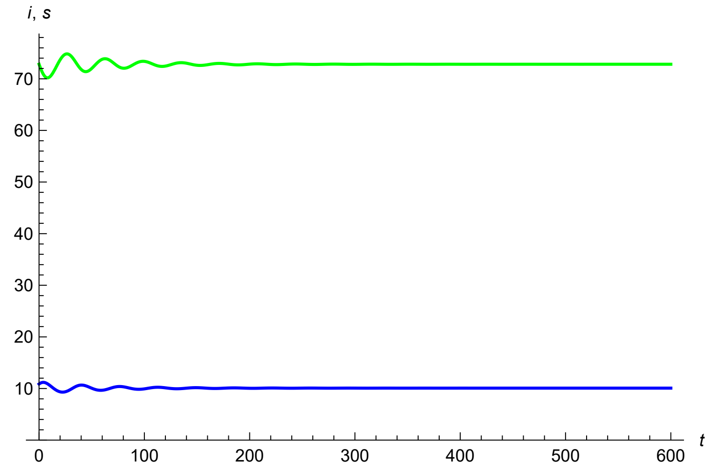
Fixed points:{{s→133.3333333333333333, i→0},
{s→133.3333333333333333, i→0}, {s→72.81753885193378788, i→10.07318548820525105}}

Eigenvalues of FP:{{-0.1200000000000000000, 0},
{-0.1200000000000000000, 0}, {-0.0171690151794508510 + 0.173631522140559829 i,
-0.0171690151794508510 - 0.173631522140559829 i}}

Time plot around E2

Out[∘]= {s → 72.8275, i → 10.8732}

Out[ ]=

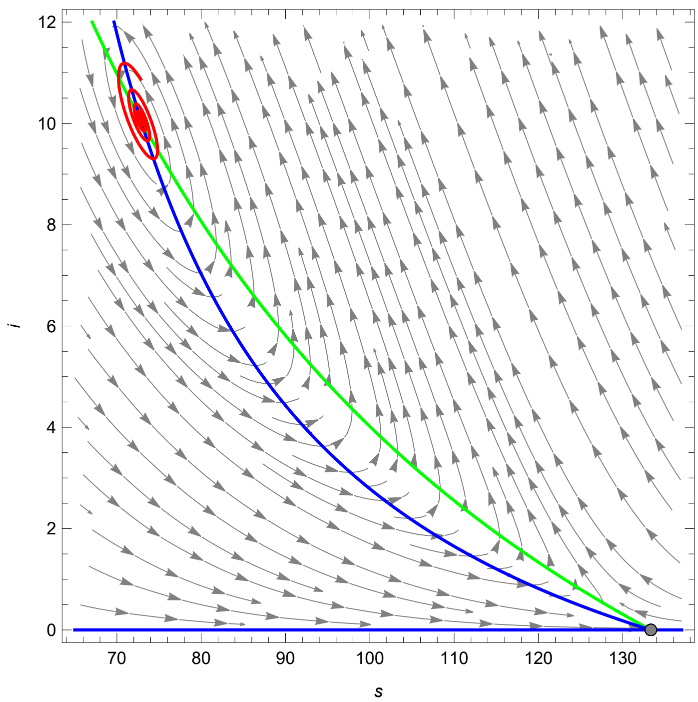


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 40.6}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 40.6}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 40.607

Out[ ]= sp0B1.pdf

Out[ ]= sol0B1.pdf

Phase plot of the boundary R0=1 between Regions III and VI (Between BT and B1):

```
ClearParameters;
T = 500;
xM = 140;
xm = 55;
yM = 22;
ym = 0;
```

$$\Lambda = 16\ ;\ \delta = \frac{2}{10};\ \gamma = \frac{12}{100};\ \beta = \frac{1}{100};\ \xi = \frac{1}{1000};\ \mu = \frac{12}{100};\ \omega = \frac{723}{137}\ //\ N$$

$$\alpha = \frac{16147}{3425}\ //\ N$$

```
(*ω=5.157354044656745;α=4.607236279893359;*)

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq⟦2, 1⟧⟦2⟧], i → IntegerPart[eq⟦2, 2⟧⟦2⟧]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB12aE.pdf", sp]
Export["solT12a.pdf", psol]
```

Out[◦]= 5.27737

Out[◦]= 4.71445

Fixed points:

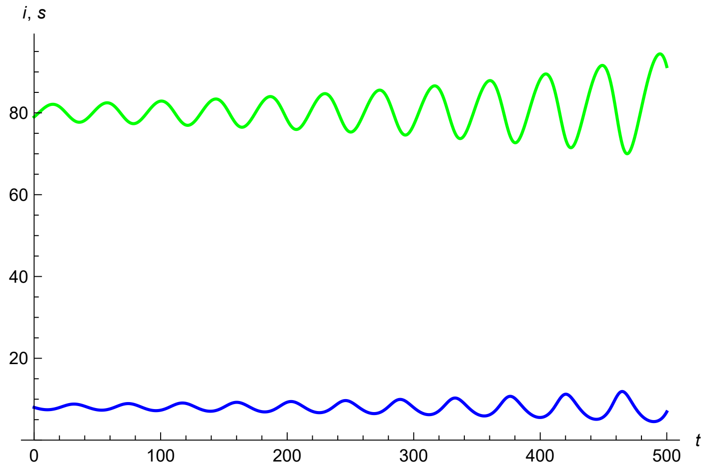
Out[◦]= $\{\{s \to 133.333,\ i \to 0.\},\ \{s \to 79.9285,\ i \to 8.0827\},\ \{s \to 133.333,\ i \to 1.55832 \times 10^{-15}\}\}$

Eigenvalues of FP:

Out[◦]= $\{\{-0.12,\ 0\},\ \{0.00347509 + 0.146926\, i,\ 0.00347509 - 0.146926\, i\},\ \{-0.12,\ 0\}\}$

Time plot around E2

Out[∘]=

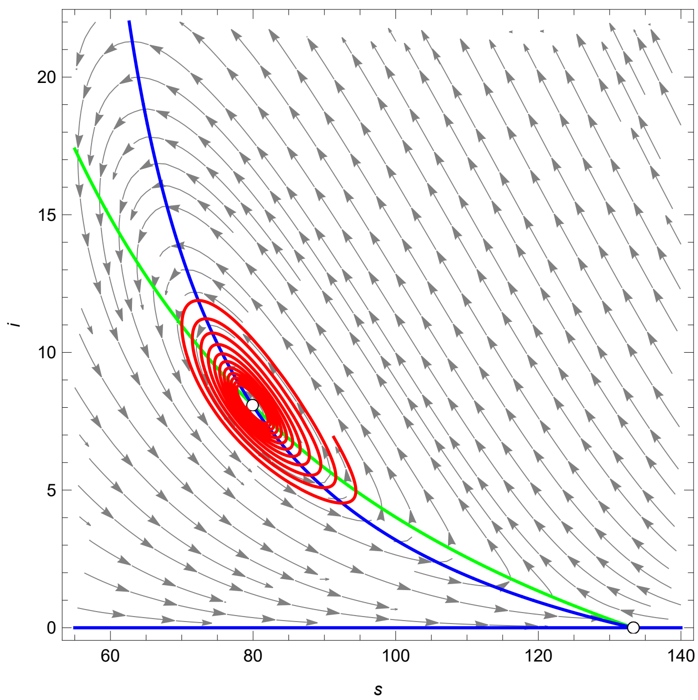



```
Finding the cycle using time slice
```

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 101.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 101.}} Output: scalar]}

Out[∘]=




```
Final time slice is
```

Out[∘]= 101.348

```
The floquet exponents are
```

Out[∘]= $\{8.51007 \times 10^{-8}, -0.0337781\}$

Out[∘]= spB12aE.pdf

Out[∘]= solT12a.pdf

Phase plot of the boundary R0=1 between Regions III and VI (Between BT and B1):

```
ClearParameters;
T = 600;
xM = 140;
xm = 60;
yM = 15;
ym = 0;
```

$$\Lambda = 16\ ;\ \delta = \frac{2}{10};\ \gamma = \frac{12}{100};\ \beta = \frac{1}{100};\ \xi = \frac{1}{1000};\ \mu = \frac{12}{100};\ \omega = \frac{25}{4}\ \ //\ N$$

