Comparative Analysis between DCT and DWT Technique of Image Compression

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Image Compression Overview

Purpose of Image Compression

- Reduce storage space for images/videos
- Improve storage efficiency and transmission performance
- Maintain image quality and information while reducing size

Research Focus

- Simulation of two compression techniques:
  - Discrete Cosine Transform (DCT)
  - Discrete Wavelet Transform (DWT)
- Comparison of quality parameters
- Testing on various images

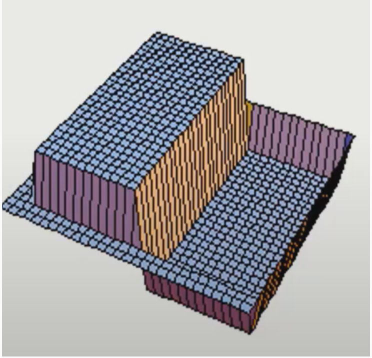
Expected Outcome
Comprehensive analysis of DCT vs DWT performance in image compression

Image Compression Overview

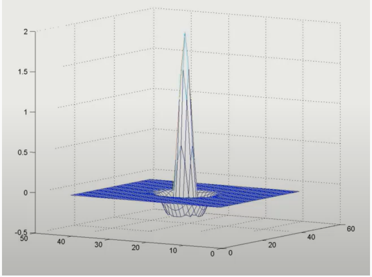
- Image compression is a technique used to reduce the storage space required for images and videos.
- It helps improve both storage efficiency and transmission performance.
- The goal is not just to reduce size, but also to preserve the quality and information of the image.
- This study simulates two image compression techniques:
    - Discrete Cosine Transform (DCT)
    - Discrete Wavelet Transform (DWT)
- The techniques are applied to various images.
- Simulation results are compared based on different image quality parameters.

Wavelets for 2-D Signals

- Images are obviously two dimensional data [i.e., $f(x, y)$].
- To transform images, we need two dimensional wavelets [i.e., $\psi(x, y)$].



2D Haar Wavelet



2D Max Hat Wavelet

Wavelets for 2D Signals

- We can apply one dimensional transform to the rows and the columns image successively as separable two dimensional transform
- We have used this approach as it has low computational complexity

Encoding System

- Input: Original image
- Output: Compressed image

Discrete Cosine Transform (DCT)

- The **Discrete Cosine Transform (DCT)** is a method used to convert data from the spatial domain to the frequency domain.
- It breaks down an image into its basic building blocks of different cosine waves.
- The beauty of DCT lies in its energy compaction property — most of the meaningful information in an image is packed into the low-frequency components.
- Because of this, DCT is widely used in **image and video compression**.
- DCT is at the **core of the JPEG compression standard**.

Discrete Cosine Transform (DCT)

The DCT compression is an orthogonal transform where the pixels of the image are converted into sets of spatial frequencies.

The compressed matrix of the image is obtained by

$$D(i,j) = \frac{2}{\sqrt{nm}} C(i)C(j) \sum_{x=0}^{n-1} \sum_{y=0}^{m-1} I(x,y) \cos\left[\frac{(2x+1)i\pi}{2n}\right] \cos\left[\frac{(2y+1)j\pi}{2m}\right] \tag{1}$$

Where m and n are the number of rows and columns of the plain image I, respectively, $i, x = 0, 1, \ldots, n-1$, and $j, y = 0, 1, \ldots, m-1$, and

Discrete Cosine Transform (DCT)

$$C(i), C(j) = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } i, j = 0 \\ 1 & \text{otherwise} \end{cases} \tag{2}$$

The inverse DCT (IDCT) can be given by:

$$I(x,y) = \frac{2}{\sqrt{nm}} \sum_{k=0}^{n-1} \sum_{v=0}^{m-1} C(i)C(j)D(i,j) \cos\left[\frac{(2x+1)i\pi}{2n}\right] \cos\left[\frac{(2y+1)j\pi}{2m}\right] \tag{3}$$

- **Lossy Compression**:
  - Output image is not an exact copy but perceptually similar
  - Irreversible information loss occurs

DCT Simulation Results

For Simulation, we apply DCT technique on two different images by choosing 8x8 block size. These two original images and output images are shown below.

