Q.1(A)

P(X=2)
P(X=2, Y=1) = 0.1
P(X=2, Y=2) = 0.2
P(X=2, Y=4) = 0

P(X=2)= 0.1 + 0.2 + 0
P(X=2)= 0.3
The total probability for (X=2) is 0.3

Q.1(B)

P(X=0,Y=2) = 0.3
P(X=2,Y=2) = 0.2
P(X=3,Y=2) = 0
P(X=0,Y=4) = 0.1
P(X=2,Y=4) = 0
P(X=3,Y=4) = 0

P(Y>1) = 0.3 + 0.2 + 0 + 0.1 + 0 + 0 = 0.6
The total probability for (Y>1) is 0.6

Q.1(C)

P(X+Y=4) = P(X=0,Y=4) + P(X=2,Y=2) + P(X=3,Y=1)
P(X+Y=4) = 0.1 + 0.2 + 0.15
P(X+Y=4) = 0.45

Q.1(D)

$E[XY] = \sum (X_i \cdot Y_i \cdot P(X_i, Y_i))$

$E(XY) = (0\times1\times0.15) + (2\times1\times0.1) + (3\times1\times0.15) + (0\times2\times0.3) + (2\times2\times0.2) + (3\times2\times0) + (0\times4\times0.1) + (2\times4\times0) + (3\times4\times0)$ $E(XY) = 0 + 0.2 + 0.45 + 0 + 0.8 + 0 + 0 + 0 + 0$

$E(XY) = 1.45$

$E(X) = 1.05$
$E(Y) = 1.8$

$Cov(X,Y) = E(XY) - E(X)\cdot E(Y)$
$Cov(X,Y) = 1.45-(1.05 \times 1.8) = 1.45-1.89$
$Cov(X,Y) = -0.44$

Q.2(A)

P(X = 3, Y = 2) = P(X = 3) · P(Y = 2)

P(X = k) = $\frac{e^{-\lambda}\ \lambda^{k}}{k!}$

P(X = 3) = $\frac{e^{-1}\ 1^{3}}{3!}$, P(Y = 2) = $\frac{e^{-3}\ 3^{2}}{2!}$

P(X = 3, Y = 2) = $\frac{9e^{-4}}{12}$

P(X = 3, Y = 2) = 0.0137

Q.2(B)

E(4X - Y)

E(4X - Y) = 4E(X) - E(Y)

E(X) = λ_X = 1

E(Y) = λ_Y = 3

E(4X - Y) = 4 · 1 - 3

E(4X - Y) = 1

Q.2(C)

Var(4X - Y)

Var(4X - Y) = Var(4X) + Var(-Y)

Var(X) = λ_X = 1

Var(Y) = λ_Y = 3

Var(4X - Y) = 4^2Var(X) + $(-1)^2$ Var(Y)

Var(4X - Y) = (4^2 · 1) + (1 · 3)

Var(4X–Y) = 16 + 3

Var(4X–Y) = 19

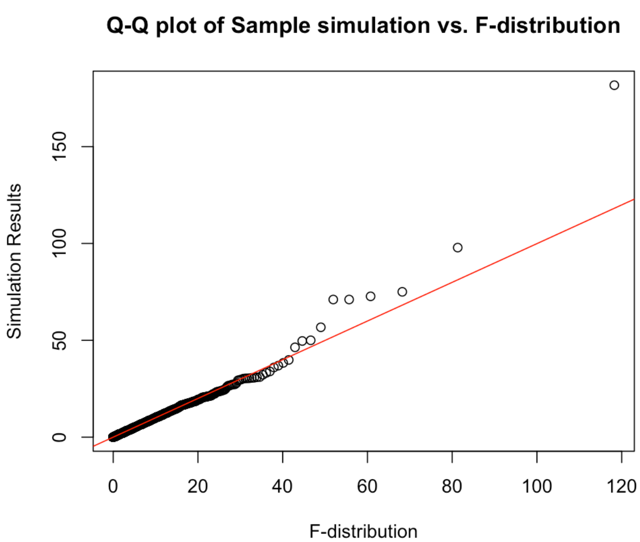
Q.3(R-Commands)

```
> # Number of simulations
> nsim <- 50000
> # Random samples from chi-square distributions
> X1 <- rchisq(nsim, df = 3)
> X2 <- rchisq(nsim, df = 6)
> # Define Y1 and Y2
> Y1 <- (1/3) * X1
> Y2 <- (1/6) * X2
> # Define Z
> Z <- Y1 / Y2
> # Calculate the mean
> mean_Z <- mean(Z)
> # Estimated mean result
> cat("Estimated mean of Z:", mean_Z, "\n")
Estimated mean of Z: 1.509085
> # Create a Q-Q plot comparing Z to the F-distribution
> qqplot(qf(ppoints(nsim), df1 = 3, df2 = 6), Z, main = "Q-Q plot of Sample simulation vs. F-distribution",
+         xlab = "F-distribution", ylab = "Simulation Results")
> # Add a reference line to the Q-Q plot
> qqline(Z, distribution = function(p) qf(p, df1 = 3, df2 = 6), col = " red")
```

Q.3(A)

Estimated mean of Z : 1.509085

Q.3(B)



Q.4(R-Commands)

```
> rm(list=ls())
> mu <- -1   # E(X)
> sd <- sqrt(12)  # Standard deviation of X
> n <- 300  # Number of samples
> # (a)
> mean_X <- mu
> sigma_X <- sd / sqrt(n)  # Standard deviation of Xbar
> # Print the results
> cat("E(X) =", mean_X, "\n")
E(X) = -1
> cat("Standard Deviation of The Sample Mean =", sigma_X, "\n")
Standard Deviation of The Sample Mean = 0.2
> # (b) Calculate P(X ≤ -0.9)
> z <- (-0.9 - mu) / sigma_X
> # Calculate the probability
> prob <- pnorm(z)
> # Print the result
> cat("P(X ≤ -0.9) =", prob, "\n")
P(X ≤ -0.9) = 0.6914625
```

Q.4(A)

$E(\bar{X}) = E(X) = -1$

Var(X) = 12

n = 300

$$\sigma\bar{X} = \sqrt{\frac{Var(X)}{n}} = \sqrt{\frac{12}{300}}$$

$\sigma\bar{X} = 3.4641/17.320 = 0.2$

Q.4(B)

$P(\bar{X}) \leq -0.9) = 0.6914625$

Q.5(R-Commands)

```
> nsim <- 10000
> XmeanLess <- rep(NA, nsim)
> for (i in 1:nsim) {
+   data300 <- runif(300, min = -7, max = 5)
+   xbar <- mean(data300)
+
+   # Check if the sample mean is less than or equal to -0.9
+   if (xbar <= -0.9) {
+     XmeanLess[i] <- 1
+   } else {
+     XmeanLess[i] <- 0
+   }
+ }
> # Estimate the probability
> probability <- mean(XmeanLess)
> cat("Monte Carlo estimate of P(X ≤ -0.9):", probability, "\n")
Monte Carlo estimate of P(X ≤ -0.9): 0.687
```

Q.5

Monte Carlo estimate of P($\overline{X}$ ≤ -0.9): 0.687