$$\alpha = \frac{67}{12}\ //\ N$$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]]]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB12E.pdf", sp]
Export["solT12.pdf", psol]
```

Out[]= 6.25

Out[]= 5.58333

Fixed points:

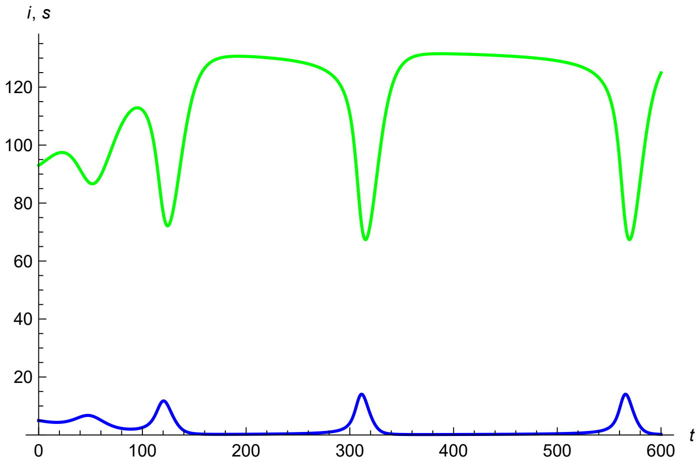
Out[]= {{s → 133.333, i → 0.}, {s → 93.5174, i → 5.13535}}

Eigenvalues of FP:

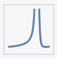
Out[]= {{-0.12, 0}, {0.0226741 + 0.0987278 i, 0.0226741 - 0.0987278 i}}

Time plot around E2

Out[∘]=

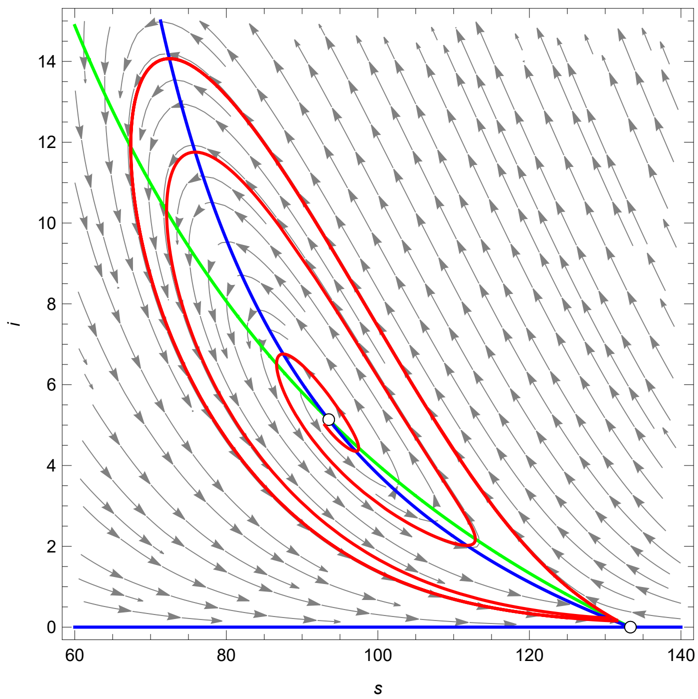


Finding the cycle using time slice

Out[∘]= {i → InterpolatingFunction[ Domain: {{0., 254.}} Output: scalar],

s → InterpolatingFunction[ Domain: {{0., 254.}} Output: scalar]}

Out[∘]=



Final time slice is

Out[∘]= 254.473

The floquet exponents are

Out[∘]= $\{-4.59451\times 10^{-8}, -0.0659782\}$

Out[∘]= spB12E.pdf

Out[∘]= solT12.pdf

Phase plot of the boundary R0=1 between Regions III and VI (between B1 and BT):

```
ClearParameters;
T = 800;
xM = 140;
xm = 50;
yM = 20;
ym = 0;
```

$\Lambda = 16\ ;\ \delta = \frac{2}{10};\ \gamma = \frac{12}{100};\ \beta = \frac{1}{100};\ \xi = \frac{1}{1000};\ \mu = \frac{12}{100};\ \omega = \frac{73}{13}$ // N

$\alpha = \frac{4891}{975}$ // N

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq⟦2, 1⟧⟦2⟧], i → IntegerPart[eq⟦2, 2⟧⟦2⟧]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spBP1E.pdf", sp]
Export["solBP1.pdf", psol]
```

Out[∘]= 5.61538

Out[∘]= 5.01641

Fixed points:

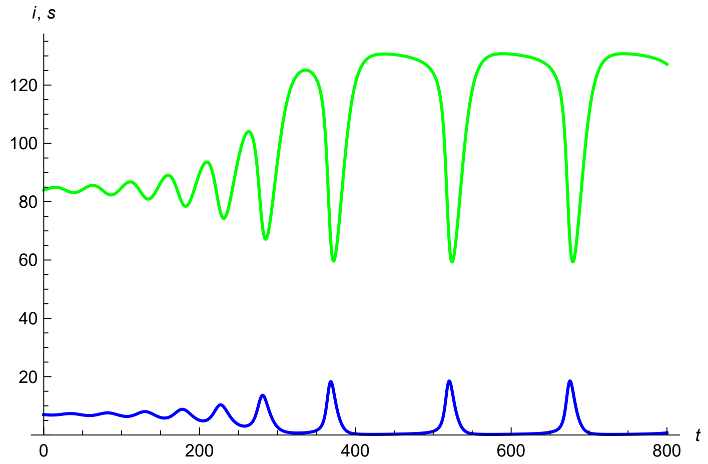
Out[∘]= {{s → 133.333, i → 0.}, {s → 84.1709, i → 7.05842}}

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {0.0122452 + 0.131269 i, 0.0122452 - 0.131269 i}}

Time plot around E2

Out[ ]=

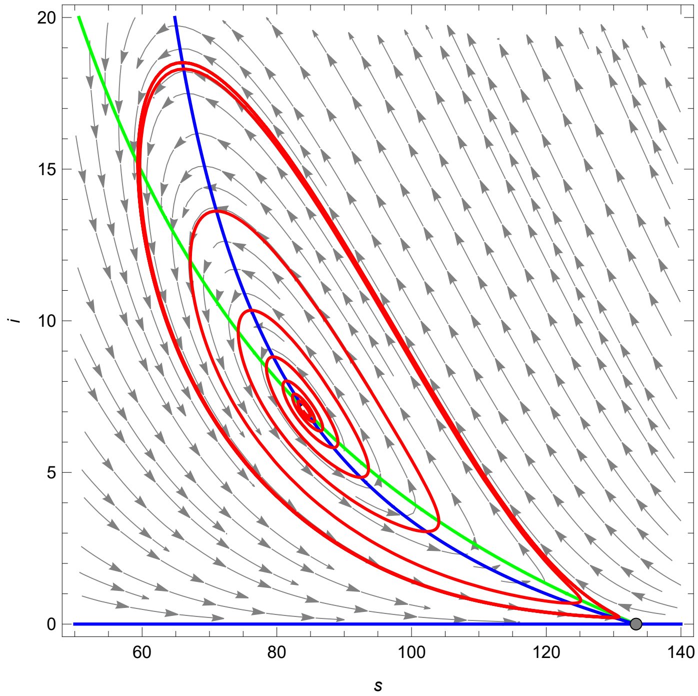


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 155.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 155.}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 155.023

The floquet exponents are

Out[ ]= $\{2.50718 \times 10^{-8}, -0.0534859\}$

Out[ ]= spBP1E.pdf

Out[ ]= solBP1.pdf

Phase plot of the boundary R0=1 between Regions III and VI (another between B1 and BT):

```
ClearParameters;
T = 800;
xM = 140;
xm = 50;
yM = 20;
ym = 0;

Λ = 16 ; δ = 2/10; γ = 12/100; β = 1/100; ξ = 1/1000; μ = 12/100; ω = 195/32 (*11/2*) // N
α = 871/160 (*737/150*) // N

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]]]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spBP2E.pdf", sp]
Export["solBP2.pdf", psol]
```

Out[•]= 6.09375

Out[•]= 5.44375

Fixed points:

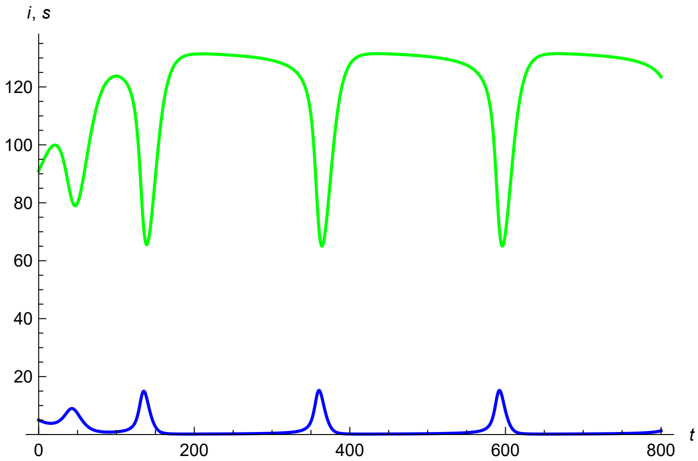
Out[•]= {{s → 133.333, i → 0.}, {s → 91.0252, i → 5.60883}}

Eigenvalues of FP:

Out[•]= {{-0.12, 0}, {0.0210629 + 0.10705 i, 0.0210629 - 0.10705 i}}

Time plot around E2

Out[•]=

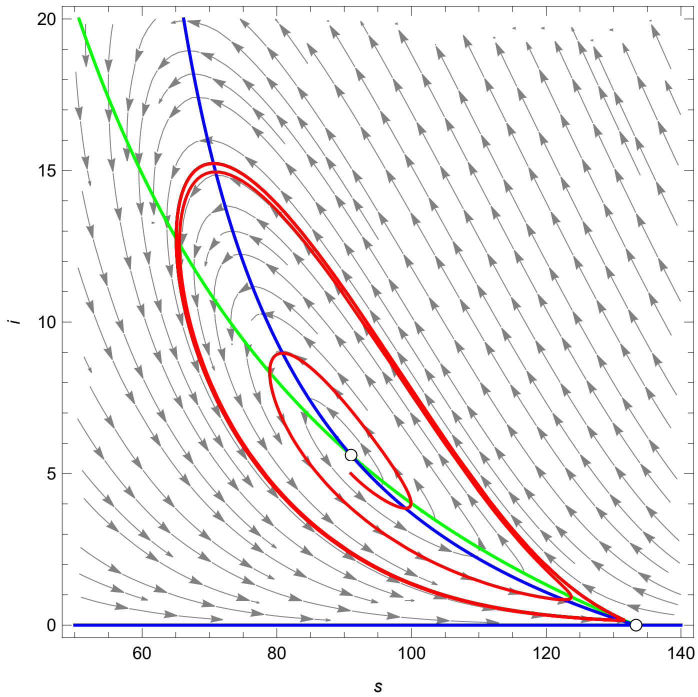


Finding the cycle using time slice

Out[•]= {i → InterpolatingFunction[Domain: {{0., 232.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 232.}} Output: scalar]}