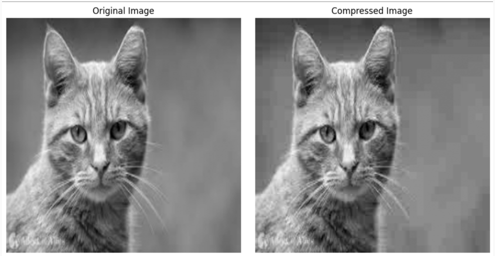


Figure 1: DCT Simulation Result using 8×8 block size

DCT simulation results

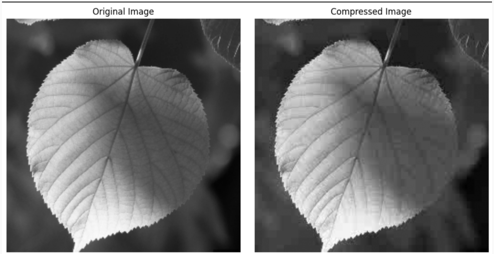


Figure 2: DCT Simulation Result using 8×8 block size

We can see the reconstructed image is not exact as the original image. But all are identical to their original image. DCT has block artifacts. If we choose a small size of block then the block artifacts is minimized. By using 8x8 block size and applying quantization, we minimized the each pixel value 0 to 32 from 0 to 256. So one pixel needs 5 bits to represent its value on behalf of 8 bits.

Discrete Wavelet Transform (DWT)

Key Concept
The Discrete Wavelet Transform (DWT) is a mathematical tool used to decompose a signal (e.g., an image) into different frequency components with spatial resolution. DWT represents image pixels using functions localized in both:

- Space (or Time) domain
- Frequency domain

2D-DWT Process

1. Apply 1D-DWT to image rows → Low (L) and High (H) frequency bands
2. Apply 1D-DWT to columns of each → 4 sub-bands:
    - **cA** – Approximation (Low-Low)
    - **cH** – Horizontal details (High-Low)
    - **cV** – Vertical details (Low-High)
    - **cD** – Diagonal details (High-High)

Mathematical Representation

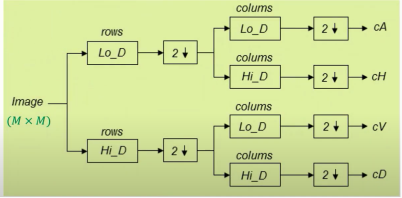
$$\psi_{jm}(t) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{t - 2^j n}{2^j}\right)$$

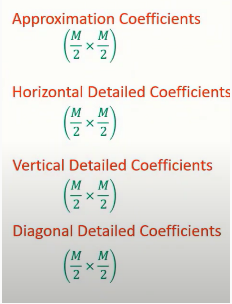
where:

- $j, n \in \mathbb{Z}$
- ψ: Wavelet function
- Haar wavelets commonly used

Filter Bank Theory

The wavelet decomposition of an image based on the multi-resolution theory can be done using digital filters.





DWT Encoding Process

1. **Row Filtering:** Apply Low-pass (Lo_D) and High-pass (Hi_D) filters to each row.
2. **Row Downsampling:** Downsample each filtered row by 2 $\rightarrow$ produces l_1 and h_1.
3. **Column Filtering:** Apply Lo_D and Hi_D to columns of l_1 and h_1.
4. **Column Downsampling:** Downsample filtered columns by 2.
5. **Form Subbands:**
   - **cA** = Lo_D (row) $\rightarrow$ Lo_D (col)
   - **cH** = Hi_D (row) $\rightarrow$ Lo_D (col)
   - **cV** = Lo_D (row) $\rightarrow$ Hi_D (col)
   - **cD** = Hi_D (row) $\rightarrow$ Hi_D (col)
6. **Result:** Four subbands = compressed image representation

