EnvZ-OmpR (7,6,5,1)

by F. Avram

ABSTRACT (original article):
Keywords:

CITATION (original article):

In[26]:=
```
Format[x[1]]:=Subscript[x,1];Format[x[2]]:=Subscript[x,2];
Format[x[3]]:=Subscript[x,3];Format[x[4]]:=Subscript[x,4];
Format[x[5]]:=Subscript[x,5];Format[x[6]]:=Subscript[x,6];Format[x[7]]:=Subscript[x,7];
Format[x[8]]:=Subscript[x,8];
Format[k1]:=Subscript[k,1];Format[k2]:=Subscript[k,2];
Format[k3]:=Subscript[k,3];Format[k4]:=Subscript[k,4];Format[k5]:=Subscript[k,5];
Format[k6]:=Subscript[k,6];
Format[k7]:=Subscript[k,7];Format[k8]:=Subscript[k,8];Format[k9]:=Subscript[k,9];
Format[k10]:=Subscript[k,10];
Format[k11]:=Subscript[k,11];Format[k14]:=Subscript[k,14];Format[k12]:=Subscript[k,12];
Format[k13]:=Subscript[k,13];
Format[Xt]:=Subscript[x,2];Format[X1]:=Subscript[x,1];Format[Y]:=Subscript[x,3];
Format[Xp]:=Subscript[y,1];Format[XpY]:=Subscript[x,4];Format[Yp]:=Subscript[y,2];
Format[XtYp]:=Subscript[x,5];
```

```
ClearAll["Global‘*"]
SetDirectory[NotebookDirectory[]];SetOptions[$FrontEndSession, NotebookAutoSave → True];
NotebookSave[];AppendTo[$Path, FileNameJoin[{$HomeDirectory, "Dropbox", "Codes"}]];
<<EpidCRN`;(*Names["EpidCRN`*"]*)Needs["ReactionKinetics`"];

cDFE={X1→0,Xt→0,Y→0,XpY→0,XtYp→0};
 RNJ={"X"→"Xt","Xt" → "Xp",
"Xp" + "Y" →"XpY", "XpY" →"X" + "Yp" ,
"Xt" + "Yp" →"XtYp",  "XtYp"→"Xt" + "Y"};
var={X1,Xt, Xp,Y,XpY, Yp , XtYp};
RND=ReactionsData[RNJ];
Γ= RND["ɣ"]//Normal;
expo= RND["α"]//Normal//Transpose;con=cons[Γ];
{comp,r,nR,spec,nS,vol,vars,defi}=
RND["complexes","reactionsteps","R","species","M","volpertgraph","variables","deficiency"];
Print["EnvZ Cap",Γ//MatrixForm, " has rank ",MatrixRank[Γ],
" and deficiency ", RND["deficiency"], " and ",con//Length," cons ",conx=con.var]


Print[RND["variables"]//Length," variables=",RND["variables"],"=",var,{var//Length}]
rvM=expM[var,expo];

tk=Array[Symbol["k" <> ToString[#]] &, nR];
cp=Thread[tk>0];Rv=tk*rvM;

cv=Thread[var≥0];ct=Join[cp,cv];
fl=cons[Γ//Transpose]//Transpose;
Print["EnvZJ  has  fluxes",fl//MatrixForm]
(*ShowFHJGraph[RNJ,Rv,DirectedEdges→True,VertexLabeling→True,ImageSize→330]*)
RHS=Γ.Rv//FullSimplify;
Print["EnvZ Cap RHS:",RHS//MatrixForm," has Rv= ",Rv//Transpose//MatrixForm]
fp=Solve[Thread[RHS==0],var]//Factor;
Print[fp//Length," fixed points  are",fp]
(*rap={x_1\to \frac{k_2 x_2}{k_1},
x_3\to \frac{k_2 x_2}{k_3 y_1},x_4\to \frac{k_2 x_2}{k_4},y_2\to \frac{k_2}{k_5},
x_5\to \frac{k_2 x_2}{k_6}}*)
Print["Check fp",(RHS//.fp)//FullSimplify]
Print["Y from cons"]
cXY=Solve[Thread[conx==xT,yT}],{Xp,Yp}]//Flatten
```

EnvZ Cap $\begin{pmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$ has rank 5 and deficiency

δ=N-L-S=9-3-5=1 and 2 cons $\{x_1 + y_1 + x_4 + x_2 + x_5,\ x_4 + x_5 + x_3 + y_2\}$

7 variables=$\{c_X, c_{Xt}, c_{Xp}, c_Y, c_{XpY}, c_{Yp}, c_{XtYp}\} = \{x_1, x_2, y_1, x_3, x_4, y_2, x_5\}\{7\}$

EnvZJ has fluxes $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

EnvZ Cap RHS: $\begin{pmatrix} -k_1 x_1 + k_4 x_4 \\ k_1 x_1 + k_6 x_5 - x_2 (k_2 + k_5 y_2) \\ k_2 x_2 - k_3 y_1 x_3 \\ k_6 x_5 - k_3 y_1 x_3 \\ -k_4 x_4 + k_3 y_1 x_3 \\ k_4 x_4 - k_5 x_2 y_2 \\ -k_6 x_5 + k_5 x_2 y_2 \end{pmatrix}$ has Rv= $\begin{pmatrix} k_1 x_1 \\ k_2 x_2 \\ k_3 y_1 x_3 \\ k_4 x_4 \\ k_5 x_2 y_2 \\ k_6 x_5 \end{pmatrix}$

Solve: Equations may not give solutions for all "solve" variables.