Out[•]=



Final time slice is

Out[•]= 231.915

The floquet exponents are

Out[•]= $\{8.82425 \times 10^{-10}, -0.0647755\}$

Out[•]= spBP2E.pdf

Out[•]= solBP2.pdf

Phase plot of the boundary R0=1 between Regions III and VI (between BT and B1 and closer to BT):

```
ClearParameters;
T = 2000;
xM = 140;
xm = 60;
yM = 12;
ym = 0;

Λ = 16 ; δ = 2/10; γ = 12/100; β = 1/100; ξ = 1/1000; μ = 12/100; (*w=235/36;α=3149/540;*)
ω = 18362/2969; α = 1230254/222675;
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]]]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
(*Export["spHPE.pdf",sp]
 Export["solHP.pdf",psol]*)
```

Fixed points:

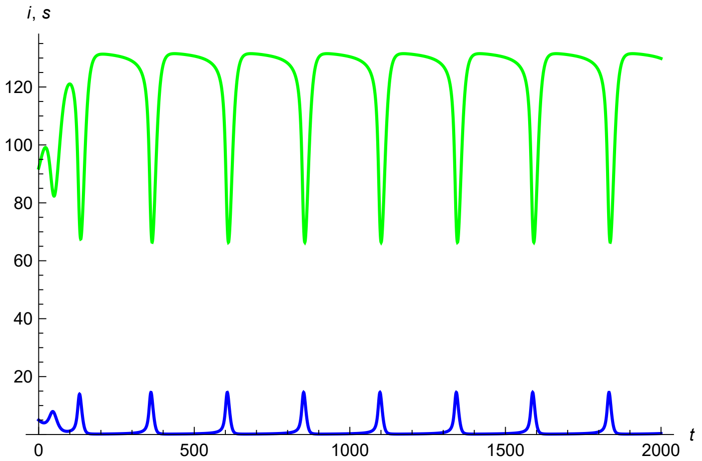
Out[∘]= {{s → 133.333, i → 0.}, {s → 92.4572, i → 5.33361}, {s → 133.333, i → 0.}}

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {0.022091 + 0.10223 i, 0.022091 - 0.10223 i}, {-0.12, 0}}

Time plot around E2

Out[ ]=

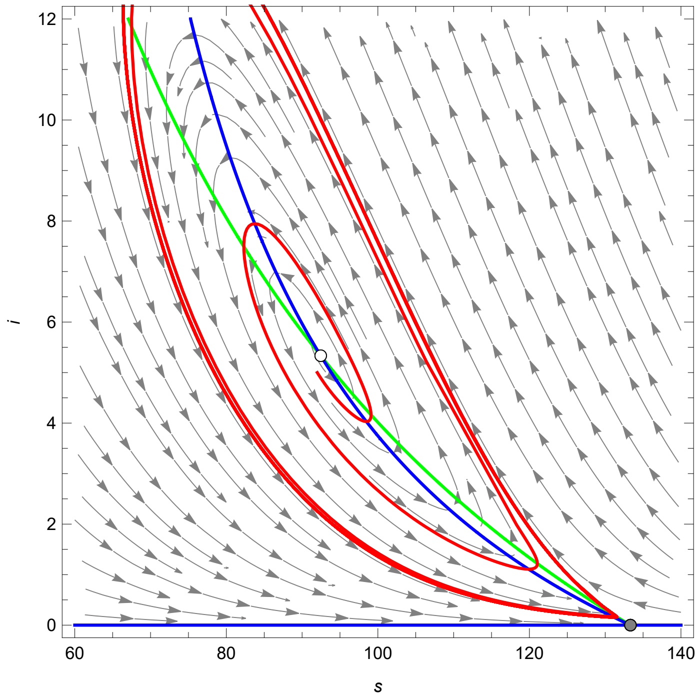


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 245.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 245.}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 245.34

The floquet exponents are

Out[ ]= $\{3.41435\times10^{-8}, -0.0656078\}$

Phase plot of the boundary R0=1 between Regions IV and II (Between B2 and BT):

```
ClearParameters;
T = 1000;
xM = 140;
xm = 90;
yM = 6;
ym = 0;
```

$$\Lambda = 16\;;\;\delta = \frac{2}{10};\;\gamma = \frac{12}{100};\;\beta = \frac{1}{100};\;\xi = \frac{1}{1000};\;\mu = \frac{12}{100};\;\omega = \frac{981}{137}\;//\;N$$

$$\alpha = \frac{21\,909}{3425}\;//\;N$$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]]]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB12bE.pdf", sp]
Export["solT12b.pdf", psol]
```

Out[•]= 7.16058

Out[•]= 6.39679

Fixed points:

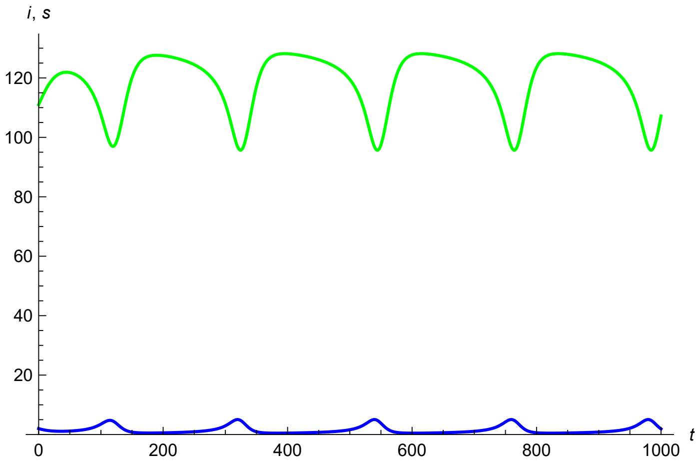
Out[•]= {{s → 133.333, i → 0.}, {s → 111.34, i → 2.376}}

Eigenvalues of FP:

Out[•]= {{-0.12, 0}, {0.0103906 + 0.0502167 ⅈ, 0.0103906 - 0.0502167 ⅈ}}

Time plot around E2

Out[∘]=

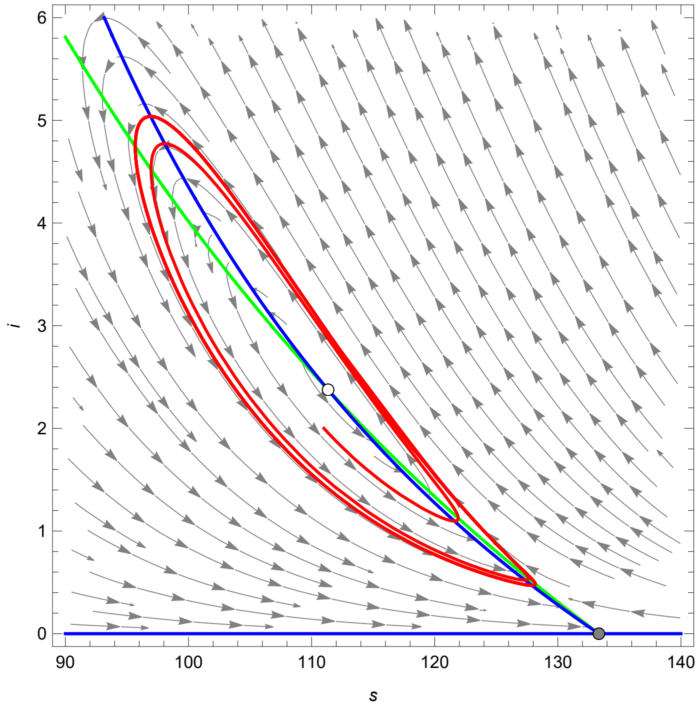


Finding the cycle using time slice

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 220.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 220.}} Output: scalar]}

Out[∘]=



Final time slice is

Out[∘]= 219.938

The floquet exponents are

Out[∘]= $\{-2.73386\times10^{-8}, -0.0298991\}$

Out[∘]= spB12bE.pdf

Out[∘]= solT12b.pdf

Phase plot of the boundary R0=1 (Between B2 and H):

```
ClearParameters;
T = 700;
xM = 140;
xm = 100;
yM = 5;
ym = 0;
```

$$\Lambda = 16\,;\ \delta = \frac{2}{10};\ \gamma = \frac{12}{100};\ \beta = \frac{1}{100};\ \xi = \frac{1}{1000};\ \mu = \frac{12}{100};\ \omega = \frac{8139}{1028};\ \alpha = \frac{181\,771}{25\,700};$$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq⟦2, 1⟧⟦2⟧], i → IntegerPart[eq⟦2, 2⟧⟦2⟧] + 1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB2H.pdf", sp]
Export["solB2H.pdf", psol]
```

Out[∘]= 7.91732

Out[∘]= 7.0728

Fixed points:

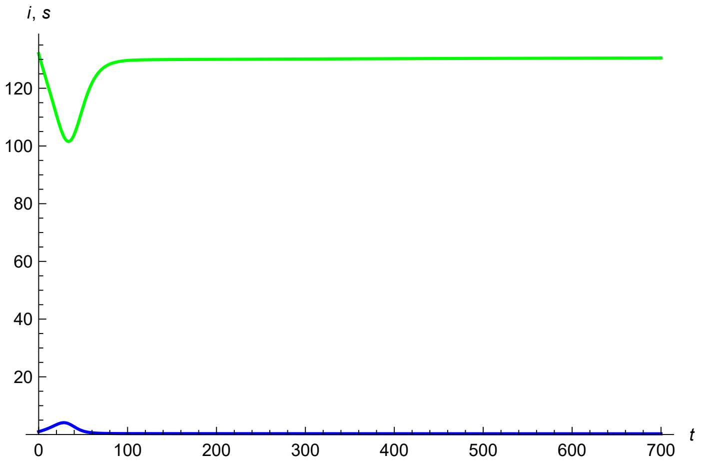
Out[∘]= {{s → 133.333, i → 0.}, {s → 132.419, i → 0.0828753}, {s → 133.333, i → 0.}}

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {-0.111746, -0.0000342071}, {-0.12, 0}}

Time plot around E2

Out[ ]=

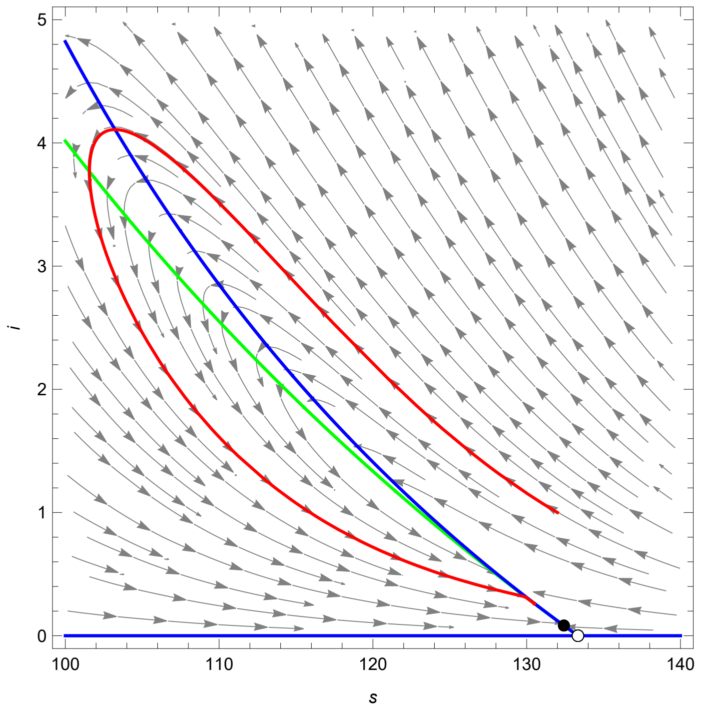


Finding the cycle using time slice

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[ ]= $Failed

Out[ ]=



Final time slice is

Out[ ]= FinalTime[$Failed]

The floquet exponents are

Out[ ]= EcoEigenvalues[$Failed]

Out[ ]= spB2H.pdf

Out[ ]= solB2H.pdf

Phase plot of the boundary R0=1 (at BT):

```
In[ ]:= ClearParameters;
T = 10^3;
xM = 140;
xm = 110;
yM = 2.5;
ym = 0;

Λ = 16 ;
δ = 2/10;
γ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω = Root[-36 408 000 000 000 + 1 969 099 000 000 #1 + 431 356 651 000 #1^2 + 8 566 772 533 #1^3 &, 3];
α = 1/545 531 250 × (-1 017 750 000 +
    70 750 Root[-837 384 000 000 000 + 7 783 000 000 #1 + 293 000 #1^2 + #1^3 &, 3, 0] +
    Root[-837 384 000 000 000 + 7 783 000 000 #1 + 293 000 #1^2 + #1^3 &, 3, 0]^2);

Print["Fixed points:"]
eq = N[equi, 14] (*Numeric fixed points*)

Print["Eigenvalues of FP:"]
eig = EcoEigenvalues[eq]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq // N, PlotMarkers → EcoStableQ[eq // N]]];
Print["Time plot around E2"]
E2n = RuleListTweak[eq[[2]], {s, i}, {0.1, 0.03}]
sol = EcoSim[E2n, T];
psol = PlotDynamics[sol, PlotPoints → 200]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spBTPE.pdf", sp]
Export["solBTP.pdf", psol]
```

```
Fixed points:
```

Out[∘]= {{s → 133.33333333333, i → 0}, {s → 117.95130747349, i → 1.5673723897566},
{s → 117.95130747349, i → 1.5673723897566}}

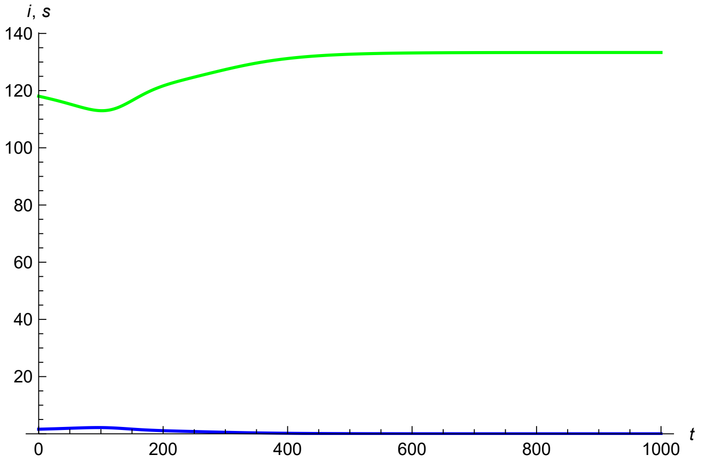
Eigenvalues of FP:

Out[∘]= {{-0.120000000000, -0.0133236764918}, {0, 0}, {0, 0}}

Time plot around E2

Out[∘]= {s → 118.051, i → 1.59737}

Out[∘]=

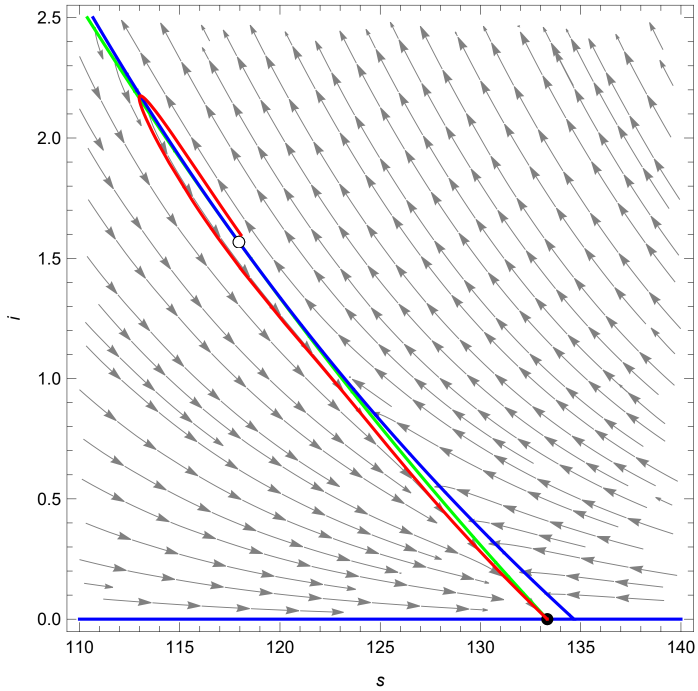


Finding the cycle using time slice

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[∘]= $Failed

Out[∘]=



Final time slice is

Out[∘]= FinalTime[$Failed]

The floquet exponents are

Out[∘]= EcoEigenvalues[$Failed]

```
Out[•]= spBTPE.pdf

Out[•]= solBTP.pdf

In[•]:=
```

Phase plot of the boundary R0=1 (at B2):

```
ClearParameters;
T = 2000;
xM = 140;
xm = 105;
yM = 3;
ym = 0;

Λ = 16 ;
δ = 2/10;
γ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω = Root[-14 273 010 000 000 + 5 043 092 010 000 #1 - 486 916 463 000 #1^2 + 8 858 269 519 #1^3 &, 2, 0];
α = 67/75
   Root[-14 273 010 000 000 + 5 043 092 010 000 #1 - 486 916 463 000 #1^2 + 8 858 269 519 #1^3 &, 2, 0];


Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]]]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB2E.pdf", sp]
Export["solB2.pdf", psol]
```

Out[∘]= 7.35966

Out[∘]= 6.57463

Fixed points:

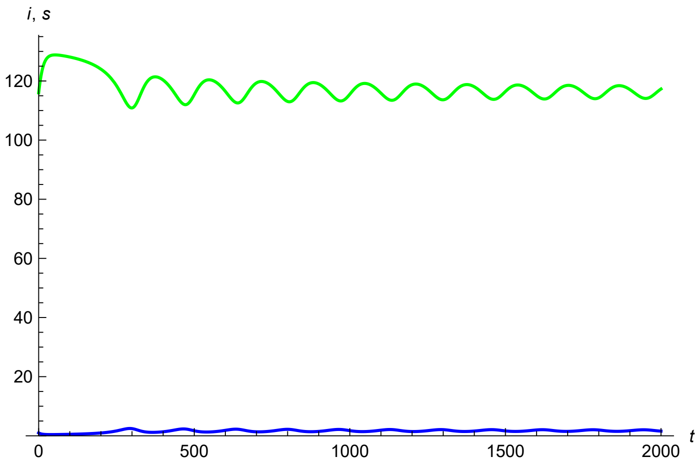
Out[∘]= {{s → 133.333, i → 0.}, {s → 116.198, i → 1.77275}, {s → 133.333, i → 0.}}