Results

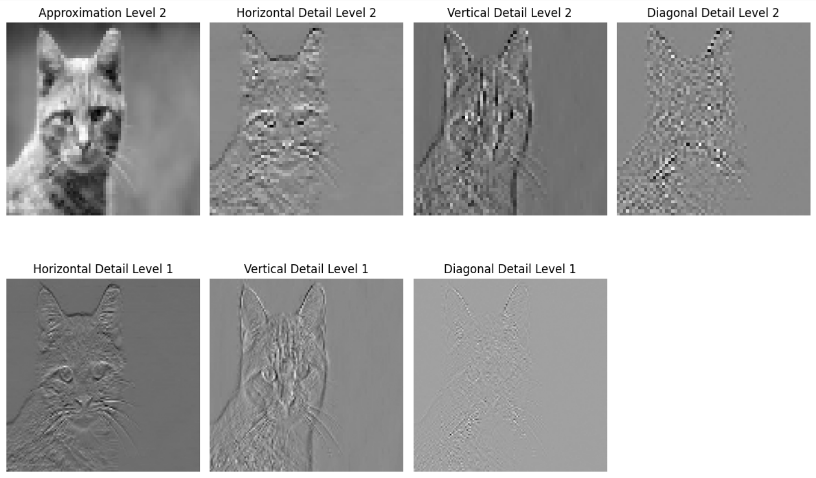


Figure 3: DWT Simulation Result using 8×8 block size

Results

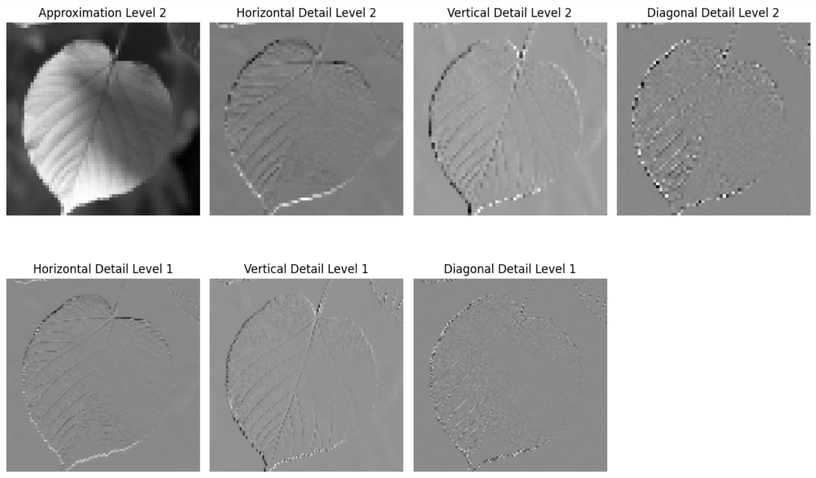


Figure 4: DWT Simulation Result using 8×8 block size

Comparison between DCT and DWT

Both techniques are **frequency-based** techniques, not spatial-based.
Both techniques have their own advantages and disadvantages:

- **DWT** provides better compression ratio without significant information loss, but requires more processing power
- **DCT** requires less processing power but suffers from block artifacts (information loss)

Research Goal
Analyze both techniques and compare their compression results

Mean Square Error (MSE)

Definition
Mean Square Error quantifies the difference between the original and reconstructed image:

$$\text{MSE} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} [x(i,j) - x'(i,j)]^2$$

where:

- M, N = image dimensions
- $x(i,j)$ = original image pixel value
- $x'(i,j)$ = reconstructed image pixel value

Interpretation

- Higher MSE $\rightarrow$ More information lost
- MSE $= 0 \rightarrow$ Perfect reconstruction
- Used to compare compression techniques

Peak Signal-to-Noise Ratio (PSNR)

PSNR is a widely used metric in image compression to evaluate the quality of the decompressed image.

- Higher PSNR indicates better image quality.
- It is computed using the Mean Squared Error (MSE) between the original and decompressed image.

Formula:

$$PSNR = 10 \log_{10} \left(\frac{255^2}{\mathrm{MSE}} \right)$$

Interpretation: Lower MSE $\Rightarrow$ Higher PSNR $\Rightarrow$ Less distortion.

MSE and PSNR

Image1	MSE	PSNR(dB)
DCT	25.24	33.87
DWT	54.35	30.54

Image2	MSE	PSNR(dB)
DCT	23.13	33.63
DWT	41.60	31.08

DCT vs. DWT Performance Analysis

Key Observations

- **DCT outperforms DWT**:
  - Lower MSE (better accuracy) for both images
  - Higher PSNR (better quality) in all cases
  - Ex: Image1: DCT (25.24/33.87) vs DWT (54.35/30.54)
- **Performance varies by image**:
  - DWT shows better relative performance on Image2
  - Image2 PSNR gap: 2.55 dB vs Image1's 3.33 dB

Conclusion
DCT demonstrates superior image quality preservation, though DWT's effectiveness depends on image characteristics.

End