3 fixed points are $\{\{x_1 \to 0, x_2 \to 0, y_1 \to 0, x_4 \to 0, x_5 \to 0\},$

$\{x_1 \to 0, x_2 \to 0, x_3 \to 0, x_4 \to 0, x_5 \to 0\}, \left\{x_1 \to \frac{k_2 x_2}{k_1}, x_3 \to \frac{k_2 x_2}{k_3 y_1}, x_4 \to \frac{k_2 x_2}{k_4}, y_2 \to \frac{k_2}{k_5}, x_5 \to \frac{k_2 x_2}{k_6}\right\}\}$

Check fp{{0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0}}

Out[89]=

$\{y_1 \to -x_1 - x_4 + xT - x_2 - x_5,\ y_2 \to -x_4 - x_5 - x_3 + yT\}$

In[216]:=

```
(*NGM  cell*)
mod={RHS,var,tk};
inf={1,2,4,5,7};
jI=Grad[RHS[[inf]],var[[inf]]];
Print["infection Jac=",jI//MatrixForm," has ch pol which does not factor"]
Collect[CharacteristicPolynomial[jI,s]//Factor,s]
ng=NGM[mod,inf];F=ng[[4]];V=ng[[5]];
Print["F,V are",F//MatrixForm,V//MatrixForm]
K=ng[[6]];
Print["NGM=",K//MatrixForm," has a quadr. non triv. factor"]
ch=-Collect[((CharacteristicPolynomial[K,s]//Factor)/(s^3)),s]
px=Numerator[ch]/.s->1+x//Factor;
Print["the subst. s->1+x yields the coefs"]
cox=FullSimplify/@CoefficientList[px,x]
Print["DFE stable when"]
so1=Assuming[ct,Reduce[cox[[1]]>0,Yp]//Refine]//Factor
```

infection Jac= $\begin{pmatrix} -k_1 & 0 & 0 & k_4 & 0 \\ k_1 & -k_2 - k_5 y_2 & 0 & 0 & k_6 \\ 0 & 0 & -k_3 y_1 & 0 & k_6 \\ 0 & 0 & k_3 y_1 & -k_4 & 0 \\ 0 & k_5 y_2 & 0 & 0 & -k_6 \end{pmatrix}$ has ch pol which does not factor

Out[220]=

$$-s^5 - k_1 k_2 k_3 k_4 k_6 y_1 + k_1 k_3 k_4 k_5 k_6 y_1 y_2 + s^4 (-k_1 - k_2 - k_4 - k_6 - k_3 y_1 - k_5 y_2) + s^3 (-k_1 k_2 - k_1 k_4 - k_2 k_4 - k_1 k_6 - k_2 k_6 - k_4 k_6 - k_1 k_3 y_1 - k_2 k_3 y_1 - k_3 k_4 y_1 - k_3 k_6 y_1 - k_1 k_5 y_2 - k_4 k_5 y_2 - k_3 k_5 y_1 y_2) + s^2 (-k_1 k_2 k_4 - k_1 k_2 k_6 - k_1 k_4 k_6 - k_2 k_4 k_6 - k_1 k_2 k_3 y_1 - k_1 k_3 k_4 y_1 - k_2 k_3 k_4 y_1 - k_1 k_3 k_6 y_1 - k_2 k_3 k_6 y_1 - k_3 k_4 k_6 y_1 - k_1 k_4 k_5 y_2 - k_1 k_3 k_5 y_1 y_2 - k_3 k_4 k_5 y_1 y_2) + s (-k_1 k_2 k_4 k_6 - k_1 k_2 k_3 k_4 y_1 - k_1 k_2 k_3 k_6 y_1 - k_1 k_3 k_4 k_6 y_1 - k_2 k_3 k_4 k_6 y_1 - k_1 k_3 k_4 k_5 y_1 y_2)$$

F,V are $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 y_1 & 0 & 0 \\ 0 & k_5 y_2 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -k_1 & 0 & 0 & k_4 & 0 \\ k_1 & -k_2 - k_5 y_2 & 0 & 0 & k_6 \\ 0 & 0 & -k_3 y_1 & 0 & k_6 \\ 0 & 0 & 0 & -k_4 & 0 \\ 0 & 0 & 0 & 0 & -k_6 \end{pmatrix}$

NGM= $\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ \frac{k_5 y_2}{k_2 + k_5 y_2} & \frac{k_5 y_2}{k_2 + k_5 y_2} & 0 & \frac{k_5 y_2}{k_2 + k_5 y_2} & \frac{k_5 y_2}{k_2 + k_5 y_2} \end{pmatrix}$ has a quadr. non triv. factor

Out[225]=

$$s^2 - \frac{k_5 y_2}{k_2 + k_5 y_2} - \frac{k_5 s y_2}{k_2 + k_5 y_2}$$

the subst. s->1+x yields the coefs

Out[228]=

$$\left\{-1 + \frac{2 k_2}{k_2 + k_5 y_2}, 1 + \frac{k_2}{k_2 + k_5 y_2}, 1\right\}$$

DFE stable when

Out[230]=

$$y_2 < \frac{k_2}{k_5}$$

In[231]:=

```
(*UPH Jacobian at EE; there is no Hopf
rap=fp[[3]];
jaE=Grad[RHS,var]//.rap;

Print["jaE at EE is",jaE//MatrixForm]
ch=-(CharacteristicPolynomial[jaE,λ]//Factor)/(λ^2)//Factor
 co=CoefficientList[ch,λ];
co//Length*)
```