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {0. + 0.0391099 i, 0. - 0.0391099 i}, {-0.12, 0}}

Time plot around E2

Out[∘]=

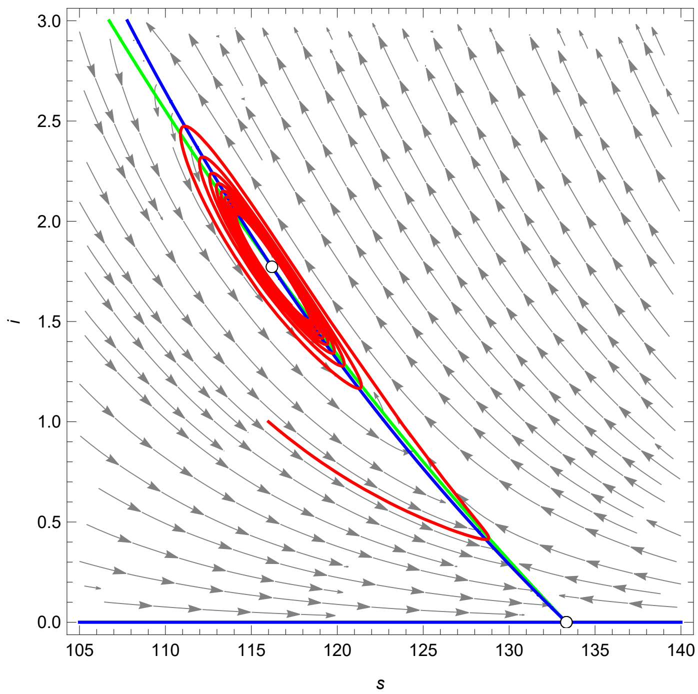


Finding the cycle using time slice

FindEcoCycle: Failed to converge to the requested accuracy within 100 iterations.

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 161.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 161.}} Output: scalar]}

Out[ ]= 

Final time slice is

Out[ ]= 160.805

The floquet exponents are

Out[ ]= {-0.0000365264, -0.000058307}

Out[ ]= spB2E.pdf

Out[ ]= solB2.pdf

Phase plot of the boundary R0=1 (at B1):

```
ClearParameters;
T = 800;
xM = 140;
xm = 65;
yM = 12;
ym = 0;

Λ = 16 ;
δ = 2/10;
ɣ = 12/100;
β = 1/100;
ξ = 1/1000;
μ = 12/100;
ω =
 Root[-14 273 010 000 000 + 5 043 092 010 000 #1 - 486 916 463 000 #1^2 + 8 858 269 519 #1^3 &, 1, 0];
α = 67/75 Root[
   -14 273 010 000 000 + 5 043 092 010 000 #1 - 486 916 463 000 #1^2 + 8 858 269 519 #1^3 &, 1, 0];

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 300],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq〚2, 1〛〚2〛], i → IntegerPart[eq〚2, 2〛〚2〛]}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spBPE.pdf", sp]
Export["solBP.pdf", psol]
```

Out[•]= 5.15735

Out[•]= 4.60724

Fixed points:

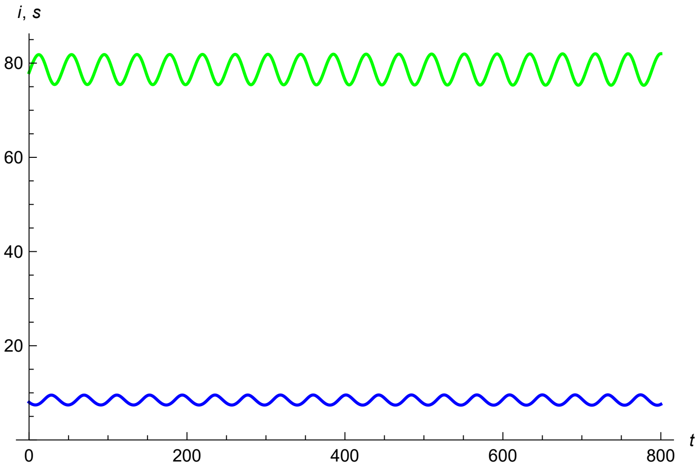
Out[∘]= {{s → 133.333, i → 0.}, {s → 78.5251, i → 8.44639}, {s → 133.333, i → 0.}}

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {0. + 0.152171 i, 0. - 0.152171 i}, {-0.12, 0}}

Time plot around E2

Out[∘]=



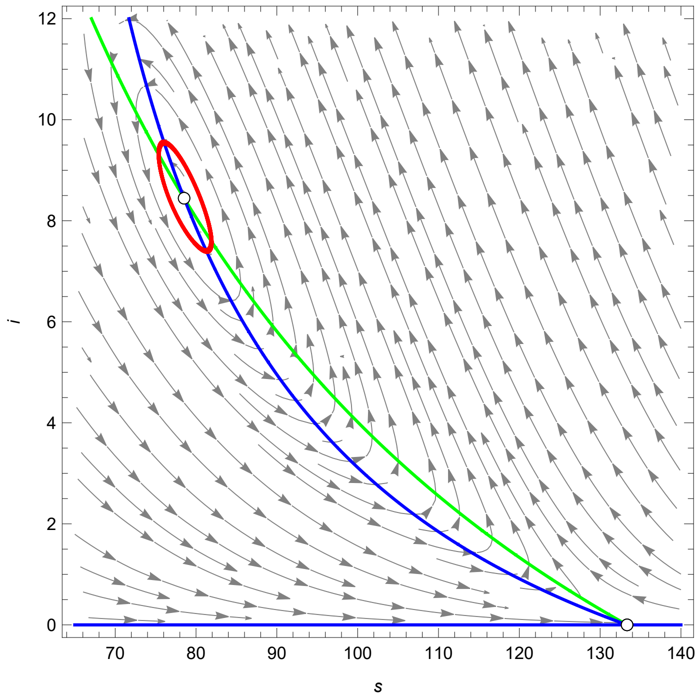
Finding the cycle using time slice

FindEcoCycle: Failed to converge to the requested accuracy within 100 iterations.

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 42.2}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 42.2}} Output: scalar]}

Out[∘]=



Final time slice is

Out[∘]= 42.2252

```
The floquet exponents are
```

Out[∘]= {0.00181877, -0.000205883}

Out[∘]= spBPE.pdf

Out[∘]= solBP.pdf

Phase plot of the boundary R0=1 (at H):

In[∘]:=
```
ClearParameters;
T = 180;
xM = 150;
xm = 65;
yM = 15;
ym = 0;
```

$\Lambda = 16$; $\delta = \frac{2}{10}$; $\gamma = \frac{12}{100}$; $\beta = \frac{1}{100}$; $\xi = \frac{1}{1000}$; $\mu = \frac{12}{100}$; $\omega = \frac{2010}{253}$;

$\alpha = \frac{8978}{1265}$; $\eta = \alpha / \omega$;

```
Print["TrE2="]
trE2 // N
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq⟦2, 1⟧⟦2⟧] + 2, i → IntegerPart[eq⟦2, 2⟧⟦2⟧] + 2}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spHPE.pdf", sp]
Export["solHP.pdf", psol]
```

```
TrE2=
```

Out[∘]= -0.12

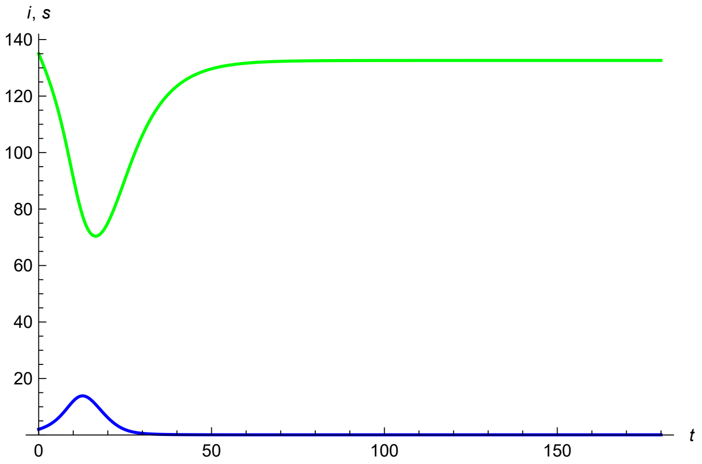
```
Fixed points:
```

Out[∘]= {{s → 133.333, i → 0.}, {s → 133.333, i → 0.}, {s → 133.333, i → 0.}}

```
Eigenvalues of FP:
```

Out[∘]= {{-0.12, 0}, {-0.12, 0}, {-0.12, 0}}

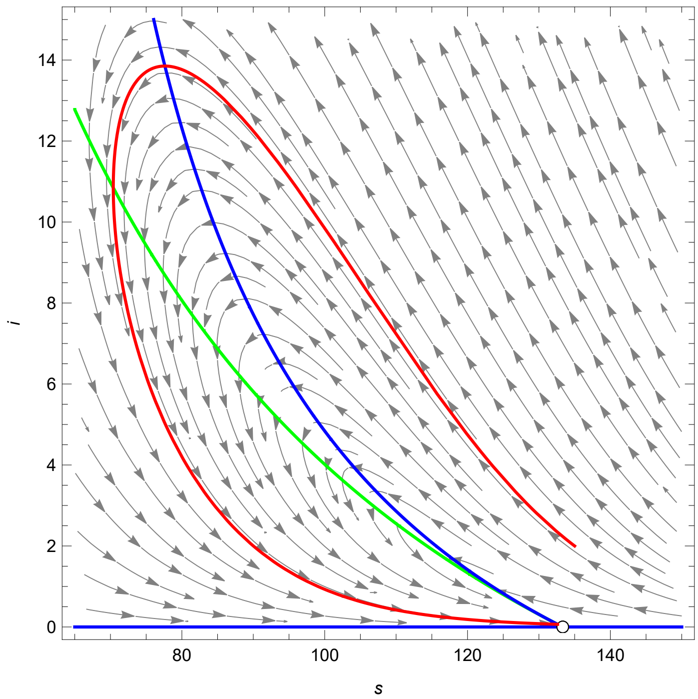
```
Time plot around E2
```

Out[ ]=


Finding the cycle using time slice

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[ ]= $Failed

Out[ ]=


Final time slice is

Out[ ]= FinalTime[$Failed]

The floquet exponents are

Out[ ]= EcoEigenvalues[$Failed]

Out[ ]= spHPE.pdf

Out[ ]= solHP.pdf

Phase plot of the boundary R0=1 (at B3):

```
ClearParameters;
T = 300;
xM = 150;
xm = 120;
yM = 3 / 2;
ym = 0;

Λ = 16 ; δ = 2/10; γ = 12/100; β = 1/100; ξ = 1/1000; μ = 12/100; w = 42.4504362201850;
α = 37.9223896900319;

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]]]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → IntegerPart[eq[[2, 1]][[2]]], i → IntegerPart[eq[[2, 2]][[2]] + 1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spB3E.pdf", sp]
Export["solB3.pdf", psol]
```

Fixed points:

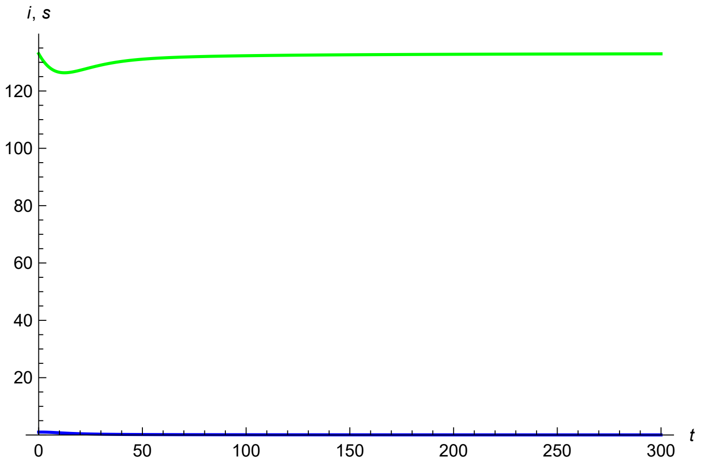
Out[∘]= $\{\{s \to 133.333,\ i \to 0.\},\ \{s \to 133.333,\ i \to 1.24666 \times 10^{-14}\}\}$

Eigenvalues of FP:

Out[∘]= {{-0.12, 0}, {-0.12, 0}}

Time plot around E2

Out[∘]=

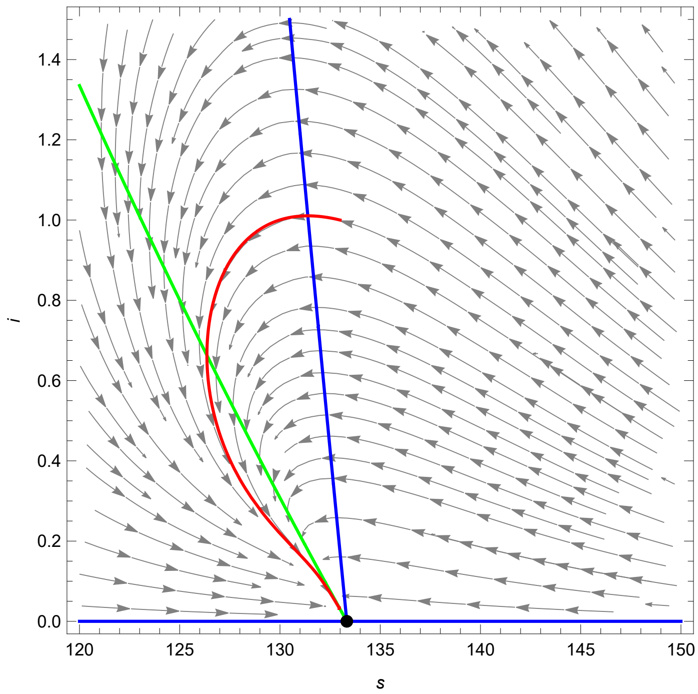



```
Finding the cycle using time slice
```

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[∘]= `$Failed`

Out[∘]=




```
Final time slice is
```

Out[∘]= `FinalTime[$Failed]`

```
The floquet exponents are
```

Out[∘]= `EcoEigenvalues[$Failed]`

Out[∘]= `spB3E.pdf`

Out[∘]= `solB3.pdf`

Phase plot of the boundary R0=1 between Regions II and IV (After HP):

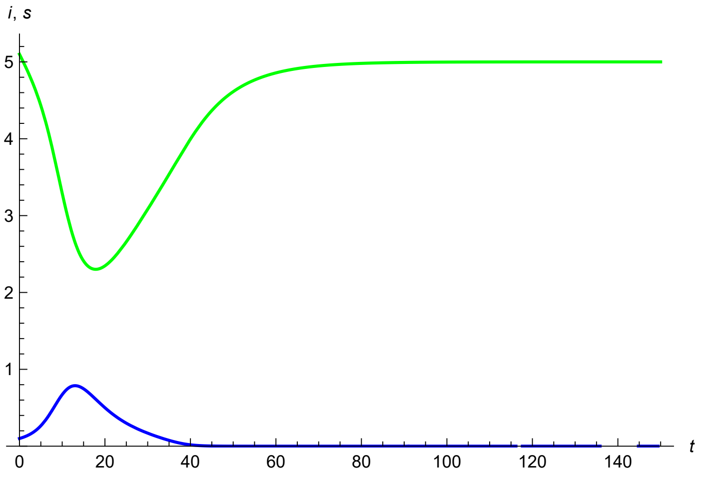
Gupta figures:

Phase plot of Fig 1 a) in Gupta:

```
ClearParameters;
T = 150;
xM = 5.1;
xm = 2;
yM = 1;
ym = 0;
Λ = 1/2; δ = 1/5; γ = 1/10; β = 1/5; ξ = 7/100; μ = 1/10;

ω = 10/99; α = 1/11;

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of E2,E0,E1:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-9]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around DFE"]
sol = EcoSim[{s → 5.1, i → 0.1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spGaE.pdf", sp]
Export["solGa.pdf", psol]
```

```
Fixed points:

Out[•]= {{s → 5., i → 0.}, {s → 3.35294, i → 0.249912}, {s → 3.87623, i → 0.146443}}

Eigenvalues of E2,E0,E1:

Out[•]= {{-0.3, -0.1}, {0.012018 + 0.0764974 i, 0.012018 - 0.0764974 i}, {0.121831, -0.0411935}}

Time plot around DFE
```

Out[ ]=

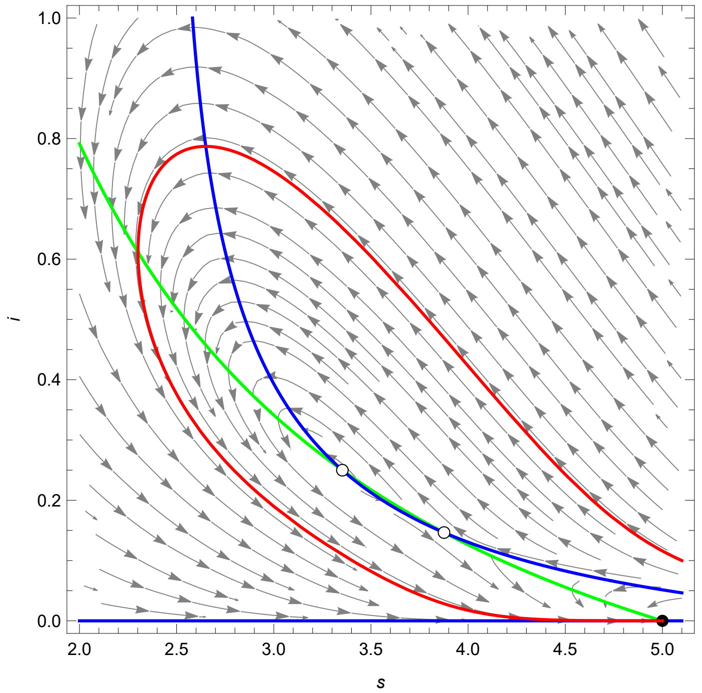


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 454.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 454.}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 453.519

The floquet exponents are

Out[ ]= {−∞, −∞}

Out[ ]= spGaE.pdf

Out[ ]= solGa.pdf

Phase plot of Fig 1 b) in Gupta:

```
T = 350;
xM = 8.1;
xm = 2;
yM = .8;
```

$\Lambda = \frac{1}{2}$; $\delta = \frac{1}{5}$; $\gamma = \frac{1}{10}$; $\beta = \frac{1}{5}$; $\xi = \frac{7}{100}$; $\mu = \frac{1}{10}$;

$\omega = \frac{10\,000}{103\,387}$; $\alpha = \frac{9000}{103\,387}$;

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of E2,E0,E1:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-12]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → 5, i → 0.1}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spGbE.pdf", sp]
Export["solGb.pdf", psol]
```

Fixed points:

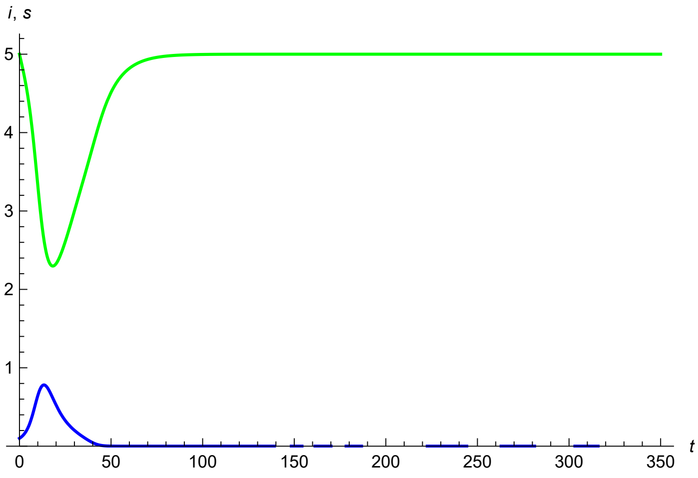
Out[•]= {{s → 5., i → 0.}, {s → 3.1916, i → 0.289038}, {s → 4.03079, i → 0.121247}}

Eigenvalues of E2,E0,E1:

Out[•]= $\{\{-0.3, -0.1\}, \{5.58554\times10^{-6} + 0.101011\,i, 5.58554\times10^{-6} - 0.101011\,i\}, \{0.144435, -0.0530554\}\}$

Time plot around E2

Out[]=

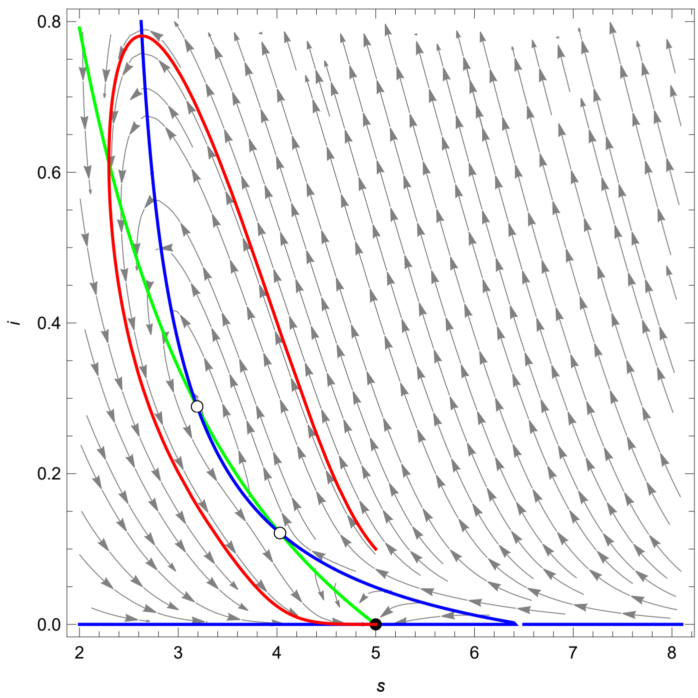


Finding the cycle using time slice

FindEcoCycle: Found no maxima in warmup #3, probably not periodic solution.

Out[]= $Failed

Out[]=



Final time slice is

Out[]= FinalTime[$Failed]

The floquet exponents are

Out[]= EcoEigenvalues[$Failed]

Out[]= spGbE.pdf

Out[]= solGb.pdf

Phase plot of Fig 1 c) in Gupta:

```
T = 400;
xm = 2.5;
yM = 0.6;
```

$\Lambda = \frac{1}{2};\ \delta = \frac{1}{5};\ \gamma = \frac{1}{10};\ \beta = \frac{1}{5};\ \xi = \frac{7}{100};\ \mu = \frac{1}{10};$

$\omega = \frac{5}{54};\ \alpha = \frac{1}{12};$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-5]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around DFE"]
sol = EcoSim[{s → 3, i → 0.4}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spGcE.pdf", sp]
Export["solGc.pdf", psol]
```

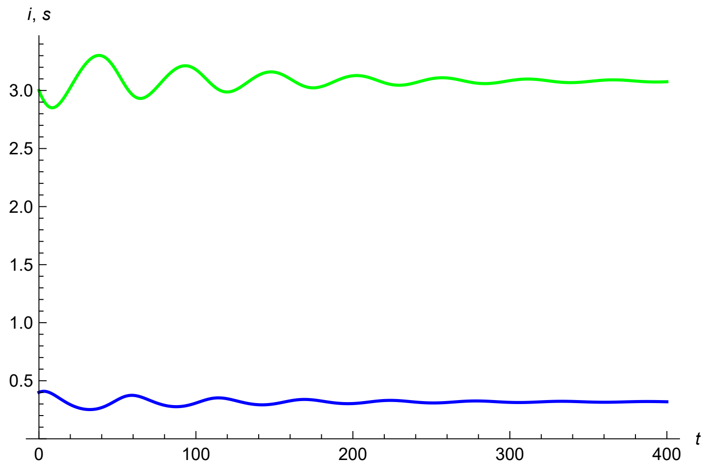
Fixed points:

Out[•]= {{s → 5., i → 0.}, {s → 3.08116, i → 0.318321}, {s → 4.13484, i → 0.105391}}

Eigenvalues of FP:

Out[•]= {{-0.3, -0.1},
{-0.00915663 + 0.115223 i, -0.00915663 - 0.115223 i}, {0.156612, -0.0594875}}

Time plot around DFE

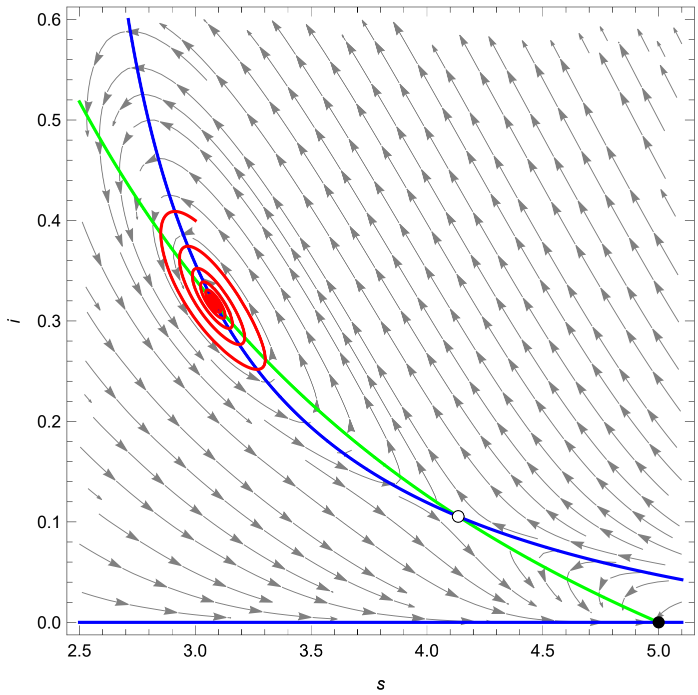
Out[•]=


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: $\{\{0., 1.00\times10^3\}\}$ Output: scalar],

s → InterpolatingFunction[Domain: $\{\{0., 1.00\times10^3\}\}$ Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 1000.

The floquet exponents are

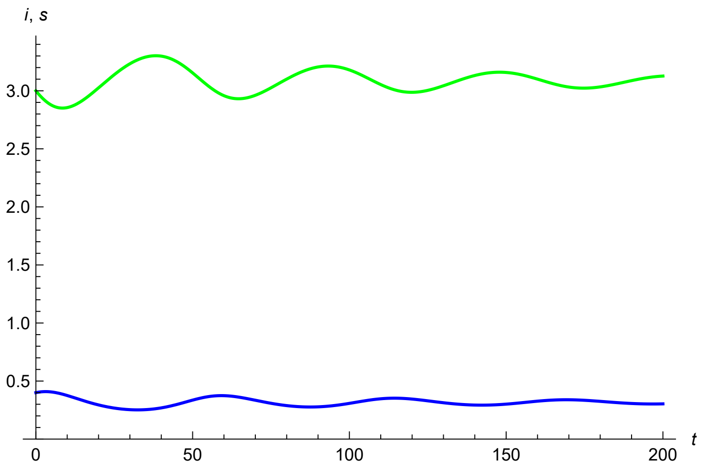
Out[ ]= {-0.00915701 + 0.00212584 i, -0.00915701 - 0.00212584 i}

Out[ ]= spGcE.pdf

Out[ ]= solGc.pdf

Time plot around DFE

Out[ ]=

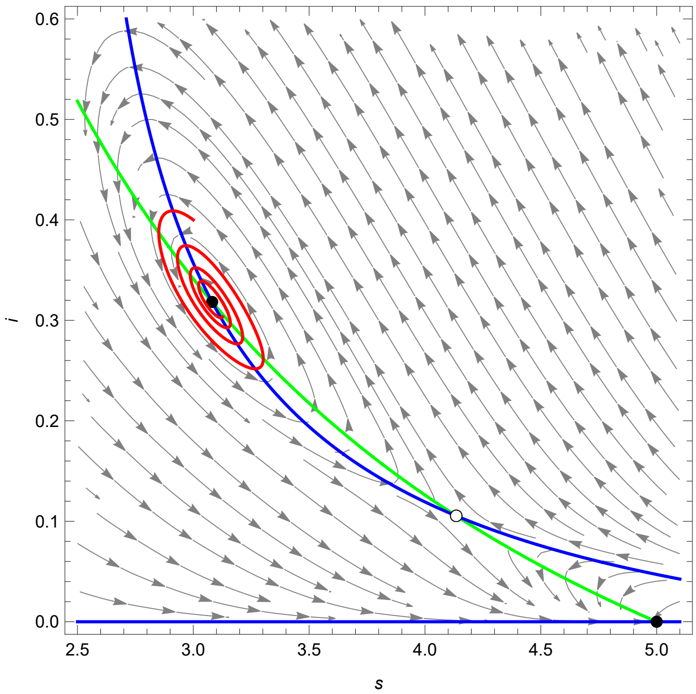


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 19.1}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 19.1}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 19.1064

The floquet exponents are

Out[ ]= {-0.00915663 + 0.115223 i, -0.00915663 - 0.115223 i}

Out[ ]= spGcE.pdf

Out[ ]= solGc.pdf

Phase plot of Fig 1 d) in Gupta:

```
T = 180;
xm = 2.5;
yM = 0.6;
ω = 1/11; α = 9/110;

Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of FP:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-5]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → 3, i → 0.4}, T];
psol
Print["Finding the cycle using time slice"]
lc
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spGdE.pdf", sp]
Export["solGd.pdf", psol]
```

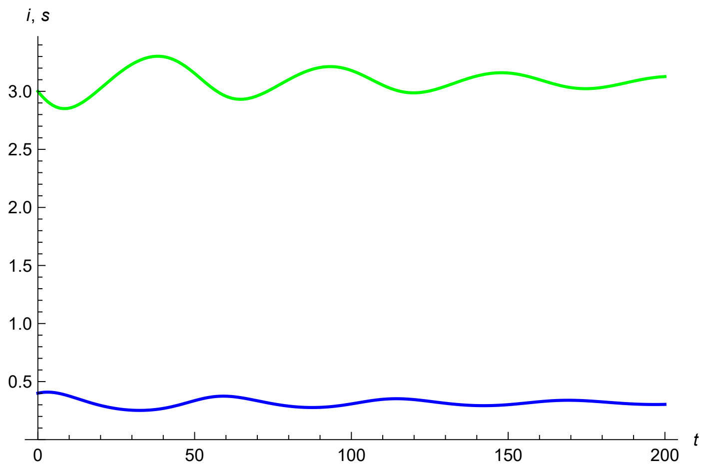
Fixed points:

Out[∘]= {{s → 5., i → 0.}, {s → 3.04252, i → 0.329097}, {s → 4.17091, i → 0.100086}}

Eigenvalues of FP:

Out[∘]= {{-0.3, -0.1},
{-0.0125465 + 0.119857 ⅈ, -0.0125465 - 0.119857 ⅈ}, {0.160369, -0.0615297}}

Time plot around E2

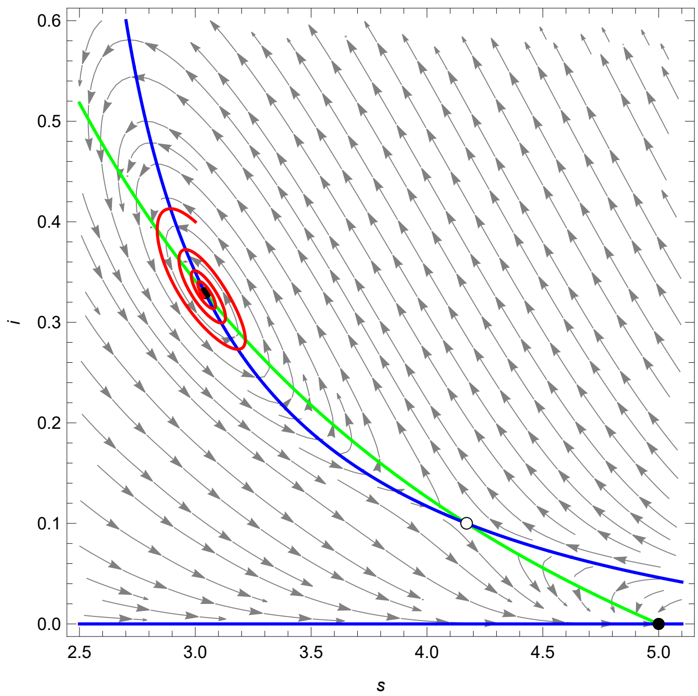
Out[∘]=


Finding the cycle using time slice

Out[ ]= {i → InterpolatingFunction[Domain: {{0., 19.1}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 19.1}} Output: scalar]}

Out[ ]=



Final time slice is

Out[ ]= 19.1064

The floquet exponents are

Out[ ]= {-0.00851491 + 0.114365 i, -0.00851491 - 0.114365 i}

Out[ ]= spGdE.pdf

Out[ ]= solGd.pdf

Phase plot of Fig 3a) in Gupta:

```
T = 250;
xM = 8.1;
xm = 0;
yM = 1;
```

$$\omega = \frac{100\,000\,000}{1\,063\,265\,757};\ \alpha = \frac{30\,000\,000}{354\,421\,919};$$

```
Print["Fixed points:"]
eq = SelectValid[equi] // N
Print["Eigenvalues of E2,E0,E1:"]
Chop[Evaluate[EcoEigenvalues[eq]], 10^-12]
pp = Show[PlotEcoPhasePlane[{s, xm, xM}, {i, ym, yM}, PlotPoints → 200],
   RuleListPlot[eq, PlotMarkers → EcoStableQ[eq]]];
Print["Time plot around E2"]
sol = EcoSim[{s → 4, i → 0.3}, T];
psol = PlotDynamics[sol]
Print["Finding the cycle using time slice"]
lc = FindEcoCycle[FinalSlice[sol]]
sp = Show[pp, RuleListPlot[sol, PlotStyle → Red]]
Print["Final time slice is"]
FinalTime[lc]
Print["The floquet exponents are"]
Chop[Evaluate[EcoEigenvalues[lc]]]
Export["spG3aE.pdf", sp]
Export["solG3a.pdf", psol]
```

```
Fixed points:
```

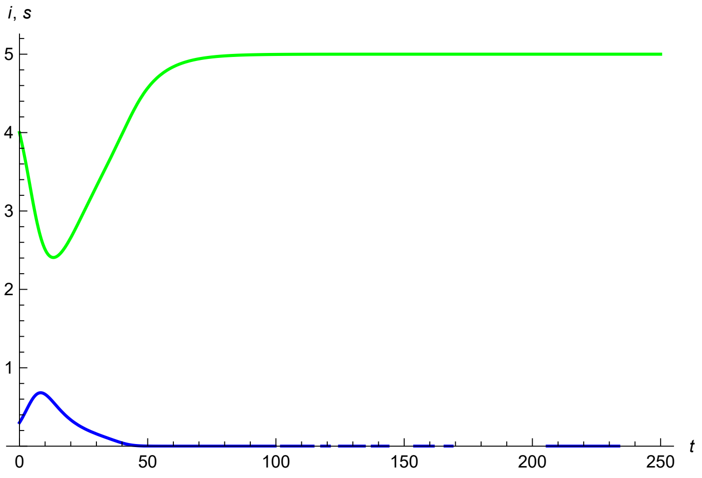
Out[ ]= `{{s → 5., i → 0.}, {s → 3.11718, i → 0.308529}, {s → 4.10105, i → 0.110447}}`

```
Eigenvalues of E2,E0,E1:
```

Out[ ]= `{{-0.3, -0.1},`
`{-0.00608274 + 0.110756 i, -0.00608274 - 0.110756 i}, {0.152885, -0.0574955}}`

```
Time plot around E2
```

Out[∘]=

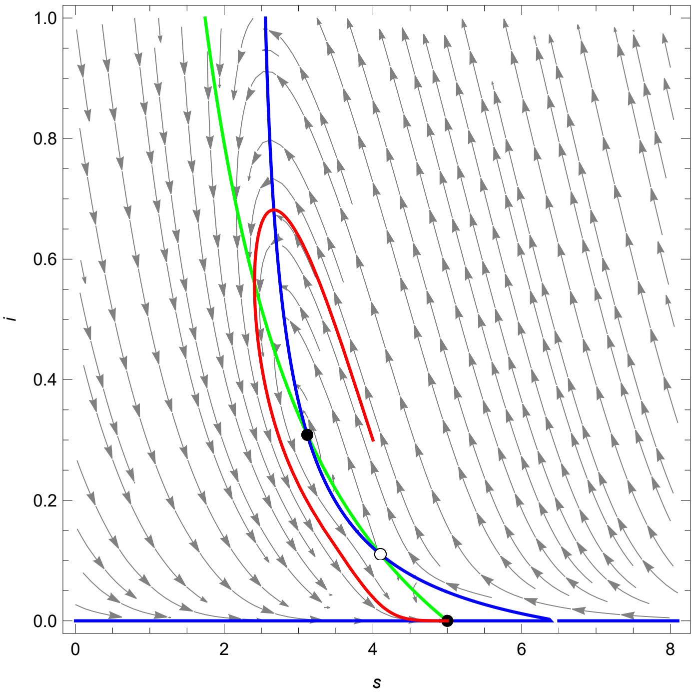



```
Finding the cycle using time slice
```

Out[∘]= {i → InterpolatingFunction[Domain: {{0., 649.}} Output: scalar],

s → InterpolatingFunction[Domain: {{0., 649.}} Output: scalar]}

Out[∘]=




```
Final time slice is
```

Out[∘]= 648.762

```
The floquet exponents are
```

Out[∘]= {−∞, −∞}

Out[∘]= spG3aE.pdf

Out[∘]= solG3a.pdf